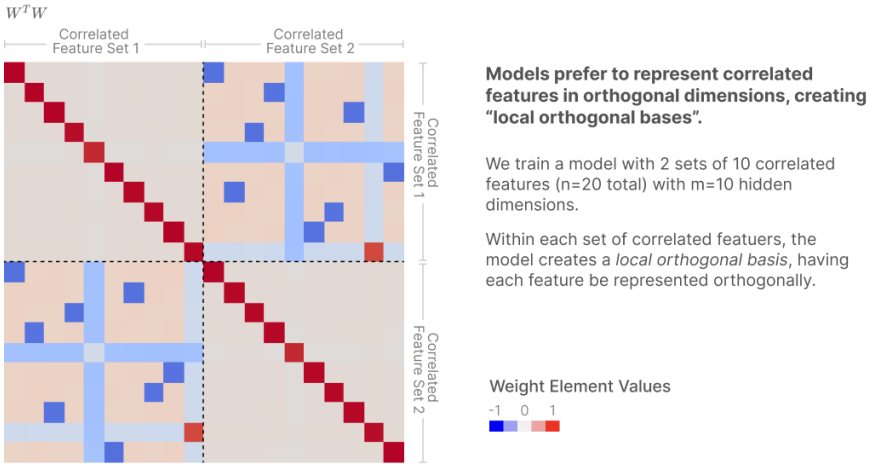


Models prefer to represent correlated features in orthogonal dimensions, creating "local orthogonal bases".

We train a model with 2 sets of 10 correlated features (n=20 total) with m=10 hidden dimensions.

Within each set of correlated featuers, the model creates a *local orthogonal basis*, having each feature be represented orthogonally.

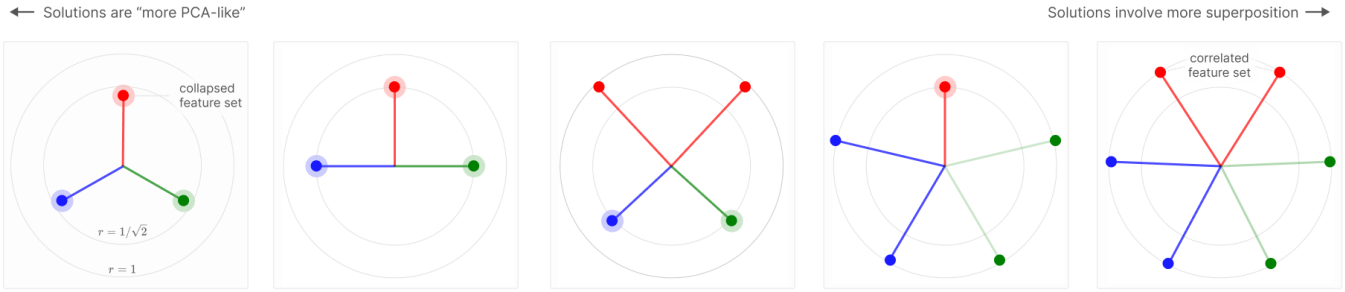
If this result holds in real neural networks, it suggests we might be able to make a kind of "local non-superposition" assumption, where for certain sub-distributions we can assume that the activating features are not in superposition. This could be a powerful result, allowing us to confidently use methods such as PCA which might not be principled to generally use in the context of superposition.

COLLAPSING OF CORRELATED FEATURES

One of the most interesting properties is that there seems to be a trade off with Principal Components Analysis (PCA) and superposition. If there are two correlated features a and b, but the model only has capacity to represent one, the model will represent their principal component $(a+b)/\sqrt{2}$, a sparse variable that has more impact on the loss than either individually, and ignore the second principal component $(a-b)/\sqrt{2}$.

As an experiment, we consider six features, organized into three sets of correlated pairs. Features in each correlated pair are represented by a given color (red, green, and blue). The correlation is created by having both features always activate together – they're either both zero or neither zero. (The exact non-zero values they take when they activate is uncorrelated.)

As we vary the sparsity of the features, we find that in the very sparse regime, we observe superposition as expected, with features arranged in a hexagon and correlated features side-by-side. As we decrease sparsity, the features progressively "collapse" into their principal components. In very dense regimes, the solution becomes equivalent to PCA.



These results seem to hint that PCA and superposition are in some sense complementary strategies which trade off with one another. As features become more correlated, PCA becomes a better strategy. As features become sparser, superposition becomes a better strategy. When features are both sparse and correlated, mixtures of each strategy seem to occur. It would be nice to more deeply understand this space of tradeoffs.

It's also interesting to think about this in the context of continuous equivariant features, such as features which occur in different rotations.

Superposition and Learning Dynamics

The focus of this paper is how superposition contributes to the functioning of fully trained neural networks, but as a brief detour it's interesting to ask how our toy models – and the resulting superposition – evolve over the course of training.

