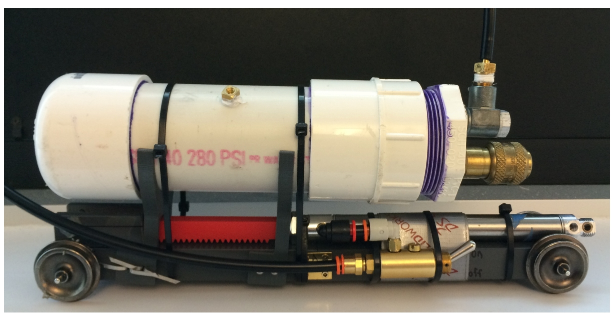
WARNING: If the design parameters in this document differ from parameters in the project description, use the project description.

Final Design Project
Simulating the motion of a compressed air train

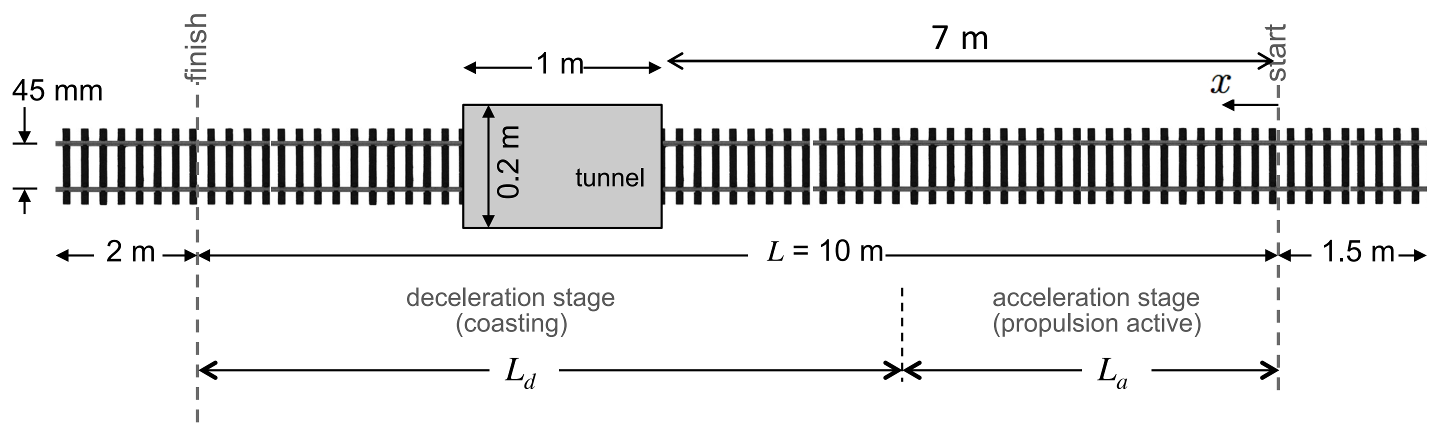


Numerical Methods

Project

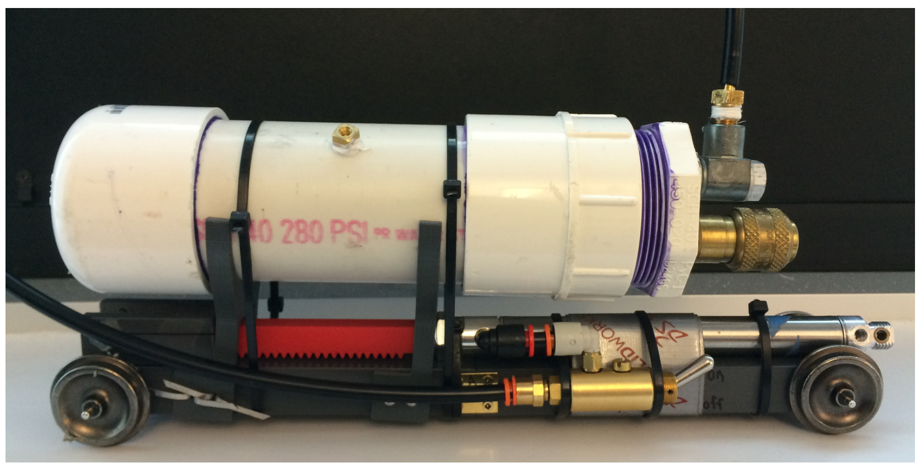
- **Objective:** Implement a fourth-order Runge-Kutta technique to solve the differential equations describing the motion of the train.
- Today we will go over the equations that will be solved and you start your coding.
- The assignment for this lab will be to:
  - (i) write down a plan (flow chart) for solving the basic equation of motion for the train one time. Include all important decision variables (if/then)
  - (ii) integrate the basic equation of motion for the train position using the parameters given to you by the TA in lab. You will turn in your plot of position versus time before lab the week of April 11th. You will upload a plot and your m-files.

