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REFERENCES

- Shun-ichi Amari. Backpropagation and stochastic gradient descent method. *Neurocomputing*, 5(4-5): 185–196, 1993.
- Rohan Anil, Andrew M Dai, Orhan Firat, Melvin Johnson, Dmitry Lepikhin, Alexandre Passos, Siamak Shakeri, Emanuel Taropa, Paige Bailey, Zhifeng Chen, et al. Palm 2 technical report. *arXiv preprint arXiv:2305.10403*, 2023.
- David Applegate, Ribert Bixby, Vasek Chvatal, and William Cook. Concorde tsp solver, 2006.
- Thomas Bäck and Hans-Paul Schwefel. An overview of evolutionary algorithms for parameter optimization. *Evolutionary computation*, 1(1):1–23, 1993.
- Yuntao Bai, Saurav Kadavath, Sandipan Kundu, Amanda Askell, Jackson Kernion, Andy Jones, Anna Chen, Anna Goldie, Azalia Mirhoseini, Cameron McKinnon, et al. Constitutional ai: Harmlessness from ai feedback. *arXiv preprint arXiv:2212.08073*, 2022.
- Tianle Cai, Xuezhi Wang, Tengyu Ma, Xinyun Chen, and Denny Zhou. Large language models as tool makers. *arXiv preprint arXiv:2305.17126*, 2023.
- Angelica Chen, David M Dohan, and David R So. Evoprompting: Language models for code-level neural architecture search. *arXiv preprint arXiv:2302.14838*, 2023a.
- Angelica Chen, Jérémie Scheurer, Tomasz Korbak, Jon Ander Campos, Jun Shern Chan, Samuel R Bowman, Kyunghyun Cho, and Ethan Perez. Improving code generation by training with natural language feedback. *arXiv preprint arXiv:2303.16749*, 2023b.
- Juhai Chen, Lichang Chen, Heng Huang, and Tianyi Zhou. When do you need chain-of-thought prompting for chatgpt? *arXiv preprint arXiv:2304.03262*, 2023c.
- Lichang Chen, Juhai Chen, Tom Goldstein, Heng Huang, and Tianyi Zhou. Instructzero: Efficient instruction optimization for black-box large language models. *arXiv preprint arXiv:2306.03082*, 2023d.
- Xinyun Chen and Yuandong Tian. Learning to perform local rewriting for combinatorial optimization. *Advances in Neural Information Processing Systems*, 32, 2019.
- Xinyun Chen, Maxwell Lin, Nathanael Schärli, and Denny Zhou. Teaching large language models to self-debug. *arXiv preprint arXiv:2304.05128*, 2023e.
- Yutian Chen, Xingyou Song, Chansoo Lee, Zi Wang, Richard Zhang, David Dohan, Kazuya Kawakami, Greg Kochanski, Arnaud Doucet, Marc'aurelio Ranzato, et al. Towards learning universal hyperparameter optimizers with transformers. *Advances in Neural Information Processing Systems*, 35:32053–32068, 2022.
- Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, et al. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.
- Mingkai Deng, Jianyu Wang, Cheng-Ping Hsieh, Yihan Wang, Han Guo, Tianmin Shu, Meng Song, Eric P Xing, and Zhiting Hu. Rlprompt: Optimizing discrete text prompts with reinforcement learning. *arXiv preprint arXiv:2205.12548*, 2022.
- Michel Deudon, Pierre Courtnut, Alexandre Lacoste, Yossiri Adulyasak, and Louis-Martin Rousseau. Learning heuristics for the tsp by policy gradient. In *International Conference on the Integration of Constraint Programming, Artificial Intelligence, and Operations Research*, pp. 170–181. Springer, 2018.

- Chrisantha Fernando, Dylan Banarse, Henryk Michalewski, Simon Osindero, and Tim Rocktäschel. Promptbreeder: Self-referential self-improvement via prompt evolution. *arXiv preprint arXiv:2309.16797*, 2023.
