Acknowledgements
The authors thank John B. Ketterson for insightful discussions about this research. This research was funded by ARO Grant No. W911NF-15-1-0272 and by Northwestern University's Finite Earth Initiative (supported by Leslie and Mac McQuown).

Author contributions
F.M., T.N., and A.E.M. designed the research and contributed to the modeling. F.M. performed the experiments and simulations. F.M., T.N., and A.E.M. analyzed the results and wrote the paper. All authors approved the final manuscript.

Competing interests
The authors declare no competing interests.

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Supplementary Information

Network experiment demonstrates converse symmetry breaking

S1 Derivation of coupled oscillator network model

Our derivation of the deterministic part of the model in Eq. (1) of the main text is based on the so-called classical model of a generator. For completeness, we first reproduce a derivation of the classical model starting from Newton's second law written in terms of the total torque accelerating the rotor of generator i:

$$J_i \ddot{\phi}_{\mathrm{m},i} = -d_i \dot{\phi}_{\mathrm{m},i} + T_{\mathrm{m},i} - T_{\mathrm{e},i}, \tag{S1}$$

where $\phi_{\mathrm{m},i}$ is the mechanical (rotor shaft) angle of generator i (relative to a stationary axis), J_i is the total moment of inertia of the rotor, $T_{\mathrm{m},i}$ is the mechanical torque provided to the rotor, $T_{\mathrm{e},i}$ is the electrical torque (load), and d_i is the damping-torque coefficient accounting for windage and friction. Changing to a frame of reference rotating at the synchronous (mechanical) frequency ω_{sm} of the rotor through $\widetilde{\phi}_{\mathrm{m},i} \equiv \phi_{\mathrm{m},i} - \omega_{\mathrm{sm}} t$, Eq. (S1) becomes

$$J_i \ddot{\widetilde{\phi}}_{\mathrm{m},i} = -d_i (\dot{\widetilde{\phi}}_{\mathrm{m},i} + \omega_{\mathrm{sm}}) + T_{\mathrm{m},i} - T_{\mathrm{e},i}. \tag{S2}$$

Multiplying this by the angular velocity $\omega_{\mathrm{m},i} \equiv \dot{\phi}_{\mathrm{m},i}$, we can write

$$\begin{aligned} \omega_{\mathrm{m},i} J_i \ddot{\widetilde{\phi}}_{\mathrm{m},i} &= -\omega_{\mathrm{m},i} d_i (\dot{\widetilde{\phi}}_{\mathrm{m},i} + \omega_{\mathrm{sm}}) + \omega_{\mathrm{m},i} T_{\mathrm{m},i} - \omega_{\mathrm{m},i} T_{\mathrm{e},i} \\ &= -\omega_{\mathrm{m},i} d_i (\dot{\widetilde{\phi}}_{\mathrm{m},i} + \omega_{\mathrm{sm}}) + P_{\mathrm{m},i} - P_{\mathrm{e},i}, \end{aligned} \tag{S3}$$

where $P_{\mathrm{m},i}$ is the mechanical power supplied to the rotor and $P_{\mathrm{e},i}$ is the electrical power drawn from the rotor. When the system is close to a frequency-synchronous state, we have $\omega_{\mathrm{m},i} \approx \omega_{\mathrm{sm}}$, and we can write

$$\omega_{\mathrm{sm}} J_i \ddot{\widetilde{\phi}}_{\mathrm{m},i} = -\omega_{\mathrm{sm}} d_i \dot{\widetilde{\phi}}_{\mathrm{m},i} + P_{\mathrm{m},i} - \omega_{\mathrm{sm}}^2 d_i - P_{\mathrm{e},i}. \tag{S4}$$

