

Algorithm for Exercise 1

1. Input:

- A directed graph `graph`.
- Two vertices: `source` and `target`.

2. Initialize:

- Let `V` be the number of vertices in `graph`.
- Create a `visited` array of size `V`, initialized to `false`.
- Create an empty `queue` (FIFO) to manage BFS traversal.
- Mark `source` as visited and enqueue it.

3. BFS Traversal:

- While the `queue` is not empty:
 1. Dequeue the front vertex `v` from the `queue`.
 2. If `v` equals `target`, return `true` (a path exists).
 3. For each neighbor `w` of `v`:
 - If `w` is not visited:
 - Mark `w` as visited.
 - Enqueue `w` into the `queue`.

4. Termination:

- If the `queue` is empty and the `target` has not been reached, return `false` (no path exists).

Example Walkthrough

Graph:

```
0 -> 1 -> 3
|           ^
|           |
|           |
-> 2 -----
```

Input:

- `source = 0, target = 3.`

Execution:

1. Initialize:

- `visited = [false, false, false, false]`.
- `queue = []`.
- Mark `0` as visited, enqueue it: `queue = [0]`.

2. BFS Traversal:

- Dequeue `0`: Mark neighbors `1` and `2` as visited, enqueue them: `queue = [1, 2]`.
- Dequeue `1`: Mark neighbor `3` as visited, enqueue it: `queue = [2, 3]`.
- Dequeue `2`: No unvisited neighbors.
- Dequeue `3`: Target found. Return `true`.

Algorithm for `hasCycle` Function

1. **Input:**
 - A directed graph `graph`.
2. **Initialize:**
 - Let `V` be the number of vertices in the graph.
 - Create a `visited` array of size `V`, initialized to `false`, to track whether a vertex has been visited.
 - Create an `onStack` array of size `V`, initialized to `false`, to track whether a vertex is part of the current DFS path.
3. **Outer Loop:**
 - For each vertex `v` in the graph:
 - If `v` is not visited:
 - Perform DFS starting from `v`.
4. **DFS Traversal:**
 - Use the reverse postorder (`DepthFirstOrder`) to traverse the vertices in a specific order.
 - For each vertex in reverse postorder:
 - Mark the vertex as visited (`visited[vertex] = true`).
 - Mark the vertex as part of the current DFS stack (`onStack[vertex] = true`).
5. **Cycle Detection:**
 - For each neighbor `adj` of the current vertex:
 - If `adj` is not visited:
 - Mark `adj` as visited and continue the DFS.
 - If `adj` is already in the current DFS stack (`onStack[adj] = true`):
 - A cycle is detected. Return `true`.
6. **Backtracking:**
 - After all neighbors of a vertex are explored, mark it as no longer part of the current DFS stack (`onStack[vertex] = false`).
7. **Termination:**
 - If no cycles are detected after exploring all vertices, return `false`.

Example Walkthrough

Graph:

Vertices: 0 -> 1 -> 2 -> 3 -> 4 -> 1 (cycle: 4 -> 1).

Execution:

1. Initialize:
 - `visited = [false, false, false, false, false]`.
 - `onStack = [false, false, false, false, false]`.
2. Process Vertex 0:
 - Mark 0 as visited and on the stack.
 - Process neighbor 1.
3. Process Vertex 1:
 - Mark 1 as visited and on the stack.
 - Process neighbor 2.
4. Process Vertex 2:
 - Mark 2 as visited and on the stack.
 - Process neighbor 3.
5. Process Vertex 3:
 - Mark 3 as visited and on the stack.
 - Process neighbor 4.
6. Process Vertex 4:
 - Mark 4 as visited and on the stack.
 - Process neighbor 1.
 - Neighbor 1 is already on the stack. A cycle is detected. Return `true`.