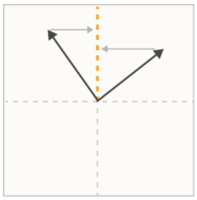
From this perspective, it only makes sense to ask if a *neuron* is interpretable when it is in a privileged basis. In fact, we typically reserve the word "neuron" for basis directions which are in a privileged basis. (See longer discussion here.)

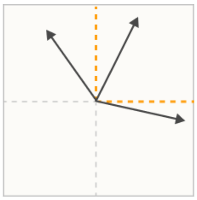
Note that having a privileged basis doesn't guarantee that features will be basis-aligned – we'll see that they often aren't! But it's a minimal condition for the question to even make sense.

The Superposition Hypothesis

Even when there is a privileged basis, it's often the case that neurons are "polysemantic", responding to several unrelated features. One explanation for this is the *superposition hypothesis*. Roughly, the idea of superposition is that neural networks "want to represent more features than they have neurons", so they exploit a property of high-dimensional spaces to simulate a model with many more neurons.



Polysemanticity is what we'd expect to observe if features were not aligned with a neuron, despite incentives to align with the privileged basis.

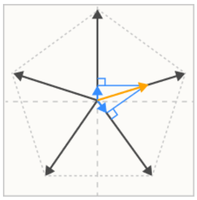


In the **superposition hypothesis**, features can't align with the basis because the model embeds more features than there are neurons. Polysemanticity is inevitable if this happens.

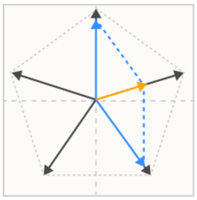
Several results from mathematics suggest that something like this might be plausible:

- **Almost Orthogonal Vectors.** Although it's only possible to have n orthogonal vectors in an n-dimensional space, it's possible to have $\exp(n)$ many "almost orthogonal" ($< \epsilon$ cosine similarity) vectors in high-dimensional spaces. See the Johnson–Lindenstrauss lemma.
- **Compressed sensing.** In general, if one projects a vector into a lower-dimensional space, one can't reconstruct the original vector. However, this changes if one knows that the original vector is sparse. In this case, it is often possible to recover the original vector.

Concretely, in the superposition hypothesis, features are represented as almost-orthogonal directions in the vector space of neuron outputs. Since the features are only almost-orthogonal, one feature activating looks like other features slightly activating. Tolerating this "noise" or "interference" comes at a cost. But for neural networks with highly sparse features, this cost may be outweighed by the benefit of being able to represent more features! (Crucially, sparsity greatly reduces the costs since sparse features are rarely active to interfere with each other, and non-linear activation functions create opportunities to filter out small amounts of noise.)



Even if only **one sparse feature** is active, using linear dot product projection on the superposition leads to **interference** which the model must tolerate or filter.



If the features aren't as sparse as a superposition is expecting, **multiple present features** can additively interfere such that there are multiple possible nonlinear reconstructions of an **activation vector**.

One way to think of this is that a small neural network may be able to noisily "simulate" a sparse larger model: