

ing custom software for phasor measurement, which implements the following calculations. For each data point, we took the last three oscillatory periods of the voltage signal and calculated the least-squares fit of a sinusoidal function with its amplitude, offset, phase shift, and frequency as the fitting parameters. The resulting amplitude, phase shift, and frequency were used as estimates of the terminal voltage peak magnitude, phasor angle, and frequency, respectively, for that data point. To eliminate measurement noise from the terminal voltage phasor and frequency time series, we applied a 3rd-order Savitzky-Golay filter⁶⁰, with a window size of 0.4 s. The generators' internal voltage phasors $E_i = |E_i| \exp(j\delta_i)$ were then obtained using the Kirchoff laws, along with the calculated terminal voltage phasors, the instantaneous synchronous frequency (computed as the average of the instantaneous AC frequencies of the generator terminals), and each component's capacitance and inductance.

Identifying time-series segments of splay states. Because some deviation from the exact splay state is unavoidable, the steady-state phase angles δ_i^* are not necessarily separated by exactly 120 degrees. We thus identified in each time series the set of maximal segments that satisfy the following criteria: 1) for each data point in the segment, one generator is ahead by an angle Δ^+ and another is behind by Δ^- relative to the other generator (taken to be generator 1 here), while satisfying $|\Delta^\pm - 120^\circ| < 10^\circ$; and 2) the length of the segment is at least 0.1 s (approximately ten cycles of voltage). In total, we have obtained 275 segments for the β_A configuration and 190 segments for the β_B configuration. Supplementary Fig. 3 shows the length distribution of these segments, which are within the timescale for which Eq. (1) is valid. Within these segments, we verified that the generator frequencies were synchronized: the maximum (instantaneous) frequency difference among the three generators was < 1 Hz, and the standard deviation of each generator's frequency was < 0.25 Hz in 93% of the $465 = 275 + 190$ identified segments. We also verified that all parameters in the deterministic part of Eq. (1) were constant within experimental noise (see Supplementary Information, Sec. S1 and Supplementary Fig. 1 for details). This further confirmed the validity of Eq. (1) for describing the δ_i dynamics in each time-series segment.

Data Availability. All data that support results in this article are available from the corresponding author upon reasonable request.

Code Availability. The custom code used for the analysis of the data from the experiment is available from the corresponding author upon reasonable request.

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