

Descriptive Statistics

- Describing data with *tables* and *graphs*
(quantitative or categorical variables)
- Numerical descriptions of *center*, *variability*, *position*
(quantitative variables)
- *Bivariate* descriptions (In practice, most studies have several variables)

1. Tables and Graphs

Frequency distribution: Lists possible values of variable and number of times each occurs

Example: Student survey ($n = 60$)

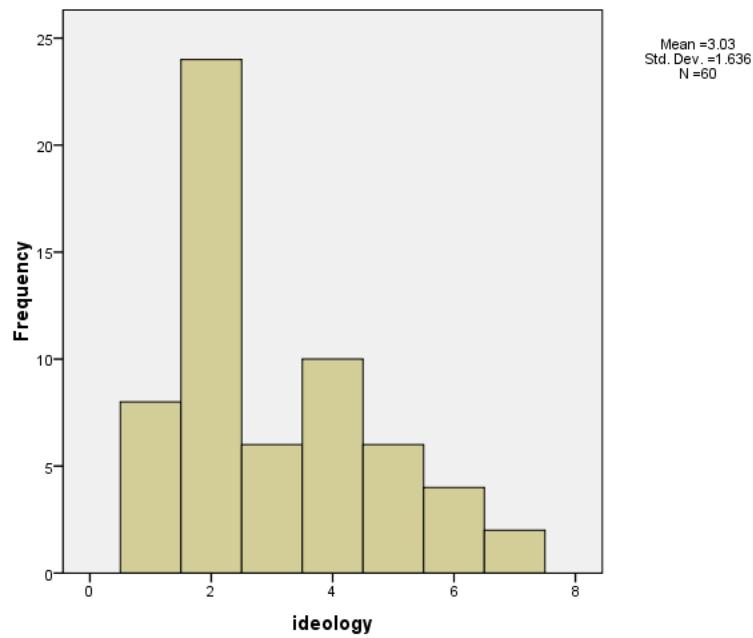
www.stat.ufl.edu/~aa/social/data.html

“political ideology” measured as ordinal variable with 1 = very liberal, ..., 4 = moderate, ..., 7 = very conservative

ideology

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	8	13.3	13.3	13.3
	2	24	40.0	40.0	53.3
	3	6	10.0	10.0	63.3
	4	10	16.7	16.7	80.0
	5	6	10.0	10.0	90.0
	6	4	6.7	6.7	96.7
	7	2	3.3	3.3	100.0
	Total	60	100.0	100.0	

Histogram: Bar graph of frequencies or percentages



Shapes of histograms (for *quantitative* variables)

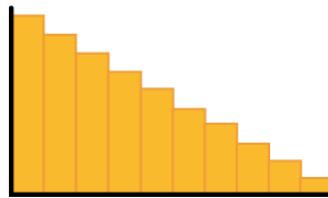
- *Bell-shaped* (IQ, SAT, political ideology in all U.S.)
- *Skewed right* (annual income, no. times arrested)
- *Skewed left* (score on easy exam)
- *Uniform*
- *Bimodal* (polarized opinions)

Shapes of histograms (for quantitative variables)

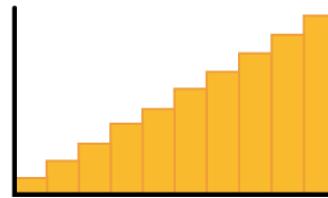
Symmetric (normal) vs skewed and uniform distributions



Normal distribution
(unimodal, symmetric,
the “bell curve”)



Right-skewed distribution
(Positively-skewed)



Left-skewed distribution
(Negatively-skewed)

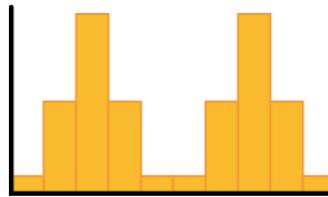


Uniform distribution
(equal spread,
no peaks)

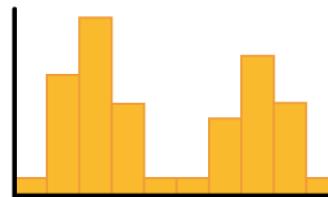
Unimodal vs bimodal distributions



Normal distribution
(unimodal, symmetric,
the “bell curve”)



Symmetric bimodal distribution
(two modes)



Non-symmetric bimodal distribution
(two modes)

Shapes of histograms (for *quantitative* variables) **Left Skewed**

- **Dataset:** 24, 45, 56, 71, 78, 80, 81, 81, 82, 83, 84, 85, 85, 89, 91, 91, 92, 93, 96, 97
- Here are the mean and median values of this dataset:
- **Mean:** 79.2
- **Median:** 83.5