- Deep Ganguli, Amanda Askell, Nicholas Schiefer, Thomas Liao, Kamilé Lukošiūtė, Anna Chen, Anna Goldie, Azalia Mirhoseini, Catherine Olsson, Danny Hernandez, et al. The capacity for moral self-correction in large language models. *arXiv preprint arXiv:2302.07459*, 2023.
- Tianyu Gao, Adam Fisch, and Danqi Chen. Making pre-trained language models better few-shot learners. *arXiv preprint arXiv:2012.15723*, 2020.
- Bruce Golden, Lawrence Bodin, T Doyle, and W Stewart Jr. Approximate traveling salesman algorithms. *Operations research*, 28(3-part-ii):694–711, 1980.
- Qingyan Guo, Rui Wang, Junliang Guo, Bei Li, Kaitao Song, Xu Tan, Guoqing Liu, Jiang Bian, and Yujiu Yang. Connecting large language models with evolutionary algorithms yields powerful prompt optimizers. *arXiv preprint arXiv:2309.08532*, 2023.
- Gregory Gutin and Abraham P Punnen. *The traveling salesman problem and its variations*, volume 12. Springer Science & Business Media, 2006.
- Keld Helsgaun. An extension of the lin-kernighan-helsgaun tsp solver for constrained traveling salesman and vehicle routing problems. *Roskilde: Roskilde University*, 12, 2017.
- Michael Jünger, Gerhard Reinelt, and Giovanni Rinaldi. The traveling salesman problem. *Handbooks in operations research and management science*, 7:225–330, 1995.
- Geunwoo Kim, Pierre Baldi, and Stephen McAleer. Language models can solve computer tasks. *arXiv preprint arXiv:2303.17491*, 2023.
- Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *International Conference on Learning Representations*, 2015.
- Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. Large language models are zero-shot reasoners. *arXiv preprint arXiv:2205.11916*, 2022.
- Wouter Kool, Herke van Hoof, and Max Welling. Attention, learn to solve routing problems! In *International Conference on Learning Representations*, 2019. URL <https://openreview.net/forum?id=ByxBF5RqYm>.
- Joel Lehman, Jonathan Gordon, Shawn Jain, Kamal Ndousse, Cathy Yeh, and Kenneth O Stanley. Evolution through large models. *arXiv preprint arXiv:2206.08896*, 2022.
- Brian Lester, Rami Al-Rfou, and Noah Constant. The power of scale for parameter-efficient prompt tuning. *arXiv preprint arXiv:2104.08691*, 2021.
- Xiang Lisa Li and Percy Liang. Prefix-tuning: Optimizing continuous prompts for generation. *arXiv preprint arXiv:2101.00190*, 2021.
- Wang Ling, Dani Yogatama, Chris Dyer, and Phil Blunsom. Program induction by rationale generation: Learning to solve and explain algebraic word problems. *arXiv preprint arXiv:1705.04146*, 2017.
- Xiao Liu, Yanan Zheng, Zhengxiao Du, Ming Ding, Yujie Qian, Zhilin Yang, and Jie Tang. Gpt understands, too. *arXiv preprint arXiv:2103.10385*, 2021.
- Yao Lu, Max Bartolo, Alastair Moore, Sebastian Riedel, and Pontus Stenetorp. Fantastically ordered prompts and where to find them: Overcoming few-shot prompt order sensitivity. *arXiv preprint arXiv:2104.08786*, 2021.
- Xiao Ma, Swaroop Mishra, Ahmad Beirami, Alex Beutel, and Jilin Chen. Let’s do a thought experiment: Using counterfactuals to improve moral reasoning. *arXiv preprint arXiv:2306.14308*, 2023.

- Aman Madaan and Amir Yazdanbakhsh. Text and patterns: For effective chain of thought, it takes two to tango. *arXiv preprint arXiv:2209.07686*, 2022.
- Aman Madaan, Niket Tandon, Prakhar Gupta, Skyler Hallinan, Luyu Gao, Sarah Wiegreffe, Uri Alon, Nouha Dziri, Shrimai Prabhumoye, Yiming Yang, et al. Self-refine: Iterative refinement with self-feedback. *arXiv preprint arXiv:2303.17651*, 2023.
