

# ICPO: ILLOCUTION-CALIBRATED POLICY OPTIMIZATION FOR MULTI-TURN CONVERSATION

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## ABSTRACT

Large Language Models (LLMs) in multi-turn conversations often suffer from a “lost-in-conversation” phenomenon, where they struggle to recover from early incorrect assumptions, particularly when users provide ambiguous initial instructions. We find that standard post-training techniques like Reinforcement Learning with Verifiable Rewards (RLVR) exacerbate this issue by rewarding confident, direct answers, thereby inducing overconfidence and discouraging the model from seeking clarification. To address this, we propose ILLocution-Calibrated Policy Optimization (ICPO), a novel training framework that sensitizes the model to instruction ambiguity. ICPO augments the training corpus with underspecified prompts and conditions the reward signal on the user’s illocutionary intent, rewarding the model for expressing uncertainty or asking for clarification when faced with ambiguity. Experiments demonstrate that ICPO fosters appropriate humility, yielding a substantial average improvement of 75% in multi-turn conversation, while preserving robust performance on single-turn benchmarks. Our work presents a practical path toward more robust and collaborative conversational AI that can better navigate the nuances of human interaction.

**Index Terms**— Large Language Models, Reinforcement Learning, Multi-turn Conversation, Conversational AI

## 1. INTRODUCTION

Today’s Large Language Models (LLMs) primarily interact with users through multi-turn conversation, such as in graphical chat interfaces and Agent-based applications [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. In practice, users often begin with ambiguous or underspecified instructions and gradually clarify their needs through iterative exchanges—a process that is susceptible to the model’s conversational priors [13]. Recent studies report a pronounced “lost-in-conversation” phenomenon in multi-turn settings: once the model adopts an incorrect assumption or produces an inappropriate response early on, it often struggles to recover, exhibiting degraded performance and increased instability [14].

Unfortunately, to our knowledge, there have not been studies aiming to address the “lost-in-conversation” problem. Existing tech-

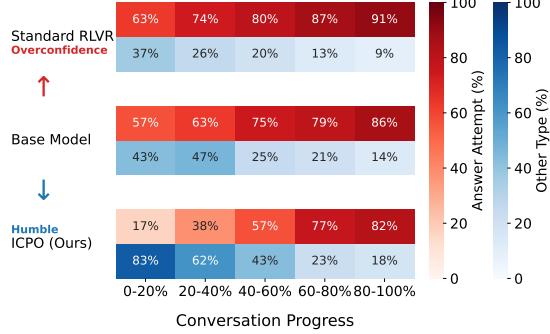
niques like Reinforcement Learning (RL) [15, 16, 17, 18] can only improve single-turn performance, lacking the ability to handle multi-turn conversations. For example, it can induce overconfidence and mode collapse by compressing policy entropy, as the model concentrates probability mass on a few high-reward responses to maximize rewards. This reduces response diversity, impairs exploration, and ultimately results in rigid, unchanging response patterns [19]. Besides, RL diminishes the tendency to seek clarification or express uncertainty in multi-turn conversations. This exacerbates the tendency to get “lost”: once the trajectory deviates, low-entropy, high-stickiness strategies accumulate over multi-turn conversations, rendering the model unable in subsequent turns to overturn prior assumptions and leading to excessive reliance on previously erroneous strategies.

To address these limitations, we argue that multi-turn optimization must be sensitive to the illocution of the user’s instruction, rather than treating all turns as requests for definitive solutions. Specifically, many degradation phenomena observed in multi-turn conversation [14] stem from the mismatch between users’ ambiguous instructions and low-entropy strategies. As a result, if a model knows when to clarify, question, or negotiate, it can preserve exploratory space to avoid hasty conclusions and improve recoverability.

Based on the above analysis, we propose ICPO: ILLocution-Calibrated Policy Optimization. ICPO introduces an ambiguity-aware training loop that (i) augments the training corpus with underspecified prompts representative of real-world usage, and (ii) conditions the reinforcement signal on the illocutionary status. ICPO is a plug-in, model-agnostic addition to Reinforcement Learning with Verifiable Rewards (RLVR), applicable to various downstream tasks. Experiments show that ICPO promotes calibrated humility, a trait manifested as sustained high entropy and greater generative diversity. This yields a substantial average improvement of 75% in multi-turn conversation, while preserving robust performance on single-turn benchmarks. Our key contributions in this paper are as follows.

