Table 1: Empirical validation of non-identifiability across models, traits and sample sizes. Cohen's d measures effect size between v and $v + v_{\perp}$ (lower = more equivalent). All values show negligible differences ($d < 0.2$), confirming observational equivalence.

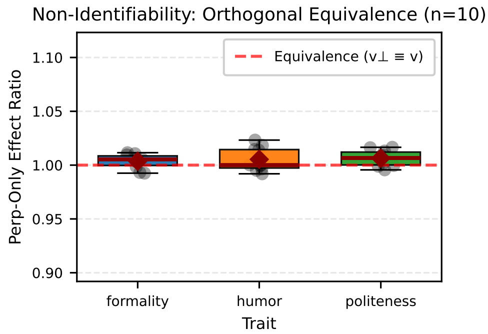
Model	Trait	Seeds	Cohen's d	Correlation	Perp-Only
Qwen2.5-3B-Instruct	Formality	$n = 5$	0.144 ± 0.095	0.210 ± 0.113	98.8%
		$n = 10$	0.075 ± 0.058	0.285 ± 0.087	100.4%
	Politeness	$n = 5$	0.112 ± 0.077	0.319 ± 0.070	101.5%
		$n = 10$	0.092 ± 0.069	0.414 ± 0.083	100.6%
	Humor	$n = 5$	0.103 ± 0.077	0.276 ± 0.177	99.7%
		$n = 10$	0.072 ± 0.061	0.044 ± 0.097	100.5%
Llama-3.1-8B-Instruct	Formality	$n = 5$	0.052 ± 0.029	0.164 ± 0.044	95.6%
		$n = 10$	0.096 ± 0.068	0.192 ± 0.085	96.8%
	Politeness	$n = 5$	0.074 ± 0.041	0.324 ± 0.058	101.2%
		$n = 10$	0.085 ± 0.043	0.347 ± 0.077	100.4%
	Humor	$n = 5$	0.159 ± 0.109	-0.032 ± 0.116	98.4%
		$n = 10$	0.119 ± 0.119	0.016 ± 0.104	95.9%

confidence intervals: formality ($0.144 \to 0.075$), politeness ($0.112 \to 0.092$), humor ($0.103 \to 0.072$). For Llama-3.1-8B, we observe similar stability: formality ($0.052 \to 0.096$), politeness ($0.074 \to 0.085$), humor ($0.159 \to 0.119$). This consistency demonstrates that the observed equivalence is not a sampling artifact but a stable property of the steering geometry.

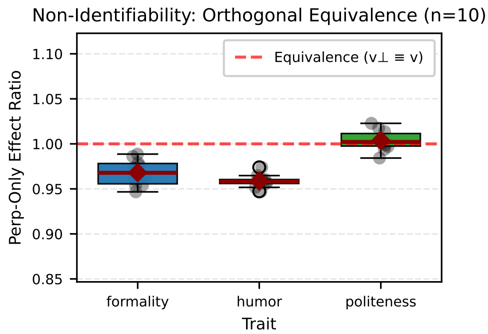
Cross-model consistency. The close agreement between Qwen2.5-3B ($d = 0.080$) and Llama-3.1-8B ($d = 0.100$) demonstrates that observational equivalence is not model-specific. Despite differences in architecture, scale (3B vs. 8B parameters) and hidden dimension ($d = 2048$ vs. $d = 4096$), both models exhibit nearly identical patterns of non-identifiability. The consistency across traits within each model further confirms that the phenomenon is general rather than an artifact of specific semantic domains or model implementations.

Visualizing orthogonal component equivalence. Figure 1 illustrates the perp-only effect ratios across all traits and models using $n = 10$ orthogonal seeds. The tight clustering around the perfect equivalence line (dashed red at 1.0) demonstrates that pure orthogonal components achieve nearly identical steering efficacy to the extracted vectors.

Qwen2.5-3B shows remarkable consistency across all three traits, with median ratios within 1% of perfect equivalence. Llama-3.1-8B exhibits slightly lower ratios for formality and humor (96–97%), yet orthogonally steering still retains over 95% efficacy, far exceeding what would be expected if v were uniquely identifiable.



(a) Qwen2.5-3B-Instruct



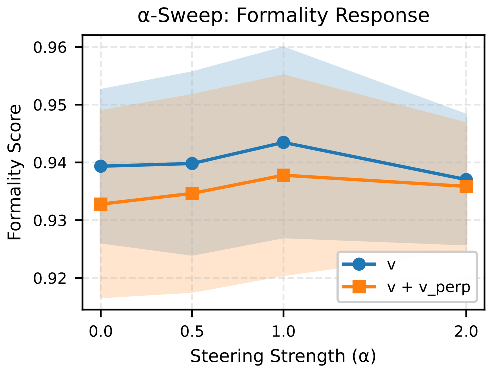
(b) Llama-3.1-8B-Instruct

Figure 1: Perp-only effect ratios ($v_{\perp}$ efficacy / v efficacy) for $n = 10$ orthogonal seeds. Values near 1.0 (dashed red line) indicate perfect equivalence.