Goal of the Race: The goal of the competition is to design a train that completes the coarse from start to finish in the least amount of time without damaging the tunnel and without running off the end of the track.



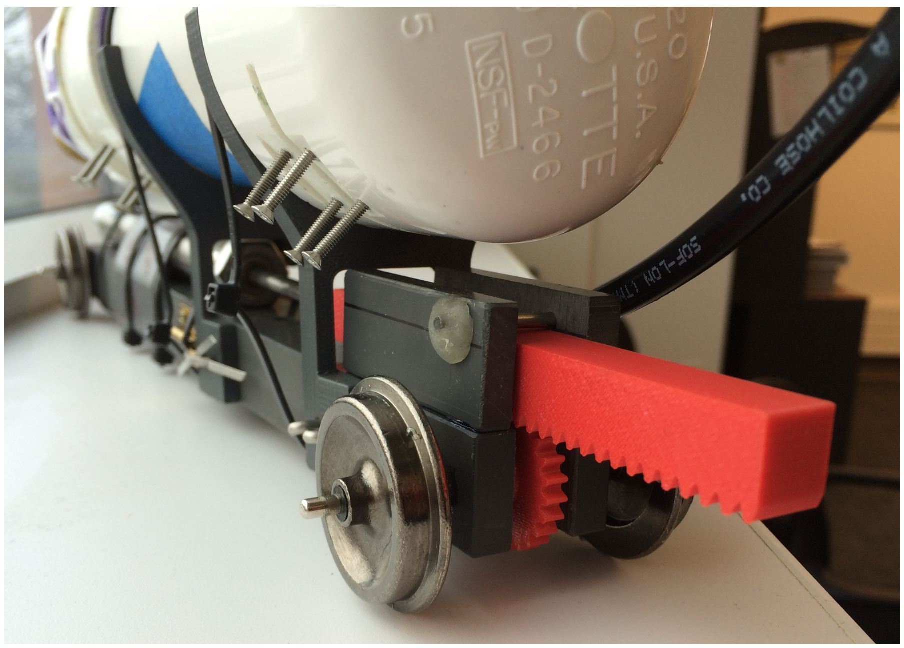
Note: Completes means that the rearmost section of the train passes the finish line.

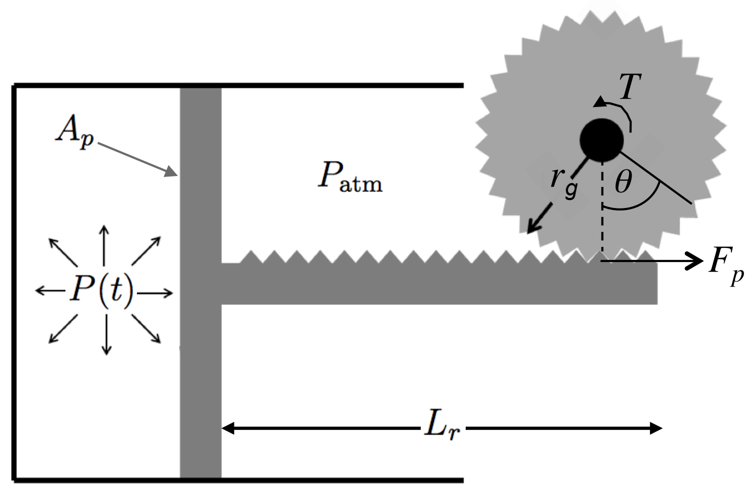
Power Source: The train is to be powered by compressed air stored on-board in a pressurized tank having a cylindrical geometry. The tank will be filled prior to the start of the race. According to the rules of the competition, the initial tank pressure must not exceed 100 psi.



