

that could introduce any bias in the LLM. We work with open-source datasets and hence produce no additional bias than what might already be part of it. During our manual evaluations, annotators were asked to correct math problems, where there is only one logically correct answer, reducing the suspect of biased annotations. We mostly used existing algorithms for supporting our analysis, hence making our findings more reliable. For reproducibility, we release the hyper-parameters used in our experiments.

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