are related. Realizing this can lead to new control approaches designed to manipulate these parameters for further optimization of stability. We suggest that the fresh opportunities for network optimization and control revealed in this study apply to network systems in general and thus have the potential to inspire new discoveries in many different disciplines.

Methods

Power-grid datasets. Here, we describe the sources of data for the six power-grid networks considered (the 3-generator system in Fig. 2; the New England, NPCC, U.K., and German systems in Figs. 3, 4, and 6; and the 4-generator system in Fig. 5). For each system, the data provide the net injected real power at all generator nodes, the power demand at all non-generator nodes, and the parameters of all power lines and transformers. These parameters are sufficient to determine all active and reactive power flows in the system using a standard power flow calculation. The data also provide the generators' dynamic parameters H_i, D_i, and $x_{\mathrm{int},i}$ used in our stability calculations. The parameters H_i and D_i are the inertia and damping constants, respectively, that define the effective damping parameter through the relation $\beta_i = D_i/(2H_i)$. The parameter $x_{\mathrm{int},i}$ represents the internal reactance of generator i and is used in the calculation of the parameters a_i, c_{ij}, and γ_{ij}. In each system, nodes are indexed as in the original data source (except for the German power grid; see below).

- **3-generator test system (3-gen):** For this IEEE 3-generator, 9-node test system, which appeared in Ref. 20, we used the data file (data3m9b.m) available in the PST toolbox[29]. This system represents the Western System Coordinating Council (WSCC), which was part of the region now called the Western Electricity Coordinating Council (WECC) in the North American power grid. The data file provides all necessary dynamical parameters for each generator.

- **New England test system (10-gen):** For the IEEE 10-generator, 39-node test system, as described in Refs. 30 and 31, we used the data file (case39.m) available in the MATPOWER toolbox[32], with dynamic parameters added manually from Ref. 30. This is a reduced model representing the New England portion of the Eastern Interconnection in the North American power grid, with one generator representing the connection to the rest of the grid.

- **NPCC power grid (48-gen):** For the 48-generator, 140-node NPCC power grid[33], we used the data file (data48em.m) available in the PST toolbox[29]. The system represents the former NPCC region of the Eastern Interconnection in the North American power grid and includes an equivalent generator/load node representing the rest of the Interconnection. The data file provides H_i and $x_{\mathrm{int},i}$ for all generators (while it assumes $D_i = 0$). We generated D_i randomly by sampling from the uniform distribution on the interval $[1, 3]$ (in per unit on the system base, as specified by the data file). The geographic coordinates of the nodes used in Fig. 3a were extracted from Ref. 34, and the coastline and boundary data used to draw the map were obtained from Natural Earth[35].

- **U.K. power grid (66-gen):** For the 66-generator, 29-node U.K. power grid, we used the data file (GBreducednetwork.m) available from Ref. 36. The system represents a reduced model for the power grid of Great Britain. The dynamical parameters, H_i, D_i, and $x_{\mathrm{int},i}$, were generated randomly by sampling from the uniform distribution on the intervals, $[1, 5]$, $[1, 3]$, and $[0.001, 0.101]$, respectively. The generated parameters values for each generator are in per unit on its own machine base, i.e., normalized by the reference values computed from the power base for the generator (chosen to be 1.5 times the maximum real power generation provided in the data file). For stability calculation, we converted these values to the corresponding values in per unit on a common system base.

- **German power grid (69-gen):** For the 69-generator, 228-node German power grid, we created the data from the ENTSO-E 2009 Winter model[37]. The ENTSO-E model is a DC power flow model of the continental Europe and contains 1,486 nodes and 565 generators. We first created a dynamical model for the entire ENTSO-E network by solving the DC power flow and converting it to an AC power flow solution (assuming a 0.95 power factor at each node), and then generating dynamical parameters using the same method as for the U.K. grid. For any node with multiple generators attached, the net reactive power injection was distributed among these generators in proportion to their real power generation. From this full ENTSO-E model, we extracted the German portion by eliminating (using Kron reduction) all the nodes outside Germany (identified using the country label "D" representing Germany in the dataset). We re-indexed the extracted nodes consecutively, preserving the original ordering. The geographic coordinates of the nodes used in Fig. 3b were extracted from the PowerWorld data files available from Ref. 37, and the coastline and boundary data used to draw the map were obtained from Natural Earth[35].

- **4-generator example system:** For the 4-generator, 5-node example system used in Fig. 5, we show a full system diagram in Supplementary Fig. 1, indicating the main parameters of the components. When the damping parameters of generators 2 and 3 are equal (i.e., $\beta_2 = \beta_3$), the system is symmetric under the permutation of these generators. MATLAB code for running simulations on this system, which includes the full set of parameters and uses the MATPOWER toolbox[32], is available from our GitHub repository[38].

Aggregation of generators and effective damping parameter $\boldsymbol{\beta_i}$. If a subset of generators are synchronized in the sense that $\delta_i - \delta_j$ is constant in time for any two generators i and j in the subset, then they can be represented by a single equivalent generator using a Zhukov-based aggregation method similar to that described in Ref. 33. In this method, the equivalent generator has inertia constant $\sum_i H_i$ and damping constant $\sum_i D_i$, where the sums are taken over the generators i in the subset. The effective damping parameter of the equivalent generator is then $\sum_i D_i/(2\sum_i H_i) = \bar{D}/(2\bar{H})$, where $\bar{D}$ and $\bar{H}$ are respectively the average of the inertia and damping constants of the generators in the subset. Thus, the aggregation does not introduce any artifactual heterogeneity.

Data Availability

Data on all six systems we consider (described in Methods) and detailed data of the core results presented in the figures are available from our GitHub repository[38].

Code Availability

Essential code for reproducing the core results in all figures, as well as scripts for generating plain versions of the figures, is available from the GitHub repository[38].

References

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2. Backhaus, S. & Chertkov, M. Getting a grip on the electrical grid. *Phys. Today* **66**, 42–48 (2013).