are indistinguishable in terms of their interactions. In network systems, this equates to stating that there are situations in which the nodes need to be suitably non-identical in order for them to stably converge to identical dynamical states even if all nodes occupy structurally equivalent positions in the network. This leads to the counter-intuitive conclusion that node heterogeneity across a network can help—rather than inhibit—convergence to a homogeneous dynamical state, as required for numerous processes including synchronization, consensus, and herding.

But what enables converse symmetry breaking? Symmetry breaking itself can be immediately appreciated by considering a one-dimensional Mexican hat potential, where the state with reflection symmetry (the system's symmetry) is unstable and the two stable states are asymmetric. This simple example illustrates the fundamental tenet that a symmetric model can describe asymmetric observations. Conversely, the phenomenon demonstrated here shows that symmetric observations may require an asymmetric model. While potentially less immediate to visualize, which might be the reason it was not demonstrated earlier, converse symmetry breaking can be interpreted as arising from the following trade-off: system asymmetry can reduce the likelihood of having a symmetric solution, but it can also increase the likelihood of having one such solution stable. This realization opens the door for new control approaches to manipulate system parameters and optimize the stability of symmetric states in networks whose function benefits from the symmetry of these states. While we purposely designed our experiment with identically coupled oscillators to isolate the phenomenon, the opportunities for optimization and control are even more evident when the identical coupling constraint is lifted, since the stabilizing effect of breaking the system symmetry with node heterogeneity is expected to be common in such cases. We thus suggest that the results presented here will naturally extend to real systems with tunable node parameters, such as networks of logic gates, neuronal systems, coupled lasers, and networks of mechanical, electrical, and chemical oscillators.

## Methods

**Experimental design.** The AC generators used in the experiment are of $2.5\,\mathrm{V}$ and $0.5\,\mathrm{W}$ because they are easily available and provide the desirable low-voltage, low-power output. While larger generators could synchronize with less noise as they are machined with higher precision relative to their sizes, they require power electronics, and leave less room for errors that could cause physical damage. To ensure that the generators are as identical as possible, we obtained eight generators from the same manufacturer, and selected three of them with the closest terminal voltage for a given rotational speed. The procedure we used to measure the generator parameters is detailed in Supplementary Information, Sec. S2.

We chose a specific AC frequency, $\omega_{\mathrm{s}} = 100\,\mathrm{Hz}$, which equals $100/3 \approx 33.3$ rotations per second for the generator shaft (our generators have three pole pairs). The generators provide $1.55\,\mathrm{V}$ terminal (r.m.s.) voltage at this speed (without load), and we verified that the voltage was proportional to speed, up to $\approx 55$ rotations per second. However, we found that the terminal voltage dropped to $\approx 1.4\,\mathrm{V}$ when the generator was connected to the network with a typical configuration we considered. We thus used $1.4\,\mathrm{V}$ as the base (a reference value) for normalizing voltages in all per-unit calculations. The impedance base ($3.48\,\Omega$) was determined in the process of making a choice of electrical component parameters, as described below, and the power base ($0.56\,\mathrm{W}$) was determined accordingly from the chosen voltage and impedance bases.

For any given combination of capacitance, inductance, and resistance that make up a rotationally symmetric circuit of the same form as in Fig. 1b (and our choice of $\omega_{\mathrm{s}} = 100\,\mathrm{Hz}$), the theoretical prediction of $\lambda^{\max}$ for the splay states for any $\boldsymbol{\beta}$ can be computed using the procedure described in the section 'Calculation of $\lambda^{\max}$' below. The specific combination shown in Fig. 1b was identified through multiple iterations of system design in which we made refinements in our choice of electrical components, construction of the experimental modules, and measurement apparatus. The refinement of component parameters was performed using their normalized, per-unit values (for example, the capacitance was chosen such that the shunt susceptance of the $\pi$ model representing each link is $\approx 1.04$ in per unit). Ultimately, we chose a parameter combination in which the predicted stability of the splay states is sufficiently strong for $\widetilde{\boldsymbol{\beta}}$ and improves significantly when changing to $\boldsymbol{\beta}_g$, while ensuring that system components operate within their rated capacities and the $\boldsymbol{\beta}$ change is experimentally realizable by friction adjustment. The actual inductors ($0.32$ H and $7.8\,\Omega$, according to the manufacturer's specification) and capacitors ($475\,\mu$F) were selected from those that were readily available to closely match the values in Fig. 1b.

