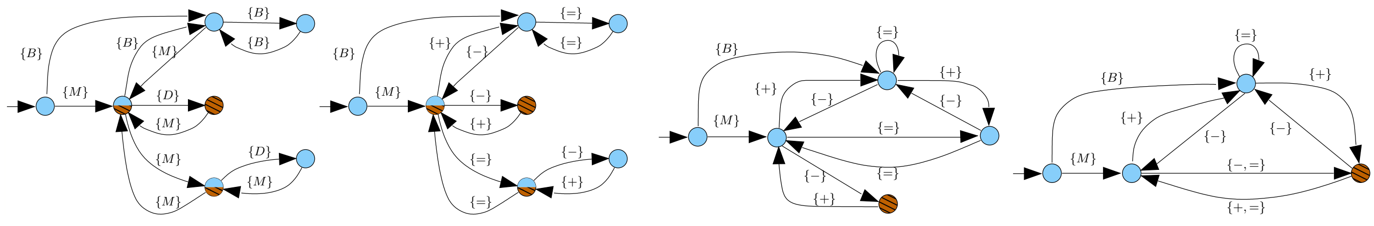

(a) (b) (c) (d)

Fig. 13: Sensori-computational devices to choose appropriate locomotion modes for the driving drone in Figure 12, with blue=fly, brown=drive; (a) works in the original signal space. The other three are derivatives that operate in the space of changes as expressed via variator D_ℓ. Applying $\textsc{Delta}_{D_\ell}$(a) gives (b). Both (c) and (d) choose a single output for each vertex; (c) is output stable, while (d) is not.

Remark 10. The concept of vertex stability is related to, but distinct from, the concept of the $\mathcal{N}$-pump (from Definition 22). For instance, a vertex stable sensori-computational device may not always be ready for an element from $\mathcal{N}$. On the other hand, simulating modulo the $\mathcal{N}$-pump relation need not imply the device is vertex stable; Figure 14 provides such an example.

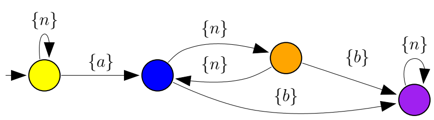

Fig. 14: A device that output simulates F_{tiny} modulo $\mathrm{P}_{\{n\}}$. (Recall that device F_{tiny} is the one in Figure 7.) It is neither vertex stable nor output stable with respect to $\{n\}$. The device can be made output stable, without becoming vertex stable, by altering the orange to become blue. Thereupon, vertex stability is possible if the two blue vertices are merged.

Figure 13d is another derivative for the driving drone scenario (and is smaller even than Figure 13c). It has not only a multi-state cycle on the '=' symbol, but this time the oscillatory behavior produces fluctuations in the output stream, with values going: blue, brown, blue, brown, …. Given that these describe actions for the robot to take-off, and then land, then take-off, … this is highly undesirable behavior. One naturally asks whether a device exists which avoids this issue:

Definition 37 (output stable)**.** For an $F = (V, V_0, Y, \tau, C, c)$ and a set $\mathcal{N} \subseteq Y(F)$, we say F is *output stable with respect to* $\mathcal{N}$ when, for all $s_1 s_2 \ldots s_{n-1} s_n \in \mathscr{L}(F)$ with $s_n \in \mathcal{N}$, $\mathscr{C}(F, s_1 s_2 \ldots s_{n-1}) = \mathscr{C}(F, s_1 s_2 \ldots s_n)$ and $|\mathscr{C}(F, s_1 s_2 \ldots s_n)| = 1$.

This concept is related to the notion of chatter in switched systems, and work that schedules events of a switched system in order to be non-chattering [2].

Further we note, both Figures 13c and 13d differ from Figure 13b in that they provide a single output per state. One is always free to make a singleton prescription:

Property 38. For any device $F = (V, V_0, Y, \tau, C, c)$, suppose one constructs $F_{\text{sing}} = (V, V_0, Y, \tau, C, c_{\text{sing}})$, where c_{sing} is a version of c restricted to singleton choices, viz., picked so that for all $v \in V$, $c_{\text{sing}}(v) \subseteq c(v)$ and $|c_{\text{sing}}(v)| = 1$. Then $F_{\text{sing}} \sim F \pmod{\mathbf{id}}$.

