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Data Structures I

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**Assignment 1**

This assignment implements a splay tree for analyzing its algorithmic complexity. There are two types of splay trees implemented: (1) a standard splay tree which utilizes double rotations, and (2) a naïve version which utilizes only single rotations up to the root node. The following graphs plot the average path length for a find operation as a function of the tree size.

**Uniform Subset Test**

The uniform subset test picks keys uniformly at random from a fixed subset of size of the inserted keys and issues a find operation for those keys. The graph below plots curves of the average path length needed to find a key in a standard splay tree for a given tree size. Each of the six curves plots a different subset size .

**Uniform Subset Test (Standard)**

As increases, so does the path length. This is simply due to the fact there are more keys being splayed to the root of the tree, further increasing the average number of steps needed to find a key. As standard splay trees have been proved to perform in amortized time, all curves here must be logarithmic up to a constant factor. As increases, the constant factor increases. The curves for appear linear because the constant factor is very small due to the small subset size (and hence perceived tree size). However, as approaches the size of the tree , so too does the curve approach the true asymptotic bound of .

The next graph displays a naïve version of the same test, where the splay tree can only perform single rotations on the path to the root.

**Uniform Subset Test (Naïve)**

Like in the previous graph, this graph shows that naïve trees have similar asymptotic complexity, approaching as and increase. However, it is immediately visible that naïve splay trees have much more variance in their average path length.

This can be proved by using amortized analysis. The only difference between a standard and naïve splay tree is in the zig-zig step.

The graph below joint plots the standard and naïve curves of the previous two graphs.

**Uniform Subset Test (Joint)**

For all values of , the naïve tree performs asymptotically worse than the standard tree, i.e., the average path length is greater in naïve trees for large values of . The naïve tree appears to have a slight advantage over the standard tree for small values of , but that advantage disappears as increases.

**Sequential Test**

The sequential test picks keys in sequential order and issues a find operation for those keys. The final graph below plots the curves of the average path length needed to find a key in a standard and naïve splay tree for a given tree size.

**Sequential Test (Joint)**

The sequential test is where the standard splay tree vastly outperforms the naïve splay tree. The standard tree has a nearly constant average path length of ~5.2, while the naïve tree increases its path length by a factor of 0.5 for each additional searched key.