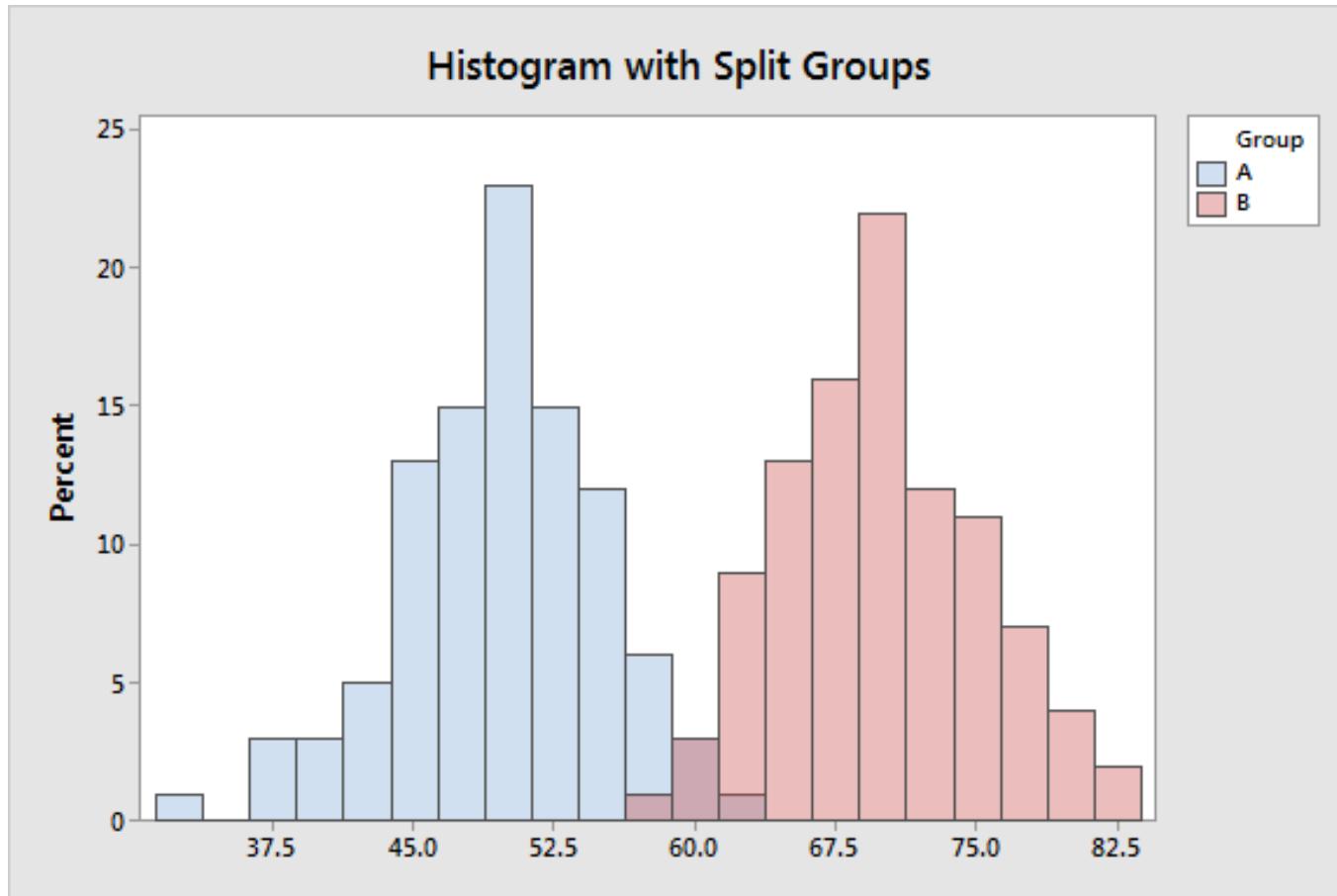
Shapes of histograms (for *quantitative* variables) **Right Skewed**

- **Dataset 2:** \$30k, \$35k, \$35k, \$40k, \$50k, \$55k, \$55k, \$70k, \$90k, \$2.5 million
- Here are the mean and median values of this dataset:
- **Mean:** \$296k
- **Median:** \$52.5k

Shapes of histograms (for *quantitative* variables) **Bimodal**

- Bimodal literally means “two modes” and is **typically used to describe distributions of values that have two centers.**
- For example, the distribution of heights in a sample of adults might have two peaks, one for women and one for men.

Shapes of histograms (for quantitative variables) **Bimodal**



2. Numerical descriptions

Let y denote a quantitative variable, with observations $y_1, y_2, y_3, \dots, y_n$

a. Describing the center

Median: Middle measurement of ordered sample

Mean:

$$\bar{y} = \frac{y_1 + y_2 + \dots + y_n}{n} = \frac{\sum y_i}{n}$$

Example: Annual per capita carbon dioxide emissions (metric tons) for $n = 8$ largest nations in population size

Bangladesh 0.3, Brazil 1.8, China 2.3, India 1.2, Indonesia 1.4, Pakistan 0.7, Russia 9.9, U.S. 20.1

Ordered sample:

Median =

Mean \bar{y} =

Example: Annual per capita carbon dioxide emissions (metric tons) for $n = 8$ largest nations in population size

Bangladesh 0.3, Brazil 1.8, China 2.3, India 1.2, Indonesia 1.4, Pakistan 0.7, Russia 9.9, U.S. 20.1

Ordered sample: 0.3, 0.7, 1.2, 1.4, 1.8, 2.3, 9.9, 20.1

$$\text{Median} = (1.4 + 1.8)/2 = 1.6$$

$$\text{Mean } \bar{y} = (0.3 + 0.7 + 1.2 + \dots + 20.1)/8 = 4.7$$

Properties of mean and median

- For symmetric distributions, mean = median
- For skewed distributions, mean is drawn in direction of longer tail, relative to median
- Mean valid for interval scales, median for interval or ordinal scales
- Mean sensitive to “outliers” (median often preferred for highly skewed distributions)

Examples:

- New York Yankees baseball team, 2006
 - mean salary = \$7.0 million
 - median salary = \$2.9 million

How possible? Direction of skew?