Locomotion System:

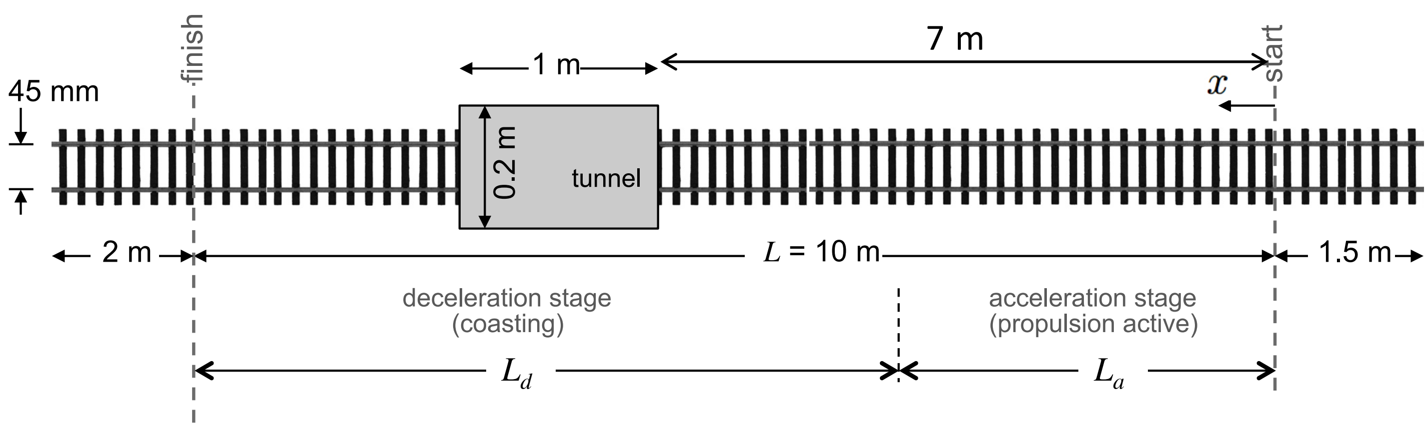
The locomotion system will consist of a pneumatic piston connected to a rack and pinion gear that drives the rear axle of the train. <u>Note:</u> we will restrict the actuation of the piston to a single stroke.





Locomotion System:

The locomotion system will provide an ``initial push'' to accelerate the train up to its maximum velocity; after which time, the train coasts (decelerates) to the finish line. In this manner, the propulsion system is only active during a portion of the race.

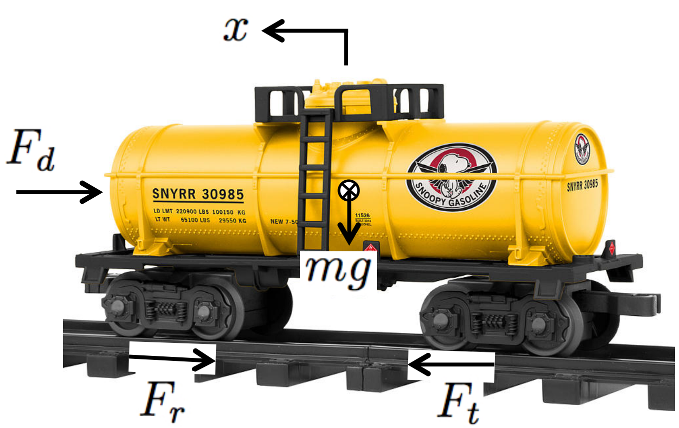


Train Dynamic Model:

Newton's 2nd Law: F = ma

$$\text{acceleration:} \quad m\frac{d^2x}{dt^2} = F_t - F_d - F_r \qquad 0 \geq x \leq L_a$$

$$\text{deceleration:} \quad m\frac{d^2x}{dt^2} = -F_d - F_r \ . \qquad L_a < x \leq L$$



