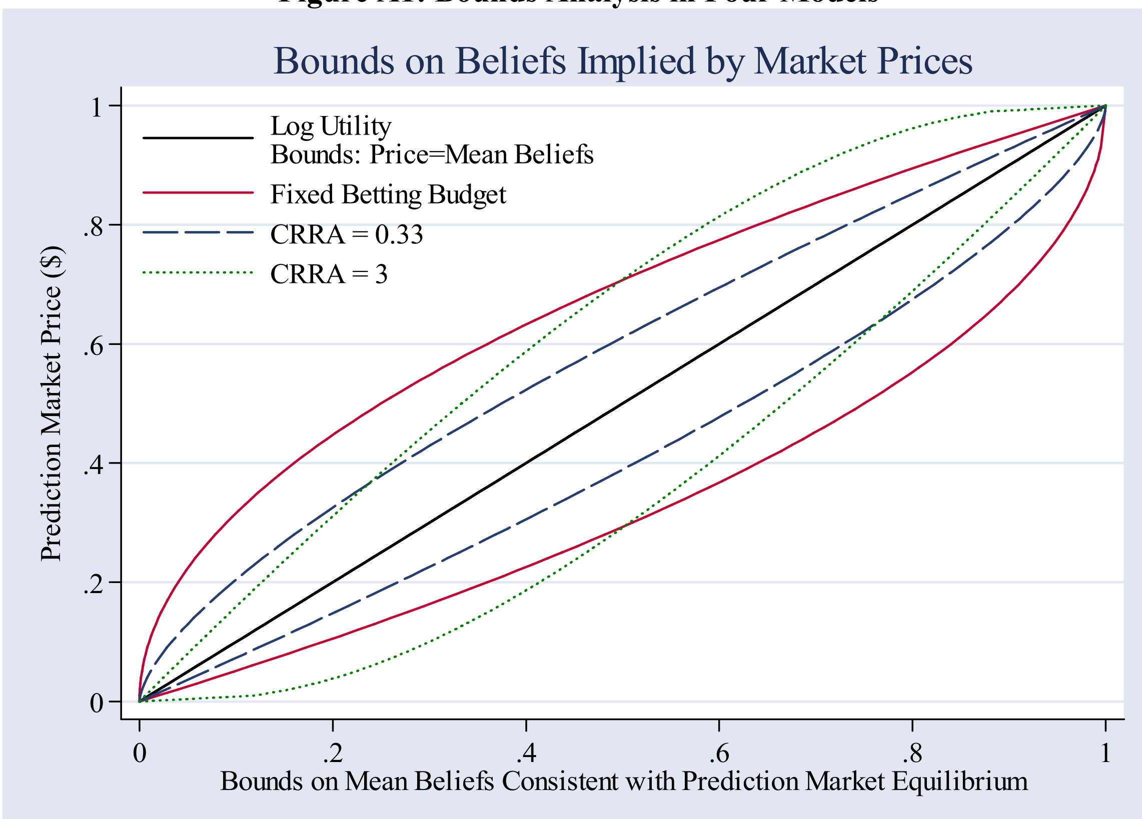
Appendix A: Bounds on Mean Beliefs

Manski considers the bounds on mean beliefs implied by a market price based on his fixed bet size model. Specifically, recall the result that the market price, π coincides with the $1\text{-}\pi^{th}$ percentile of the distribution of beliefs. Thus this price is consistent with many different distributions of beliefs: the lower bound reflects the two-point distribution $f(0)=1-\pi$ and $f(\pi+\varepsilon)=\pi$; the upper bound is generated by the distribution $f(\pi-\varepsilon)=\pi$ and $f(1)=1-\pi$ (for $\varepsilon\rightarrow 0$). As such, mean beliefs are bounded by $(\pi^2, 2\pi-\pi^2)$. These bounds are shown as solid lines in Figure A1.