- We first uncover a unique phenomenon that models trained via standard RLVR tend to become overconfident: when faced with ambiguous or under-specified instructions, they default to confident responses, exacerbating the tendency to get “lost” and degrading multi-turn performance.
- We propose ICPO, a novel framework that dynamically modulates the reward signal based on the illocution of the user’s instruction, thereby maintaining humility during the RLVR process.
- Compared to standard RLVR approaches, ICPO improves reasoning ability significantly while maintaining appropriate humility and achieves a 75% enhancement in multi-turn conversation performance.

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**Fig. 1:** Average distribution of response type during multi-turn conversations starting with ambiguous instructions, in which the instruction conditions gradually become complete as the conversation turns progress.

## 2. RLVR IMPAIRS MULTI-TURN CONVERSATION

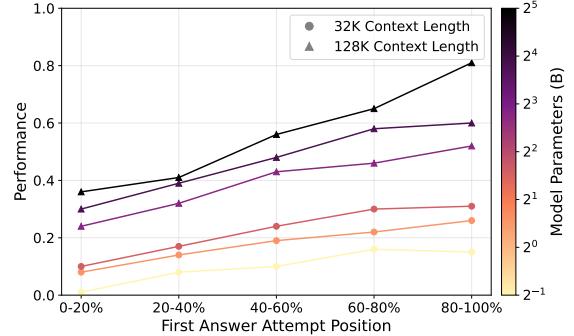
To test the effect of RLVR on a model’s multi-turn conversation capabilities in simulations of math tasks, we conduct experiments to examine the distribution of its response types before and after training. Furthermore, we analyze the impact of the distribution of its first answer attempt on overall performance. The experimental configuration is described in Sec. 4.1.

**Overconfidence in Standard RLVR.** A key challenge in applying RLVR to LLMs stems from the vast solution space of these models contrasted with sparse reward signals, which impedes existing RLVR methods from effectively guiding them toward comprehensive improvements. Because the reward depends entirely on the success or failure of the entire reasoning chain, models are driven toward exploitation of existing capabilities—continuously refining their knowledge and strategies—which constricts exploration and induces gradual capability degradation. As illustrated in Fig. 1, models trained via standard RLVR, when faced with underspecified or ambiguous queries, tend to produce overconfident responses rather than seeking clarification or engaging in further user interaction.

**Why do LLMs need to be humble.** In conversations initiated with under-specified queries, the language model operates with the least amount of information, facing maximum ambiguity. Consequently, the likelihood of formulating a correct solution attempt is at its lowest. Previous studies have demonstrated that an early attempt at a solution can implant flawed assumptions, thereby exerting a misleading influence on subsequent conversational interactions. As illustrated in Fig. 2, we delineate five intervals: if the first answer attempt occurs within the first 20% of conversation turns, it is categorized into the 0-20% interval; analogous divisions are applied to the subsequent intervals of 20-40%, 40-60%, 60-80%, and 80-100%. Our key observation is that for all models, conversations with a later first answer attempt lead to higher performance. Furthermore, a longer context length enhances multi-turn conversation capabilities. This phenomenon thus imposes a key requirement on models: when confronted with ambiguous instructions, they must be humble.

## 3. ICPO

To address the aforementioned limitations, we propose ICPO. The core principle of ICPO is to sensitize the model’s policy to the illocutionary intent of user instructions, particularly by enabling it to iden-



**Fig. 2:** A breakdown of the averaged multi-turn performance of the Qwen2.5-Instruct series, categorized by how early the LLM make its first answer attempt in the conversation.

tify and respond to the ambiguity implicit within them. We present the overview of ICPO in Fig. 3.

### 3.1. Scenario Simulation

First, we utilize an expert model  $\mathcal{M}_{\text{sim}}$  to process questions  $q$ , with the objective of transforming it into a situation that might be encountered in real-world conversation. In the context of a conversation scenario involving an underspecified instruction, we prompt the expert model to randomly remove one or two conditions from the question, thereby rendering it an unsolvable question:

$$q^* = \mathcal{M}_{\text{sim}}(q), \quad (1)$$

### 3.2. Illocutionary Judgment

Subsequently, we employ an expert model  $\mathcal{M}_{\text{jud}}$  to determine the response types  $\{I_i\}_{i=1}^G$  corresponding to the  $G$  candidate responses  $\{o_i\}_{i=1}^G$  generated from  $q^*$ . Motivated by previous studies [13, 14], we similarly define seven response categories: Answer attempt, Clarification, Interrogation, Discussion, Hedging, Refusal, and Missing.