Examples:

- Give an example for which you would expect
mean < median

Consider the **wealth distribution in a low-income neighbourhood**. Most residents might have incomes clustered around a median value, but a few individuals might have significantly lower incomes (perhaps unemployed or working part-time).

b. Describing *variability*

Range: Difference between largest and smallest observations

(but highly sensitive to outliers, insensitive to shape)

Standard deviation: A “typical” distance from the mean

The *deviation* of observation i from the mean is

$$y_i - \bar{y}$$

The **variance** of the n observations is

$$s^2 = \frac{\sum (y_i - \bar{y})^2}{n-1} = \frac{(y_1 - \bar{y})^2 + \dots + (y_n - \bar{y})^2}{n-1}$$

The **standard deviation** s is the square root of the variance,

$$s = \sqrt{s^2}$$

Example: Political ideology

- For those in the student sample who attend religious services at least once a week ($n = 9$ of the 60),
- $y = 2, 3, 7, 5, 6, 7, 5, 6, 4$

$$\bar{y} = 5.0,$$

$$s^2 = \frac{(2-5)^2 + (3-5)^2 + \dots + (4-5)^2}{9-1} = \frac{24}{8} = 3.0$$

$$s = \sqrt{3.0} = 1.7$$

For entire sample ($n = 60$), mean = 3.0, standard deviation = 1.6, tends to have similar variability but be more liberal

- **Properties of the standard deviation:**

- $s \geq 0$, and only equals 0 if all observations are equal
 - s increases with the amount of variation around the mean
 - Division by $n - 1$ (not n) is due to technical reasons (later)
 - s depends on the units of the data (e.g. measure euro vs \$)
 - Like mean, affected by outliers
- *Empirical rule:* If distribution is approx. bell-shaped,
- about 68% of data within 1 standard dev. of mean
 - about 95% of data within 2 standard dev. of mean
 - all or nearly all data within 3 standard dev. of mean

Example: SAT with mean = 500, $s = 100$

(sketch picture summarizing data)

Example: $y = \text{number of close friends you have}$

GSS: The variable ‘frinum’ has mean 7.4, $s = 11.0$

Probably highly skewed: right or left?

Empirical rule fails; in fact, median = 5, mode=4

Example: $y = \text{selling price of home in Syracuse, NY.}$

If mean = \$130,000, which is realistic?

$s = 0, \ s = 1000, \ s = 50,000, \ s = 1,000,000$

c. Measures of *position*

p^{th} percentile: p percent of observations below it, $(100 - p)\%$ above it.

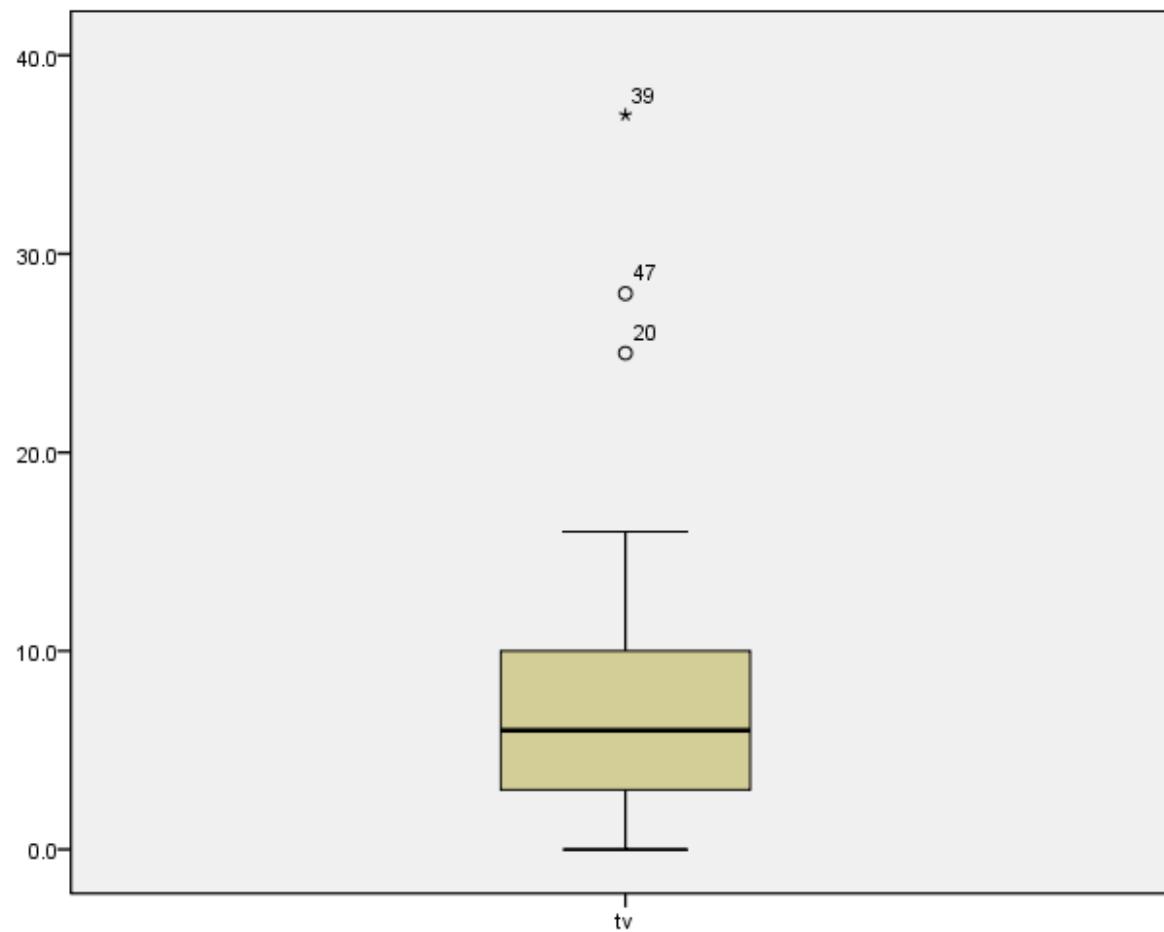
- $p = 50$: *median*
- $p = 25$: *lower quartile* (LQ)
- $p = 75$: *upper quartile* (UQ)