Figure A1: Bounds Analysis in Four Models



We extend the spirit of Manski's bounds analysis to our other models. That is, we solve:

$$Lower\ bound = \underset{\{f(q)\}}{Min} \int_0^1 qf(q)dq \quad Upper\ bound = \underset{\{f(q)\}}{Max} \int_0^1 qf(q)dq$$

$$subject\ to\ prediction\ market\ equilibrium: \int_0^p x(q)f(q)dq = -\int_p^1 x(q)f(q)dq$$

It is easy to show that these bounds must be generated by two-point distributions. We map these bounds for each prediction market prices between \$0 and \$1, for each utility function. Log utility yields prices that always coincide with mean beliefs. For other utility functions these two-point distributions yield some divergence between prices and probabilities, but the divergence is typically smaller than in the fixed bet size model. (Increasing risk aversion substantially makes extreme prices somewhat less informative.) By construction these bounds reflect extreme bimodal distributions of beliefs; for results with more standard distributions, see Figures 2-4.

6. References

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Table 1: Utility Functions and Demand for Prediction Securities

Utility Function	Utility	Demand
Log utility (CRRA with γ = 1)	$u(y)=\ln(y)$	$\frac{y}{\pi(1-\pi)}(q-\pi)$
Constant relative risk aversion (CRRA) (γ > 0, γ ≠ 1)	$u(y)=\frac{y^{1-\gamma}}{1-\gamma}$	$\frac{y}{\pi}\cdot\left(\frac{\pi\{[\frac{q(1-\pi)}{\pi(1-q)}]^{\frac{1}{\gamma}}-1\}}{1+\pi\{[\frac{q(1-\pi)}{\pi(1-q)}]^{\frac{1}{\gamma}}-1\}}\right)$
Constant absolute risk aversion (CARA)	$u(y)=-e^{-ry}$	$r^{-1}\cdot[\ln(\frac{q}{1-q})-\ln(\frac{\pi}{1-\pi})]$
Quadratic utility	$u(y)=-\frac{1}{2}(y^{\max}-y)^2$	$(y^{\max}-y)\frac{q-\pi}{q(1-\pi)-\pi(q-\pi)}$
Hyperbolic absolute risk aversion (HARA)*	$u(y)=\frac{1-\gamma}{\gamma}(\frac{ay}{1-\gamma}+b)^{\gamma}$	$\frac{y+b(1-\gamma)a^{-1}}{\pi}\cdot\left(\frac{\pi\{[\frac{\pi(1-q)}{q(1-\pi)}]^{\frac{1}{\gamma-1}}-1\}}{1+\pi\{[\frac{\pi(1-q)}{q(1-\pi)}]^{\frac{1}{\gamma-1}}-1\}}\right)$

* The HARA utility function nests the others as special cases. (For log utility γ→0; risk neutral: γ→1; quadratic: γ = 2; CRRA: γ < 1 and b=0; CARA: γ → -∞ and b > 0).

Table 2. Belief distributions consistent with market price and poll results under different utility functions and distributional assumptions

	Normal [μ,σ]	**Beta (α,β)**	**Uniform (q_L,q_H)**
Implied Distribution of Beliefs			
Fixed bet size (Limit; γ→0)	0.578 [0.584,0.278]	0.571 [2.112,1.589]	0.586 [0.229,0.942]
CRRA; γ=⅓	0.560 [0.561,0.201]	0.558 [3.370,2.675]	0.575 [0.252,0.897]
Log Utility (γ=1)	0.550 [0.550, 0.163]	0.550 [4.640, 3.804]	0.550 [0.342,0.758]
CRRA; γ=3	0.546 [0.546,0.149]	0.547 [5.337,4.432]	0.549 [0.343,0.755]
CRRA; γ=20	0.544 [0.544,0.144]	0.546 [5.640,4.707]	0.548 [0.345,0.752]
CARA; ρ=3	.544 [0.544,.0144]	0.545 [5.692,4.754]	0.547 [0.351,0.743]
Quadratic; y^{max}=3	.542 [0.542,0.138]	0.542 [6.568,5.553]	0.546 [0.351,0.742]

Notes: Table shows mean of distribution. [Parameters of the belief distribution shown in parentheses]
Source: Authors' calculations. Note that beliefs outside (0,1) were treated as lim. q→0 or 1, respectively.