chronized state the states of different nodes are equal and can be permuted without altering the state of the system. For power grids, CSB would translate to a stability enhancement mechanism in which maintaining the stability of synchronous (and thus symmetric) states requires the generator parameters to be heterogeneous (thus making the system asymmetric). By systematically removing all the other system heterogeneities and isolating the effect of the generator heterogeneity, we establish that CSB is responsible for a significant portion of the stability improvement observed in the power grids we consider. This offers insights into mechanisms underlying the parameter heterogeneity that arises when the generators are tuned to damp oscillations[18,19] (e.g., by adjusting devices called power system stabilizers). Our results are of particular relevance given that CSB has thus far not been observed in any real-world system outside controlled laboratory conditions, let alone power-grid networks.

Results

Power-grid dynamics and stability. To describe the dynamics of n generators in a power-grid network, we represent each generator node as a constant voltage source behind a reactance (the so-called classical model) and their interactions through intermediate non-generator nodes as effective impedances (a process known as Kron reduction)[20]. We assume that the system is operating near a synchronous state in which the voltage frequencies of the n generators are all equal to a constant reference frequency ω_{s}, and we examine whether the homogeneous state is stable against dynamical perturbations. Such perturbations, whether they are small or large, may come for instance from sudden changes in generation and/or demand due to shifting weather condition at wind or solar farms, variations in power consumption, switching on/off connections to microgrids, etc. The short-term dynamics (of the order of one second or less) are then governed by the so-called swing equation[20,21]:

$$\ddot{\delta}_i + \beta_i \dot{\delta}_i = a_i - \sum_{k \neq i} c_{ik} \sin\left(\delta_i - \delta_k - \gamma_{ik}\right), \tag{1}$$

where δ_i is the phase angle variable for generator i (representing the generator's internal electrical angle, relative to a reference frame rotating at the reference frequency ω_{s}); $\beta_i \equiv D_i/(2H_i)$ is an effective damping parameter (corresponding to b_i in the mass-spring system of Fig. 1), with constant D_i capturing both mechanical and electrical damping and constant H_i representing the generator's inertia; a_i is a parameter representing the net power driving the generator (i.e., the mechanical power provided to the generator, minus the power demanded by the network, including loss due to damping); and c_{ik} and γ_{ik} are respectively the coupling strength and phase shift characterizing the electrical interactions between the generators. The parameters in Eq. (1) for a given system are determined by computing the active and reactive power flows between network nodes from system data and using them to calculate the complex-valued effective interaction (and thus its magnitude c_{ik} and angle γ_{ik}) between every pair of generators. In real power grids, stable system operation is ensured by a hierarchy of controllers that adjust generator power outputs and thus the parameters in Eq. (1). Here, however, these parameters can be regarded as constants, since the lowest level of control (known as the primary control) is modeled as a damping-like effect captured by the β_i term in Eq. (1), while the upper-level controls (known as the secondary and tertiary controls) act on time scales much longer than that of the short-term generator dynamics described by the model. In addition, fluctuations in power generation and demand on the time scales of minutes or longer (which can come, e.g., from renewable energy sources) do not affect the short-term dynamics. Equation (1) has recently been studied extensively in the network dynamics community[3,6,22–25].

We first analyze the stability of the synchronous state against small perturbations. The synchronous state corresponds to a fixed point of Eq. (1) given by $\delta_i = \delta_i^*$ and $\dot{\delta}_i = 0$, which represents frequency synchronization because $\dot{\delta}_i$ is the frequency relative to the reference ω_{s}. The Jacobian matrix of Eq. (1) at this point can be written as

$$\mathbf{J} = \begin{pmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{P} & -\mathbf{B} \end{pmatrix}, \tag{2}$$

where $\mathbf{O}$ and $\mathbf{I}$ denote the $n \times n$ null and identity matrices, respectively; $\mathbf{P} = (P_{ik})$ is the $n \times n$ matrix defined by

$$P_{ik} = \begin{cases} -c_{ik}\cos(\delta_i^* - \delta_k^* - \gamma_{ik}), & i \neq k, \\ -\sum_{k' \neq i} P_{ik'}, & i = k, \end{cases} \tag{3}$$

