B Supplementary Case

Question:
Among the 900 residents of Aimeville, there are 195 who own a diamond ring, 367 who own a set of golf clubs, and 562 who own a garden spade. In addition, each of the 900 residents owns a bag of candy hearts. There are 437 residents who own exactly two of these things, and 234 residents who own exactly three of these things. Find the number of residents of Aimeville who own all four of these things.

Response:
Step by Step Solution:
1. Let's define the following variables:
Let A be the set of residents who own a diamond ring.
Let B be the set of residents who own a set of golf clubs.
Let C be the set of residents who own a garden spade.
Let D be the set of residents who own a bag of candy hearts. Since all 900 residents own a bag of candy hearts, $|D| = 900$.
Let a_i denote the number of residents who own exactly i out of the 4 items.
Let's use the principle of inclusion-exclusion for the union of 4 sets A, B, C, D.
...
$a_4 = \frac{-219}{-4} = \frac{219}{4} = 54.75$.
Since a_4, the number of people owning all four items, must be a whole number, our current approach needs to be reconsidered in terms of the combinatorial counting of people owning different combinations of the items.
Instead of directly using the inclusion-exclusion principle, we can use a different approach involving the number of people owning combinations of the items A, B, C, and D.
Let x_i be the number of people owning exactly i item among A, B, C (excluding the item D which everyone owns).
Then the total number of people owning exactly k items among the A, B, C, and D can be broken down:
...

Table 3 A case of reflective behavior emergence.