- *Interquartile range* IQR = UQ - LQ

Quartiles portrayed graphically by *box plots*

(John Tukey)

Example: weekly TV watching for $n=60$ from student survey data file, 3 outliers



Box plots have box from LQ to UQ, with median marked. They portray a *five-number summary* of the data:

Minimum, LQ, Median, UQ, Maximum

except for outliers identified separately

Outlier = observation falling

below $LQ - 1.5(IQR)$

or above $UQ + 1.5(IQR)$

Ex. If $LQ = 2$, $UQ = 10$, then $IQR = 8$ and outliers above $10 + 1.5(8) = 22$

3. Bivariate description

- Usually we want to study *associations* between two or more variables (e.g., how does number of close friends depend on gender, income, education, age, working status, rural/urban, religiosity...)
- Response variable: the outcome variable
- Explanatory variable(s): defines groups to compare

Ex.: number of close friends is a response variable, while gender, income, ... are explanatory variables

Response var. also called “dependent variable”

Explanatory var. also called “independent variable”

Summarizing associations:

- Categorical var's: show data using *contingency tables*
- Quantitative var's: show data using *scatterplots*
- Mixture of categorical var. and quantitative var. (e.g., number of close friends and gender) can give numerical summaries (mean, standard deviation) or side-by-side box plots for the groups

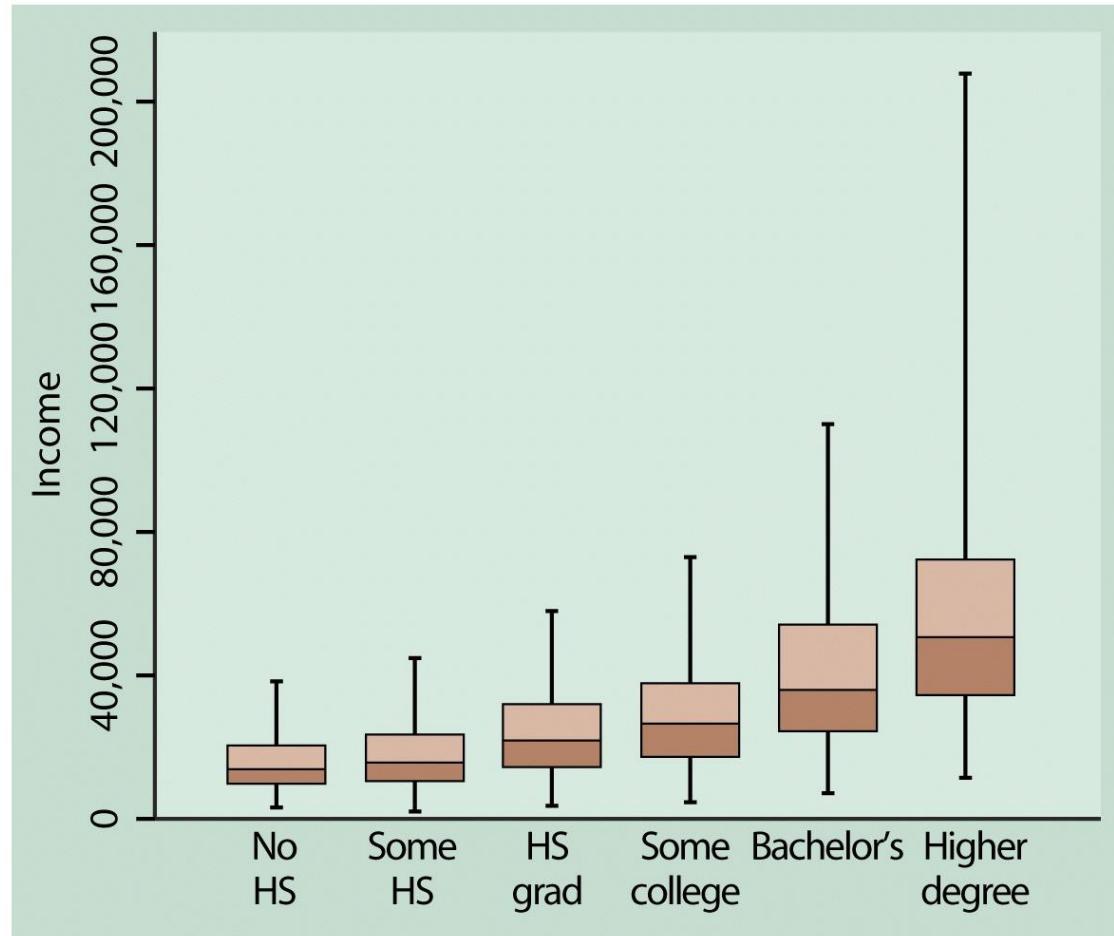
- **Ex.** General Social Survey (GSS) data

Men: mean = 7.0, $s = 8.4$

Women: mean = 5.9, $s = 6.0$

Shape? Inference questions for later chapters?