### 3.3. Reward Modeling

For standard questions  $q$ , we adopt the verifiable reward defined in the standard RLVR framework:

$$r = \begin{cases} 1, & \text{if correct,} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

For simulation scenario questions  $q^*$ , we define a set of expected response types, denoted as

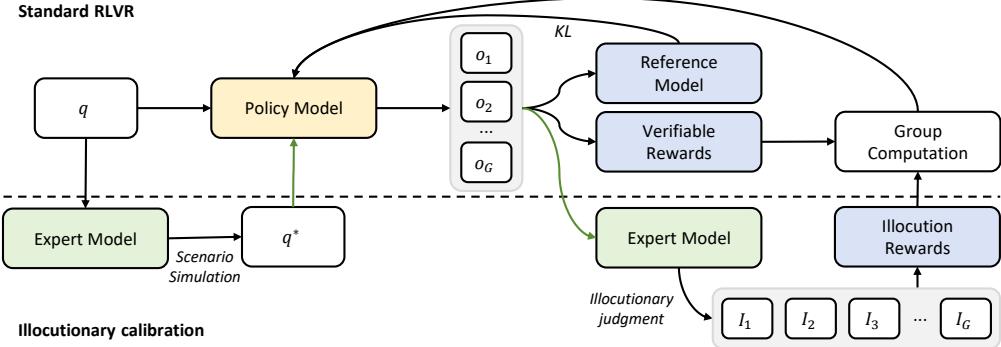
$$\mathcal{T} = \{\text{Clarification, Interrogation, Discussion, Hedging, Refusal}\}, \quad (3)$$

If the model’s response type belongs to the expected type set  $\mathcal{T}$ , a positive reward is assigned; otherwise, a zero reward is given:

$$r^* = \begin{cases} 1, & \text{if the response type } \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The final reward function is:

$$\tilde{r}(o|\tilde{q}) = \begin{cases} r(o|\tilde{q}), & \tilde{q} = q, \\ r^*(\mathcal{M}_{\text{jud}}(o)|\tilde{q}), & \tilde{q} = q^*. \end{cases} \quad (5)$$



**Fig. 3:** Overview of ICPO. Unlike standard RLVR which rewards only final outcomes, ICPO conditions the reward signal on the user’s illocutionary intent. By incorporating underspecified prompts via **Scenario Simulation** and rewarding calibrated humility through **Illocutionary Judgment**, the model learns to avoid overconfident hallucinations.

### 3.4. Reinforcement Learning Algorithms

We use Group Relative Policy Optimization (GRPO) [20] as our standard RLVR algorithm. Each question  $\tilde{q} = \{q, q^*\}$  corresponds to a set of candidate responses  $\{o_i\}_{i=1}^G$ . We define:

$$\rho_i = \frac{\pi_\theta(o_i | \tilde{q})}{\pi_{\theta_{\text{old}}}(o_i | \tilde{q})}, \quad A_i = \frac{\tilde{r}_i - \text{mean}(\tilde{r})}{\text{std}(\tilde{r})}, \quad (6)$$

The final loss is then:

$$\begin{aligned} \mathcal{L}_{\text{ICPO}}(\theta) = & \mathbb{E}_{\substack{\tilde{q} \sim \mathcal{D} \\ \{o_i\} \sim \pi_{\theta_{\text{old}}}}} \left[ \frac{1}{G} \sum_{i=1}^G \min(\rho_i A_i, \text{clip}(\rho_i, 1 - \alpha, 1 + \alpha) A_i) \right. \\ & \left. - \beta D_{\text{KL}}(\pi_\theta \| \pi_{\text{ref}}) \right]. \quad (7) \end{aligned}$$

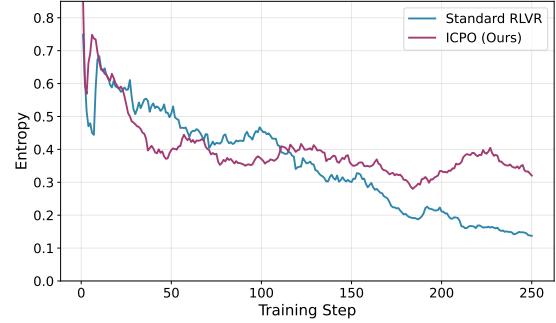
## 4. EXPERIMENTS

### 4.1. Experimental Setups

**Model.** We chose the Qwen2.5-Instruct series [21] as our backbone, with model sizes ranging from 0.5B to 32B parameters. For scenario simulation and illocutionary judgment, we use DeepSeek-V3-250324 [22] as the expert model.