**Modeling dynamical noise.** We analyze the stability of Eq. (1) in the presence of the noise term $\varepsilon\xi_i(t)$, described either by a discrete-time model or a continuous-time model. In the discrete-time model, $\varepsilon\xi_i(t)$ is modeled as random impulse perturbations (i.e., as a sum of Dirac delta functions with random magnitudes located at random times). We can account for any distributions of perturbation magnitudes and times that are bounded in the sense that there is a maximum magnitude $M$ for the impulse perturbations and a minimum time interval $\tau$ between consecutive perturbations (which corresponds to a maximum rate at which the system is perturbed). When the system is close to a splay state with maximum Lyapunov exponent $\lambda^{\max} < 0$, the dynamics approximately follows the linearized equation (as described in the main text) and is characterized by an exponential convergence to the splay state at a rate of $\lambda^{\max}$ between any two consecutive perturbations. Thus, if the deviation of the system state from the splay state immediately after the $k$th perturbation at time $t_k$ is $\Delta\mathbf{x}(t_k)$, the deviation after the next perturbation is given by $\Delta\mathbf{x}(t_{k+1}) = e^{\lambda^{\max}\Delta t_k} \cdot \Delta\mathbf{x}(t_k) + \boldsymbol{\xi}_{k+1}$, where $\Delta t_k$ is the (random) time interval between the $k$th and $(k+1)$th perturbations, and $\boldsymbol{\xi}_k$ is the (random) displacement resulting from the $k$th impulse perturbation. Since $\Delta t_k \geq \tau$ and $\|\boldsymbol{\xi}_k\| \leq M$, we have $\|\Delta\mathbf{x}(t_{k+1})\| \leq e^{\tau\lambda^{\max}}\|\Delta\mathbf{x}(t_k)\| + M$. By recursively applying this inequality, we obtain

$$\begin{aligned} \|\Delta\mathbf{x}(t_k)\| &\leq (e^{\tau\lambda^{\max}})^k \|\Delta\mathbf{x}(t_0)\| + M \cdot \frac{1-(e^{\tau\lambda^{\max}})^{k+1}}{1-e^{\tau\lambda^{\max}}} \\ &\to \frac{M}{1-e^{\tau\lambda^{\max}}} \quad \text{as } k \to \infty, \end{aligned} \tag{4}$$

which indicates that the deviation from the splay state is bounded by $M/(1-e^{\tau\lambda^{\max}})$ in the limit of large $k$ (and thus large $t$). Using $\|\Delta\mathbf{x}(t_k)\| \leq \Delta_{\text{sync}}$, $\forall k$, where $\Delta_{\text{sync}}$ is a constant, we see that the system would stay synchronized in the splay state if

$$\frac{M}{1-e^{\tau\lambda^{\max}}} < \Delta_{\text{sync}}. \tag{5}$$

It follows that the splay state is stable in the presence of noise if the noise magnitude $M$ is sufficiently small to satisfy this condition. On the other hand, if the asymptotic bound $M/(1-e^{\tau\lambda^{\max}})$ is larger than $\Delta_{\text{sync}}$, there is a non-zero probability that the deviation $\|\Delta\mathbf{x}(t_k)\|$ exceeds $\Delta_{\text{sync}}$ for sufficiently large $k$, implying that the splay state is unstable. Therefore, given $M$, $\tau$, and $\Delta_{\text{sync}}$, there is a (negative) threshold $\lambda^{\max}$ value

$$\lambda^{\max}_{\text{th}} \equiv \frac{1}{\tau}\ln(1-M/\Delta_{\text{sync}}) \tag{6}$$

corresponding to the stability transition for the noisy system: the splay state is stable in the presence of noise if $\lambda^{\max} < \lambda^{\max}_{\text{th}}$ and unstable if $\lambda^{\max} > \lambda^{\max}_{\text{th}}$. Note that each value of $\lambda^{\max}_{\text{th}}$ represents a range of combinations of $M$, $\tau$, and $\Delta_{\text{sync}}$. In general, for any pair of configurations $\boldsymbol{\beta}_{\text{I}}$ and $\boldsymbol{\beta}_{\text{II}}$ with $\lambda^{\max}(\boldsymbol{\beta}_{\text{I}}) < \lambda^{\max}(\boldsymbol{\beta}_{\text{II}}) < 0$, there is a range of combinations of $M$, $\tau$, and $\Delta_{\text{sync}}$ for which $\lambda^{\max}(\boldsymbol{\beta}_{\text{I}}) < \lambda^{\max}_{\text{th}} < \lambda^{\max}(\boldsymbol{\beta}_{\text{II}})$, i.e., the splay state is stable for the $\boldsymbol{\beta}_{\text{I}}$ configuration, while it is unstable for the $\boldsymbol{\beta}_{\text{II}}$ configuration.