ii) The distribution of beliefs is symmetric: $f(q-\overline{q}) = f(\overline{q}-q)$

then *f(q)x(q)* will be symmetric and supply will equal demand if and only if price is the mean of beliefs.

These *dual symmetry* assumptions are sufficient to imply that equlibrium prices are equal to the mean beliefs of traders. Thus, the assumption of log utility is sufficient, but not necessary for this result. These dual symmetry conditions are likely to be met for distributions of beliefs symmetric around $\overline{q}$ =½, as long as traders are not affected by framing issues.[2] This suggests that we might generally expect prediction markets to be particularly accurate for prices close to ½. (We illustrate this in section 3.)

Beyond these special assumptions though, different utility functions and distributions of beliefs will yield less analytically neat expressions, and raise the possibility that market prices diverge from mean beliefs. Our aim is to evaluate the extent to which plausible parameters yield important divergence.

In the general case, the trader's utility maximization problem becomes:

$$\underset{\{x\}}{Max}\ EU_j = q_j U(y + x_j(1-\pi)) + (1-q_j)U(y - x_j\pi)$$

$$FOC: \quad \frac{U'(y + x - \pi x)}{U'(y - \pi x)} = \frac{\pi}{1-\pi}\frac{(1\text{-}q)}{q} \tag{6}$$

which yields an interior solution when the trader is risk averse and a corner solution of betting one's entire wealth when the trader is risk neutral or risk loving. Specific functional forms allow us to solve for individual demand functions, and by specifying the distribution of beliefs, to solve for the relationship between equilibrium prices and mean beliefs. Table 1 shows the individual demand functions for a few widely used functional forms.

Figure 1 plots demand as a function of beliefs for π = \$0.33 for a range of interesting utility functions. Four features are evident in the chart. First, all involve investors increasing their demand as the divergence between their beliefs and the market price increase. Second, the aggressiveness with which traders respond to perceived profit opportunities is dictated by their risk aversion. Third, all functions involve traders

[2] That is, we assume that investors are indifferent between economically equivalent long and short trades, such as buying a security at π with belief q, and shorting at $1\text{-}\pi$ with belief $1\text{-}q$, so that $x(\pi,q)=-x(1\text{-}\pi,1\text{-}q)$. At a price of ½, this implies $x(½,q)=-x(½,1\text{-}q)$ and hence demand is symmetric.

investing nearly all of their wealth as the investment approaches a perceived "sure thing". Fourth, while only log utility yields an exactly linear demand function, almost all of these functions are *approximately* linear over the range in which beliefs are within about 10-20 percentage points of the price. This suggests that for distributions of beliefs that are not too disperse, market prices will be quite close to the mean belief across traders.[3]

The only exception to this approximate local linearity is the risk-neutral investor, who always invests her entire wealth whenever market prices diverge from her beliefs. Interestingly, this is the only case considered by Manski (2004), adapting the example in Ali (1977).[4] Not surprisingly, these strong assumptions yield strong implications. Specifically for \$0.33 to be an equilibrium requires twice as many sellers as buyers at this price (because risking one's entire wealth allows each buyer to purchase twice as many contracts as a seller). Noting that a risk-neutral investor switches from buying to selling when the price falls below her belief, this implies that a price of \$0.33 corresponds to the 67^{th} percentile of the distribution of beliefs. Manski shows that this logic extends through the price distribution, and hence in that case, the equilibrium price corresponds not with the mean of beliefs, but rather with the $100\text{-}\pi^{th}$ percentile of the distribution of beliefs.

As noted, mean beliefs and prices exactly coincide for the log utility case. While we will solve for equilibrium prices below, we can first use the demand curves shown in Figure 1 to provide some intuition about the sign of any divergence between prices and mean beliefs. Specifically, the chart shows that traders with low risk aversion ($\gamma<1$) trade particularly aggressively if they believe the event to be closer to 50-50. Further, investors on the short side of a longshot are more likely to be wealth-constrained than those on the long side. Both of these forces suggest that if beliefs are distributed symmetrically that prices will be biased toward ½. Further, if the mean beliefs are accurate predictors, this implies a "favorite-longshot bias", in which longshots win less often than might be expected given the prediction market price. Analogously, the demand functions for more risk-averse investors ($\gamma>1$) suggest that investors with extreme beliefs have a greater

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[3] As Manski notes, if traders update their beliefs based on the market price, this typically leads the distribution of beliefs to become less disperse, suggesting that these conditions are often met.

[4] Similarly, risk-loving investors also invest their entire wealth, although given that they value the opportunity to gamble, the discontinuity in their demand function can occur at a different point.

effect on prices than in the log utility case. As such, prices will tend to be biased toward zero or one, and a "reverse favorite-longshot bias" may occur.

Mapping Prices to Probabilities

Given the individual demand functions derived above, all that is required to solve for equilibrium prices is a distribution of beliefs. We start by assuming that beliefs are drawn from a uniform distribution with a range of 10 percentage points, and solve for the mapping between mean beliefs and prices implied by each of the utility functions shown in Figure 1. (We rescale beliefs outside the (0,1) range to 0 or 1.)

Figure 2 shows that for moderately dispersed beliefs, prediction market prices tend to coincide fairly closely with the mean beliefs. While there is some divergence, it is typically within a percentage point, although the risk neutral model yields larger differences. Greater risk-aversion leads to a bias toward more extreme prices, while lesser risk aversion leads prices to be biased toward \$0.50. The divergence between prices and average beliefs is greatest for prices closest to \$0 and \$1, although behavior at the extremes partly reflects the distribution of beliefs becoming increasingly skewed as the mass point at 0 or 1 grows.

Figure 3 shows the mapping from prices to probabilities when beliefs are more disperse (in this case the standard deviation and range were doubled). As the dispersion of beliefs widens, the number of traders with extreme beliefs increases, and hence the non-linear response to the divergence between beliefs and prices is increasingly important. As such, the biases evident in Figure 2 become even more evident as the distribution of beliefs widens. Even so, for utility functions with standard levels of risk-aversion, these biases are small.

Finally, in Figure 4 we show that alternative utility functions yield similar implications. We have also experimented with uniform, beta and log-normal distributions of beliefs, and these results are also similar.

Cumulatively, figures 2-4 show six main patterns. First, under all conditions, log utility yields prices that coincide exactly with mean beliefs. Second, for other utility functions the divergence between prices and mean beliefs is generally quite small. Third,