Example: Income by highest degree



Contingency Tables

- Cross classifications of categorical variables in which rows (typically) represent categories of explanatory variable and columns represent categories of response variable.
- Counts in “cells” of the table give the numbers of individuals at the corresponding combination of levels of the two variables

Happiness and Family Income (GSS 2008 data: “happy,” “finrela”)

Income	Happiness			Total
	Very	Pretty	Not too	
<hr/>				
Above Aver.	164	233	26	423
Average	293	473	117	883
Below Aver.	132	383	172	687
<hr/>				
Total	589	1089	315	1993

Can summarize by percentages on response variable
(happiness)

Example: Percentage “very happy” is

39% for above aver. income ($164/423 = 0.39$)

33% for average income ($293/883 = 0.33$)

19% for below average income (??)

Happiness

Income	Very	Pretty	Not too	Total
<hr/>				
Above	164 (39%)	233 (55%)	26 (6%)	423
Average	293 (33%)	473 (54%)	117 (13%)	883
Below	132 (19%)	383 (56%)	172 (25%)	687
<hr/>				

Inference questions for later chapters? (i.e., what can we conclude about the corresponding population?)

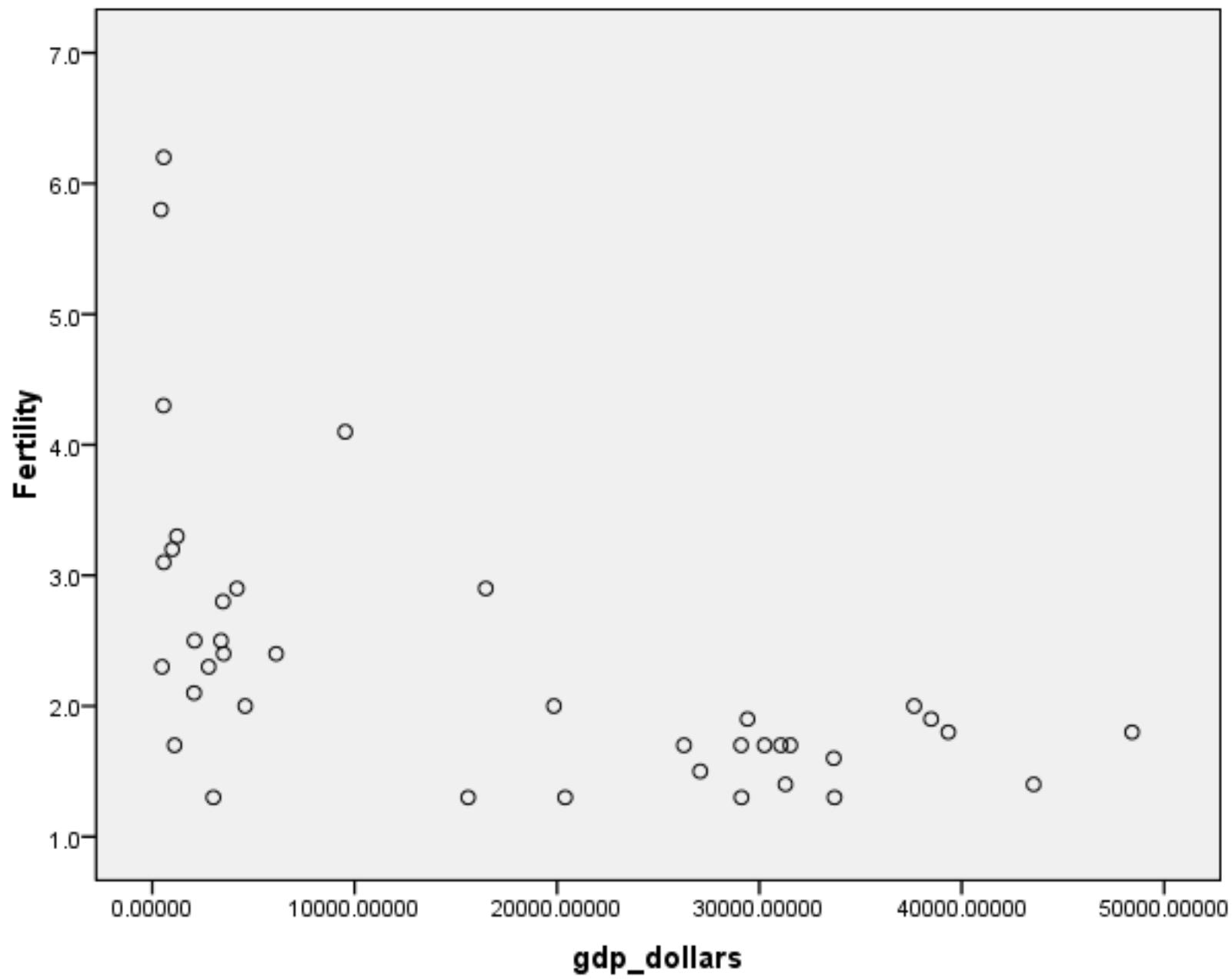
Scatterplots (for quantitative variables)

plot response variable on vertical axis,
explanatory variable on horizontal axis

Example: Table 9.13 (p. 294) shows UN data for several nations on many variables, including fertility (births per woman), contraceptive use, literacy, female economic activity, per capita gross domestic product (GDP), cell-phone use, CO₂ emissions