- Elliot Meyerson, Mark J Nelson, Herbie Bradley, Arash Moradi, Amy K Hoover, and Joel Lehman. Language model crossover: Variation through few-shot prompting. *arXiv preprint arXiv:2302.12170*, 2023.
- Suvir Mirchandani, Fei Xia, Pete Florence, Brian Ichter, Danny Driess, Montserrat Gonzalez Arenas, Kanishka Rao, Dorsa Sadigh, and Andy Zeng. Large language models as general pattern machines. *arXiv preprint arXiv:2307.04721*, 2023.
- Varun Nair, Elliot Schumacher, Geoffrey Tso, and Anitha Kannan. Dera: Enhancing large language model completions with dialog-enabled resolving agents. *arXiv preprint arXiv:2303.17071*, 2023.
- MohammadReza Nazari, Afshin Oroojlooy, Lawrence Snyder, and Martin Takac. Reinforcement learning for solving the vehicle routing problem. In *Advances in Neural Information Processing Systems*, pp. 9861–9871, 2018.
- Theo X Olausson, Jeevana Priya Inala, Chenglong Wang, Jianfeng Gao, and Armando Solar-Lezama. Demystifying gpt self-repair for code generation. *arXiv preprint arXiv:2306.09896*, 2023.
- Gurobi Optimization et al. Gurobi optimizer reference manual, 2020.
- Archiki Prasad, Peter Hase, Xiang Zhou, and Mohit Bansal. Grips: Gradient-free, edit-based instruction search for prompting large language models. *arXiv preprint arXiv:2203.07281*, 2022.
- Reid Pryzant, Dan Iter, Jerry Li, Yin Tat Lee, Chenguang Zhu, and Michael Zeng. Automatic prompt optimization with "gradient descent" and beam search. *arXiv preprint arXiv:2305.03495*, 2023.
- Ning Qian. On the momentum term in gradient descent learning algorithms. *Neural networks*, 12(1): 145–151, 1999.
- Guanghui Qin and Jason Eisner. Learning how to ask: Querying lms with mixtures of soft prompts. *arXiv preprint arXiv:2104.06599*, 2021.
- Colin R Reeves. *Modern heuristic techniques for combinatorial problems*. John Wiley & Sons, Inc., 1993.
- Laria Reynolds and Kyle McDonell. Prompt programming for large language models: Beyond the few-shot paradigm. In *Extended Abstracts of the 2021 CHI Conference on Human Factors in Computing Systems*, pp. 1–7, 2021.
- Luis Miguel Rios and Nikolaos V Sahinidis. Derivative-free optimization: a review of algorithms and comparison of software implementations. *Journal of Global Optimization*, 56:1247–1293, 2013.
- Daniel J Rosenkrantz, Richard E Stearns, and Philip M Lewis, II. An analysis of several heuristics for the traveling salesman problem. *SIAM journal on computing*, 6(3):563–581, 1977.
- Subhro Roy and Dan Roth. Solving general arithmetic word problems. *arXiv preprint arXiv:1608.01413*, 2016.
- Timo Schick, Jane Dwivedi-Yu, Roberto Dessì, Roberta Raileanu, Maria Lomeli, Luke Zettlemoyer, Nicola Cancedda, and Thomas Scialom. Toolformer: Language models can teach themselves to use tools. *arXiv preprint arXiv:2302.04761*, 2023.
- Taylor Shin, Yasaman Razeghi, Robert L Logan IV, Eric Wallace, and Sameer Singh. Autoprompt: Eliciting knowledge from language models with automatically generated prompts. *arXiv preprint arXiv:2010.15980*, 2020.
- Noah Shinn, Beck Labash, and Ashwin Gopinath. Reflexion: an autonomous agent with dynamic memory and self-reflection. *arXiv preprint arXiv:2303.11366*, 2023.