Theorem 2 (Corollary 3.1 in [49]). Given a $k \times n$ matrix A with the restricted isometry property, a sparse recovery algorithm find a k-sparse approximation $\hat{x}$ of $x \in \mathbb{R}^n$ from Ax such that

$$\|x - \hat{x}\|_1 \leq C(k) \min_{x', \|x'\|_0 \leq k} \|x - x'\|_1$$

for an approximation factor $C(k)$. If $C(k) = O(1)$, then a sparse recovery algorithm exists only if $m = \Omega(k \log(n/k))$.

Theorem 1 follows directly from Lemma 1 and Theorem 2.