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AGR5201
Advanced Statistical Methods

Factorial experiments

Topic outline

1.0 Factorial experiment

1.1 Two factors experiment

1.2 Interaction

1.3 Result presentation

1.4 Three factors experiment

1.5 Two-factorial example

1.6 Review question

Reference:
Gomez and Gomez (page 84)

1.0 Factorial experiment

So far...

- We have seen experiments with single factor only (rate of nitrogen, variety, etc.)
- The result can only be recommended in condition used in those experiment.
- If a different rate of fertilizer were tested using a different variety, the result will be different.
- Thus, a factorial experiment is conducted to study two (or more) factors at once.

1.0 Factorial experiment

Single factor experiment

In a single factor ANOVA, the effect of nitrogen and variety is determined using separate ANOVA table as shown below:

Nitrogen experiment

- Blocks = 4
- Nitrogen rates: 5
- Variety = 1

Source	DF
Blocks	3
Nitrogen	4
Error	12
Total	19

Variety experiment

- Blocks = 7
- Variety : 3
- Nitrogen rate: 1

Source	DF
Blocks	6
Variety	2
Error	12
Total	20

1.0 Factorial experiment

Definition

- A method for designing treatments, which allows testing and estimation of **main effects** and **interaction effects**.
- An experiment which looks at the effects of more than one factor (independent variable=IV)
- The interaction effects is measured from experiment that involves at least two factors with at least two levels each factor.
- Example:
  - i. Factor A (a1, a2)
  - ii. Factor B (b1, b2)

1.0 Factorial experiment

FACTOR

A specific type of treatment.

- Alfalfa cultivar (A)
- Methods of weed control (W)
- Rates of N fertilizer (N)
- Diet feds to animal (D)

LEVEL

A specific state of a factor.

- Alfalfa cultivars (A)→ Saranac, Arc, Oneida (a=3)
- Methods of weed control (W)→ herbicide, tillage, none, hand (w=4)
- Rates of N fertilizer (N)→ 0, 50, 100, 150, 200 kg/ha (n=5)
- Diets fed to animals (D) → 100% hay, 70% hay, 30% hay (d=3)

1.0 Factorial experiment

- **Single factor** – only one factor (independent variable)
  - effect of watering: 100% FC, 50% FC, 20% FC

- **Two factors** – two factors (independent variables)
  - Effect of watering: 100% FC, 50% FC, 20% FC
  - Effect of Nitrogen: 0 kg/ha, 50 kg/ha, 100 kg/ha

- **Three factors** – three factors (independent variables)
  - Effect of watering: 100% FC, 50% FC, 20% FC
  - Effect of Nitrogen: 0 kg/ha, 50 kg/ha, 100 kg/ha
  - Effect of variety: A, B, C.

Factorial experiment

1.0 Factorial experiment

Factorial treatment

- A treatment defined by the combination of one level of each factor.
- Factorial treatment designs have one main purpose:

→ **TO INCREASE THE SCOPE OF INFERENCE.**

- Identify important factors.
- Identify optimum or suboptimum levels of a factor.
- Identify and study interactions between factors.

to increase the scope of inferencre

1.0 Factorial experiment

Single factor ANOVA

1 factor: 3 levels 4 reps = 12 Experimental units (3 levels x 4 reps)

Source	df	SS	MS	F	Pr>F
Water	2				
Error	9				
Total	11				

1.1 Two factors experiment

- Water: 3 levels; Nitrogen: 3 levels; Rep = 4.
- Design RCBD = 3 x 3 x 4 = 36 experimental units

Nitrogen

Water

	N 0	**N 50**	**N 100**
20% FC	N0W1	N50W1	N100W1
50% FC	N0W2	N50W2	N100W2
100% FC	N0W3	N50W3	N100W3

Block 1

The same treatment combinations are repeated in block 2,3 and 4 with different randomizations

Block 2

Block 3

Block 4

1.1 Two factors experiment

Linear model (RCBD)

$$Y_{ijk} = \mu + \rho_k + \nu_i + \pi_j + (\nu\pi)_{ij} + \varepsilon_{ijk}$$

Where,

Y_{ijk} = observation

μ = overall mean

ρ_k = the effect of the k^{th} block

ν_i = the effect of the i^{th} level of factor 1

π_j = the effect of the j^{th} level of factor 2

$(\nu\pi)_{ij}$ = the interaction effect of the ij^{th} factorial treatment.

ε_{ijk} = random error

$\nu_i + \pi_j + (\nu\pi)_{ij} = \tau_i$ if we were to ignore the factorial structure.

1.1 Two factors experiment

ANOVA table

- Water (3 levels) Nitrogen (3 levels) Blocks (4)

	Source	df	SS	MS	F	Pr>F
	Block	3				
Main effect →	Water (W)	2				
Main effect →	Nitrogen (N)	2				
Interaction effect →	W x N	4				
	Error	24				
	Total	35				

1.1 Two factors experiment

ANOVA table - CRD

Linear model:
$$Y_{ij} = \mu + \nu_i + \pi_j + (\nu\pi)_{ij} + \varepsilon_{ij}$$

Source	df	SS	MS	F
Factor A	a-1	SSA	MSA=SSA/(a-1)	F_A= MSA/MSE
Factor B	b-1	SSB	MSB=SSB/(b-1)	F_B= MSB/MSE
A x B (interaction)	(a-1)(b-1)	SSAB	MSAB= SSAB/(a-1)(b-1)	F_{AB}= MSAB/MSE
Error	ab(r-1)	SSE	MSE= SSE/(r-1)(ab-1)	
Total	rab-1	SSTot		

1.1 Two factors experiment

ANOVA table - RCBD

Linear model:
$$Y_{ijk} = \mu + \rho_k + \nu_i + \pi_j + (\nu\pi)_{ij} + \varepsilon_{ijk}$$

Source	df	SS	MS	F
Block	r-1	SSR	MSR=SSR/(r-1)	F_R= MSR/MSE
Factor A	a-1	SSA	MSA=SSA/(a-1)	F_A= MSA/MSE
Factor B	b-1	SSB	MSB=SSB/(b-1)	F_B= MSB/MSE
A x B (interaction)	(a-1)(b-1)	SSAB	MSAB= SSAB/(a-1)(b-1)	F_{AB}= MSAB/MSE
Error	(r-1)(ab-1)	SSE	MSE= SSE/(r-1)(ab-1)	
Total	rab-1	SSTot		

1.1 Two factors experiment

Definition formulae

$$SSTot = \sum_i \sum_j \sum_k \left(Y_{ijk} - \overline{\overline{Y}}\right)^2$$

$$SSR = ab\sum_k \left(\overline{Y}_{..k} - \overline{\overline{Y}}\right)^2$$

$$SSA = rb\sum_i \left(\overline{Y}_{i..} - \overline{\overline{Y}}\right)^2$$

$$SSB = ra\sum_j \left(\overline{Y}_{.j.} - \overline{\overline{Y}}\right)^2$$

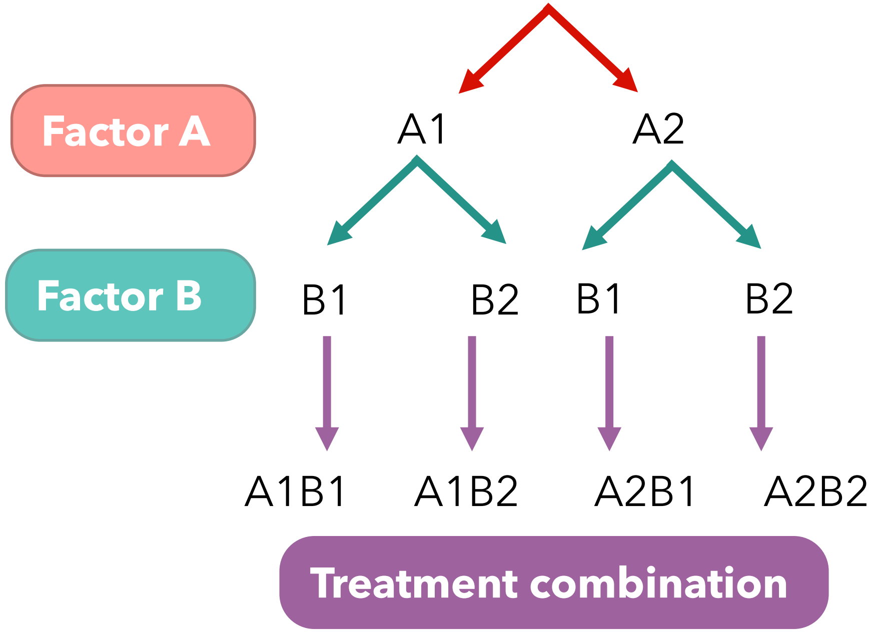
$$SSAB = r\sum_i \sum_j \left(\overline{Y}_{ij.} - \overline{Y}_{i..} - \overline{Y}_{.j.} + \overline{\overline{Y}}\right)^2$$

$$SSE = SSTot - SSR - SSA - SSB - SSAB$$

$$SS_{treatment} = SSA + SSB + SSAB$$

1.1 Two factors experiment

- **If there are interactions**, we should be able to measure and test them.
  - We **cannot** do this if we vary only one factor at a time
- We can combine two or more factors at two or more levels of each factor
  - Each level of every factor occurs together with each level of every other factor
  - Total number of treatments = the product of the levels of each factor



This has to do with the selection of treatments:

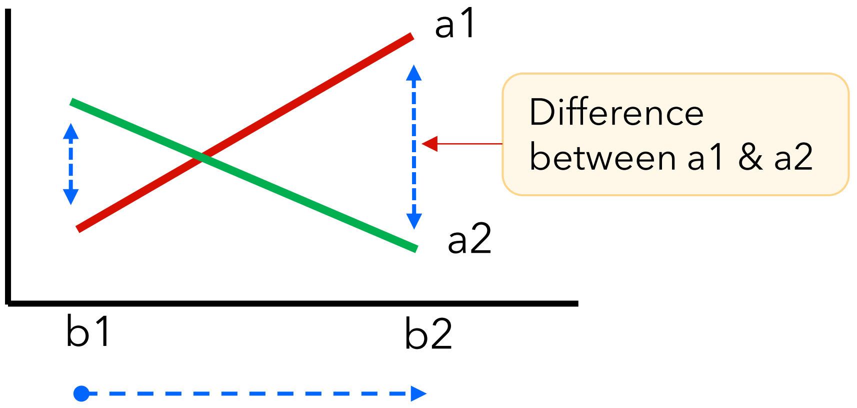
- Can be used in any design - CRD, RCBD, Latin Square - etc.
- "Designs" generally refer to the layout of replications or blocks in an experiment
- A "factorial" refers to the treatment combinations

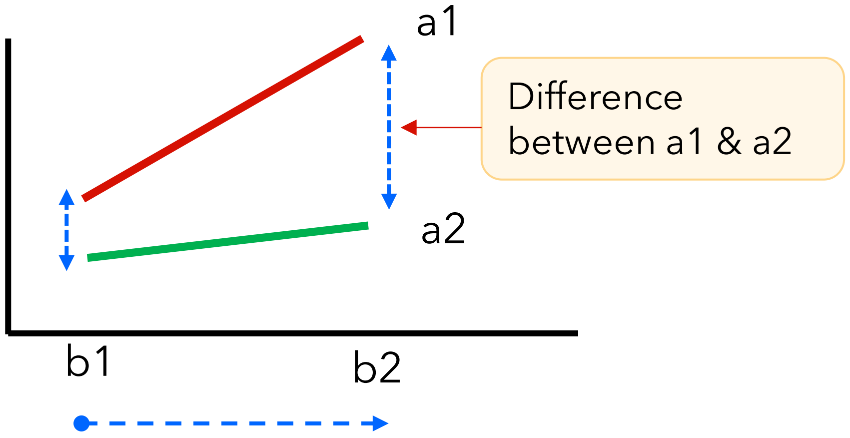
1.2 Interaction

Interaction between two factors:

The effect of one factor changes as the level of the other factor changes

Example:
As we move from b1 to b2, there is a change in the differences between a1 and a2 (blue arrow)





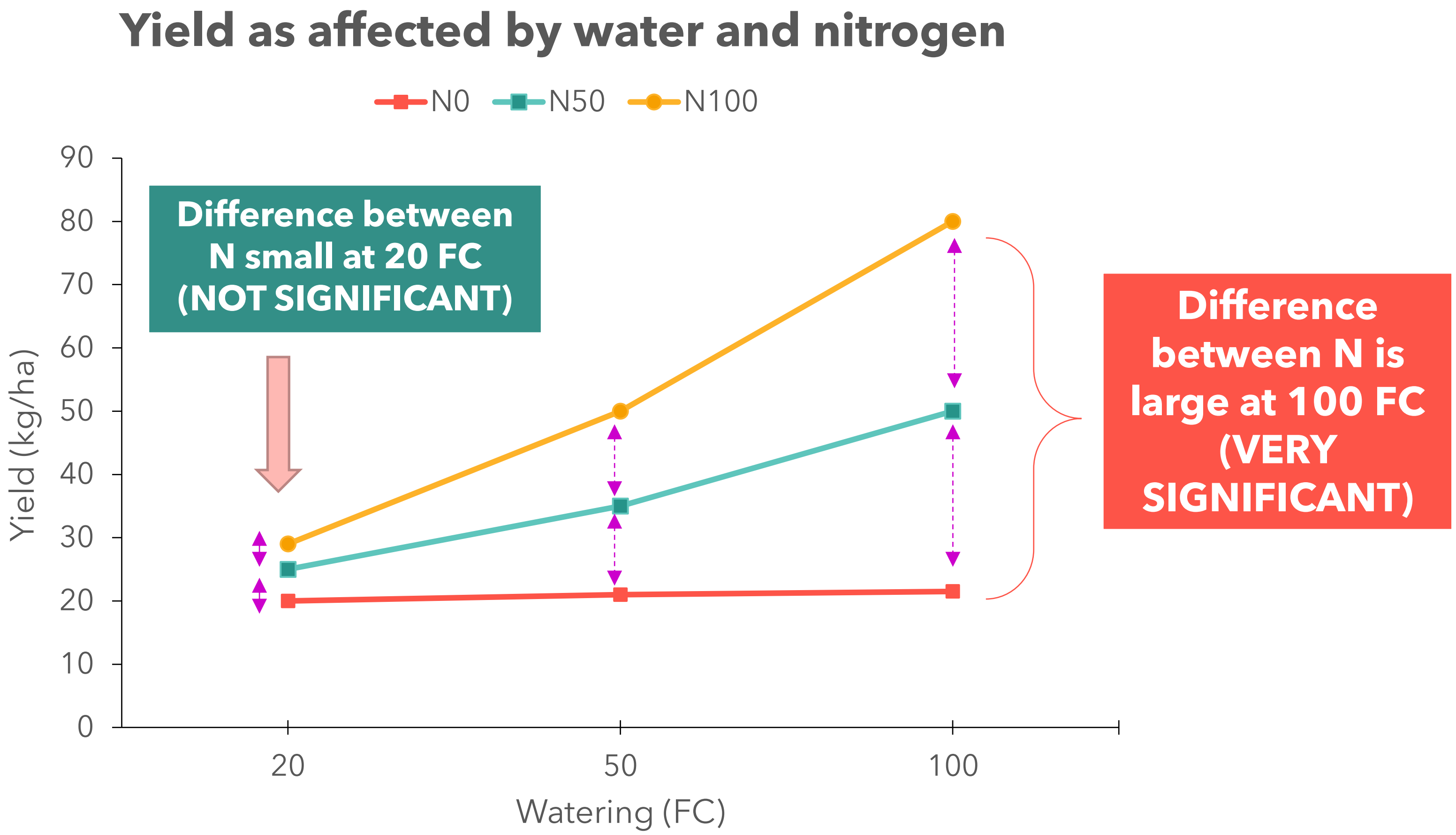
The interaction can be measured from a two factor experiment with at least 2 levels for each factor. Example:

- Factor A (a1, a2)
- Factor B (b1, b2)

1.2 Interaction

INTERACTION

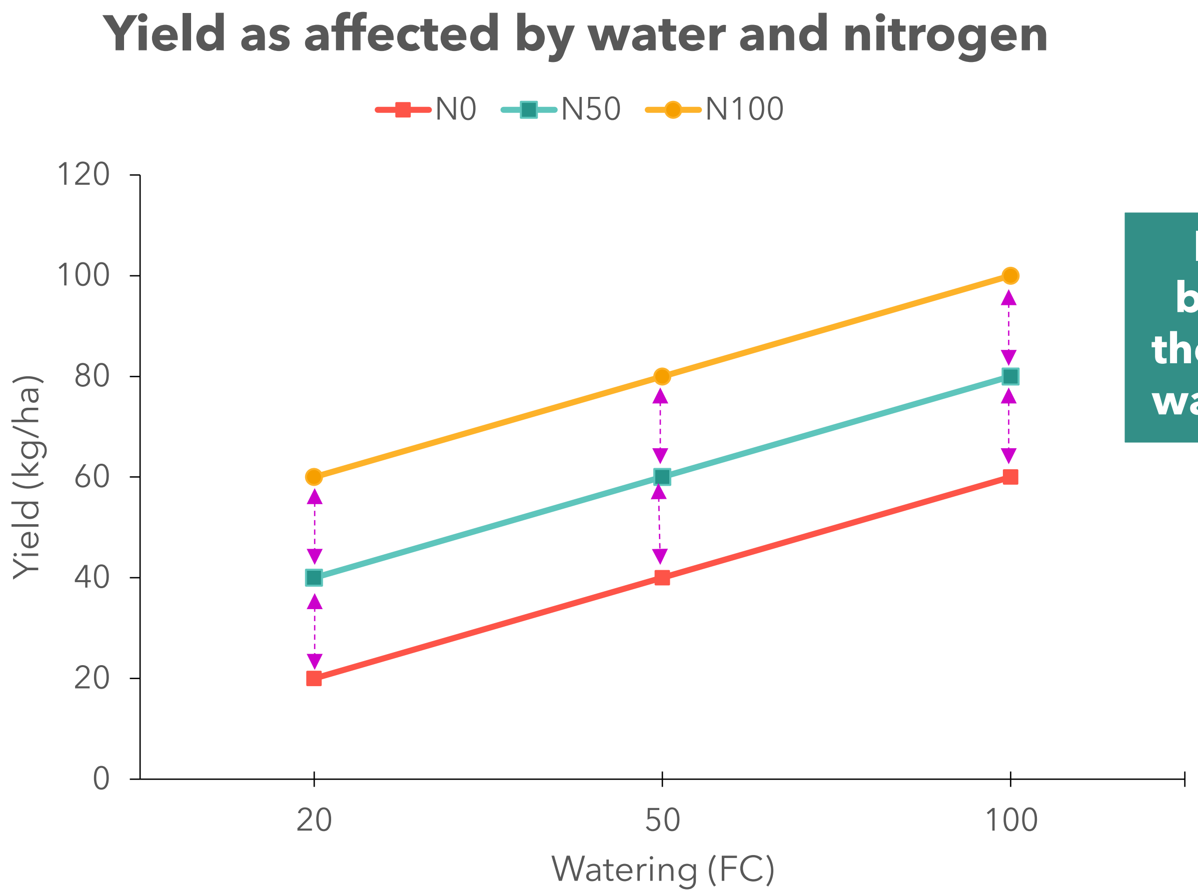
- The effect of one factor ***DOES*** affect the other factor



