

Numerical Analysis Notes

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Chapter 1

Error Analysis

Definition 1.0.1: Truncation Error

Error between the true value and approximate value due to chopping of the number from the series.

Example 1.0.1 (Taylor Series)

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2!}f''(x) + \dots$$

$x_0 \neq 0$

Example 1.0.2 (Maclaurin Series)

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots$$

$x = 0$

Definition 1.0.2: Round-off Error

Error between the true value and approximate value due to rounding off.

Definition 1.0.3: Absolute True Error

Error between the true value and the approximate value

$$\epsilon_t = |p - p^*|$$

ϵ_t is the Absolute True Error

p is the True Error

p^* is the Approximate Value

Definition 1.0.4: Absolute Relative Error

The relative error between the true value and the approximate value

$$\epsilon_r = \left| \frac{p - p^*}{p} \right| \cdot 100$$

ϵ_r is the Absolute Relative Error

p is the True Error

p^* is the Approximate Value

Definition 1.0.5: Absolute Approximation Error

The relative error between the previous approximation and current approximation

$$\epsilon_a = \left| \frac{p_0 - p_1}{p_0} \right| \cdot 100$$

ϵ_a is the Absolute Approximation Error

p_0 is the Previous Approximation

p^* is the Current Approximation

Definition 1.0.6: Tolerance

The maximum expected error in approximating error

$$\epsilon = 0.5 \cdot 10^{2-n} \%$$

ϵ is the Tolerance

n is the required Significant figures or decimal places

$$\epsilon = 10^{-n}$$

ϵ is the Tolerance

n is the required Significant figures or decimal places

Note:-

Use ϵ_a instead of ϵ_r when true value is not available

Algorithm 1: Approximating $f(x)$

- 1 Find the series representing $f(x)$
 - 2 Find tolerance
 - 3 Find first approximating using only first two numbers of the series
 - 4 **while** $\epsilon_a > \epsilon$ **do**
 - 5 | Calculate new approximate using another number from the series
 - 6 | Calculate new ϵ_a
 - 7 This value is the approximation
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Chapter 2

Solutions of Non-Linear Equations

2.1 Bisection Method

Theorem 2.1.1 Intermediate Value Theorem

If f is continuous on $[a, b]$ and if $f(a)$ and $f(b)$ are non-zero and have opposite signs, then there is at least one solution of $f(x) = 0$ in the interval (a, b)