Chebyshev distance

	a	b	c	d	e	f	g	h	
8	5	4	3	2	2	2	2	2	8
7	5	4	3	2	1	1	1	2	7
6	5	4	3	2	1	♔	1	2	6
5	5	4	3	2	1	1	1	2	5
4	5	4	3	2	2	2	2	2	4
3	5	4	3	3	3	3	3	3	3
2	5	4	4	4	4	4	4	4	2
1	5	5	5	5	5	5	5	5	1
	a	b	c	d	e	f	g	h	

The Chebyshev distance between two spaces on a chess board gives the minimum number of moves a king requires to move between them. This is because a king can move diagonally, so that the jumps to cover the smaller distance parallel to a rank or column is effectively absorbed into the jumps covering the larger. Above are the Chebyshev distances of each square from the square f6.

In mathematics, **Chebyshev distance** (or **Tchebychev distance**), **Maximum metric**, or L_∞ metric[1] is a metric defined on a vector space where the distance between two vectors is the greatest of their differences along any coordinate dimension.[2] It is named after Pafnuty Chebyshev.

It is also known as **chessboard distance**, since in the game of chess the minimum number of moves needed by a king to go from one square on a chessboard to another equals the Chebyshev distance between the centers of the squares, if the squares have side length one, as represented in 2-D spatial coordinates with axes aligned to the edges of the board.[3] For example, the Chebyshev distance between f6 and e2 equals 4.

Definition

The Chebyshev distance between two vectors or points *p* and *q*, with standard coordinates p_i and q_i, respectively, is

$$D_{\text{Chebyshev}}(p,q) := \max_i(|p_i - q_i|).$$

This equals the limit of the L_p metrics:

$$\lim_{k\to\infty} \left(\sum_{i=1}^{n} |p_i - q_i|^k \right)^{1/k},$$

hence it is also known as the L_∞ metric.

Mathematically, the Chebyshev distance is a metric induced by the **supremum norm** or **uniform norm**. It is an example of an injective metric.

In two dimensions, i.e. plane geometry, if the points *p* and *q* have Cartesian coordinates $(x_1, y_1)$and (x_2, y_2), their Chebyshev distance is

$$D_{\text{Chess}} = \max\left(|x_2 - x_1|, |y_2 - y_1|\right).$$

Under this metric, a circle of radius *r*, which is the set of points with Chebyshev distance *r* from a center point, is a square whose sides have the length 2*r* and are parallel to the coordinate axes.

On a chess board, where one is using a *discrete* Chebyshev distance, rather than a continuous one, the circle of radius r is a square of side lengths $2r$, measuring from the centers of squares, and thus each side contains $2r+1$ squares; for example, the circle of radius 1 on a chess board is a 3×3 square.

Properties

In one dimension, all L_p metrics are equal – they are just the absolute value of the difference.

The two dimensional Manhattan distance also has circles in the form of squares, with sides of length $\sqrt{2}r$, oriented at an angle of $\pi/4$ (45°) to the coordinate axes, so the planar Chebyshev distance can be viewed as equivalent by rotation and scaling to the planar Manhattan distance.

However, this equivalence between L_1 and L_∞ metrics does not generalize to higher dimensions. A sphere formed using the Chebyshev distance as a metric is a cube with each face perpendicular to one of the coordinate axes, but a sphere formed using Manhattan distance is an octahedron: these are dual polyhedra, but among cubes, only the square (and 1-dimensional line segment) are self-dual polyhedra.

The Chebyshev distance is sometimes used in warehouse logistics.[4]

On a grid (such as a chessboard), the points at a Chebyshev distance of 1 of a point are the Moore neighborhood of that point.

References

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External links

- Example of Chebyshev distance (http://people.revoledu.com/kardi/tutorial/Similarity/ChebyshevDistance.html)

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