1.2 Interaction

NO INTERACTION

- the effect of one factor DOES NOT affect of the other factor



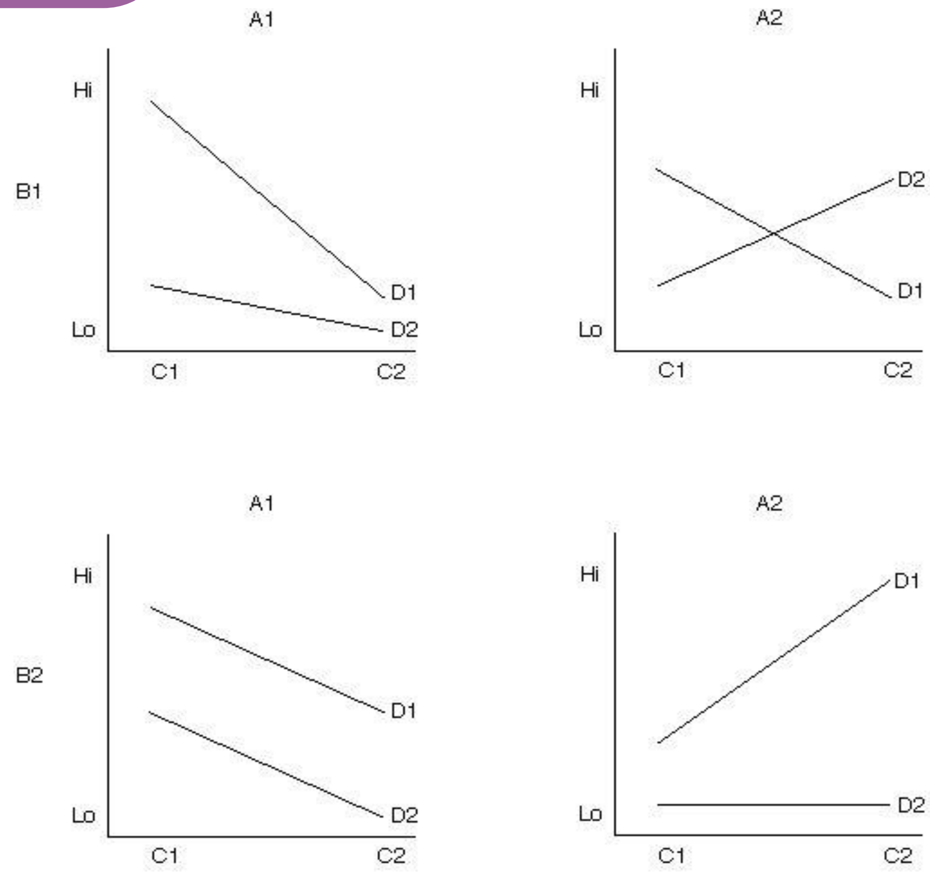
Differences between N is the same for all watering levels

1.2 Interaction

Types of interaction

Non-crossover 1

Crossover



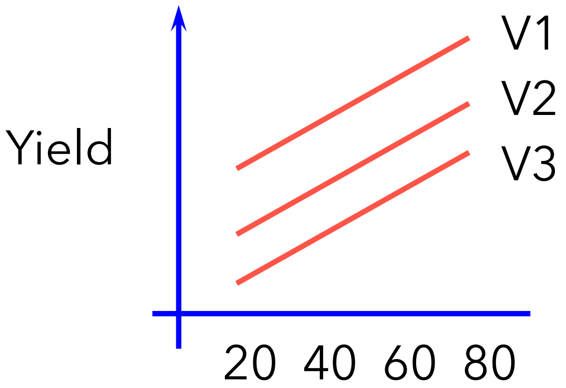
No interaction

Non-crossover 2

1.2 Interaction

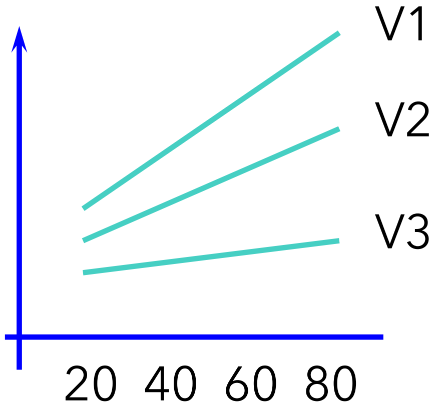
Interaction graphs

Consider 3 varieties at four rates of nitrogen



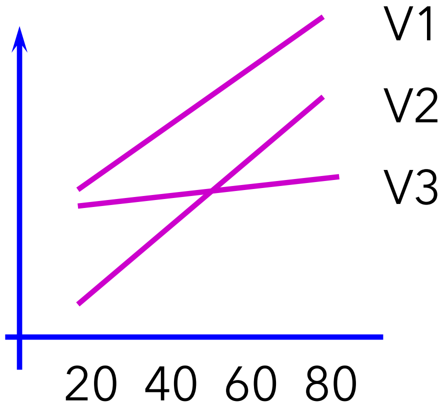
No interaction

Relative yield of varieties is the **same** at all fertilizer levels



Non-crossover Interactions

Magnitude of differences among varieties depends on fertilizer level



Crossover Interactions

Ranks of varieties depend on fertilizer level

1.2 Interaction

Numerical examples of interaction

Effect of two levels of phosphorus and potassium on crop yield

- ✓ Main effects are determined from the marginal means
- ✓ Simple effects refer to differences among treatment means at a single level of another factor