Data available at

<http://www.stat.ufl.edu/~aa/social/data.html>



Example: Survey in Alachua County, Florida, on predictors of mental health

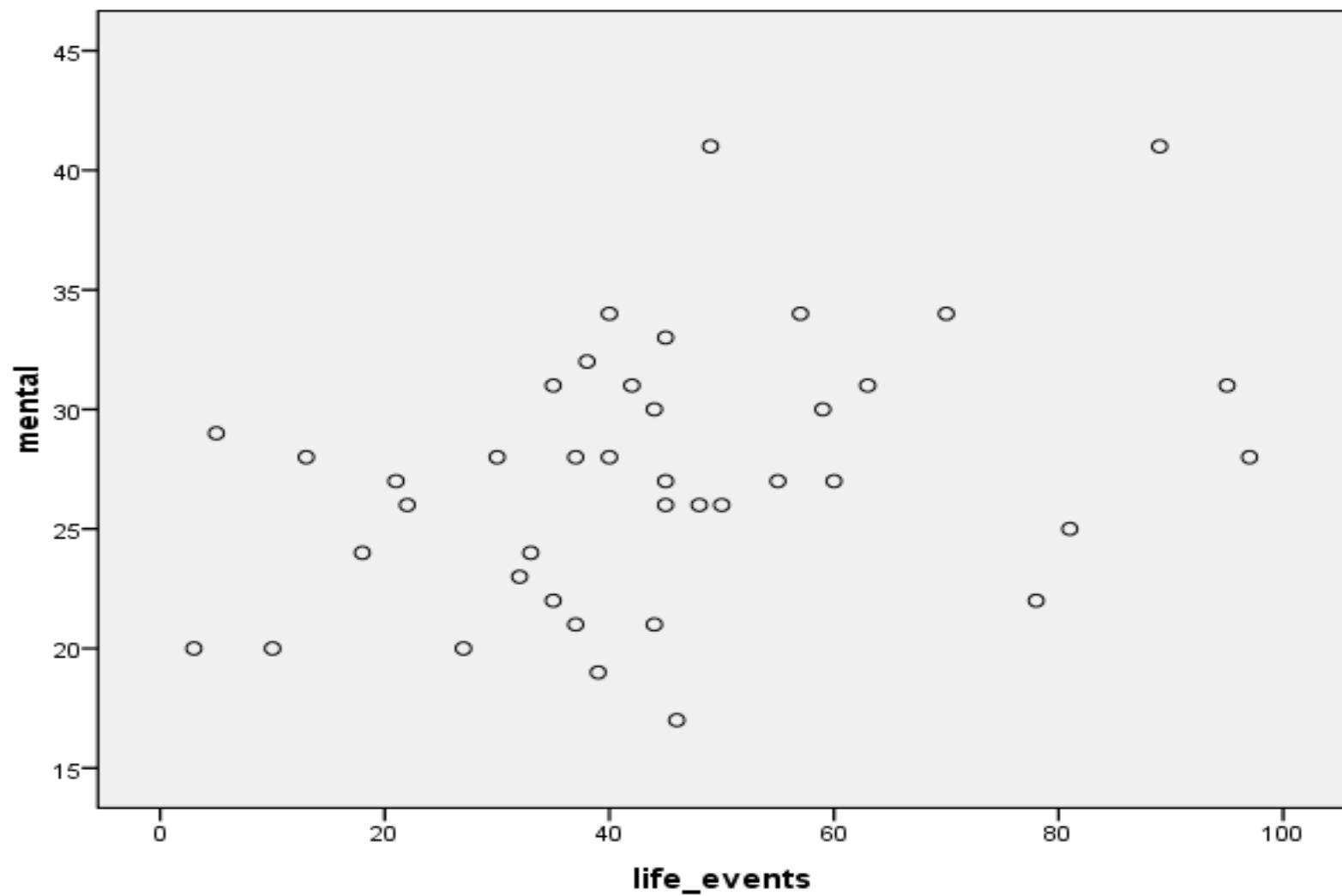
(data for $n = 40$ on p. 327 of text and at
www.stat.ufl.edu/~aa/social/data.html)

y = measure of mental impairment (incorporates various dimensions of psychiatric symptoms, including aspects of depression and anxiety)

(min = 17, max = 41, mean = 27, $s = 5$)

