effective coupling between generators. Substituting this into Eq. (S7) and defining

$$
\begin{aligned}
\beta_i &\equiv \frac{D_i}{2H_i} = \frac{d_i}{J_i}, \\
a_i &\equiv \frac{\omega_{\mathrm{s}}\left[\widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i} - |E_i^2 Y_{ii}|\cos(\alpha_{ii})\right]}{2H_i}, \\
c_{ik} &\equiv \frac{\omega_{\mathrm{s}}|E_i E_k Y_{ik}|}{2H_i}, \\
\gamma_{ik} &\equiv \alpha_{ik} - \pi/2,
\end{aligned}
\tag{S10}
$$

we obtain the oscillator network model in the form of Eq. (1) of the main text (with $\varepsilon = 0$).

Note that $\beta_i$ is constant because $d_i$ and $J_i$ are both physical constants characterizing the corresponding generator. Note also that, for a synchronous state with a given (constant) frequency $\omega_{\mathrm{s}}$, the parameter $H_i = \frac{1}{2}J_i\omega_{\mathrm{sm}}^2/P_{\mathrm{base}} = \frac{1}{2}J_i\omega_{\mathrm{s}}^2/(p^2 P_{\mathrm{base}})$ and the admittances $Y_{ik} = |Y_{ik}|\exp(j\gamma_{ik})$ are constant. Thus, if the parameters $\widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i}$ and $E_i$ are constant, then all the parameters defined in Eq. (S10) would be constant. In our experiment, we validated the constancy of $\omega_{\mathrm{s}}$, $\widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i}$, and $E_i$ directly from the measurements for each time-series segment (Supplementary Fig. 1). The linearization of Eq. (1) involving the matrix $\mathbf{P}$ in Eq. (2) assumes that all parameters in Eq. (S10) are constant, which is valid when $\omega_{\mathrm{s}}$, $\widetilde{P}^{(\mathrm{pu})}_{\mathrm{m},i}$, and $E_i$ are constant.

# S2 Measurement of generator parameters

We experimentally measured all generator parameters required for our analysis, i.e., the parameter $\beta_i$, the moment of inertia $J_i$, and the internal impedance $z_{\mathrm{int},i}$ for each generator ($i = 1, 2, 3$). To simplify the notations, the generator index $i$ in the subscript are omitted in the following subsections.

## S2.1 Effective damping parameter $\beta$

The adjustable parameter $\beta$ for each generator combines the moment of inertia and all damping effects for the given generator, $\beta = d/J$. When both the mechanical and the electrical torques are turned off, Eq. (S1) becomes $\ddot{\phi}_{\mathrm{m}} = -\frac{d}{J}\dot{\phi}_{\mathrm{m}} = -\beta\dot{\phi}_{\mathrm{m}}$, which can be expressed as $\dot{\omega}_{\mathrm{m}} = -\beta\omega_{\mathrm{m}}$. Thus, the rotor decelerates at an exponential rate of $\beta$. We can directly measure this rate by fitting an exponential decay curve to the experimentally measured rotor speed, after the generator is disconnected from the circuit and the DC motor driving it is turned off. We thus equipped the rotating shaft of each generator with a reflective infrared phototransistor that detects markings on the shaft, providing direct measurements of the rotor frequency. The function we used to fit the frequency measurement $\omega_{\mathrm{m}}(t)$ is the following:

$$y(t) = \begin{cases} A, & t < t_0, \\ A\exp(-\beta t), & t_0 \leq t < t_1, \end{cases} \tag{S11}$$

where the steady-state speed $A$, the turn-off time $t_0$, and the rate of deceleration $\beta$ are fitting parameters. The turn-off time $t_0$ needs to be fitted because the switching is initiated manually rather than by the microcontroller. The measured values of $\omega_{\mathrm{m}}(t)$ were taken up to $t = t_1$, at which $\omega_{\mathrm{m}}(t) \approx 20$ Hz; below this threshold the exponential decay is not a good approximation.

A typical fitting scenario is shown in Supplementary Fig. 4a, and a single set of measurements of $\beta$ in the $\boldsymbol{\beta}_A$ and $\boldsymbol{\beta}_B$ configurations are shown in panels b–d of the same figure. The standard deviation of these measurements was $\approx 0.1$, indicating the precision of the $\beta$ values that we could configure and confirm by experimental measurements. At the beginning of each experimental run, we adjusted the breaks if necessary to ensure that the average of $\beta$ (estimated from at least ten measurements) was within $\pm 0.1$ of the corresponding designed value for the given $\boldsymbol{\beta}$ configuration.

