each corresponding to a circuit equivalent to the so-called π model[43]. Each generator is fixed on its own platform to minimize mechanical coupling with the other generators via vibrations.

We focus on frequency-synchronous states of the system in which the voltage frequencies of the generators are all equal to a constant frequency ω_s. On short timescales, of the order of $1\,\mathrm{s}$ or less, the dynamics of the generators when the system is close to such a state can be described by a coupled oscillator model[44,45]. When written for an arbitrary number n of generators, the model equation reads:

$$\ddot{\delta}_i + \beta_i \dot{\delta}_i = a_i - \sum_{k\neq i} c_{ik} \sin\left(\delta_i - \delta_k - \gamma_{ik}\right) + \varepsilon \xi_i(t), \tag{1}$$

where δ_i is the internal electrical angle for generator i (a state variable related to the rotor shaft angle by a factor determined by the number of poles in the generator), relative to a reference frame rotating at the synchronous frequency ω_s; the constant β_i is an effective damping parameter (capturing both mechanical and electrical damping, normalized by the generator's inertia); a_i is a parameter representing the net power accelerating the generator's rotor (i.e., the mechanical power provided by the DC motor driving the generator, minus the power consumed by the network components and the power lost to damping); c_{ik} and γ_{ik} are the coupling strength and phase shift characterizing the electrical interactions between the generators; $\xi_i(t)$ is a random function representing dynamical noise; and ε is the noise amplitude (see Supplementary Information, Sec. S1 for a derivation of the deterministic part, Methods for details on modeling dynamical noise, and Supplementary Fig. 1 for validation). In the co-rotating frame, the frequency-synchronous states correspond to the fixed-point solutions of Eq. (1) with $\varepsilon = 0$, characterized by $\omega_i \equiv \dot{\delta}_i = 0$, $\forall i$. Note that uniform angle shifts of one such solution represent the same state, as all angle differences remain unchanged. The deterministic part of Eq. (1) has the same form as that used to model the dynamics of generators in power grids[20,21], which have recently been studied extensively in the network dynamics community[46–50], but here the equation is used to model coupled electromechanical oscillators that are tunable and not constrained to operating states of power grids.

Linearizing Eq. (1) for $\varepsilon = 0$ around a fixed point with $\delta_i = \delta_i^*$, we obtain $\dot{\mathbf{x}} = \mathbf{J}\mathbf{x}$, where $\mathbf{x} = \left(\begin{smallmatrix} \Delta\boldsymbol{\delta} \\ \Delta\boldsymbol{\omega} \end{smallmatrix}\right)$ and $\mathbf{J} = \left(\begin{smallmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{P} & -\mathbf{B} \end{smallmatrix}\right)$. Here, $\Delta\boldsymbol{\delta}$ and $\Delta\boldsymbol{\omega}$ are the n-dimensional vectors of angle and frequency deviations, $\delta_i - \delta_i^*$ and $\omega_i - 0 = \omega_i = \dot{\delta}_i$, respectively. The $n \times n$ matrix $\mathbf{P} = (P_{ik})$ is given by

$$P_{ik} = \begin{cases} -c_{ik}\cos(\delta_i^* - \delta_k^* - \gamma_{ik}), & i \neq k, \\ -\sum_{k'\neq i} P_{ik'}, & i = k, \end{cases} \tag{2}$$