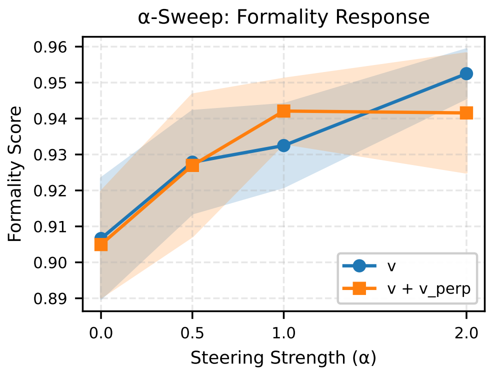
6.3 Scale Invariance Analysis

To verify that observational equivalence holds across different steering strengths, we evaluate the formality trait at four steering magnitudes $\alpha \in \{0.0, 0.5, 1.0, 2.0\}$ for both models. Figure 2 shows the response curves for the extracted vector v and the observationally equivalent vector $v + v_{\perp}$.

The curves track closely with overlapping confidence bands, demonstrating that v and $v + v_{\perp}$



(a) Qwen2.5-3B-Instruct



(b) Llama-3.1-8B-Instruct

Figure 2: Scale invariance of observational equivalence. Formality scores across steering strengths $\alpha \in \{0.0, 0.5, 1.0, 2.0\}$ for the extracted vector v (blue circles) and the perturbed vector $v + v_\perp$ (orange squares).

produce statistically indistinguishable behavioral effects at all tested steering strengths. For Qwen2.5-3B, mean differences remain below 0.022 across all α values (less than 2.5% deviation in formality scores); for Llama-3.1-8B, differences remain below 0.023 ($< 2.5\%$ deviation). This confirms that non-identifiability is a structural property: any vector in the equivalence class $v + \ker(J)$ produces identical observations regardless of scaling.

7 Conclusion

Rather than viewing non-identifiability as a limitation, we frame it as a necessary foundation for principled steering research. Our characterization clarifies what behavioral observations can reveal about representations and provides a framework for designing interventions that are both empirically effective and theoretically grounded. The empirical validation demonstrates that these are not abstract concerns: non-identifiability affects real steering interventions in deployed models, with typical steering vectors containing substantial observationally irrelevant components. However, the constructive characterization in Proposition 2 shows that identifiability is achievable through principled design choices, enabling alignment methods that clearly specify their affordances and limitations.

8 Limitations

Our empirical validation covers two models at mid-network layers across three semantic traits (formality, politeness, humor). While our theoretical results apply generally, the empirical magnitude of non-identifiability, including null space dimensionality and the fraction of vector norm therein, may vary across model families, scales, architectures and layer positions. Our semantic evaluation uses lexical heuristics rather than full distributional metrics or human judgments, providing a conservative test of observational equivalence. The primary theoretical mechanism relies on local linear approximation, formally valid in a neighborhood of the reference distribution, though our empirical validation confirms large equivalence classes exist in practice.

Proposition 2 characterizes sufficient conditions for identifiability but comprehensive empirical evaluation of each regime across multiple models, tasks and experimental designs remains future work. The practical applicability of identifiable regimes involves trade-offs: ICA requires carefully curated contrastive prompts, sparsity regularization may reduce efficacy if true factors are not sparse and multi-environment validation requires substantial additional data. Understanding these trade-offs and extending our framework to other intervention modalities (activation patching, prompt based steering) are important directions for the representational alignment community.

9 Future Work

Systematic evaluation of identifiable regimes, comparing ICA-based extraction, sparse recovery, multi-environment training and cross-layer validation against standard contrastive methods, would clarify which structural assumptions are most effective in practice. Hybrid methods combining multiple identifiability conditions may achieve stronger identifiability than individual approaches, while layer-wise analysis could reveal whether certain network positions afford more identifiable steering and decomposing the Jacobian by architectural component could identify which elements contribute to non-identifiability.

The multi-environment identifiability condition connects to causal representation learning, where adapting invariant risk minimization to train steering vectors that maintain consistent effects across distributions could filter spurious correlations. Investigating adversarial robustness and out-of-distribution generalization would test whether identifiable methods exhibit superior robustness. Characterizing scaling laws for identifiability, including how null-space dimensionality evolves as models scale, would inform long-term strategy for controlling future systems.

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