## S2.2 Moment of inertia $J$

The measurement of $J$ is based on the measurement of $\beta$, which was performed by estimating the exponential rate of deceleration as the DC motor driving the generator was turned off, while no load was attached to the generator. If we repeat this experiment but with a load connected to the terminal of the generator, then the generator would decelerate slightly faster. Using a resistor with $R = 2\,\Omega$ as the load (rated for a maximum of 10 W dissipation) would result in a time-dependent electric torque,

$$T_{\mathrm{e}}(t) = \frac{P_{\mathrm{e}}(t)}{\omega_{\mathrm{m}}(t)} = \frac{V^2(t)}{R\omega_{\mathrm{m}}(t)}, \tag{S12}$$

where $V(t)$ is the instantaneous r.m.s. voltage measured across the generator terminals. Substituting into Eq. (S1) and using $\omega_{\mathrm{m}} = \dot{\phi}_{\mathrm{m}}$ and $T_{\mathrm{m}} = 0$, we obtain

$$\dot{\omega}_{\mathrm{m}} = -\beta\omega_{\mathrm{m}} - \frac{V^2(t)}{JR\omega_{\mathrm{m}}(t)}, \tag{S13}$$

whose solution is given by

$$\omega_{\mathrm{m}}(t) = \exp\left(-\beta t\right)\left(\omega_{\mathrm{m}}^2(0) - \frac{2}{J}\int_0^t \frac{V^2(t')}{R}\exp(2\beta t')dt'\right)^{\frac{1}{2}}. \tag{S14}$$

With this and the measured time series of $V(t)$, we can calculate $\omega_{\mathrm{m}}(t)$ numerically for any $t$. The parameter $J$ can then be estimated by minimizing the sum of squared differences between these calculated values of $\omega_{\mathrm{m}}(t)$ and the corresponding direct measurements for all $t$ (taken up to a time at which $\omega_{\mathrm{m}}(t) \approx 20\,\mathrm{Hz}$).

The necessary voltage values $V(t)$ for Eq. (S14) were provided by direct measurement of the terminal voltages, which were taken simultaneously with the rotor speed measurements (using a reflective infrared phototransistor, as was done for the $\beta$ measurements; see Sec. S2.1 above). Since the voltage measurements were taken at a much higher rate than the rotor speed measurements ($3{,}320$ samples per second for the voltage vs. $480$ samples per second for the rotor speed at $\omega_{\mathrm{m}} = 40 \times 2\pi$ rad/s, which decreases as the rotor slows down), we used the sampling rate of the voltages for the time discretization of the integral in Eq. (S14). For the $\beta$ value in Eq. (S14), we substituted the estimate from Sec. S2.1 above, which is an average over many measurements (see, for example, Supplementary Fig. 4). The estimate of $J$ for each generator was then obtained by repeating this procedure several times (using the same average $\beta$ value) and taking the average. A typical fitting curve, as well as the estimated values of $J$ are provided in Supplementary Fig. 5.

## S2.3 Internal impedance $z_{\mathrm{int}}$

The measurement of the generator's internal impedance $z_{\mathrm{int}} = r_{\mathrm{int}} + jx_{\mathrm{int}}$ requires the open-circuit voltage ($V_{\mathrm{oc}}$) and the short-circuit current ($I_{\mathrm{sc}}$) to be measured at the same rotor speed. We can take these readings directly using the voltage and current sensors. Our experimental setup has a set of ACS712 current sensors that can measure the current passing through each generator's terminal. In large industrial synchronous generators the rotating magnetic field is induced by an adjustable field current, thus making the $V_{\mathrm{oc}}$ and $I_{\mathrm{sc}}$ values dependent on this current as well. Our permanent-magnet generators, however, have constant magnetic field, and thus $V_{\mathrm{oc}}$ and $I_{\mathrm{sc}}$ are functions of the rotor speed only. Both $V_{\mathrm{oc}}$ and $I_{\mathrm{sc}}$ generally increase linearly with the speed until the air gap flux starts to saturate the iron parts of the generator. This linear regime was observed up to $55$ rotations per second for our generators. We performed all our experiments and measurements in this regime.