$\mathbf{B}$ is the $n \times n$ diagonal matrix with β_i as its diagonal elements, and $\mathbf{0}$ and $\mathbf{1}$ denote the null and identity matrices of size n, respectively. The stability of the corresponding frequency-synchronous state of Eq. (1) with $\varepsilon = 0$ is thus determined by the eigenvalues λ_i of the Jacobian matrix $\mathbf{J}$, excluding the identically zero eigenvalue (which we denote by λ_1) present only because of the zero row-sum property of $\mathbf{P}$. Specifically, if the maximal real part of these eigenvalues is negative, i.e., the Lyapunov exponent $\lambda^{\max} \equiv \max_{i\geq 2} \mathrm{Re}(\lambda_i)$ is negative, then the state is asymptotically stable, and smaller $\lambda^{\max}$ implies stronger stability. (The zero eigenvalue λ_1 is excluded because it is associated with perturbations that uniformly shift phases, which lead to equivalent fixed points corresponding to the same frequency-synchronous state.) The problem of maximizing this stability with respect to system parameters can thus be formulated as the optimization of $\lambda^{\max}$. This is similar to the problem of optimizing the largest real part of the eigenvalues of a matrix, known as the spectral abscissa (relevant when the matrix has no identically null eigenvalues)[51–53], and the problem of optimizing the second largest eigenvalue of a Laplacian matrix, known as the algebraic connectivity[54,55]. Some previous studies have considered problems concerning the optimization of damping parameters[52,53], but they do not focus on relations to system symmetry and typically exclude the class of non-positive definite non-symmetric coupling matrices relevant to the experiment considered here.

In the presence of dynamical noise (i.e., when $\varepsilon > 0$), we can show that, for general classes of discrete- and continuous-time noise models, there is a (negative) threshold $\lambda^{\max}$ value for the stability of the frequency-synchronous state. We denote this stability threshold by $\lambda^{\max}_{\text{th}}$; the frequency synchronization is stable below this threshold and unstable above it (see Methods for details).

We now use our stability analysis to derive a condition for observing converse symmetry breaking in this system. Note that $\lambda^{\max}$ is a function of $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$, and that, even though Eq. (1) does not depend on β_i when restricted to the synchronous states $\dot{\delta}_i = 0$, the corresponding variational equation $\dot{\mathbf{x}} = \mathbf{J}\mathbf{x}$ does. This implies that making β_i heterogeneous, which breaks the symmetry of the system, generally leads to symmetry breaking in the eigenmodes around the synchronous state (Fig. 2). We see that, if two values of $\boldsymbol{\beta}$ correspond to distinct values of $\lambda^{\max}$, then there is a range of noise intensities (and thus of $\lambda^{\max}_{\text{th}}$) for which the state is stable for one $\boldsymbol{\beta}$ value and unstable for the other (see Methods for details). Thus, a condition for exhibiting converse symmetry breaking is that there exists $\boldsymbol{\beta}^*$ representing a non-uniform β_i assignment for which

$$\lambda^{\max}(\boldsymbol{\beta}^*) < \min\{0, \lambda^{\max}(\widetilde{\boldsymbol{\beta}})\}, \tag{3}$$

