A Multi-objective Sequential Quadratic Programming Algorithm Based on Low-order Smooth Penalty Function

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Abstract: In this paper, we propose a Multi-Objective Sequential Quadratic Programming (MOSQP) algorithm for constrained multi-objective optimization problems, basd on a low-order smooth penalty function as the merit function for line search. The algorithm constructs single-objective optimization subproblems based on each objective function, solves quadratic programming (QP) subproblems to obtain descent directions for expanding the iterative point set within the feasible region, and filters non-dominated points after expansion. A new QP problem is then formulated using information from all objective functions to derive descent directions. The Armijo step size rule is employed for line search, combined with Powell's correction formula (1978) for B^j iteration updates. If QP subproblems is infesible, the negative gradient of the merit function is adopted as the search direction. The algorithm is proven to converge to an approximate Pareto front for constrained multi-objective optimization. Finally, numerical experiments are performed for specific multi-objective optimization problems

Keywords: Constrained multi-objective optimization; Low-order smooth penalty function; MOSQP algorithm; Approximate Pareto front

0 Introduction

In 2015, Fliege and Vaz [1] using the idea of single target SQP method, developed the SQP method to solve the constraint multi-objective optimization problem, the algorithm is divided into two stages, by solving the secondary planning subproblem obtain search direction, it uses the L1 accurate penalty function as a value function line search step, to obtain approximate Pareto frontier with constraint multi-objective optimization problem. Fliege and Vaz discuss the problem of its convergence to a local Pareto optimal solution under appropriate differential assumptions. Extensive numerical experiments also confirm the superiority of this algorithm compared to the state-of-the-art multi-objective optimization solver NSGA-II and the classical scalarization method MOScalar. However, the value function in the paper uses the most classical L1 exact penalty function, and only the symmetric positive definite matrix is the Hesse matrix or the unit matrix is discussed. Later, we can consider correcting the symmetric positive definite matrix, and avoid a large calculation of Hesse matrix, and avoid the disadvantages such as using only the unit matrix. And the sub-problems in the algorithm are not necessarily feasible at every iterative step. In 2019, Gebken et al. [2] also discussed the solution of unconstrained and constrained multi-objective optimization problems by solving quadratic planning subproblems for multi-objective optimization problems. The iterative process in the paper does not use the penalty function, and only positive constraints are used in the subproblems. However, the SQP method in the paper requires a feasible initial approximation, which is very difficult to implement in the nonlinear constraint problem. In 2021, Ansary, Panda [3] [4] proposed a SQP algorithm different from [1] by proposing different secondary planning sub-problems and penalty functions, and had good computing performance compared with the original algorithm and classical weights and methods.

In this paper, we propose a new MOSQP algorithm based on the multi-objective optimization problems with inequality constraints, and prove that it converges to the Pareto critical point of the original multi-objective optimization problem under certain conditions. Finally, numerical experiments are conducted on multi-objective optimization problems with boundary constraints or nonlinear constraints, and the algorithm performance is compared by NSGAII and MOSQP(F).

1 Preliminaries

1.1 Basic definition

Gradient: let $f: R^n \to R$, and $f(x)$ is a continuous differentiable function, then the first derivative is defined as:

$$\nabla_x f(x) = (\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}, \cdots \frac{\partial f(x)}{\partial x_n})^T$$

Hessian Matrix: let $f: R^n \to R$, and $f(x)$ is a second-order continuous differentiable function, then say:

$$\nabla_{xx} f(x)=\begin{bmatrix} \frac{\partial^2 f(x)}{\partial x_1{}^2} & \frac{\partial^2 f(x)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(x)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(x)}{\partial x_2 \partial x_1} & \frac{\partial^2 f(x)}{\partial x_2{}^2} & \cdots & \frac{\partial^2 f(x)}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f(x)}{\partial x_n \partial x_1} & \frac{\partial^2 f(x)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(x)}{\partial x_n{}^2} \end{bmatrix},$$

is the Hesse matrix of $f(x)$ at $x=(x_1,x_2,\cdots x_n)^T \in X \subset R^n$.

Jacobi Matrix: let $F: R^n \to R^m$, then say:

$$J_F(x_{1,}x_2,\cdots x_n)=\begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \cdots & \frac{\partial f_2(x)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m(x)}{\partial x_1} & \frac{\partial f_m(x)}{\partial x_2} & \cdots & \frac{\partial f_m(x)}{\partial x_n} \end{bmatrix},$$

is the Jacobi matrix of $F(x)=(f_1(x),\cdots,f_m(x))^T, m\geq 2$ at $x=(x_1,x_2,\cdots x_n)^T \in X \subset R^n$

1.2 Problem

This paper focuses on the following multi-objective optimization problems of inequality constraints：

$$\text{(CMOP)} \quad \min \ F(x)=(f_1(x),\cdots,f_m(x))^T, m\geq 2$$
$$s.t. \quad g_i(x)\leq 0, i\in I=\{1,2,\cdots a\}$$

where $x=(x_1,x_2,x_3,\cdots,x_n)^T \in X \subset R^n$, $f_i(x): R^n \to R$, $g_i(x): R^n \to R$ are second-order continuous differentiable functions， $X=\left\{x \middle| g_i(x)\leq 0 \ , \ i\in I\right\}$ is the feasible domain of the problem. Define the following symbols:

$I_0(x)=\left\{i\in I \middle| g_i(x)=0\right\}$， $I_+(x)=\left\{i\in I \middle| g_i(x)>0\right\}$， $I_-(x)=\left\{i\in I \middle| g_i(x)<0\right\}$

Globally optimal solution: If $x^*=(x_1,x_2,\cdots x_n)^T \in X \subset R^n$, for each $i=1,\cdots,m, x\in X$ there are:

$$f_i(x^*)\leq f_i(x)$$

Then x^* is the global optimal solution of problem (CMOP).