Network experiment demonstrates converse symmetry breaking

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Symmetry breaking—the phenomenon in which the symmetry of a system is not inherited by its stable states—underlies pattern formation, superconductivity, and numerous other effects. Recent theoretical work has established the possibility of *converse* symmetry breaking (CSB), a phenomenon in which the stable states are symmetric only when the system itself is not. This includes scenarios in which interacting entities are required to be non-identical in order to exhibit identical behavior, such as in reaching consensus. Here we present an experimental demonstration of this phenomenon. Using a network of alternating-current electromechanical oscillators, we show that their ability to achieve identical frequency synchronization is enhanced when the oscillators are tuned to be suitably non-identical and that CSB persists for a range of noise levels. These results have implications for the optimization and control of network dynamics in a broad class of systems whose function benefits from harnessing uniform behavior.

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Synchronization[1]—perhaps the most widely studied phenomenon in network dynamics[2–4]—has been observed in many contexts, including both natural systems (e.g., circadian clock cells[5–7], ecological populations[8,9], human menstrual cycles[10], and crowds of pedestrians[11]) and engineered systems (e.g., Boolean logic gates[12], semiconductor lasers[13–15], electrochemical and nanomechanical oscillators[16–18], and power generators[19–21]). Such observations are significant because they show that approximately homogeneous dynamics can emerge in heterogeneous populations. Yet, until recently, the prevailing view had been that homogeneity in the dynamics is facilitated by increased homogeneity in the population. This view has now changed with the theoretical discovery[22] that, in numerous systems, heterogeneity can be required for stable identical synchronization—even when the entities are identically coupled to the population.

The underlying phenomenon, which we term *converse symmetry breaking*, can be elegantly described using the notion of symmetry—a fundamental property that can characterize a system and has deep implications for its dynamics[23–32]. In contrast to the well-known phenomenon of symmetry breaking, in which symmetry in the system implies broken symmetry in the stable states, converse symmetry breaking represents a scenario in which symmetry in the stable states implies broken symmetry in the system. The interaction networks of many real systems are invariant under node permutations and hence possess symmetries[33]. Symmetry breaking in networks include important examples of chimera states[34–40], in which a broken-symmetry state with coexisting groups of synchronized and non-synchronized nodes is observed even though the system is symmetric. Converse symmetry breaking, on the other hand, has been predicted for oscillator networks in which the phenomenon can be mediated, for example, by amplitude dynamics[22], couplings internal to the oscillators[41], and interaction delays[42]. However, unlike symmetry breaking, evidence for converse symmetry breaking has thus far remained theoretical.

In this Article, we present the first experimental demonstration of converse symmetry breaking, in which we account for noise and other realistic features. Our experimental system is designed to allow for frequency synchronization and consists of alternating current (AC) electromechanical oscillators, which are identically coupled in order to isolate the effect of oscillator heterogeneity from that of coupling heterogeneity. We show that, within the precision of experimental measurements, the optimal stability of frequency synchronization can be enhanced by making the values of a tunable parameter of the oscillators—the damping coefficient—suitably different from each other. Our results indicate that we can harness converse symmetry breaking in optimizing network dynamics. In potential applications to networks in which synchronization is desirable, this would translate to controlling oscillator heterogeneity to enable enhanced

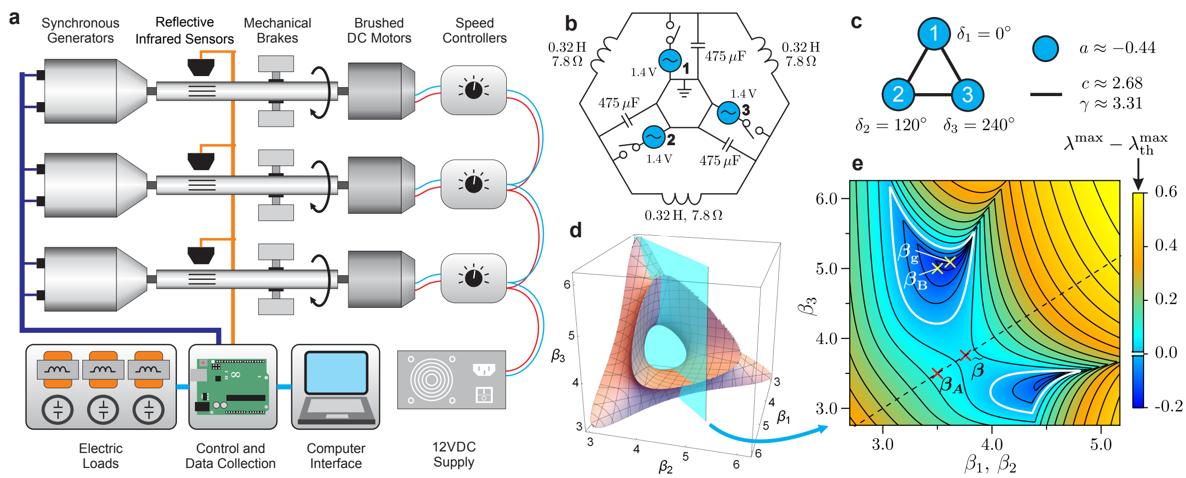

Fig. 1: **Experiment involving a network of coupled electromechanical oscillators.** **a**, Main components of the experimental setup, including three AC generators, three DC motors driving them, and the computerized data acquisition system. **b**, Diagram of the AC electrical circuit connecting the three generators, running at 100 Hz. **c**, Network representation of the circuit, where the nodes represent generators and the links represent the electrical interactions between them. The parameters characterizing the nodes and links are normalized by suitable references (i.e., given in per-unit quantities). **d**, Predicted stability of the frequency-synchronous splay states as a function of the generator parameters β_i. Inside the colored surface is the region of stability given by $\lambda^{\max} < \lambda^{\max}_{\text{th}}$ for noise corresponding to $\lambda^{\max}_{\text{th}} = -1.5$. **e**, Cross section of the stability landscape at the plane shown in **d**. Color-coded is the value of $\lambda^{\max}$ relative to $\lambda^{\max}_{\text{th}}$. The optimal uniform assignment ($\widetilde{\boldsymbol{\beta}}$) and the globally optimal non-uniform assignment ($\boldsymbol{\beta}_g$) are marked by red and yellow crosses, respectively. Also marked by crosses are the projections of $\boldsymbol{\beta}_A$ and $\boldsymbol{\beta}_B$, the nearby assignments that we realize experimentally.

stability and performance.

Figure 1a illustrates the main components of our experiment, in which three permanent-magnet generators are mechanically driven by DC motors with adjustable speed and a separate 12V DC power supply. The generators are chosen to have identical parameters (e.g., internal damping coefficient, internal impedance, and terminal voltage at various speeds) within manufacturing precision; see Methods for details. To allow for heterogeneous configurations of the generators, their shafts are equipped with mechanical brakes that can be used to adjust friction. The generators' output is connected to a set of electric loads (inductors and capacitors) forming the circuit depicted in Fig. 1b. The parameters of the circuit components are chosen for the system to be symmetric with respect to rotational permutations of the generators ($1 \to 2 \to 3 \to 1$). The pattern of coupling among the generators can thus be represented as a rotationally symmetric network of three nodes (generators) connected by three identical links,