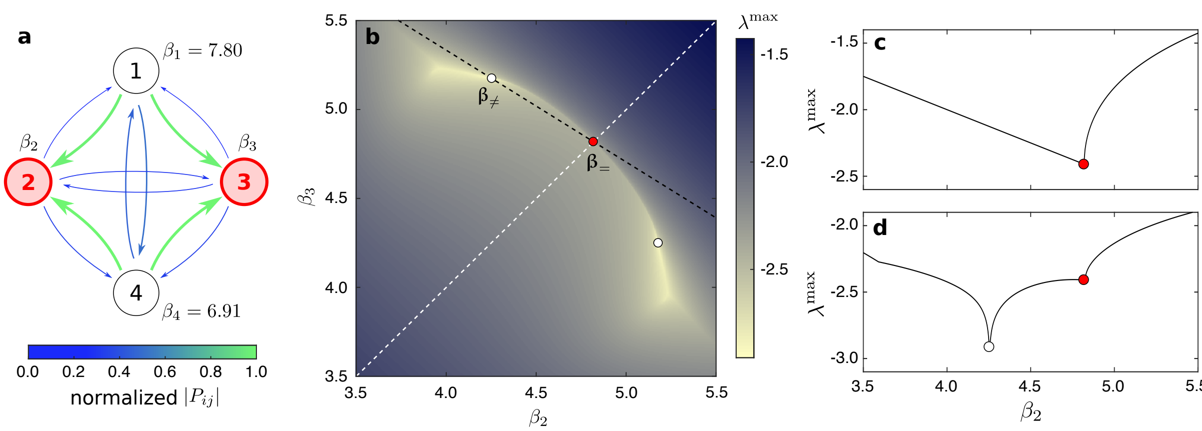

Fig. 5: **Illustrating CSB in a 4-generator example system.** **a** Effective interaction network connecting the generators and given by matrix $\mathbf{P}$. Both the thickness and color of the arrow connecting node j to node i represent the interaction strength $|P_{ij}|$, with the thickness proportional to $|P_{ij}|$ and the color encoding $|P_{ij}|$ normalized by its maximum over all i and j with $i \neq j$. **b–d** Dependence of $\lambda^{\max}$ on β_2 and β_3, with the values of β_1 and β_4 set to the values indicated in **a**. In **b**, $\lambda^{\max}$ is color coded to visualize the full 2D landscape. In **c**, $\lambda^{\max}$ is shown as a function of β_2 along the white dashed line in **b**, corresponding to $\beta_2 = \beta_3$. It attains its minimum value ≈ -2.40 at $\beta_2 = \beta_3 \approx 4.80$ (red circle). In **d**, $\lambda^{\max}$ as a function of β_2 along the black dashed line in **b** attains its minimum value ≈ -2.97 at $(\beta_2, \beta_3) \approx (4.27, 5.17)$ (white circle). Thus, the substantially improved optimal $\lambda^{\max}$ is possible only when the permutation symmetry between generators 2 and 3 is broken. For details on the system, see Methods.