There are several reasons why these models seem like a particularly interesting case for studying learning dynamics. Firstly, unlike most neural networks, the fully trained models converge to a simple but non-trivial structure that rhymes with an emerging thread of evidence that neural network learning dynamics might have geometric weight structure that we can understand. One might hope that understanding the final structure would make it easier for us to understand the evolution over training. Secondly, superposition hints at surprisingly discrete structure (regular polytopes of all things!). We'll find that the underlying learning dynamics are also surprisingly discrete, continuing an emerging trend of evidence that neural network learning might be less continuous than it seems. Finally, since superposition has significant implications for interpretability, it would be nice to understand how it emerges over training – should we expect models to use superposition early on, or is it something that only emerges later in training, as models struggle to fit more features in?

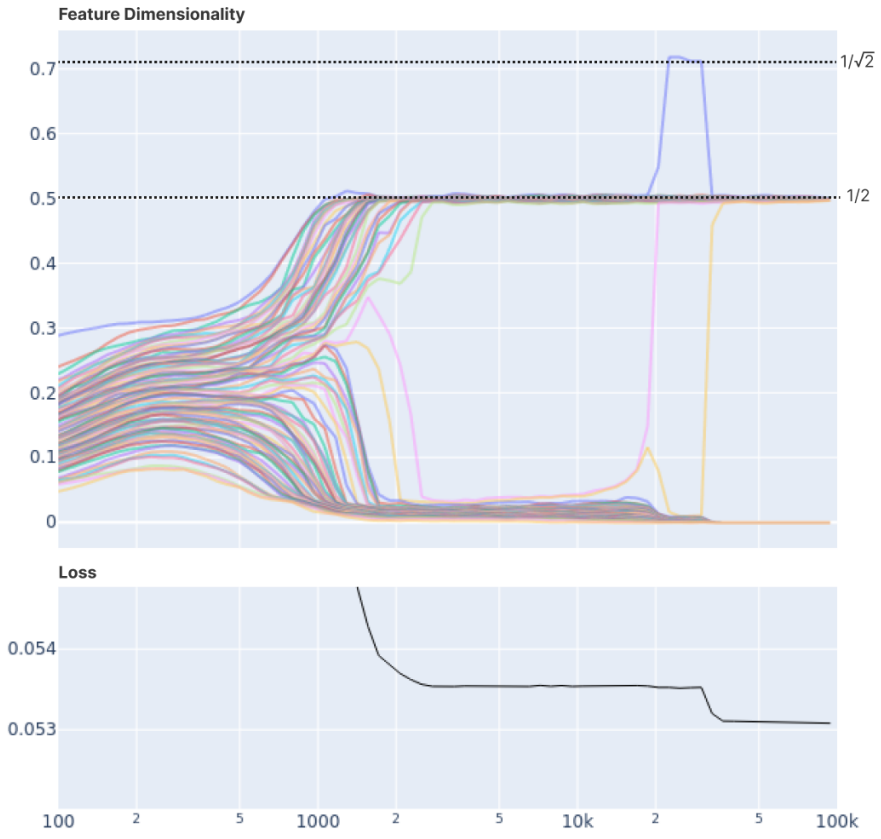
Unfortunately, we aren't able to give these questions the detailed investigation they deserve within the scope of this paper. Instead, we'll limit ourselves to a couple particularly striking phenomena we've noticed, leaving more detailed investigation for future work.

PHENOMENON 1: DISCRETE "ENERGY LEVEL" JUMPS

Perhaps the most striking phenomenon we've noticed is that the learning dynamics of toy models with large numbers of features appear to be dominated by "energy level jumps" where features jump between different feature dimensionalities. (Recall that a feature's dimensionality is the fraction of a dimension dedicated to representing a feature.)

Let's consider the problem setup we studied when investigating the geometry of uniform superposition in the previous section, where we have a large number of features of equal importance and sparsity. As we saw previously, the features ultimately arrange themselves into a small number of polytopes with fractional dimensionalities.

A natural question to ask is what happens to these feature dimensionalities over the course of training. Let's pick one model where all the features converge into digons and observe. In the first plot, each colored line corresponds to the dimensionality of a single feature. The second plot shows how the loss curve changes over the same duration.



Note how the dimensionality of some features "jump" between different values and swap places. As this happens, the loss curve also undergoes a sudden drop (a very small one at the first jump, and a larger one at the second jump).

These results make us suspect that seemingly smooth decreases of the loss curve in larger models are in fact composed of many small jumps of features between different configurations. (For similar results of sudden mechanistic changes, see Olsson *et al.*'s induction head phase change [27], and Nanda and Lieberum's results on phase changes in modular arithmetic [28]. More broadly, consider the phenomenon of grokking [29].)

PHENOMENON 2: LEARNING AS GEOMETRIC TRANSFORMATIONS

Many of our toy model solutions can be understood as corresponding to geometric structures. This is especially easy to see and study when there are only $m = 3$ hidden dimensions, since we can just directly visualize the feature embeddings as points in 3D space forming a polyhedron.

It turns out that, at least in some cases, the learning dynamics leading to these structures can be understood as a sequence of simple, independent geometric transformations!

One particularly interesting example of this phenomenon occurs in the context of correlated features, as studied in the previous section. Consider the problem of representing $n = 6$ features in superposition within $m = 3$ dimensions. If we have the 6 features be 2 sets of 3 correlated features, we observe a really interesting pattern. The learning proceeds in distinct regimes which are visible in the loss curve, with each regime corresponding to a distinct geometric transformation: