Asymmetry underlies stability in power grids

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Abstract

Behavioral homogeneity is often critical for the functioning of network systems of interacting entities. In power grids, whose stable operation requires generator frequencies to be synchronized—and thus homogeneous—across the network, previous work suggests that the stability of synchronous states can be improved by making the generators homogeneous. Here, we show that a substantial additional improvement is possible by instead making the generators suitably heterogeneous. We develop a general method for attributing this counterintuitive effect to *converse symmetry breaking*, a recently established phenomenon in which the system must be asymmetric to maintain a stable symmetric state. These findings constitute the first demonstration of converse symmetry breaking in real-world systems, and our method promises to enable identification of this phenomenon in other networks whose functions rely on behavioral homogeneity.

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Introduction

In an alternating current power grid, the generators provide electrical power that oscillates in time as sinusoidal waves. Because these waves are superimposed before reaching the consumers, they need to be synchronized to the same frequency; otherwise, time-dependent cancellation between these waves would cause the delivered power to fluctuate, which can lead to equipment malfunction and damage[1]. Maintaining frequency synchronization is challenging because the system is complex in various ways, with every generator responding differently to the continual influence of disturbances and varying conditions[2]. Adding to the challenge is the increase in perturbations resulting from the ongoing integration of energy from intermittent sources[3], the emergence of grid-connected microgrids[4], and the expansion of an increasingly open electricity market[5]. Furthermore, the inherent heterogeneities in the parameters of system components and in the structure of the interaction network are perceived as obstacles to achieving synchronization. Consistent with the view that heterogeneities may generally inhibit frequency homogeneity, an earlier study showed that homogenizing the (otherwise heterogeneous) values of generator parameters can lead to stronger stability of synchronous states than in the original system[6]. An outstanding question remains, however, as to whether there is a heterogeneous parameter assignment (different from the nominal one) that would enable even stronger stability for synchronous states than the best homogeneous parameter assignment. Though motivated by its significance for power grids, this question is broadly relevant for improving the stability of homogeneous dynamics in complex network systems in general, including consensus dynamics in networks of human or robotic agents[7,8], coordinated spiking of neurons in the brain[9,10], and synchronization in communication networks[11,12].

To gain insights into the potential role of heterogeneity in enhancing stability, it is instructive to first consider the case of damped harmonic oscillators. For a single oscillator, the optimal stability corresponds to the fastest convergence to the stable equilibrium and is achieved when the oscillator is critically damped: underdamping would lead to lingering oscillations around the equilibrium, and overdamping would lead to slowed convergence due to excess dragging. This optimization is exploited in door closers (devices that passively close doors in a controlled manner), which are designed to be critically damped for the door to close fast without slamming. When multiple damped oscillators are coupled, the damping giving rise to optimal stability will be influenced by the network interactions. More important, we can show that the optimal stability in such a network requires different oscillators to have different damping (even when their other parameters are all identical and they are positioned identically in the network), as

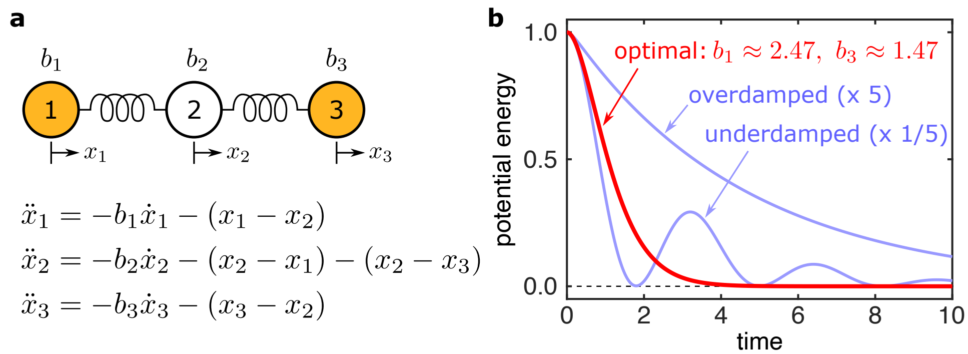

Fig. 1: Stabilizing effect of heterogeneity in a mass-spring system. **a** System consisting of a linear chain of three unit masses connected by two identical springs. The masses are constrained to move horizontally, and their dynamics are governed by the equation shown, where x_i is the displacement of mass i relative to its equilibrium and b_i is its damping coefficient. **b** Total potential energy of the springs vs. time for three different damping scenarios. The optimal damping (red), corresponding to the fastest energy decay, is achieved for $b_1 \approx 2.47$, $b_2 \approx 3.17$, $b_3 \approx 1.47$ (or equivalently, $b_1 \approx 1.47$, $b_2 \approx 3.17$, $b_3 \approx 2.47$), despite the fact that masses 1 and 3 are otherwise identical and identically coupled. Overdamping leads to a slower monotonic decay, while underdamping results in a slower oscillatory decay, as shown in blue by varying b_1 and b_3 by a factor of 5. In all cases, the initial conditions are $(x_1, x_2, x_3) = (1, 0, -1)$ and $(\dot{x}_1, \dot{x}_2, \dot{x}_3) = (0, 0, 0)$.

illustrated in Fig. 1.

In this paper, we first demonstrate that an analogous effect occurs in power-grid networks: heterogeneity in generator parameters can robustly enhance both the linear and the nonlinear stability of synchronous states in power grids from North America and Europe. Since these systems have heterogeneity in the network structure in addition to the tunable generator parameters, one possibility is that the effect arises entirely from compensation: stability reduction due to one heterogeneity is compensated by another heterogeneity, leading to a stability enhancement when the latter heterogeneity is added. An alternative, which we validate here, involves the recently established phenomenon of *converse symmetry breaking* (CSB)[13], in which the stability of a symmetric state requires the system's symmetry to be broken. Due to its counterintuitive nature, this phenomenon had not been recognized until it was recently predicted and experimentally confirmed[13,14] for synchronization in oscillator networks (a class of network dynamics widely studied in the literature[15–17]). Despite its conceptual generality and potential to underlie symmetric states of many systems, this phenomenon has not yet been observed outside laboratory settings. The symmetry relevant here is node-permutation symmetry, since in a syn-