which expresses the effective interactions between the generators; and $\mathbf{B}$ is the $n \times n$ diagonal matrix with β_i as its diagonal elements. We note that, while the form of the Jacobian matrix for coupled damped harmonic oscillators is the same as in Eq. (2), power grids are different in that they can have $\mathbf{P} \neq \mathbf{P}^T$ because $c_{ik} \neq c_{ki}$ in general and because γ_{ik} appears in Eq. (3). The stability under noiseless conditions is determined by the Lyapunov exponent defined as $\lambda^{\max} \equiv \max_{i \geq 2} \mathrm{Re}(\lambda_i)$, where λ_i are the eigenvalues of $\mathbf{J}$. The identically zero eigenvalue, which comes from the zero row-sum property of $\mathbf{P}$ and is denoted here by λ_1, is excluded because it is associated with the invariance of the equation under uniform shift of phases. If

$\lambda^{\max} < 0$, then the synchronous state is asymptotically stable, and smaller $\lambda^{\max}$ implies stronger stability. (This is known as small-signal stability analysis in power system engineering.) Since real power-grid dynamics are noisy due to power generation/demand fluctuations and various other disturbances occurring on short time scales, $\lambda^{\max}$ needs to be sufficiently negative to keep the system close to the synchronous state. Indeed, a previous study[14] showed that, for broad classes of noise dynamics, there is a (negative) threshold value of $\lambda^{\max}$ for such stability: the system is stable if and only if $\lambda^{\max}$ is below the threshold. This stability threshold depends on the noise intensity level. For impulse-like disturbances, the intensity level corresponds to the maximum deviation of δ_i that can be induced by a single disturbance, such as a sudden loss of a generator or a spike in power demand. For continual disturbances, the intensity level can be quantified by the variances of the fluctuating power generation and demand, which can be modeled by adding a randomly varying term to the parameter a_i. Since the stability threshold is generally lower for higher noise levels, the lower the value of $\lambda^{\max}$ for a given power grid, the more intense disturbances and fluctuations the system can endure without losing stability. Incidentally, the optimal damping in the mass-spring system of Fig. 1 is given precisely by minimizing $\lambda^{\max}$ for that system.

Enhancing stability with generator heterogeneity. We now study $\lambda^{\max} = \lambda^{\max}(\boldsymbol{\beta})$ as a function of $\boldsymbol{\beta} \equiv (\beta_1, \ldots, \beta_n)$ for a selection of power grids whose dynamics can be described by Eq. (1) with the parameter values based on data. Using the same model, it was previously shown[6] that, under the constraint that all β_i's have the same value, $\lambda^{\max}$ is minimized when $\boldsymbol{\beta} = \boldsymbol{\beta}_=$, where $\boldsymbol{\beta}_= \equiv (\beta_=, \ldots, \beta_=)$ and $\beta_= \equiv 2\sqrt{\alpha_2}$, with α_2 denoting the smallest non-identically zero eigenvalue of matrix $\mathbf{P}$. The eigenvalue α_2 is associated with the least stable eigenmode, and we assume that it is real and positive (as confirmed in all systems we consider). It was further shown that, at this homogeneous optimal point $\boldsymbol{\beta}_=$, the function $\lambda^{\max}(\boldsymbol{\beta})$ is non-differentiable (which precludes the use of a standard derivative test), but its one-sided derivative along any given straight-line direction is positive, i.e., the directional derivative $D_{\boldsymbol{\beta}'}\lambda^{\max}(\boldsymbol{\beta}_=)$ is positive in the direction of any n-dimensional vector $\boldsymbol{\beta}'$. Thus, moving away from $\boldsymbol{\beta}_=$ along any straight line would necessarily increase $\lambda^{\max}$ from the local minimum value $\lambda^{\max}(\boldsymbol{\beta}_=) = -\sqrt{\alpha_2}$ and hence only reduce the stability of the synchronous state.

Despite the apparent impossibility of improving on $\lambda^{\max}(\boldsymbol{\beta}_=)$ locally, we first show that there can be curved paths starting at $\boldsymbol{\beta}_=$ along which $\lambda^{\max}$ can be further minimized with heterogeneous β_i. Indeed, Fig. 2a illustrates using a 3-generator system that such curved paths