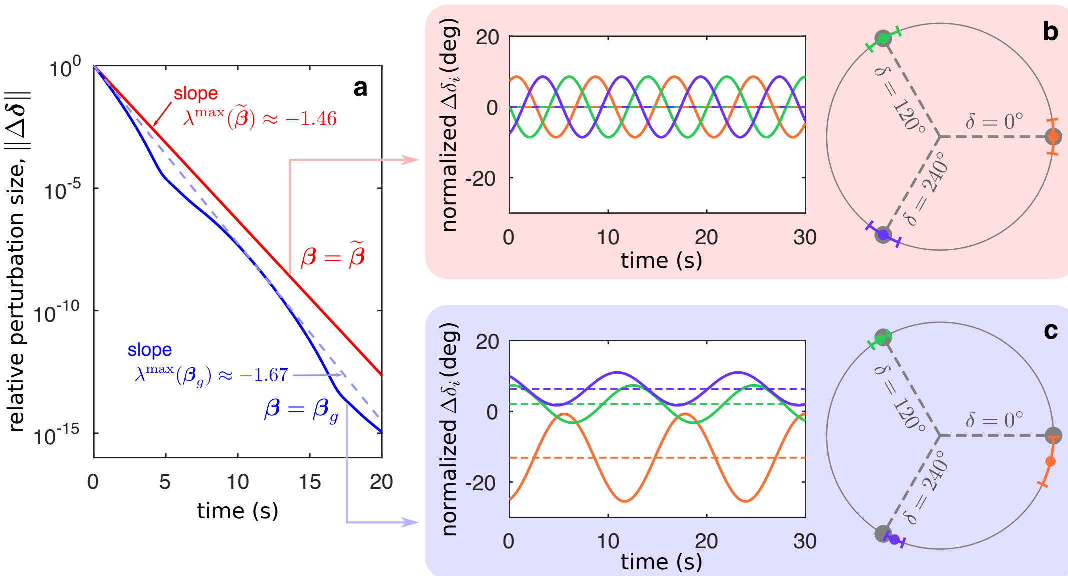

Fig. 2: **Oscillator heterogeneity breaks the symmetry of the dominant eigenmodes.** The predicted slowest decaying eigenmodes of deviations from the splay state are visualized for the system corresponding to Fig. 1b–e. **a**, Magnitude $\|\Delta\boldsymbol{\delta}\|$ of the eigenmodes, which decay at an overall exponential rate $\lambda^{\max}(\boldsymbol{\beta})$, for $\boldsymbol{\beta} = \widetilde{\boldsymbol{\beta}}$ (red) and for $\boldsymbol{\beta} = \boldsymbol{\beta}_g$ (blue). For $\boldsymbol{\beta} = \widetilde{\boldsymbol{\beta}}$, the decay shown is for a combination of two oscillatory eigenmodes associated with complex conjugate eigenvalues corresponding to $\lambda^{\max}(\widetilde{\boldsymbol{\beta}})$, whereas for $\boldsymbol{\beta} = \boldsymbol{\beta}_g$, the decay is for a combination of three eigenmodes corresponding to $\lambda^{\max}(\boldsymbol{\beta}_g)$, of which two oscillatory modes are associated with complex conjugate eigenvalues and one non-oscillatory mode is associated with a real eigenvalue. **b**, Dynamics of individual oscillators (orange, green, and purple for oscillator $i = 1, 2, 3$, respectively) given by the eigenmodes for $\boldsymbol{\beta} = \widetilde{\boldsymbol{\beta}}$, after normalization that removes the exponential decay shown in **a**. (Left) Dynamics with respect to time. (Right) Amplitude of eigen-oscillations indicated by the bars drawn along the unit circle (normalized such that $\|\Delta\boldsymbol{\delta}\| = 0.5$). The gray dots in the background represent the splay state, in which δ_i for different i are $120°$ apart from each other. **c**, Same as in **b**, but for $\boldsymbol{\beta} = \boldsymbol{\beta}_g$. In this case, the bars for perturbation amplitudes are shifted by offsets corresponding to the non-oscillatory eigenmode. We observe that the dominant eigenmodes are rotationally symmetric for $\boldsymbol{\beta} = \widetilde{\boldsymbol{\beta}}$, but this symmetry is broken for $\boldsymbol{\beta} = \boldsymbol{\beta}_g$. For an animated version of the plots in **b** and **c**, see https://youtu.be/R_BOIWXYtSk

where we define $\widetilde{\boldsymbol{\beta}} \equiv (\widetilde{\beta}, \ldots, \widetilde{\beta})$ to represent the (uniform) β_i assignment that minimizes $\lambda^{\max}$ under the constraint that $\beta_1 = \cdots = \beta_n$.

In the design of our experiment, the electrical parameters of the AC circuit (indicated in Fig. 1b) were chosen to ensure that the circuit is rotationally symmetric and the frequency-synchronous states that inherit that symmetry have $\lambda^{\max} < 0$ at frequency $\omega_{\mathrm{s}} = 100\,\mathrm{Hz}$ (see Methods for details). The specific states we focus on are known as *splay states*[56], which for a ring of n phase oscillators are defined as states in which phase differences between consecutive