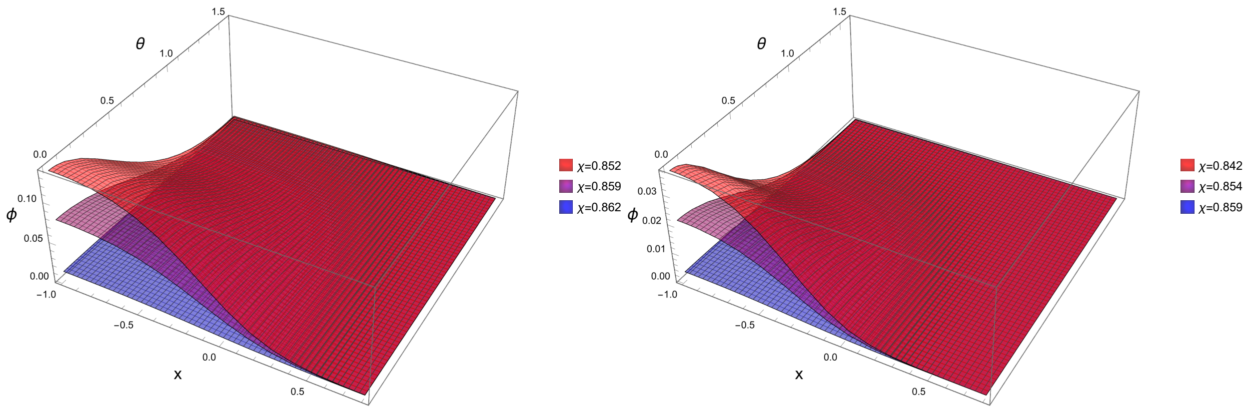

FIG. 2. Scalar field wave function $\phi(x,\theta)$ for representative scalarized black holes along the black solid lines in Fig. 1 for $\alpha = -100$ (**Left Panel**) and $\alpha = -1000$ (**Right Panel**). The scalar field corresponding to the critical scalarized black hole at point B_1 is highlighted in red and exhibits the largest amplitude. As the black hole solution approaches the bifurcation line, the scalar field amplitude decreases. In all cases, the scalar field wave functions are concentrated near the black hole poles and decay with increasing distance from the event horizon.

same q and χ, suggesting that KN black holes are slightly entropically favored over scalarized black holes. In contrast, for $\alpha > 0$, scalarized KN black holes coexist with unstable KN black holes and are always entropically favored within the coexistence region [39].

Fig. 2 presents representative scalar wave functions $\phi(x,\theta)$ for $\alpha = -100$ and -1000. Three scalarized black hole solutions along the constant-q line connecting B_1 and B_2, are selected. All scalar wave functions exhibit maxima near the black hole poles and decay outward from the event horizon, consistent with the fundamental scalar cloud wave functions reported in [55]. Notably, rapidly rotating KN black holes with $\alpha > 0$ develop equatorial-plane scalar cloud concentrations [57], contrasting the polar concentrations observed here. As the black hole spin decreases (i.e., approaching the critical line), scalar field amplitudes intensify near the horizon and poles. The scalarized black hole on the critical line exhibits the largest scalar field amplitude. Additionally, the amplitude of the scalar field is significantly higher for $\alpha = -100$ compared to $\alpha = -1000$.

To quantify the energy stored in the scalar field outside the event horizon, we define

$$E_\phi = \int_{r_H}^{\infty} n^\mu n^\nu T^\phi_{\mu\nu} dV, \tag{18}$$

where $T^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(\partial\phi)^2/2$ is the stress-energy tensor of the scalar field, and n^α is the unit normal vector to spatial hypersurfaces in the $3+1$ decomposition of spacetime. In Fig. 3, we

