Numerical Analysis Notes

Abdullah Faraz

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Chapter 1

Error Analysis

Definition 1.0.1: Truncation Error

Error between the true value and approximate value due to chopping of the number from the series.

Example 1.0.1 (Taylor Series)

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2!}f''(x) + \cdots$$
$$x_0 \neq 0$$

Example 1.0.2 (Maclaurin Series)

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$$
$$x = 0$$

Definition 1.0.2: Round-off Error

Error between the true value and approximate value due to rounding off.

Definition 1.0.3: Absolute True Error

Error between the true value and the approximate value

$$\epsilon_t = |p - p^*|$$

ϵ_t is the Absolute True Error
p is the True Error
p^* is the Approximate Value

Definition 1.0.4: Absolute Relative Error

The relative error between the true value and the approximate value

$$\epsilon_r = \left|\frac{p - p^*}{p}\right| \cdot 100$$

ϵ_r is the Absolute Relative Error
p is the True Error
p^* is the Approximate Value

Definition 1.0.5: Absolute Approximation Error

The relative error between the previous approximation and current approximation

$$\epsilon_a = \left|\frac{p_0 - p_1}{p_0}\right| \cdot 100$$

ϵ_a is the Absolute Approximation Error
p_0 is the Previous Approximation
p^* is the Current Approximation

Definition 1.0.6: Tolerance

The maximum expected error in approximating error

$$\epsilon = 0.5 \cdot 10^{2-n} \ \%$$

ϵ is the Tolerance
n is the required Significant figures or decimal places

$$\epsilon = 10^{-n}$$

ϵ is the Tolerance
n is the required Significant figures or decimal places

Note:-

Use ϵ_a instead of ϵ_r when true value is not available

Algorithm 1: Approximating $f(x)$

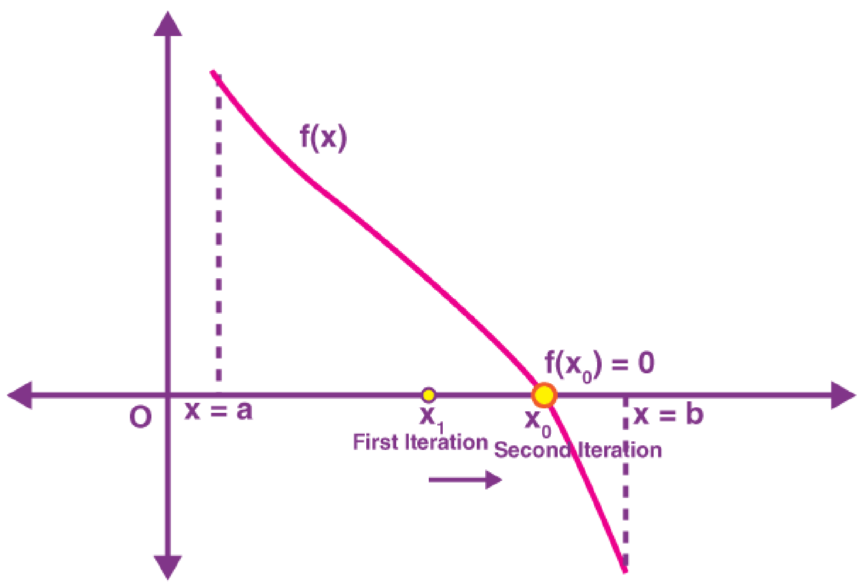
Find the series representing $f(x)$
Find tolerance
Find first approximating using only first two numbers of the series
while $\epsilon_a > \epsilon$ **do**
    Calculate new approximate using another number from the series
    Calculate new ϵ_a
This value is the approximation

Chapter 2

Solutions of Non-Linear Equations

2.1 Bisection Method

> **Theorem 2.1.1** Intermediate Value Theorem
>
> If f is continuous on $[a, b]$ and if $f(a)$ and $f(b)$ are non-zero and have opposite signs, then there is at least one solution of $f(x) = 0$ in the interval (a, b)



Algorithm 2: Finding root of $f(x)$ using Bisection Method

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For any continuous function f(x), find a closed interval [a, b] such that f(a).f(b) < 0.
Find the midpoint of a, b. Let x1 = (a + b)/2
if f(x1) = 0 then
    then x1 is the root.
if f(x1) ≠ 0 then
    if f(a).f(x1) < 0 then
        Root of f(x) lies in [a, x1], continue the above steps for interval [a, x1]
    else
        Root of f(x) lies in [x1, b], continue the above steps for interval [x1, b].
Continue the process repeatedly until we find a point xo in [a, b] for which f(xo) = 0.
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Theorem 2.1.2 Error Analysis of Bisection Method

If f is a continuous function on $[a, b]$ and $f(a) \cdot f(b) < 0$ and let $\{c_n\}$ be a sequence generated using bisection method and let c be the exact root of $f(x) = 0$, then error at the n^{th} interval is

$$|c - c_n| \leqslant \frac{b-a}{2^n}$$

Theorem 2.1.3

The number of iterations n required to obtain a approximating that is less than the tolerance ϵ is given by

$$n \geqslant \frac{\log(b-a) - \log(\epsilon)}{\log(2)}$$

n is the number of iterations
a is the lower bound of the interval
b is the upper bound of the interval
ϵ is the tolerance