F_d : aerodynamic drag force

F_r : rolling friction force

F_t : traction force

Train Dynamic Model:

Newton's 2nd Law: F = ma

$$F_d = \tfrac{1}{2}\, C_d\, \rho\, A\, V^2$$ Drag Force

$$F_r = C_r\, m\, g$$ Rolling Resistance Force

acceleration: $$m\,\frac{d^2x}{dt^2} = \boxed{F_t} - \tfrac{1}{2}\, C_d\, \rho\, A \left(\frac{dx}{dt}\right)^2 - C_r\, m\, g$$

deceleration: $$m\,\frac{d^2x}{dt^2} = -\tfrac{1}{2}\, C_d\, \rho\, A \left(\frac{dx}{dt}\right)^2 - C_r\, m\, g\ .$$

Traction Force:

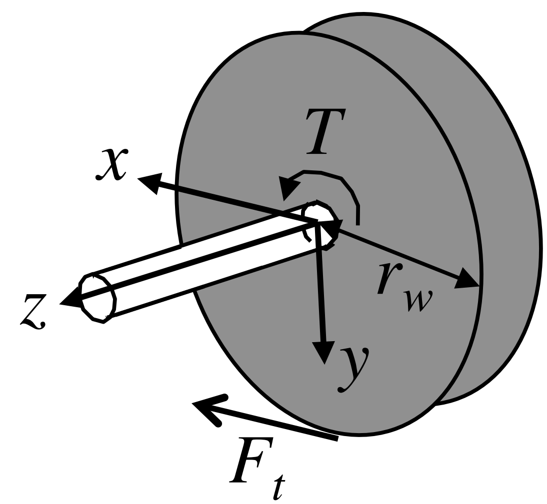
Apply conservation of Angular Momentum to the wheel (Newton's 2nd Law for rotating object)

$$\sum Torques = T - r_w F_t = I\alpha$$

$$I = \int_M r^2 dm$$

I – moment of inertia of the wheel

α - angular acceleration of the wheel



Moment of inertia for a single solid disk

$$I = \frac{1}{2} m_w r_w^2$$

$$\boxed{F_t = \frac{T}{r_w} - m_w r_w \alpha}$$

(For two disks)

Traction Force:

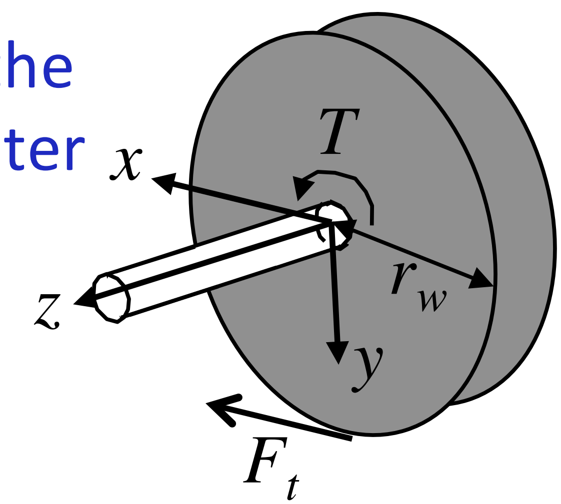
Apply conservation of Angular Momentum to the wheel

$$\sum Torques = T - r_w F_t = I\alpha$$

How do we determine if the wheel slips?

Our code must have a wheel Slip Criteria, the wheels will slip, if the traction force is greater than static friction force

$$F_t > \mu_s \frac{m}{2} g$$



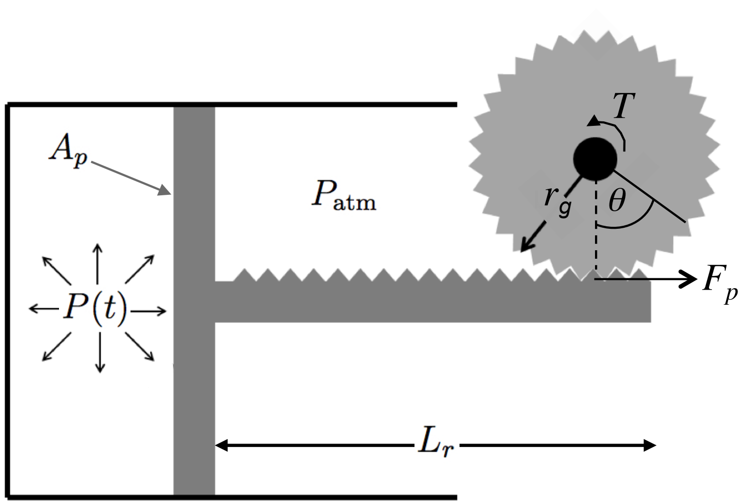
Applied Torque (T):