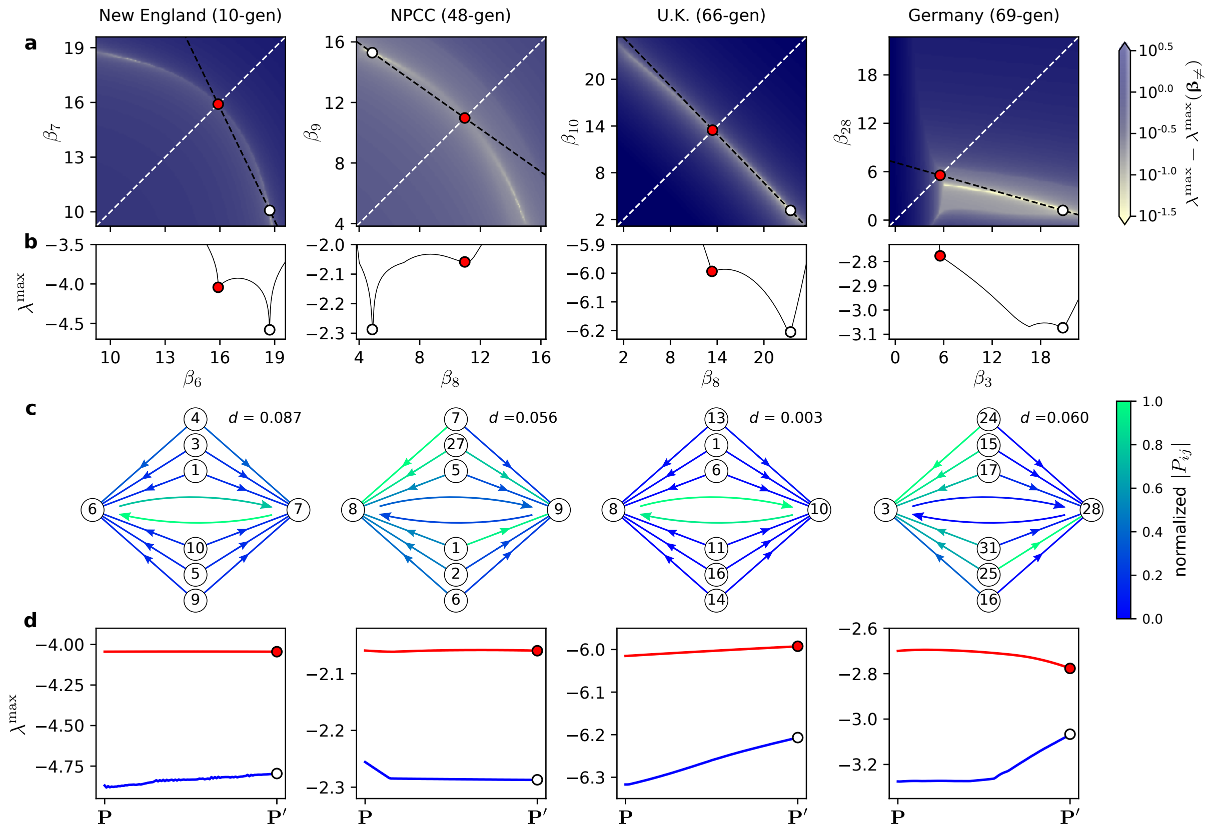

Fig. 6: **Isolating the CSB effect in power grids.** Each column synthesizes results for the system indicated at the top. **a** 2D stability landscape $\lambda^{\max}(\beta_{i_1}, \beta_{i_2})$, where i_1 and i_2 are the nodes whose permutation holds the symmetrized matrix $\mathbf{P}'$ invariant. In each panel, the red circle marks the optimal $(\beta_{i_1}, \beta_{i_2})$ on the diagonal $\beta_{i_1} = \beta_{i_2}$ (white dashed line), while the white circle marks the optimal when $\beta_{i_1} \neq \beta_{i_2}$ is allowed. The other β_i are fixed at the values in $\boldsymbol{\beta}_{\neq}$ identified for a stress level of 1 in Fig. 4. **b** Stability $\lambda^{\max}$ as a function of β_{i_1} along the black dashed line connecting the red and white circles in **a**, respectively. **c** Input strength patterns of the nodes i_1 and i_2 for the original matrix $\mathbf{P}$. The color of each arrow indicates $|P_{ij}|$, normalized by the largest value of $|P_{ij}|$ shown. For nodes i_1 and i_2, we show incoming links from the top six common neighbors in terms of the input strength. Also shown is the distance d from the original matrix $\mathbf{P}$ to its symmetrized version $\mathbf{P}'$, in which the two nodes receive identical incoming (weighted) links, defined as $d \equiv \|\mathbf{P} - \mathbf{P}'\|_2 / \|\mathbf{P}\|_2$, where $\|\cdot\|_2$ denotes the matrix 2-norm. **d** Change in the optimal $\lambda^{\max}$ with (red) and without (blue) the constraint $\beta_{i_1} = \beta_{i_2}$, as we interpolate between the original matrix $\mathbf{P}$ and its symmetrized version $\mathbf{P}'$. Node indexing is described for all four systems in Methods.

and much of the enhancement is maintained as one interpolates from the symmetrized system back to the original system (Fig. 6). This indicates that a significant portion of the stability enhancement for the original system can be attributed to CSB.

Discussion

Our demonstration that heterogeneity of generators can enhance the stability of synchronous states in a range of power grids suggests that there is large previously under-explored potential for tuning and upgrading current systems for better stability. Since larger conventional generators have larger inertia and thus larger impact on the stability of other generators, tuning of their parameters may be particularly beneficial. While we focused on the heterogeneity of a specific generator parameter here, further stability enhancement is likely to be possible by exploiting heterogeneity in other generator parameters and in the parameters of other network components as well as in the network topology. We suggest that such stability enhancement opportunities exist beyond power systems and extend to any network whose function benefits from homogeneous dynamics and whose stability depends on tunable system parameters. For example, the results presented here suggest that CSB can potentially be observed for coupled oscillatory flows in microfluidic networks and for networks of coupled chemical reactors whose oscillatory node dynamics is close to a Hopf bifurcation. It is known[13] that such systems can be parameterized so that their Jacobian matrices take a form generalizing Eq. (2), and they are thus capable of exhibiting CSB. The approach we developed here to isolate CSB is versatile and can be applied broadly to systems for which different heterogeneities co-occur. Determining how prevalent CSB is and how it depends on the properties of the system (e.g., the network size, link distribution, and node dynamics) are important questions for future research.

It is instructive to interpret our results and contrast them with past approaches in network optimization. In seeking the best approach, one may form two complementary hypotheses. One hypothesis, invoked in the past, was that the stability of the desired homogeneous states would be optimal when the system is homogeneous; the approach would thus be to limit the optimization search to the low-dimensional parameter subspace corresponding to networks with identical parameter values for all nodes. The other hypothesis, validated here, is that optimal stability of the desired homogeneous states is generally obtained with heterogeneous parameter assignment, which implies that the search for this optimum requires exploring the high-dimensional parameter space without making *a priori* assumptions on how the parameters of different nodes