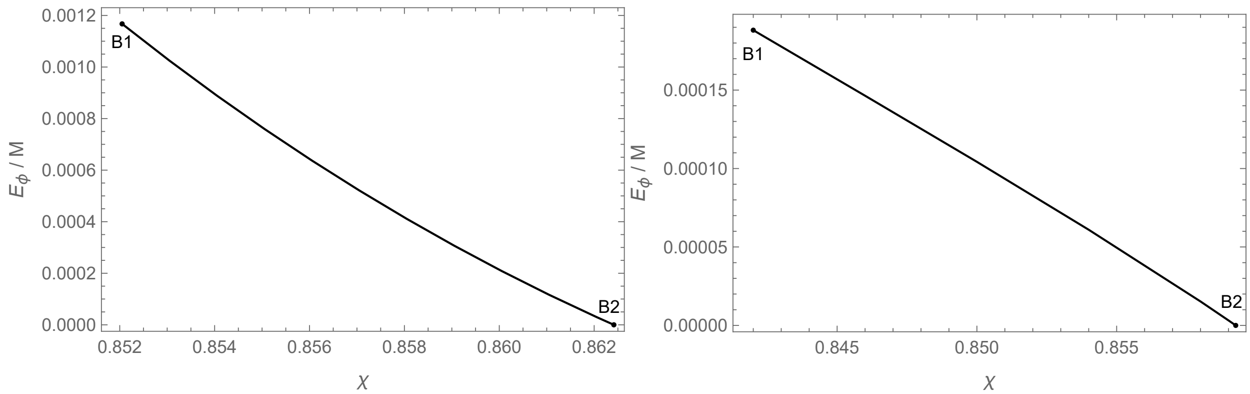

FIG. 3. Ratio of scalar field energy to black hole mass, E_ϕ/M, as a function of χ along the black solid lines in Fig. 1. **Left Panel:** $\alpha = -100$. **Right Panel:** $\alpha = -1000$. The critical scalarized black hole at point B_1 stores the largest amount of energy in the scalar field, while the black hole on the bifurcation line at point B_2 has zero scalar field energy.

present E_ϕ as a function of χ for scalarized black holes on the constant-q line connecting B_1 to B_2, for $\alpha = -100$ and -1000. The results show that the scalar field energy increases as χ decreases (i.e., approaching the critical line). Furthermore, E_ϕ constitutes only a small fraction of the black hole mass M, on the order of 10^{-3} for $\alpha = -100$ and 10^{-4} for $\alpha = -1000$. This suggests that the backreaction of the scalar field on the spacetime geometry is limited. Consequently, scalarized KN black hole solutions do not significantly deviate from KN black holes with scalar cloud along the bifurcation line. This limited deviation may help explain the considerably smaller domain of existence compared to cases with positive coupling. Moreover, the suppressed nonlinear effects of the scalar field also imply that terms beyond quadratic order in the series expansion of $f(\phi)$, given in Eq. (4), do not contribute significantly. This, in turn, suggests similarities in the features of scalarized black holes across different coupling functions. Indeed, Appendix B demonstrates that the existence domains for scalarized black holes are quite similar for both exponential and quadratic coupling functions.

Compared to scalarized KN black holes with $\alpha > 0$ discussed in [39], the $\alpha < 0$ case exhibits two notable distinctions: (1) the scalar field energy is substantially smaller, and (2) scalarized KN black holes coexist with stable KN black holes, rather than unstable ones. To understand these differences, consider the dynamical evolution of an unstable KN black hole undergoing a tachyonic instability, and assume that the end state of this evolution is a scalarized KN black hole[1]. During the evolution, the scalar field accumulates outside the event horizon, while scalar,

[1] Other final states of the dynamical evolution are also possible. For instance, an unstable KN black hole may

electromagnetic and gravitational radiation carry away energy and angular momentum to infinity. Since the scalar field is electrically neutral, the total charge of the system remains conserved—an observation supported by fully nonlinear simulations of Reissner-Nordström (RN) black holes evolving into scalarized RN black holes [61]. Furthermore, it has been shown that slower spin enhances (suppresses) the tachyonic instability for the $\alpha > 0$ ($\alpha < 0$) case [55, 57]. At least during the early stages of the evolution, the spacetime remains well approximated by a KN black hole. Consequently, in the $\alpha > 0$ case, angular momentum loss accelerates scalar field condensation, whereas in the $\alpha < 0$ case, it hinders the process. As a result, the final scalarized KN black hole in the $\alpha < 0$ scenario is expected to deviate less from the initial KN black hole, which may account for the significantly lower scalar field energy. Moreover, for $\alpha < 0$, if the final scalarized KN black hole coexists with a KN black hole having the same global charges, the coexisting KN black hole possesses less angular momentum—and therefore a weaker tachyonic instability—than the initial (unstable) KN black hole. This reduction in angular momentum may stabilize the coexisting KN black hole, explaining why scalarized KN black holes with $\alpha < 0$ can coexist with stable KN black holes.

IV. CONCLUSIONS

In this work, we have investigated the scalarization of KN black holes within the EMS models, which exhibit a spin-induced tachyonic instability for negative coupling constants, as identified in [53, 62]. By numerically constructing spin-induced scalarized KN black hole solutions for both exponential and quadratic coupling functions, we analyzed their domain of existence and physical properties in the (χ, q) parameter space. Our results show that slowly rotating stationary black holes in these models are well described by the KN metric, whereas rapidly rotating ones develop scalar hair. Additionally, we found that the scalar field energy constitutes only a small fraction of the total black hole mass, indicating suppressed nonlinear effects of the scalar field during scalarization. This provides a plausible explanation for the relatively narrow domain of existence of scalarized black hole solutions and the similarity between solutions obtained with different coupling functions.

Notably, our analysis reveals that spin-induced scalarized KN black holes coexist with linearly stable KN black holes. This coexistence may be attributed to the loss of angular momentum during the scalarization of unstable KN black holes. If scalarized KN black holes represent the

evolve into a spun-down, stable KN black hole surrounded by a scalar cloud, which is eventually depleted through electromagnetic and gravitational wave emission or other dissipative processes.