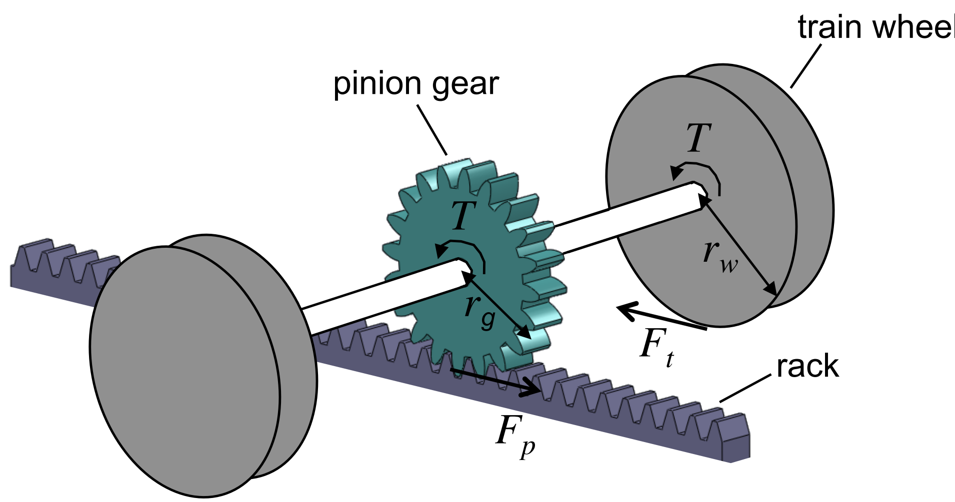
$$T = r_g \cdot F_p$$

$$F_p = P_{\text{net}} \cdot A_p$$

$$P_{net} = P - P_{atm}$$

$$F_p = (\boxed{P} - P_{\text{atm}})\, A_p$$



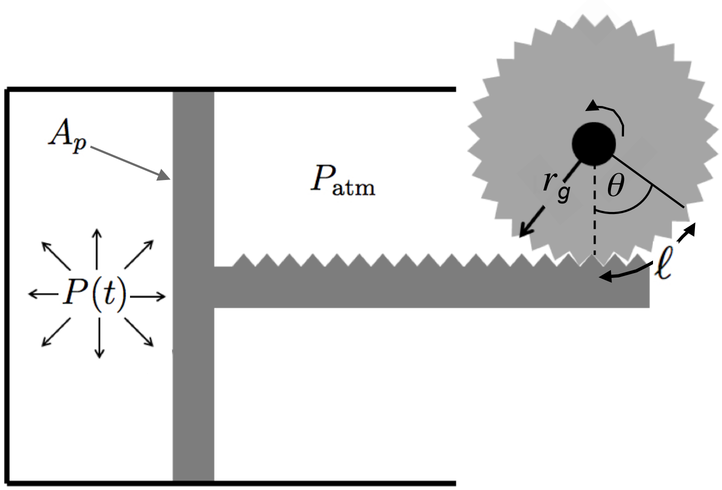


Piston Pressure (P): A bit of Thermodynamics

We will use the ideal gas Law: $P\,\mathcal{V} = n\,R\,T$

If we assume that T is constant (isothermal process): $P\,\mathcal{V} = P_0\,\mathcal{V}_0$

$$P = \frac{P_0\,\mathcal{V}_0}{\mathcal{V}_0 + \Delta\mathcal{V}}$$



Now Geometry

$$\Delta\mathcal{V} = A_p\,\ell$$

$$\ell = r_g\,\theta$$

$$P = \frac{P_0\,\mathcal{V}_0}{\mathcal{V}_0 + A_p\,r_g\,\theta}$$

Final System of ODES to Solve

$$F_t = \frac{T}{r_w} - m_w\, r_w\, \alpha$$

Need to deal with the angle and angular acceleration

$$F_t = \frac{r_g\, A_p}{r_w}\left[\frac{P_0\, \mathcal{V}_0}{\mathcal{V}_0 + A_p\, r_g\, \theta} - P_{\mathrm{atm}}\right] - m_w\, r_w\, \alpha$$