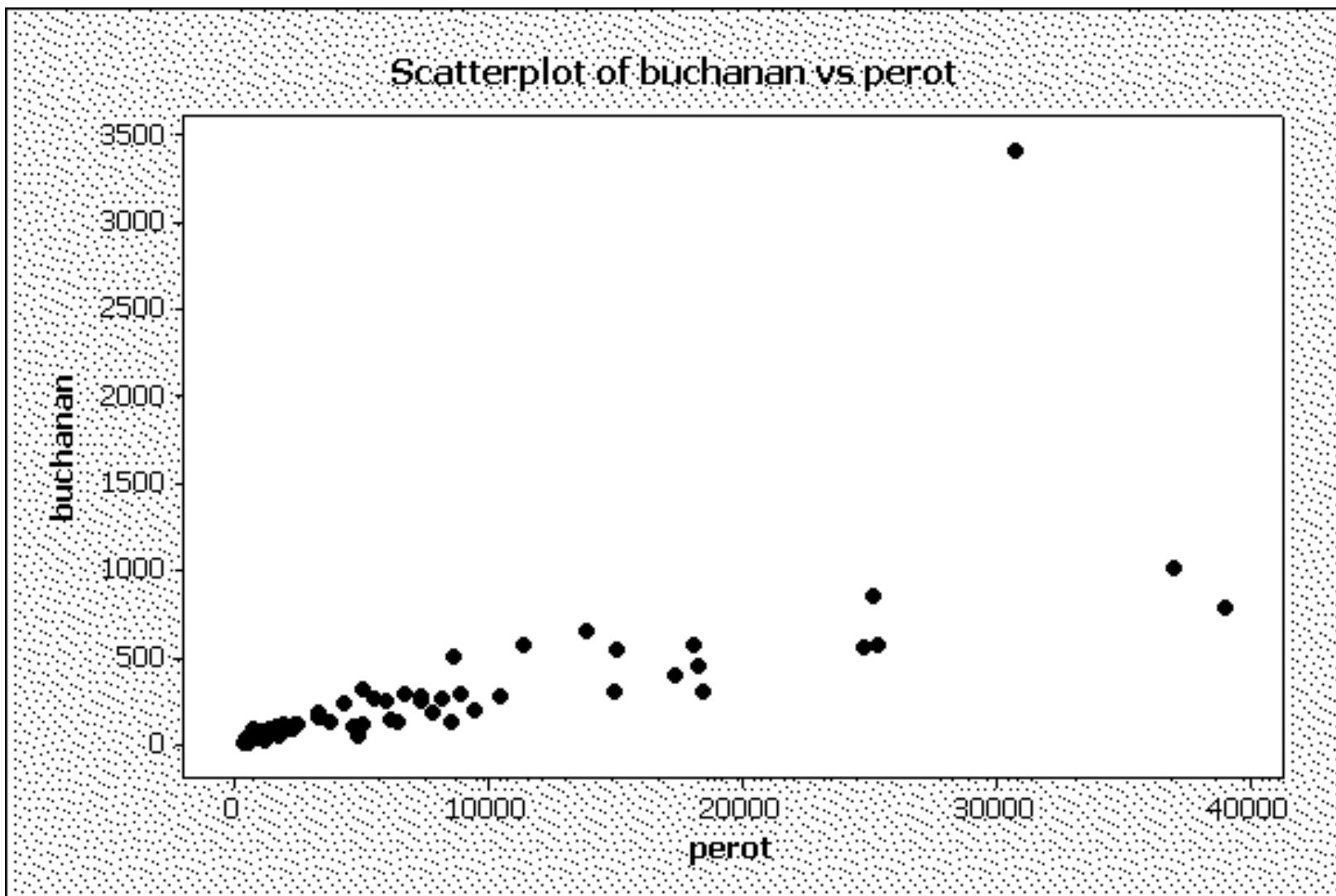
x = life events score (events range from severe personal disruptions such as death in family, extramarital affair, to less severe events such as new job, birth of child, moving)

(min = 3, max = 97, mean = 44, $s = 23$)

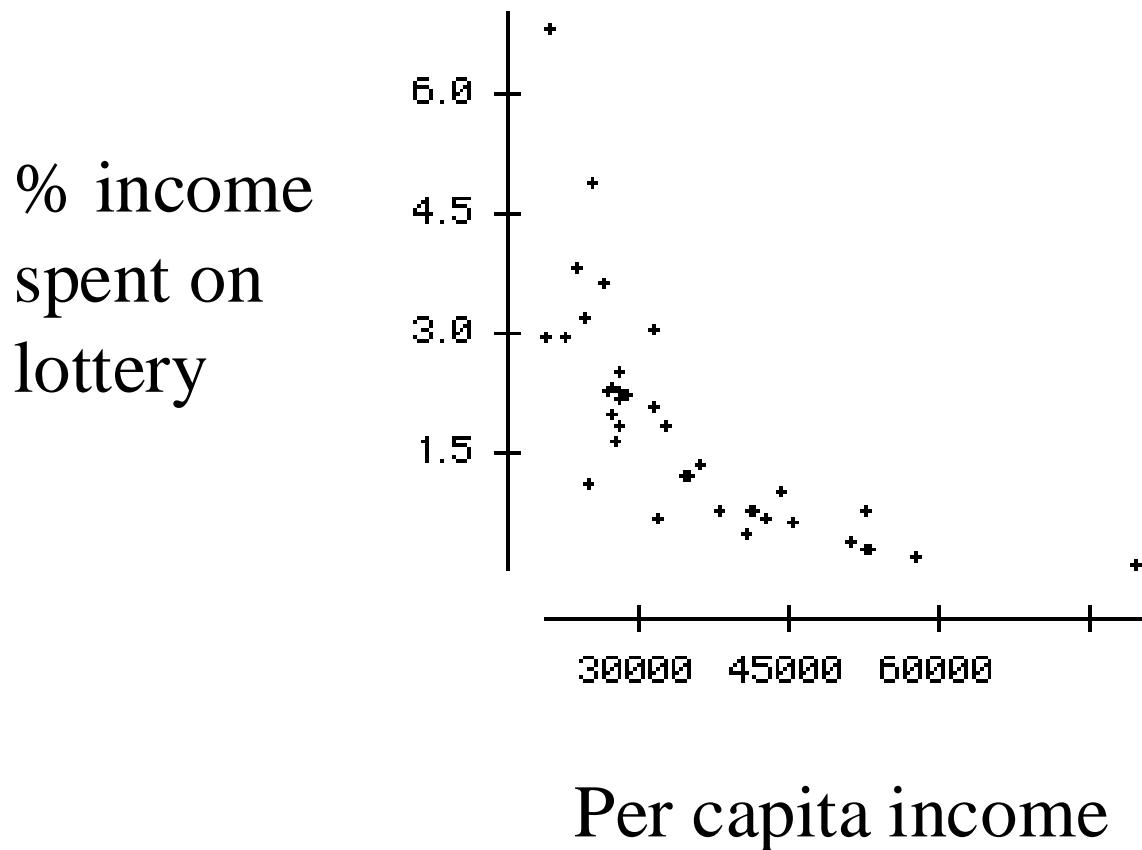


Bivariate data from 2000 Presidential election

Butterfly ballot, Palm Beach County, FL, text p.290



Example: The Massachusetts Lottery (data for 37 communities)



