quantity and quality are better than MOSQP (F), we calculated the MOSQP, MOSQP (F) and NSGAII algorithm (300) approximate Pareto front corresponding to the purity index $\overline{t}_{p,s}$, Spread index $\Gamma_{p,s}$ $\Delta_{p,s}$.

Table 6.3.2 Solve the corresponding values of each performance index of ZDT 1

|	$\overline{t}_{p,s}$	$\Gamma_{p,s}$	$\Delta_{p,s}$
MOSQP	2.3659	0.0996	0.5656
MOSQP(F)	4.2174	0.1904	0.9252
NSGAII	2.9394	0.66	0.4462

From the above indicators, it can be analyzed that based on the same initial iteration point set, the MOSQP algorithm proposed in this paper has more number of non-dominated solutions for the approximate Pareto edge obtained by Problem1, with higher quality, which is better than MOSQP (F) and NSGAII algorithms, and the divergence index is also less than MOSQP (F) algorithm. For the problem of high dimension of decision variables, the calculation of Hessian matrix is avoided, and the calculation efficiency can be greatly improved.

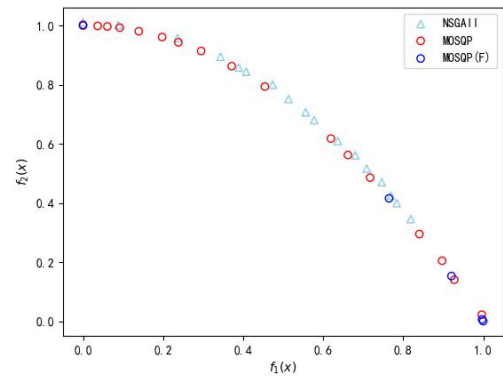ZDT 2 for two-objective boundary constraint optimization of Pareto front (n=30):

$$(ZDT2) \quad \min \ F(x)=[f_1(x)=x_1, f_2(x)=1-(\frac{f_1(x)}{g(x)})^2]^T$$

$$g(x)=1+\frac{9}{n-1}\sum_{i=2}^{n}x_i$$

$$s.t. \quad 0\leq x_i \leq 1, i=1,\cdots,n$$
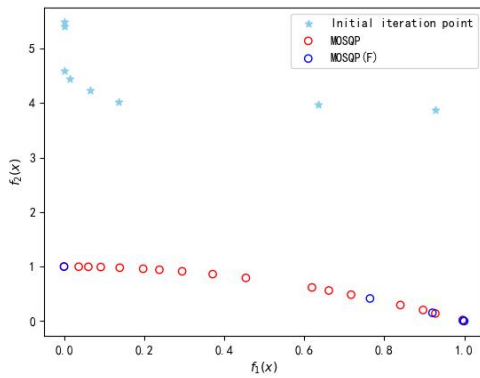
(a) Algorithm iteration process   (b) MOSQP (F) and NSGAII algorithm frontier comparison

Figure 4.2.4 MOSQP algorithm, NSGAII obtained problem (ZDT 2)

From Figure Figure 4.2.4, we can see that the MOSQP algorithm proposed here can effectively solve the frontier non-convex constrained multi-objective optimization problem ZDT 2, and improve the MOSQP algorithm of diffusion step Pareto front in
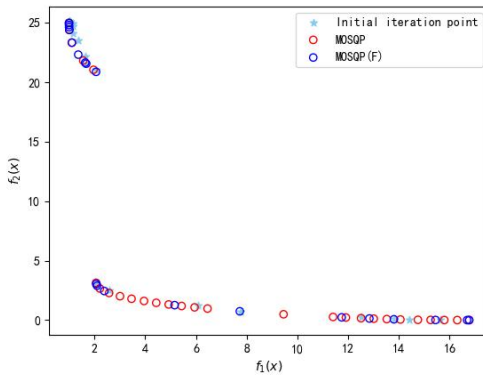
quantity and quality are better than MOSQP (F), we calculated the MOSQP, MOSQP (F) and NSGAII algorithm (300) approximate Pareto front corresponding to the purity metric $\overline{t}_{p,s}$, Spread metric $\Gamma_{p,s}$ $\Delta_{p,s}$.