Similar to the gear, wheel rotation is related to track position:

$$x = r_w\, \theta$$

$$\theta = \frac{x}{r_w} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{1}{r_w}\frac{d^2 x}{dt^2}$$

Final System of ODES to Solve

Final from of the traction force is:

$$F_t = \frac{r_g\,A_p}{r_w}\left[\frac{P_0\,\mathcal{V}_0}{\mathcal{V}_0 + A_p\,\frac{r_g}{r_w}\,x} - P_{\text{atm}}\right] - m_w\,\frac{d^2x}{dt^2}$$

Final System of ODES to Solve

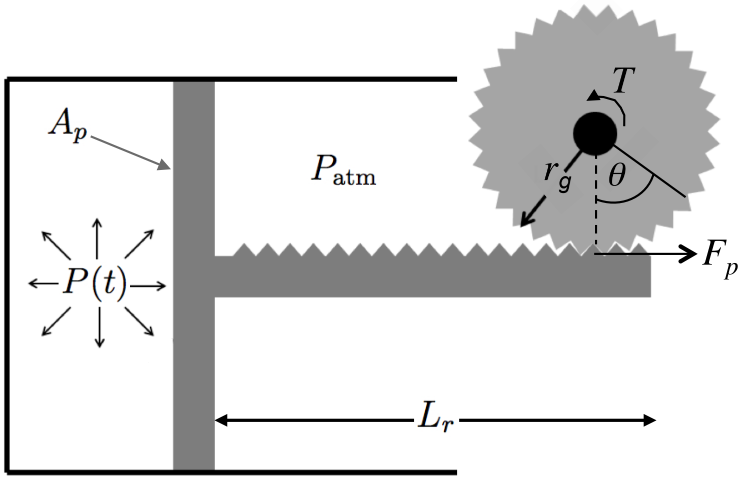
$$\text{acceleration: } \frac{d^2x}{dt^2} = \frac{1}{m+m_w}\left[\frac{r_g\,A_p}{r_w}\left(\frac{P_0\,\mathcal{V}_0}{\mathcal{V}_0 + A_p\,\frac{r_g}{r_w}\,x} - P_{\text{atm}}\right) - \frac{1}{2}\,C_d\,\rho\,A\left(\frac{dx}{dt}\right)^2 - C_r\,m\,g\right], \tag{23}$$

$$\text{deceleration: } \frac{d^2x}{dt^2} = -\frac{C_d\,\rho\,A}{2\,m}\left(\frac{dx}{dt}\right)^2 - C_r\,g\,. \tag{24}$$

Note: the acceleration phase is fixed by Lr. It must end when $r_g\theta < L_r$

$$\text{acceleration stage: } \quad x \leq L_r\,\frac{r_w}{r_g}$$

$$\text{deceleration stage: } \quad x > L_r\,\frac{r_w}{r_g}$$



Final System of ODES to Solve

$$\text{acceleration: } \frac{d^2x}{dt^2} = \frac{1}{m + m_w}\left[\frac{r_g\,A_p}{r_w}\left(\frac{P_0\,\mathcal{V}_0}{\mathcal{V}_0 + A_p\,\frac{r_g}{r_w}\,x} - P_{\text{atm}}\right) - \frac{1}{2}\,C_d\,\rho\,A\left(\frac{dx}{dt}\right)^2 - C_r\,m\,g\right], \tag{23}$$

$$\text{deceleration: } \frac{d^2x}{dt^2} = -\frac{C_d\,\rho\,A}{2\,m}\left(\frac{dx}{dt}\right)^2 - C_r\,g\;. \tag{24}$$