No interaction

	P_0	P_1	Mean
K_0	10	18	14
K_1	14	22	18
Mean	12	20	

(22-14)-(18-10) = 0

Positive interaction

	P_0	P_1	Mean
K_0	10	18	14
K_1	12	26	19
Mean	11	22	

(26-12)-(18-10) = +6

Negative interaction

	P_0	P_1	Mean
K_0	10	18	14
K_1	16	14	15
Mean	13	16	

(14-16)-(18-10) = -10

1.2 Interaction

Interpretation

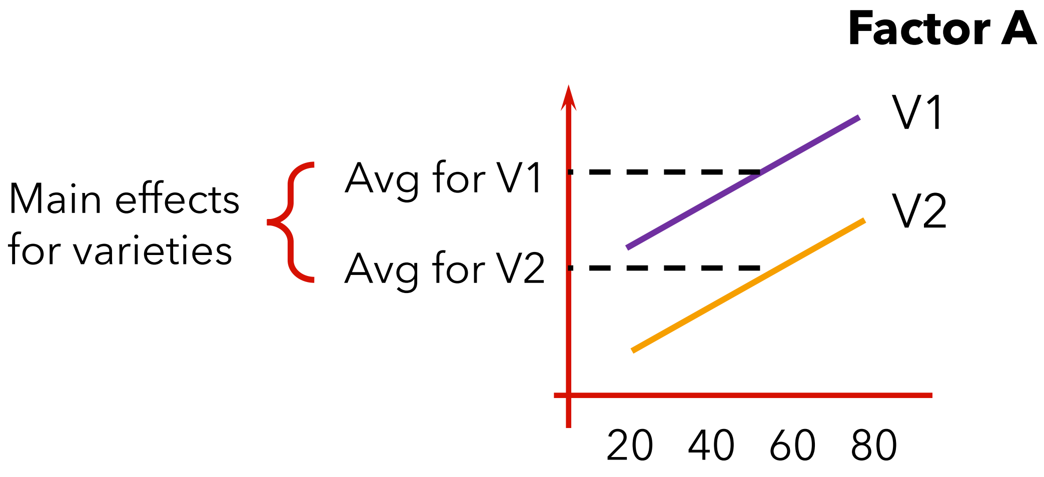
- If the **AB interaction IS significant**:
  - the main effects may have no meaning whether or not they test significant
  - Data summary:
    - Table: A two-way table of means for the various AB combinations
    - Figure: A bar chart of means for the various AB combination
- If the **AB interaction IS NOT significant**:
  - test the independent factors for significance
  - Data summary:
    - Table: A one-way table of means for the significant main effects
    - Figure: A bar chart of means for main effects only

1.2 Interaction

Interaction and statistical test

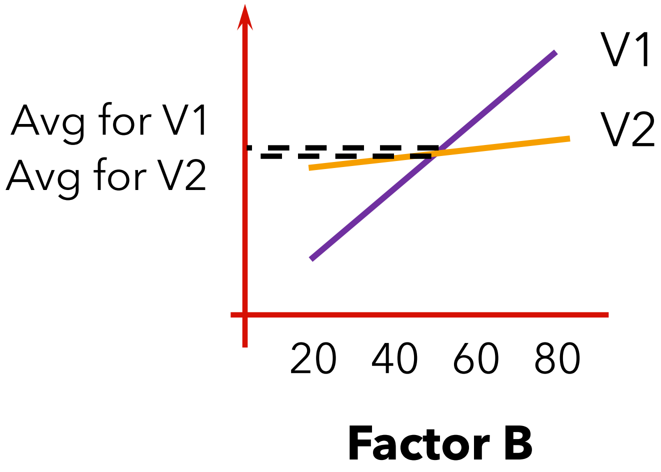
No Interaction

- Tests for **main effects are meaningful** because differences are constant across all levels of factor B



Interaction

- Tests for **main effects may be misleading**
- In this case the test would show no differences between varieties, when in fact their response to factor B is very different



1.3 Result presentation

Non-significant interaction

- Look at the **MAIN EFFECTS ONLY**

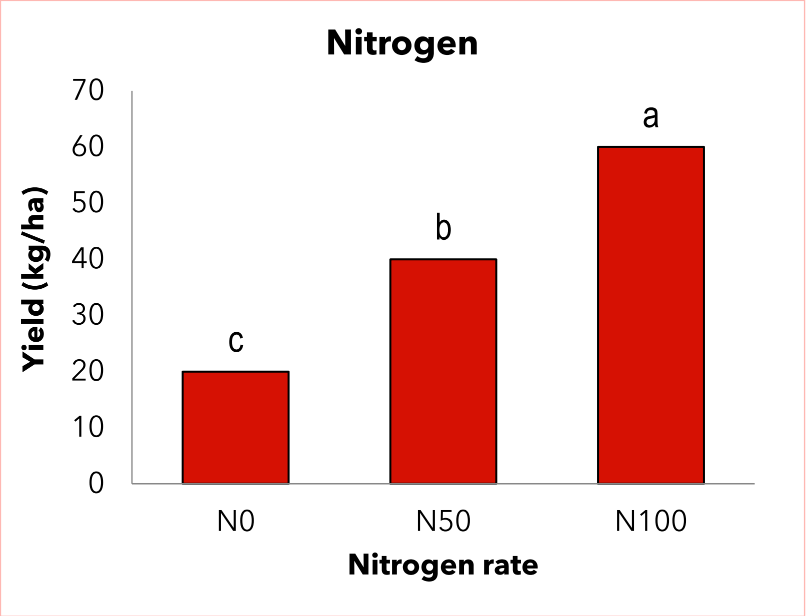


Figure 1.1. The effect of nitrogen rate on yield.

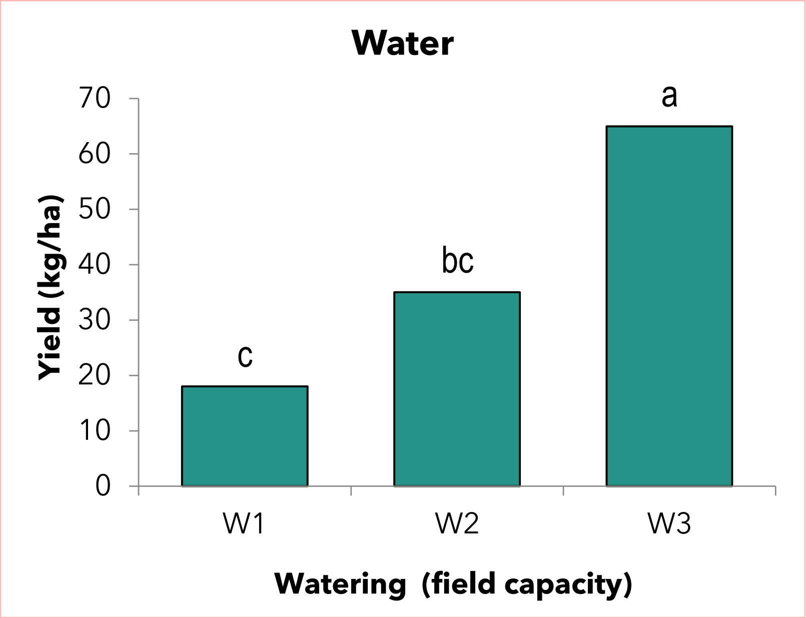
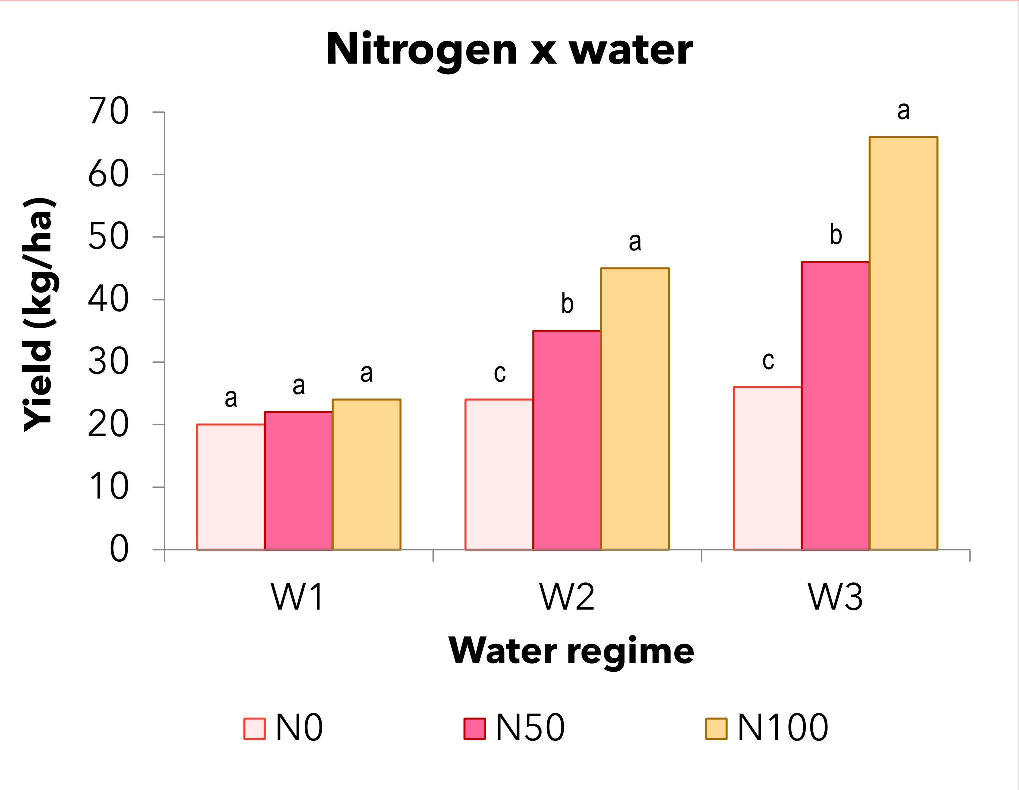


