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矩阵

2.1 矩阵的概念

设有n个未知数的m个方程的线性代数方程组$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = & b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = & b_2 \\ ...... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = & b_m \end{cases}$

其中 a_{ij}是第i个方程的第j个未知系数的系数，b_i是第i个方程的常数项，$i = 1, 2, ...m; j = 1, 2, ..., n.$ 上述方程为非齐次线性代数方程组，当 $b_1 = b_2 =. .. = b_m = 0$时

得到n元齐次线性代数方程组：$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = & 0 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = & 0 \\ ...... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = & 0 \end{cases}$

构造数表 $\begin{matrix} a_{11} & a_{12} & ... & a_{1n} & b_1 \\ a_{21} & a_{22} & ... & a_{2n} & b_2 \\ ... & ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} & b_m \end{matrix}$ 与 $\begin{matrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{matrix}$

矩阵的定义

由m×n个数 a_{ij}(i=1,2,...,m;j=1,2,...n)排成m行n列的数表，称为m行n列矩阵，简称m×n矩阵记作：

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{pmatrix}$$

这m×n个数称为矩阵A的元素，以 a_{ij}为(i,j)元的矩阵记作 (a_{ij})或$(a_{ij})_{m\times n}$ $m\times$n矩阵A也记作$A_{m\times n}$.

*只有一行的矩阵 $A = (a_1, a_2, ..., a_n)$称为行矩阵，又称行向量。只有一列的矩阵 $B = \begin{pmatrix} b_1 \\ b_2 \\ ... \\ b_m \end{pmatrix}$称为列矩阵，又称列向量。

*对于n阶方阵 $\Lambda = \begin{pmatrix} \lambda_1 & 0 & ... & 0 \\ 0 & \lambda_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \lambda_n \end{pmatrix}$称为对角矩阵记作 $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_n)$,当 $\lambda_1 = \lambda_2 =. .. = \lambda_n = 1$得到n阶单位矩阵 $E = \begin{pmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{pmatrix}$