Hence, if one seeks an output stable sensori-computational device, then finding a vertex stable one will suffice (because, formally, Theorem 8 can be applied at last step to change to a singleton version).

The universal monoid integrator (from Algorithm 5) has blocks that encode the transitions arising from the action of the monoid. Being a monoid action, the 1_D element is neutral, which manifests in a simple fact: vertices in the third layer, after $\textsc{FillLeaves}_D(\cdot)$, have self loops labeled with 1_D. This specific structure leads to the following observation.

Theorem 39. For any device F with monoidal variator $(D, \bullet)$, $1_D \notin Y(F)$, which is $\left(\nabla\big|^{D}_{F} \fatsemi \int\oplus\big|_{F}\right)$-simulatable, there exists a single F' such that:

1) $F' \sim F \left(\bmod \nabla\big|^{D}_{F} \fatsemi \partial\oplus\big|_{Y(F)}\right)$,
2) $F' \sim F \left(\bmod \nabla\big|^{D}_{F} \fatsemi \mathrm{P}_{\{1_D\}}\right)$,
3) $F' \sim F \left(\bmod \nabla\big|^{D}_{F} \fatsemi \pi_{\{1_D\}}\right)$,
4) F' is vertex stable with respect to $\{1_D\}$, and
5) F' is output stable with respect to $\{1_D\}$.

Proof: An F' obtained by applying the choice process of Property 38 to the result of $\textsc{FillLeaves}_D(\textsc{Int}_D(F))$. It will suffice for 1) owing to the previous correctness of Algorithms 4 and 5. The self loops on all vertices in the layer comprising blocks implies that 2) holds, as any string traced with 1_D added (after the first symbol) in any quantity, arrives at the same final vertex. Since $\pi_{\{1_D\}}$ cannot drop the first symbol, the omission of monoid units simply avoids some loops, hence 3) holds. This specific structure implies 4). And, also, since the $c(\cdot)$ function has been restricted to a $c_{\text{sing}}(\cdot)$ choice in producing F', 4) suffices for 5). ∎

VIII. Summary and Outlook

This paper's focus has been less on sensors as used by people currently but rather on whether some hypothetical event sensor might be useful were it produced. So: what then is an event sensor, exactly? The preceding treatment has shown that there are several distinct facets. At the very core is the need to have some signal space on which differences can be meaningfully computed. This requires some basic statefulness, even if it is very shallow (like Example 5, the single-pixel camera). We formalize this idea in the concept of an observation variator. Also important is the model of event propagation. In this paper four separate cases have been identified and distinguished, namely: tightly coupled synchronous, event-triggered, polling, and asynchronous cases. At least in our framework, some of these choices depend on variators having certain properties with which to encode or express aspects of signal differences. Our model expresses these cases through relations. The notion of output simulation modulo those relations leads to decision questions, for which we were able to provide algorithms that give solutions if they exist. There remain other properties of interest and practical importance (such as vertex and output stability) which one might like to impose as constraints on the sensori-computational devices one seeks. We can meet these constraints when the variator possesses the algebraic structure of a monoid, as our final theorem is constructive.

More work remains to be done, but a start has been made on the question of whether information conflated in the process of forming an event sensor — the process of eventification — harms input–output behavior. We especially believe that this paper's extension of the notion of output simulation, and the algorithms we describe, ought to serve as a useful foundation for future work. One important limitation of the theory developed in this paper is that, as it depends on sequences of symbols, discrete time appears from the very outset. To directly model truly analog devices (as distinct from eventified digital ones) a theory dealing with continuous time may be required. Since the vast majority of robots process streams of digitized data, this may be a question of what one decides to treat as the atomic elements that generate observations. But one might also imagine a hybrid theory, connecting a continuous time approach with the model presented in this work.

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