In the continuous-time model, the term $\varepsilon\xi_i(t)$ represents the effect of Brownian noise on the dynamics of the ith oscillator. Since the effect of the noise decays most slowly along the eigenmode associated with the eigenvalue $\lambda^{\max} < 0$, we focus on the projection of the full six-dimensional dynamics onto that eigenmode. We model this projected dynamics by the following stochastic differential equation:

$$dx(t) = \lambda^{\max} x(t)dt + \sigma d\xi(t), \tag{7}$$

where $x(t)$ represents the (one-dimensional) deviation from the frequency-synchronous state, $\xi(t)$ is the standard Brownian noise (i.e., $\xi(t) - \xi(0)$ for fixed t follows the Gaussian distribution with zero mean and variance t), and $\sigma > 0$ is a constant that can be used to tune the level of noise intensity felt by the system. The stochastic process given by Eq. (7) is called the Ornstein–Uhlenbeck (OU) process[57,58]. Given the constants σ and Δ_{sync}, we define $\Pr(\lambda^{\max})$ to be the probability that $|x(t)| \leq \Delta_{\text{sync}}$ for $t \to \infty$ (i.e., the probability that the system is synchronized in the large t limit). From the known property of the OU process[59] that $x(t)$ for a fixed t follows the Gaussian distribution with mean $e^{t\lambda^{\max}}x(0)$ and variance $\sigma^2(1-e^{2t\lambda^{\max}})/(-2\lambda^{\max})$, we compute $\Pr(\lambda^{\max}) = \text{erf}(\Delta_{\text{sync}}\sqrt{-\lambda^{\max}}/\sigma)$, where erf denotes the error function. Note that, as $\lambda^{\max} < 0$ is increased (and thus $\sqrt{-\lambda^{\max}}$ is decreased), the probability $\Pr(\lambda^{\max})$ decreases. Thus, for a given constant p_{th} (chosen close to one), we define the stability threshold $\lambda^{\max}_{\text{th}}$ as the value of $\lambda^{\max}$ for which $\Pr(\lambda^{\max}) = p_{\text{th}}$. It then follows that $\lambda^{\max}_{\text{th}} = -\sigma^2[\text{erf}^{-1}(p_{\text{th}})]^2/\Delta^2_{\text{sync}}$. Note that the threshold $\lambda^{\max}_{\text{th}} = \lambda^{\max}_{\text{th}}(\sigma)$ depends on the noise intensity level σ. For $\sigma = 0$ (noiseless case), we have $\lambda^{\max}_{\text{th}}(0) = 0$, recovering the stability threshold for deterministic systems. As σ increases, the threshold $\lambda^{\max}_{\text{th}}(\sigma)$ monotonically decreases, indicating that the decay of deviation needs to be faster to maintain the same probability $\Pr(\lambda^{\max})$ for higher noise levels. Therefore, if $\lambda^{\max}(\boldsymbol{\beta}_{\text{I}}) < \lambda^{\max}(\boldsymbol{\beta}_{\text{II}}) < 0$, there is a range of σ for which $\lambda^{\max}(\boldsymbol{\beta}_{\text{I}}) < \lambda^{\max}_{\text{th}}(\sigma) < \lambda^{\max}(\boldsymbol{\beta}_{\text{II}})$, i.e., the system stays near the frequency-synchronous state with high probability as $t \to \infty$ for the $\boldsymbol{\beta}_{\text{I}}$ configuration, while it does not for the $\boldsymbol{\beta}_{\text{II}}$ configuration.

Matrix $\mathbf{P}$ for splay states. Assuming ideal system components, the electric circuit we use (Fig. 1b) is rotationally symmetric, and thus the complex node-to-node admittances Y_{0ik} are identical for all $i \neq k$. If, in addition, the internal impedances $z_{\text{int},i}$ of the three generators are identical, then the effective admittances Y_{ik} are also identical, i.e., $Y_{ik} = y = |y|\exp(j\alpha)$ for all $i \neq k$, where j is the imaginary unit. Assuming further that the inertia constants H_i are all identical, the steady splay states satisfy $c_{ik} = c$ for all $i \neq k$, and the matrix $\mathbf{P}$ in Eq. (2) takes the form

$$\begin{pmatrix} -b-b' & b & b' \\ b' & -b-b' & b \\ b & b' & -b-b' \end{pmatrix}, \tag{8}$$

where $b = c\big(-\frac{1}{2}\cos\gamma \mp \frac{\sqrt{3}}{2}\sin\gamma\big)$, $b' = c\big(-\frac{1}{2}\cos\gamma \pm \frac{\sqrt{3}}{2}\sin\gamma\big)$, and $\gamma = \alpha - \pi/2$. The topology of the underlying network corresponds to one favoring converse symmetry breaking in the analysis of coupled oscillators presented in Ref. 41.

Calculation of $\boldsymbol{\lambda}^{\mathbf{max}}$. For both the theoretically predicted stability landscape in Fig. 1 and the estimates from the experimental measurements in Fig. 3 and Supplementary Fig. 2, we compute $\lambda^{\max}$ using the following procedure. We first apply the Kirchoff law to compute the active and reactive power injections at the terminal of each generator (P_i and Q_i, respectively) from the network's inductances and capacitances (at a given frequency) as well as the terminal voltage magnitudes and angles ($|V_i|$ and θ_i, respectively). We then apply the Kirchoff law again to the internal impedance of each generator (assuming the classical model) to compute $|E_i|$ and δ_i^* from P_i, Q_i, $|V_i|$, and θ_i. From these and the values of H_i, the parameters c_{ik} and γ_{ik} can be computed. We can then obtain $\lambda^{\max}$ for any given $\boldsymbol{\beta}$ by computing the eigenvalues of $\mathbf{J} = \big(\begin{smallmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{P} & -\mathbf{B} \end{smallmatrix}\big)$ with the matrix $\mathbf{P}$ defined in Eq. (2). For the theoretical prediction, we use $\omega_\mathrm{s} = 100\,\mathrm{Hz}$ and the terminal voltage magnitude $|V_i| = 1.4\,\mathrm{V}$. We set the angles θ_i to have exactly $120°$ differences, which ensures splay-state values for δ_i as we assume the generator parameters to be identical: $r_{\mathrm{int},i} = 1.6\,\Omega$ and $x_{\mathrm{int},i} = 3.34\,\Omega$ for all i. We compute H_i from $J_i = 5 \times 10^{-5}$ kg·m^2, which makes H_i all identical. For the experimental estimation of $\lambda^{\max}$ from a given time-series segment, we use the average of the measured values over the segment for ω_s, $|E_i|$, and δ_i^*, along with the values of J_i, $r_{\mathrm{int},i}$, and $x_{\mathrm{int},i}$ measured for each generator (see Supplementary Information, Secs. S2.2 and S2.3 for our measurement procedure).

Conducting the experiment and taking measurements. Arduino Uno and Arduino Mega microcontrollers were used to provide sufficient computational capacity and I/O channels to facilitate measurement and control of the experiment. The Arduino Uno has six analog inputs, of which three were used to measure the terminal voltages of the three generators, and the remaining three were used to measure the currents passing through the generator terminals. The same controller also has several digital outputs, four of which were used to control relays that switch the voltage inputs between measurement signals and test signals. The test signals were used to calibrate the voltage levels. We used three analog inputs on the Arduino Mega to take shaft rotational speed measurement from infrared sensors that detect markings on the shafts. This capability was used in measuring β_i for each generator (see Supplementary Information, Sec. S2.1 for details). In addition, three digital outputs of this controller were used to drive relays that switch the generators on and off in the AC circuit.

Each experimental run was performed as follows. First, while the generator terminal relays were open (i.e., disconnected from the circuit), we turned on the DC motors driving the generators, and we adjusted their speed manually to $100/3 \approx 33.3$ rotations per second, yielding terminal voltage signals with frequencies approximately at the desired $100\,\mathrm{Hz}$ (noting that the frequency is three times higher than the shaft's rotational frequency because our generators have three pole pairs). Once this was achieved, the generator terminal relays were closed, establishing the coupling between the generators. This can result in a transient voltage dynamics with large fluctuations, with the speed of each generator dropping due to the impedance of the connected network. The speed of each DC motor was adjusted once again to ensure that each generator was running at the desired speed. When the last generator approached the desired speed, the synchronous state formed spontaneously. Real-time voltage phasor readouts were displayed on the computer interface to confirm that the system had achieved a desired splay state. At this point, we started recording the voltage phasor readings.

External perturbations from vibrations and other sources continuously disturb the synchronous state, and the resulting transients prevented us from observing this state for more than a few seconds at a time. Moreover, as a result of components heating up during the experiment, the amount of friction between the shaft and its housing tends to change at different rates for different generators, leading to unbalanced net changes in the mechanical power input of the generators that distort the splay state. We mitigated this problem by re-adjusting the speed of the DC motors. These adjustments are implemented manually, aided by the real-time display of the phase angle differences of the generators. To prevent damage due to rising temperature of the components, we were limited to a maximum of 10 min of continuous run when no brakes were applied (the $\boldsymbol{\beta}_A$ configuration), and a maximum of 5 min when brakes were applied (the $\boldsymbol{\beta}_B$ configuration).

The terminal voltage was recorded at $3{,}320$ samples per second for each generator using the analog-to-digital converters of the Arduino Uno, along with the microsecond-accurate time stamps from the microcontroller's internal timer. The data were streamed to the computer interface for post-processing in which the raw readings were converted to time-dependent frequency and phasor angle and magnitude with the original resolution. This conversion was done us-