

Figure 26: The $L(N)$ scaling law, including the best 16M checkpoint, which we did not have time to train to the $L(N)$ token budget due to compute constraints.

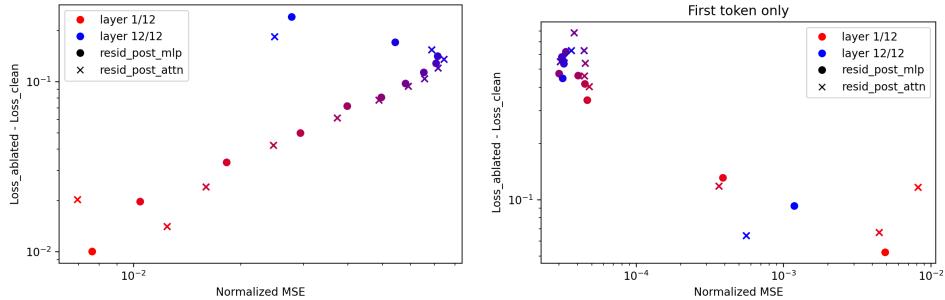


Figure 27: (a) Normalized MSE gets worse later in the network, with the exception of the last two layers, where it improves. Later layers suffer worse loss differences when ablating to reconstruction, even at the final two layers. (b) First position token loss is more severely affected by ablation than other layers despite having lower normalized MSE. Overall loss difference has no clear relation with normalized MSE across layers. In very early layers and the final layer, where residual stream norm is also more normal (Figure 30), we see a more typical loss difference and MSE. This is consistent with the hypothesis that the large norm component of the first token is primarily to serve attention operations at later tokens.

Number of tokens needed for convergence is noticeably higher for earlier layers. MSE is lowest at early layers of the residual stream, increasing until the final layer, at which point it drops. Residual stream deltas have MSE peaking around layer 6, with attention delta falling sharply at the final layer.

When ablating to reconstructions, downstream loss and KL get strictly worse with layer. This is despite normalized MSE dropping at late layers. However, there appears to be an exception at final layers (layer 11/12 and especially 12/12 of GPT-2 small), which can have better normalized MSE than earlier layers, but more severe effects on downstream prediction (Figure 27).

We can also see that choice of layer affects different metrics differently (Figure 28). While earlier layers (unsurprisingly) have better N2G explanations, later layers do better on probe loss and sparsity.

In early results with autoencoders trained on the last layer of GPT-2 small, we found the results to be qualitatively worse than the layer 8 results, so we use layer 8 for all experiments going forwards.

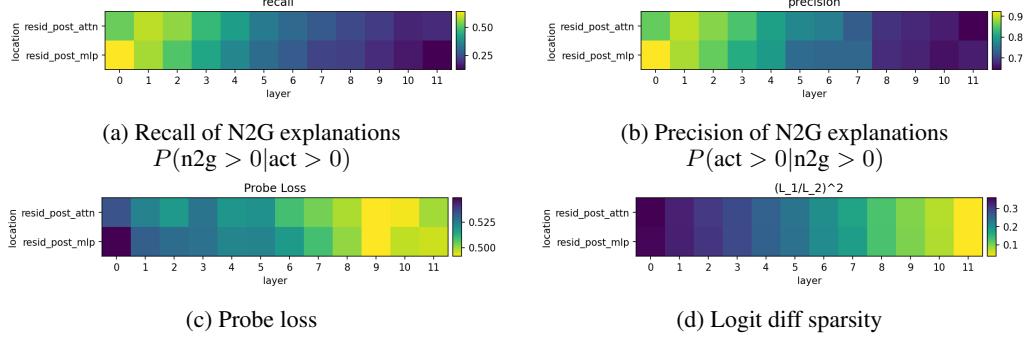


Figure 28: Metrics as a function of layer, for GPT-2 small autoencoders with $k = 32$ and $n = 32768$. Earlier layers are easier to explain in terms of token patterns, but later layers are better for recovering features and have sparser logit diffs.

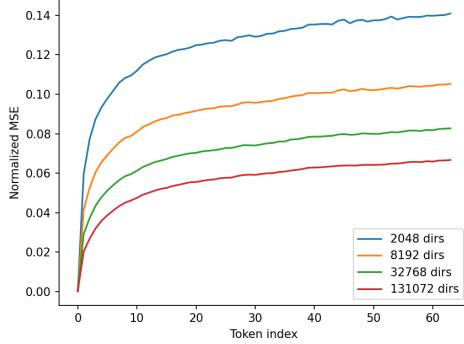


Figure 29: Later tokens are more difficult to reconstruct. (lower is better)

F.2 Impact of token position

We find that tokens at later positions are harder to reconstruct (Figure 29). We hypothesize that this is because the residual stream at later positions have more features. First positions are particularly egregiously easy to reconstruct, in terms of normalized MSE, but they also generally have residual stream norm more than an order of magnitude larger than other positions (Figure 30 shows that in GPT-2 small, the exception is only at early layers and the final layer of GPT-2 small). This phenomenon was explained in [Sun et al., 2024, Xiao et al., 2023], which demonstrate that these activations serve as crucial attention resting states.

First token positions have significantly worse downstream loss and KL after ablating to autoencoder reconstruction at layers with these large norms (Figure 27), despite having better normalized MSE. This is consistent with the hypothesis that the large norm directions at the first position are important for loss on other tokens but not the current token. This position then potentially gets subtracted back out at the final layer as the model focuses on current-token prediction.

G Irreducible loss term

In language models, the irreducible loss exists because text has some intrinsic unpredictability—even with a perfect language model, the loss of predicting the next token cannot be zero. Since an arbitrarily large autoencoder can in fact perfectly reconstruct the input, we initially expected there to be no irreducible loss term. However, we found the quality of the fit to be substantially less good without an irreducible loss.

While we don't fully understand the reason behind the irreducible loss term, our hypothesis is that the activations are made of a spectrum of components with different amount of structure. We expect

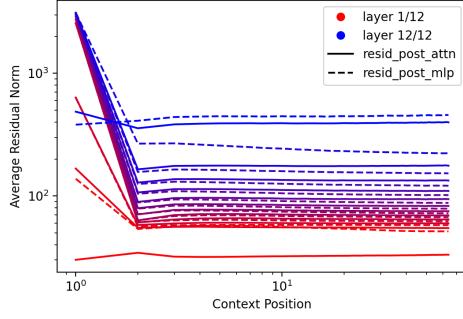


Figure 30: Residual stream norms by context position. First token positions are more than an order of magnitude larger than other positions, except at the first and last layer for GPT-2 small.

less structured data to also have a worse scaling exponent. At the most extreme, some amount of the activations could be completely unstructured gaussian noise. In synthetic experiments with unstructured noise (see Figure 31), we find an $L(N)$ exponent of -0.04 on 768-dimensional gaussian data, which is much shallower than the approximately -0.26 we see on GPT-2-small activations of a similar dimensionality.

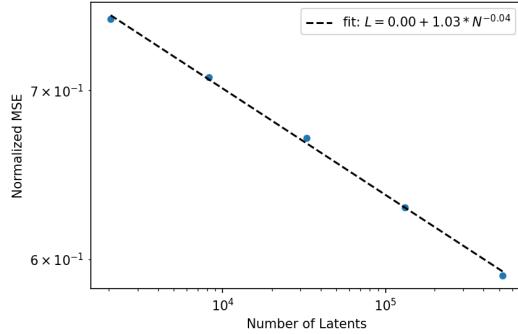


Figure 31: $L(N)$ scaling law for training on 768-dimensional random gaussian data with $k=32$

H Further probe based evaluations results