**Evaluation.** We evaluate the model’s multi-turn capability based on the framework from prior research [14]. This framework simulates real-world scenarios, where a user begins with vague instructions and progressively provides more complete conditions across conversation turns. The evaluation metric is the accuracy of the final response. In this study, we focus on assessing multi-turn performance in mathematical tasks, using a multi-turn conversation dataset adapted from GSM8K [23]. In addition, we also evaluate the model’s performance on MATH500 [24], AMC23, AIME24, AIME25, Minerva [25], and Olympiad [26]. For the AMC23, AIME24, and AIME25 datasets, we compute the average accuracy over 8 rollouts. During evaluation, we set the rollout temperature to 0.7 and the maximum generation length to 4096.

**Training Details.** Our codebase is adapted from VeRL [27]. We trained two models, Qwen2.5-1.5B-Instruct and Qwen2.5-7B-Instruct, on the OpenR1-Math-220k dataset using both standard RLVR and ICPO. Both methods are based on the GRPO [20]. The training is conducted for 600 steps with a batch size of 64, a rollout



**Fig. 4:** Entropy comparison between Standard RLVR and ICPO during training.

of 8, and a learning rate of 1e-6. The experiment was conducted on 8 × A800-SXM4-80GB.

### 4.2. Main Results

- **Greater Humility.** Fig. 1 provides a compelling illustration of the dynamic shifts in response types following model training. To ensure the analysis focuses purely on model behavior rather than technical limitations, we have excluded statistics for the “Missing” type, which can arise from context length constraints. The results clearly indicate that after ICPO training, the model adopts a more humble and cautious stance when confronted with ambiguous instructions. This is evidenced by a remarkable 93% increase in the proportion of other response types (e.g., Clarification, Refusal). In contrast, the standard RLVR approach tends to amplify the model’s propensity for direct answers, potentially increasing the risk of overconfident and incorrect responses.

- **Better multi-turn conversation.** As detailed in Tab. 1, we present a comparative analysis of model performance in both mathematical reasoning and multi-turn conversation following training with standard RLVR and our proposed ICPO method, respectively. We observe that the standard RLVR model exhibits a modest gain in multi-turn accuracy, an improvement we attribute to its enhanced mathematical capabilities. In contrast, our ICPO method not only achieves a slight competitive edge on mathematical tasks but, more critically, delivers a substantial enhancement to the model’s multi-

**Table 1:** Performance of the Qwen2.5-1.5B-Instruct and Qwen2.5-7B-Instruct models on six competition-level math benchmarks, along with average performance (Avg.) and average performance on multi-turn conversations (Mt.). Bold and underline indicate the best and second-best results, respectively.

Model	AIME24	AIME25	AMC23	MATH500	Olympiad	Minerva	Avg.	Mt.
Qwen2.5-1.5B-Instruct	2.5	1.2	25.3	43.8	18.1	10.6	16.9	17.0
+ Standard RLVR	7.8	3.7	34.4	50.4	23.9	17.0	22.9	26.1
<b>+ ICPO (Ours)</b>	<b>7.9</b>	<u>3.2</u>	<b>34.9</b>	<u>49.9</u>	<b>24.3</b>	<b>17.7</b>	<b>23.0</b>	<b>32.8</b>
Qwen2.5-7B-Instruct	11.7	6.6	36.4	67.3	29.4	14.5	27.7	35.4
+ Standard RLVR	13.3	7.9	49.8	80.7	43.3	31.5	37.8	47.2
<b>+ ICPO (Ours)</b>	<u>12.9</u>	<u>7.7</u>	<b>51.9</b>	<b>81.2</b>	<b>44.8</b>	<u>30.4</u>	<b>38.2</b>	<b>55.4</b>

**Table 2:** Comparison of Multi-turn (Mt.) performance on Qwen2.5-7B-Instruct against various entropy-regularization and anti-collapse RLVR variants.