Correlation describes strength of association

- Falls between -1 and +1, with sign indicating direction of association (formula later in Chapter 9)

The larger the correlation in absolute value, the stronger the association (in terms of a straight line trend)

Examples: (positive or negative, how strong?)

Mental impairment and life events, correlation =

GDP and fertility, correlation =

GDP and percent using Internet, correlation =

Correlation describes strength of association

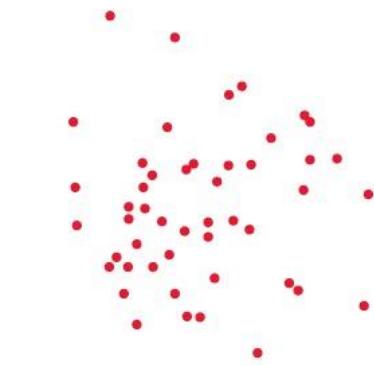
- Falls between -1 and +1, with sign indicating direction of association

Examples: (positive or negative, how strong?)

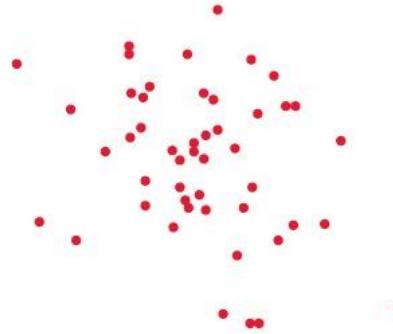
Mental impairment and life events, correlation = 0.37

GDP and fertility, correlation = - 0.56

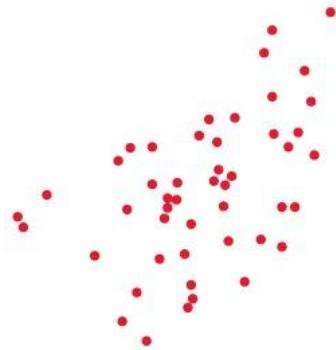
GDP and percent using Internet, correlation = 0.89



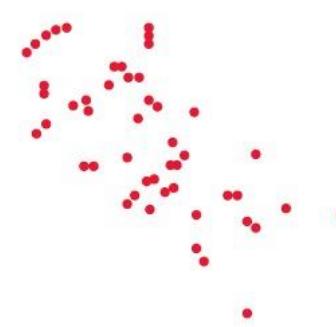
Correlation $r = 0$



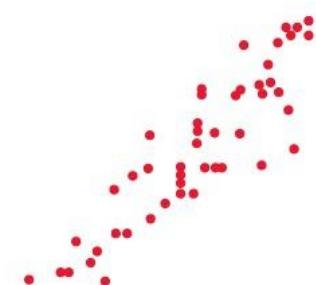
Correlation $r = -0.3$



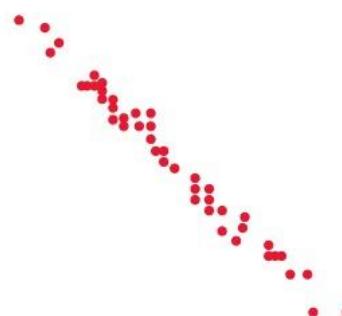
Correlation $r = 0.5$



Correlation $r = -0.7$



Correlation $r = 0.9$



Correlation $r = -0.99$

Regression analysis gives line predicting y using x

Example:

y = mental impairment, x = life events

Predicted $y = 23.3 + 0.09x$

e.g., at $x = 0$, predicted $y =$

at $x = 100$, predicted $y =$

Regression analysis gives line predicting y using x

Example:

y = mental impairment, x = life events

Predicted $y = 23.3 + 0.09x$

e.g., at $x = 0$, predicted $y = 23.3$

at $x = 100$, predicted $y = 23.3 + 0.09(100) = 32.3$

Inference questions for later chapters?

(i.e., what can we conclude about the population?)

Example: student survey

y = college GPA, x = high school GPA

(data at www.stat.ufl.edu/~aa/social/data.html)

What is the correlation?

What is the estimated regression equation?

We'll see later in course the formulas for finding the correlation and the "best fitting" regression equation (with possibly several explanatory variables), but for now, try using software such as SPSS to find the answers.

Sample statistics / Population parameters

- We distinguish between summaries of *samples (statistics)* and summaries of *populations (parameters)*.
- Common to denote statistics by Roman letters, parameters by Greek letters:

Population mean = μ , standard deviation = σ ,
proportion π are parameters.

In practice, parameter values unknown, we make
inferences about their values using sample statistics.

- The sample mean \bar{y} estimates the population mean μ (quantitative variable)
- The sample standard deviation s estimates the population standard deviation σ (quantitative variable)
- A sample proportion p estimates a population proportion \hat{p} (categorical variable)