You must solve these equations using 4th order Runge-Kutta

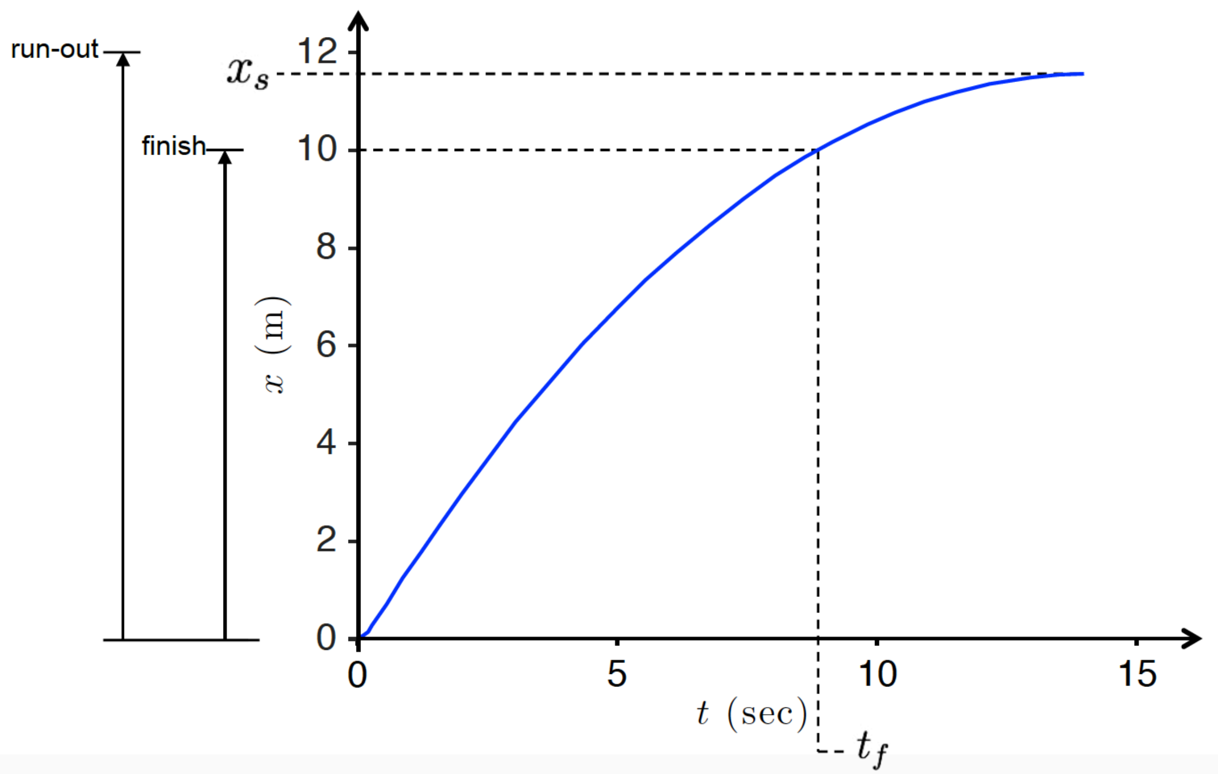
For this week

- Prepare your plan for how you will code up everything
- Solve the equation using the Physical Parameters Table provided

Table 1: Relevant Physical Parameters

parameter	symbol	range of values	units
length of train	L_t	60	cm
outer diameter of train	D_o	12	cm
density of train material	ρ_t	1400	kg/m^3
initial tank gage pressure	$P_{0_{\text{gage}}}$	255.0	kPa
pinion gear radius	r_g	3	cm
length of piston stroke	L_r	5	cm
diameter of piston	D_p	8	cm
air density	ρ_a	1	kg/m^3
atmospheric pressure	P_{atm}	85.0	kPa
drag coefficient	C_d	1.0	—
rolling friction coefficient	C_r	0.03	—
coefficient of static friction	μ_s	1.0	—
wheel diameter	D_w	5	cm
mass of wheels and axles (per axle)	m_w	100	g

Required Plot for next week



Recommended Plots

- Velocity vs. time
- Forces vs. time
- Pressure vs. time