# 4

# 向量、矩阵及其对角化

## 4.2 线性方程组的解

齐次线性方程组 $\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0 \\ \dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = 0 \end{cases}$ 即表示为 $Ax = 0$, 其中 $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

若 $x_1 = \xi_{11}, x_2 = \xi_{21}, \dots, x_n = \xi_{n1}$为齐次方程组的解，则 $x = \xi_1 = \begin{pmatrix} \xi_{11} \\ \xi_{21} \\ \vdots \\ \xi_{n1} \end{pmatrix}$称为方程组的解向量.

性质：若 $x = \xi_1, x = \xi_2$都是方程组的解，则 $x = \xi_1 + \xi_2$ 也是方程组的解.

性质：若 $x = \xi_1$是方程组的解，k为实数，则 $x = k\xi_1$ 也是方程组的解.

例题：求齐次线性方程组的解 $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ 2x_1 - 5x_2 + 3x_3 + 2x_4 = 0 \\ x_1 + x_2 - x_3 - x_4 = 0 \end{cases}$

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 2 & -5 & 3 & 2 \\ 7 & -7 & 3 & 1 \end{pmatrix} \xrightarrow[r_3 - 7r_1]{r_2 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & -7 & 5 & 4 \\ 0 & -14 & 10 & 8 \end{pmatrix} \xrightarrow{r_2 - 2r_3} \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & -7 & 5 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[r_1 - r_2]{r_2 \div (-7)} \begin{pmatrix} 1 & 0 & \frac{-2}{7} & \frac{-3}{7} \\ 0 & 1 & \frac{-5}{7} & \frac{-4}{7} \\ 0 & 0 & 0 & 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{2}{7}x_3 + \frac{3}{7}x_4 \\ \frac{5}{7}x_3 + \frac{4}{7}x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

令 $\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$、$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$对应 $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{7} \\ \frac{5}{7} \end{pmatrix}$、$\begin{pmatrix} \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}$,可得基础解系 $\xi_1 = \begin{pmatrix} \frac{2}{7} \\ \frac{5}{7} \\ 1 \\ 0 \end{pmatrix}, \xi_2 = \begin{pmatrix} \frac{3}{7} \\ \frac{4}{7} \\ 0 \\ 1 \end{pmatrix} \Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = c_1 \begin{pmatrix} \frac{2}{7} \\ \frac{5}{7} \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} \frac{3}{7} \\ \frac{4}{7} \\ 0 \\ 1 \end{pmatrix}$