

optimally-lengthed CoTs boosts performance, and proposed "Length-Filtered Vote" as a promising inference strategy. Our work underscores the critical need to calibrate CoT length, moving beyond a one-size-fits-all approach towards a principled framework where LLMs adaptively choose the right amount of thought to optimize reasoning.

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