Method	Mt.
Qwen2.5-7B-Instruct	35.4
GRPO	47.2
+ Clip-higher	48.9
+ Clip-Cov	50.2
+ KL-Cov	47.7
RL-PLUS	51.6
<b>ICPO (Ours)</b>	<b>55.4</b>

turn conversational capabilities, boasting an average improvement of approximately 75% over the RLVR baseline.

- **Higher Response Diversity.** As illustrated in Fig. 4, our proposed method fosters greater response diversity by maintaining a stable and high policy entropy, fluctuating around 0.3. This elevated entropy signifies a flatter, more varied probability distribution over the action space, which prevents the model from prematurely converging on suboptimal solutions. In stark contrast, the standard RLVR baseline suffers from mode collapse as its entropy rapidly decreases to a near-deterministic level of 0.1. This indicates an overconfident policy that has settled on a narrow set of outputs. By avoiding this, our approach preserves greater diversity in its generations and maintains the flexibility to produce more nuanced and varied responses.
- **Superiority over Entropy Regularizers.** To verify that ICPO’s effectiveness stems from specific illocutionary calibration rather than merely preventing policy collapse, we conducted controlled experiments comparing ICPO against other RLVR variants equipped with entropy-regularization or anti-collapse mechanisms, including GRPO with Clip-higher [17], Clip-Cov, KL-Cov [19], and RL-PLUS [18]. As shown in Tab. 2, while generic regularization methods alleviate the “lost-in-conversation” issue to some extent, ICPO achieves consistently larger improvements (55.4%). This demonstrates that conditioning rewards on illocutionary intent provides a more precise guidance for maintaining multi-turn robustness than indiscriminate entropy maintenance.

### 4.3. Case Study

Fig. 5 illustrates a real-world scenario initiated by an under-specified user instruction. The problem as stated is inherently unsolvable due to the absence of critical conditions, such as the value of “daily rewards”. When presented with this ambiguity, the baseline model

**[User]** Sam is hired for a 20-day period. At the end of the 20-day period, he received \$660. How many days did he not work?

**[Qwen with Standard RLVR]**

To solve this problem, we need to determine how many days Sam did not work given that he received \$660 over a 20-day period. Let's denote the number of days Sam worked as x.

.....

Solving for x:  
 $x = 660/33 = 20$

This calculation shows that Sam worked exactly 20 days, which means he did not work at all. Therefore, the number of days he did not work is:

 $20 - 20 = 0$ 

Thus, the answer is:

 $\boxed{0}$

**[Qwen with ICPO]**

To determine how many days Sam did not work, we need to know the number of days Sam worked and then subtract that from the total number of days in the period. However, the information provided does not specify the number of days Sam worked or the daily wage. Without this information, it is impossible to determine the number of days Sam did not work. If you can provide the number of days Sam worked or his daily wage, I can help you calculate the number of days Sam did not work. Otherwise, we cannot determine the answer based on the information given.

OverConfidence
Humble

**Fig. 5:** Conversation example, in which the user starts the conversation with a problem that has insufficient conditions and cannot be solved.

trained with standard RLVR produces a factually incorrect, overconfident, and hallucinatory response. In stark contrast, the model trained with our proposed ICPO method demonstrates a more robust and cautious behavior. It correctly analyzes the prompt to identify the missing prerequisites, proactively requests the necessary information from the user, and candidly admits its inability to determine a definitive answer without further details.

## 5. CONCLUSION

In this paper, we identify a critical limitation of standard RLVR: it inadvertently promotes overconfidence, causing LLMs to struggle with ambiguous multi-turn conversations. To address this, we propose ICPO, a novel framework that sensitizes the model to instruction ambiguity. By dynamically rewarding the model for seeking clarification rather than forcing direct answers, ICPO achieves a significant 75% improvement in multi-turn reasoning while preserving robust single-turn performance. Crucially, our comparative experiments demonstrate that ICPO consistently outperforms generic entropy-regularization and anti-collapse mechanisms. This finding underscores that maintaining conversational robustness requires targeted illocutionary guidance rather than merely enforcing policy di-

versity. Our work offers a practical path toward more collaborative AI agents. Future work will focus on scaling ICPO to larger models and verifying its efficacy across broader, non-mathematical conversational contexts.

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