Table4.2.3 Solve the corresponding values of each performance index of ZDT 2

	$\overline{t}_{p,s}$	$\Gamma_{p,s}$	$\Delta_{p,s}$	FE
MOSQP	2.0588	0.1906	0.6908	1148.8
MOSQP(F)	7	0.7643	0.7588	2219.6
NSGAII	2.6923	0.1476	0.5619	--

From the above indicators, it can be analyzed that based on the same initial iteration point set, the MOSQP algorithm has a higher number of non-dominant solutions for the approximate Pareto edge obtained by Problem1, which is better than MOSQP (F) and NSGAII algorithms, and the divergence indexes are less than MOSQP (F) algorithm. However, the divergence index will deviate relative to the NSGAII algorithm. However, the divergence index will be higher relative to the NSGAII algorithm.

For the two-objective boundary constraint optimization problem of the Pareto front MOP 3:

$$(\text{MOP3}) \quad \max \quad F(x)=[f_1(x)=-1-(A_1-B_1)^2-(A_2-B_2)^2,$$
$$f_2(x)=-(x_1+3)^2-(x_2+1)^2]^T$$
$$A_1=\sin 1-2\cos 1+\sin 2-1.5\cos 2$$
$$A_2=1.5\sin 1-\cos 1+2\sin 2-0.5\cos 2$$
$$B_1=\sin x_1-2\cos x_1+\sin x_2-1.5\cos x_2$$
$$B_2=1.5\sin x_1-\cos x_1+2\sin x_2-0.5\cos x_2$$
$$s.t. \quad -\pi\le x_i\le\pi,i=1,\cdots,n$$
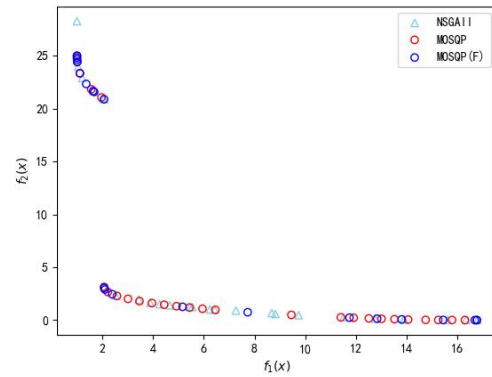
(a) Algorithm iteration process   (b) MOSQP (F) and NSGAII algorithm frontier comparison

Figure 4.2.5. MOSQP algorithm, NSGAII obtained problem (MOP3)

From Figure 4.2.5, we can see that the MOSQP algorithm proposed in this paper can effectively solve the noncontinuous constrained multi-objective optimization problem of Pareto front, MOP 3, and in this paper propose that the Pareto front obtained by MOSQP algorithm is better than MOSQP (F) algorithm in both quantity and quality.same. we calculated the MOSQP, MOSQP (F) and NSGAII algorithm (300) approximate Pareto front corresponding to the purity metric $\bar{t}_{p,s}$, Spread metric $\Gamma_{p,s}$ $\Delta_{p,s}$.

Table 4.2.4 Solving the corresponding values of each performance index of MOP 3

	$\bar{t}_{p,s}$	$\Gamma_{p,s}$	$\Delta_{p,s}$
MOSQP	2.1228	17.7978	1.4662
MOSQP(F)	6.05	17.8549	1.2833
NSGAII	2.75	17.7383	1.4027

It can be analyzed from Table 4.2.4 that, based on the same initial iteration point set, the proposed MOSQP algorithm has a larger number of non-dominant solutions for the approximate Pareto front obtained by Problem1, with higher quality, which is better than MOSQP (F) and NSGAII algorithm, and the divergence index is not much different from MOSQP (F) and NSGAII algorithm.

By solving the above different types of problems, we find that the MOSQP algorithm proposed can be effectively solved for nonlinear function constraint optimization, high-dimensional boundary constraint optimization, non-convex in Pareto front, and non-continuous multi-objective optimization in Pareto front. And the improved MOSQP is better than the MOSQP (F) algorithm and NSGAII in the number of non-dominant solutions in the Pareto front. The divergence index is inferior to NSGAII, but the MOSQP (F) algorithm has different problems. Moreover, in the multi-objective optimization problem with high dimension of decision variables, it can improve the computational efficiency.

5 Conclusion

This paper proposes a new MOSQP algorithm based on the low-order smooth penalty function as the value function of line search. The algorithm constructs a single objective optimization problem based on each objective function, and solves the secondary planning sub-problem to spread in the feasible domain, selects the non-dominant point set after diffusion, then constructs the new secondary planning problem by Armijo step rule, linear search step, and corrected by Powell in 1978. If the