Figure 1.2. The effect of watering on yield.

1.3 Result presentation

Significant interaction

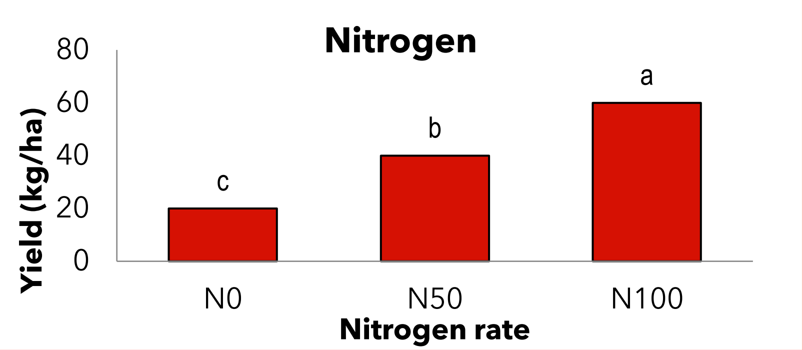
- Significant interaction means:
  - The effect of one factor influences the effect of the other factor.
- Data must be SORTED by one factor and comparisons must be made within a SORTED factor

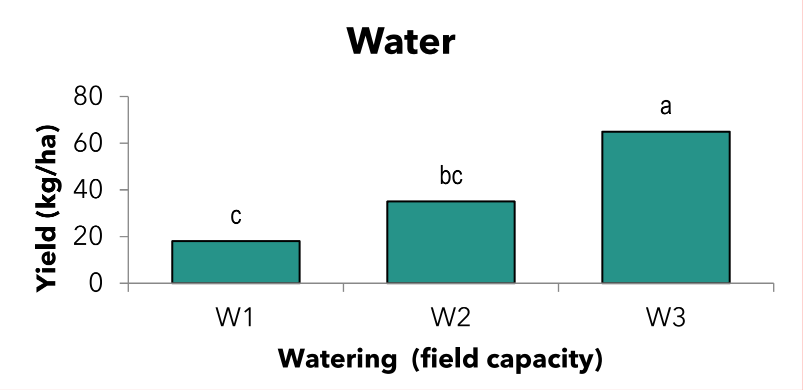


1.3 Result presentation

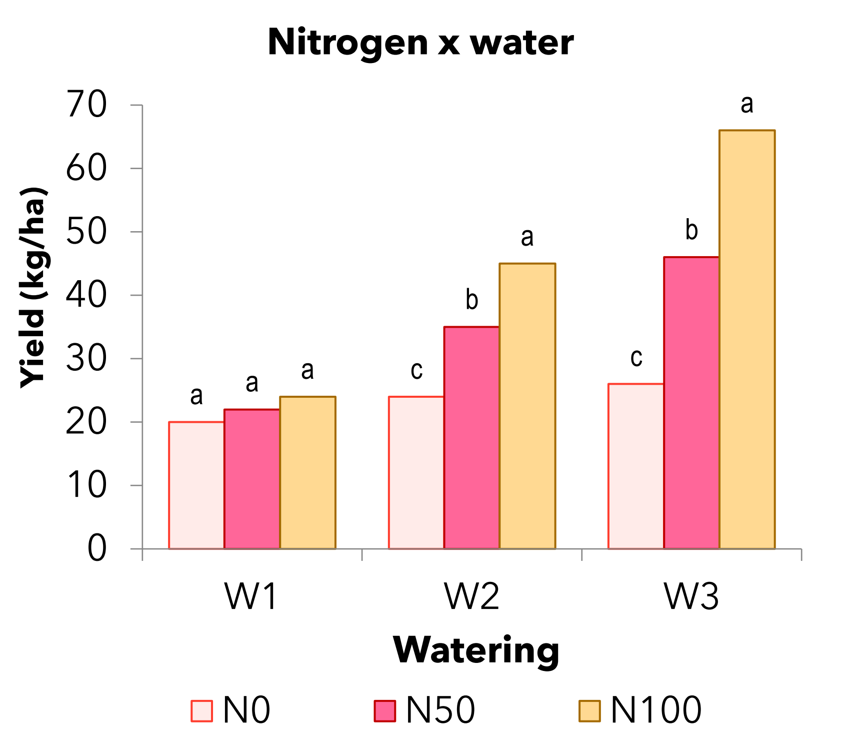
Main vs. interaction effects

Main effects





Interaction effects



1.3 Result presentation

Example

Yield and quality of two kenaf varieties as affected by harvesting age.