Thus, in a synchronous steady state in which $\dot{\widetilde{\phi}}_{\mathrm{m},i} = \ddot{\widetilde{\phi}}_{\mathrm{m},i} = 0$, the mechanical power input must balance the electrical power output plus all the losses due to damping and friction. Dividing Eq. (S4) by a power base (P_{base}) allows us to express power in per-unit quantities:

$$\frac{\omega_{\mathrm{sm}} J_i}{P_{\mathrm{base}}} \ddot{\widetilde{\phi}}_{\mathrm{m},i} = -\frac{\omega_{\mathrm{sm}} d_i}{P_{\mathrm{base}}} \dot{\widetilde{\phi}}_{\mathrm{m},i} + P_{\mathrm{m},i}^{(\mathrm{pu})} - \frac{\omega_{\mathrm{sm}}^2 d_i}{P_{\mathrm{base}}} - P_{\mathrm{e},i}^{(\mathrm{pu})}, \tag{S5}$$

which leads to

$$\frac{2H_i}{\omega_{\mathrm{sm}}}\,\ddot{\widetilde{\phi}}_{\mathrm{m},i} = -\frac{D_i}{\omega_{\mathrm{sm}}}\,\dot{\widetilde{\phi}}_{\mathrm{m},i} + \widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i} - P^{(\mathrm{pu})}_{\mathrm{e},i}, \tag{S6}$$

where $H_i \equiv \frac{1}{2}J_i\omega_{\mathrm{sm}}^2/P_{\mathrm{base}}$ is the inertia constant (which equals the kinetic energy of the rotor at the synchronous speed), $D_i \equiv d_i\omega_{\mathrm{sm}}^2/P_{\mathrm{base}}$ is the damping coefficient, and $\widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i} \equiv P^{(\mathrm{pu})}_{\mathrm{m},i} - \omega_{\mathrm{sm}}^2 d_i/P_{\mathrm{base}}$ is the net power input.

We assume that each generator i can be represented by two nodes: an internal node, with voltage phasor $E_i = |E_i|\exp(j\delta_i)$, whose magnitude $|E_i|$ is assumed to be constant; and the terminal node, whose voltage is $V_i = |V_i|\exp(j\theta_i)$ and matches the generator's terminal voltage. Note that j denotes the imaginary unit. The internal and terminal nodes are connected through an (internal) impedance, which we measured in our experiment (see Sec. S2.3 below). If the generator has p pole pairs, the mechanical and electrical angles are related by $\delta_i = p\widetilde{\phi}_{\mathrm{m},i}$ (and thus $\dot{\delta}_i = p\dot{\widetilde{\phi}}_{\mathrm{m},i}$, $\ddot{\delta}_i = p\ddot{\widetilde{\phi}}_{\mathrm{m},i}$, and the synchronous voltage frequency $\omega_{\mathrm{s}} = p\omega_{\mathrm{sm}}$), leading to a second form of the equation of motion:

$$\frac{2H_i}{\omega_{\mathrm{s}}}\ddot{\delta}_i = -\frac{D_i}{\omega_{\mathrm{s}}}\dot{\delta}_i + \widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i} - P^{(\mathrm{pu})}_{\mathrm{e},i}. \tag{S7}$$

This is equivalent to the classical model. The voltage at the terminal node is related to the terminal voltages of the other generators by the Kirchoff laws as

$$P_i = \sum_{k=1}^{n} |V_i V_k Y_{0ik}| \sin(\theta_i - \theta_k - \alpha_{0ik} + \pi/2), \tag{S8}$$

where P_i is the real power injection from generator i into the network, and $Y_{0ik} = |Y_{0ik}|\exp(j\alpha_{0ik})$ are the complex entries of the admittance matrix $\mathbf{Y}_0 = (Y_{0ik})$ representing the AC electric network. Through a procedure known as Kron reduction applied to the network (see, for example, Refs. 44 and 45 of the main text), we can also express the real power injection $P^{(\mathrm{pu})}_{\mathrm{e},i}$ at the internal node of generator i in a form similar to Eq. (S8), but in terms of the internal voltage angles δ_i:

$$P^{(\mathrm{pu})}_{\mathrm{e},i} = \sum_{k=1}^{n} |E_i E_k Y_{ik}| \sin(\delta_i - \delta_k - \alpha_{ik} + \pi/2), \tag{S9}$$

where $\mathbf{Y} = (Y_{ik})$ is the *effective* admittance matrix with $Y_{ik} = |Y_{ik}|\exp(j\alpha_{ik})$, representing the