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Factors:

1. Varieties: MHC 123 and V 36
2. Harvesting age: 8, 12, 16 and 20 weeks after planting (WAP)

1.3 Result presentation | Non-significant interaction

Table 1: Dry matter yield based on harvest age and variety

Treatment	Dry matter (kg plot^{-1})	Yield (t ha^{-1})
Harvest age (H)		
8 WAP	1.71b	8.5b
12 WAP	2.24a	11.2a
16 WAP	2.18a	10.9a
20 WAP	2.03a	10.2a
Variety (V)		
MHC 123	2.34a	11.7a
V 36	1.74b	8.7b
Significance level		
Harvest age	**	**
Variety	**	**
H x V	ns	ns
Mean	2.43	12.15
CV	13.66	13.66

Means with the same letter were not significantly different among harvest age and variety ($p>0.05$) using LSD

** $p<0.01$, ns: not significant.

Main effects

ANOVA (simplified)

Note:
The results can be presented using a table or a figure (graph).

1.3 Result presentation | Significant interaction (Crossover)

Main effects and ANOVA

Table 2: Mean crude protein content

Treatment	CP (%)
Harvest age (H)	
8 WAP	20.3a
12 WAP	14.3b
Variety (V)	
MHC 123	18.1a
V 36	16.5a
Significance level	
Harvest age	**
Variety	ns
H x V	**
Mean	17.29
CV	9.62

Means with the same letter were not significantly different among harvest age and variety p>0.05 using LSD.

Interaction effect: Graph

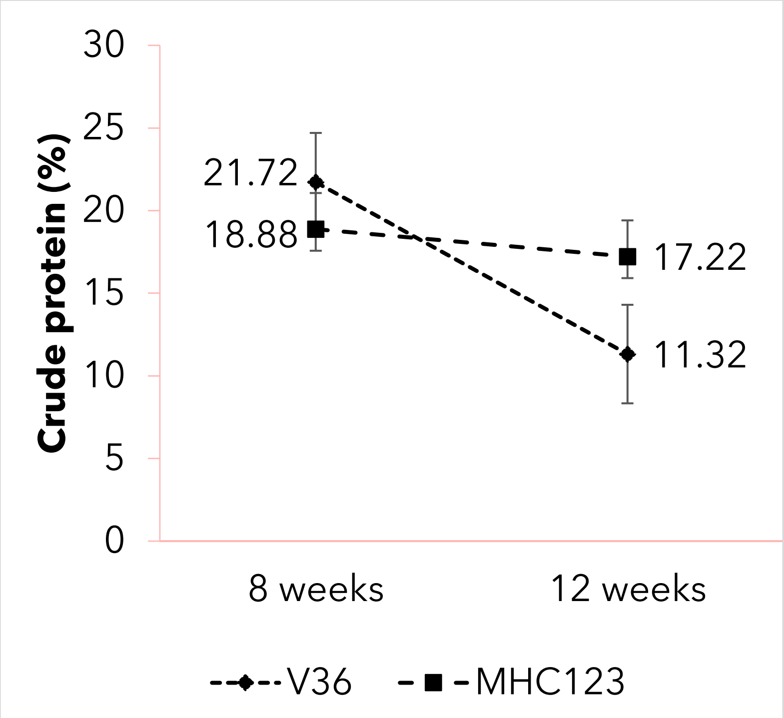


Figure 1. The effect of variety and harvest age on crude protein %.

Interpretation:

When the interaction is significant, the interpretation should focus on the interaction effect as shown in the graph.

1.3 Result presentation | Significant interaction (non-crossover)

Main effects and ANOVA

Table 3: Mean acid detergent fiber content

Treatment	ADF (%)
Harvest age (H)	
8 WAP	35.6b
12 WAP	46.2a
Variety (V)	
MHC 123	34.3b
V 36	47.6a
Significance level	
Harvest age	**
Variety	**
H x V	**
Mean	40.93
CV	7.61

Means followed by the different letters were significantly different among harvest age and variety using the LSD (P<0.05)
** P<0.0l

Interaction effect: Graph

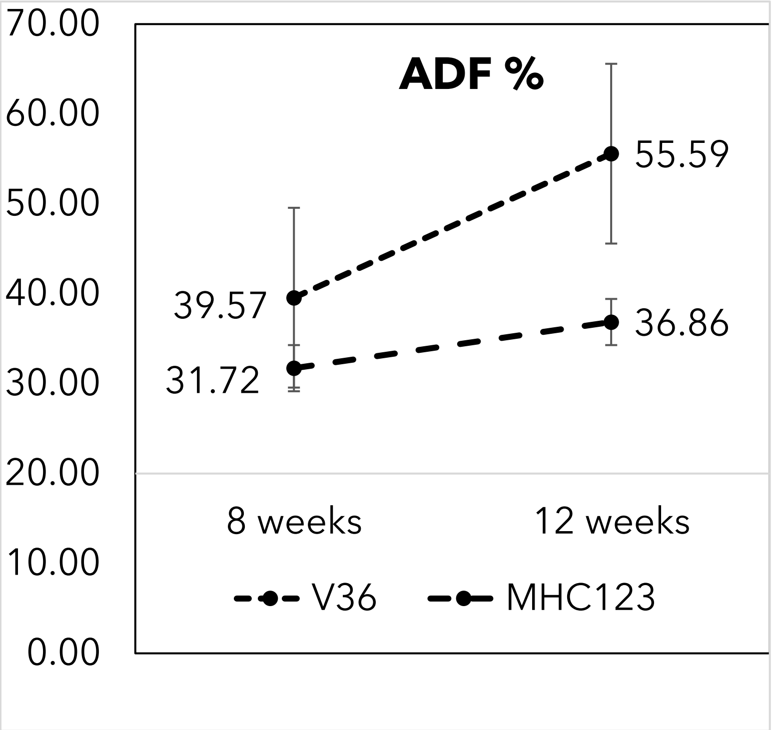


Figure 2. The effect of variety and harvest age on ADF %

Interpretation:

When the interaction is significant, the interpretation should focus on the interaction effect as shown in the graph.

1.4 Three-factor experiment

- Three factors – three IV
  - Effect of watering: 100% FC, 50% FC, 20% FC
  - Effect of Nitrogen: 0 kg/ha, 50 kg/ha, 100 kg/ha
  - Effect of variety: A, B, C.

1.4 Three factor experiment

ANOVA of 3 factor experiment - CRD

Source	df	SS	MS	F
Water (W)	2			
Nitrogen (N)	2			
Variety (V)	2			
W x N	4			
W x V	4			
N x V	4			
W x N x V	8			
Error	9			
Total	35			

3 main effects
4 interaction terms

1.5 Two-factorial example

- To study the effect of row spacing and phosphate on the yield of bush beans
  - 3 spacings: 45 cm, 90 cm, 135 cm
  - 2 phosphate levels: 0 and 25 kg/ha

S2P1 60	S1P2 45	S1P1 55
S1P1 65	S3P1 55	S3P1 51
S3P2 66	S3P2 57	S1P2 43
S3P1 59	S1P1 58	S2P1 54
S1P2 56	S2P2 50	S2P2 45
S2P2 62	S2P1 59	S3P2 50

1.5 Two-factorial example

Tables of Means

Treatment Means

Phosphate	Spacing S1	S2	S3	Mean
P1	59.3	57.7	55.0	57.3
P2	48.0	52.3	57.7	52.7
Mean	53.7	55.0	56.3	55.0

Block Means

Block	I	II	III	Mean
Mean	61.3	54.0	49.7	55.0

1.5 Two-factorial example

ANOVA table

Source	df	SS	MS	F
Block	2	417.33	208.67	31.00**
Spacing	2	21.33	10.67	1.58
Phosphate	1	98.00	98.00	14.56**
S X P	2	148.00	74.00	11.00**
Error	10	67.33	6.73	
Total	17	752.00		

** Significant at the 1% level.
CV = 4.7%
StdErr Spacing Mean = 1.059
StdErr Phosphate Mean = 0.865
StdErr Treatment (SxP) Mean = 1.498

1.5 Two-factorial example

Report of statistical analysis | Table (A two-way table)

Table 1. The effect of phosphate and spacing on yield of bush bean (kg/ha)

Phosphate (kg/ha)	Spacing (cm)		
	45	90	135
0	59.33a	57.67ab	55.00b
25	48.00c	52.33b	57.67a

Within phosphate (row), the means with different letters are significantly different at $P<0.05$ using LSD

Interpretation:

- Yield response depends on whether or not phosphate was supplied
- If no phosphate - yield decreases as spacing increases
- If phosphate is added - yield increases as spacing increases
- Blocking was effective

The explanation is based on "phosphate" level (separated by phosphate)

1.5 Two-factorial example

Report of statistical analysis | Figure (Line graph)

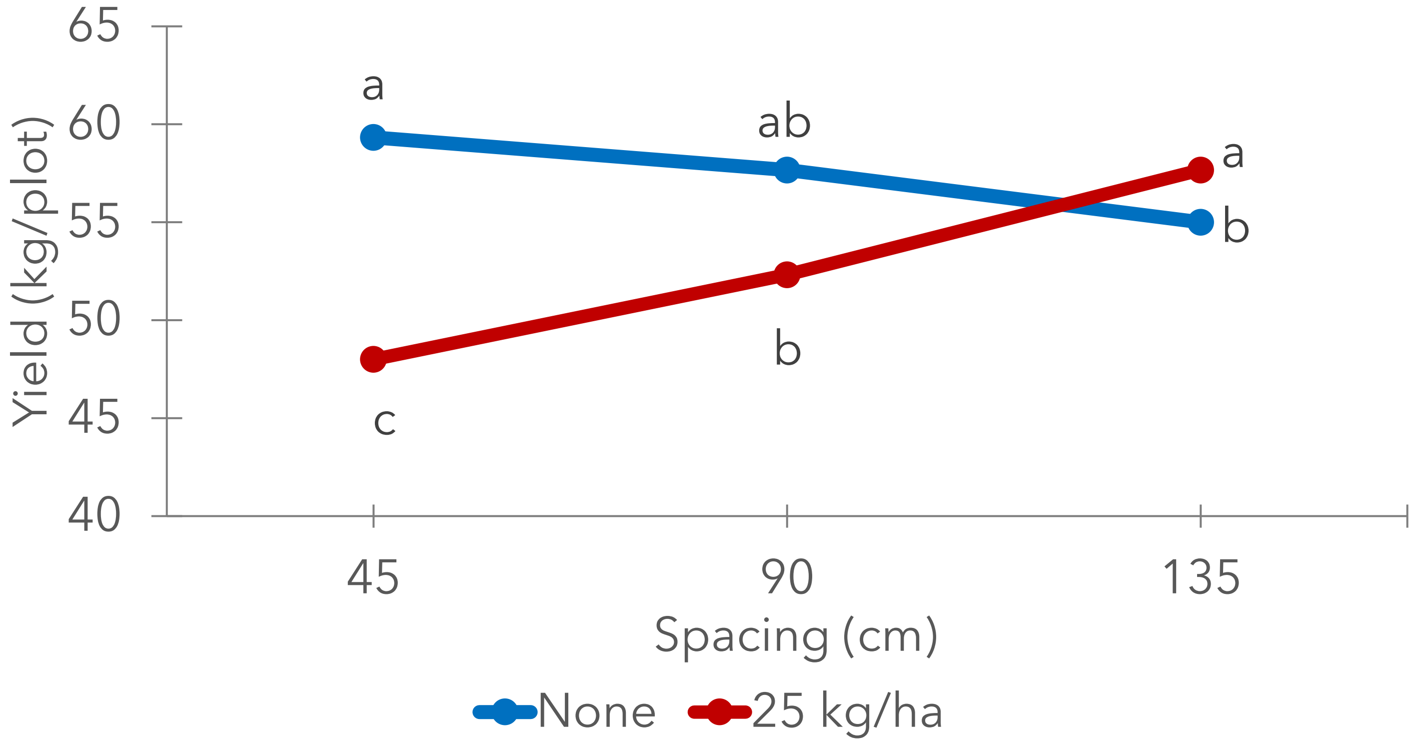
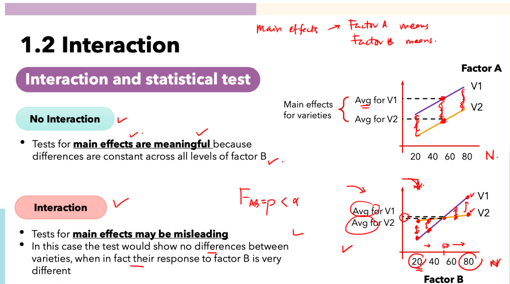


Figure 1. The interaction between spacing and rate of P on bush bean yield (kg/plot). Within phosphate, means with different letters are significantly different at P<0.05 using LSD.

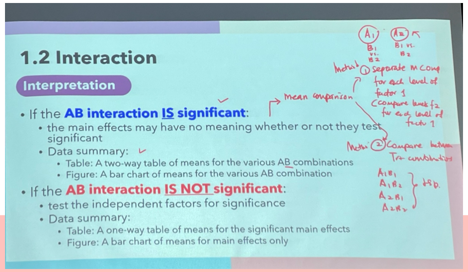
1.6 Review question

1. Two-factorial experiment is an experimental design. True of false? Explain your answer.
2. Explain the requirement in an experiment to test the interaction effects between factors.
3. List the interaction types and explain each of the interaction. see the figure below
4. How do you proceed with mean comparison if the interaction is significant in ANOVA (p-value < alpha (e.g., 0.05))? see the figure below
5. Explain why the approach for mean comparison is different when the interaction term in ANOVA is significant compared to when the interaction is not significant.

Q5



Q4



Q3