1

行列式

1.1 简单行列式

二元线性方程组与二阶行列式

二元线性方程组：$\begin{cases} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{cases}$

消去未知数 x_2得：$(a_{11}a_{22} - a_{12}a_{21})x_1 = b_1 a_{22} - a_{12} b_2$;消去未知数 x_1得: $(a_{11}a_{22} - a_{12}a_{21})x_2 = a_{11}b_2 - b_1 a_{21}$;

当 $a_{11}a_{22} - a_{12}a_{21} \neq 0$时，方程组得解为：$\begin{cases} x_1 = \dfrac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}} \\ x_2 = \dfrac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \end{cases}$

二位线性方程组系数确定数表：$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$

表达式 $a_{11}a_{22} - a_{12}a_{21}$ 称为数表所确定得二阶行列式，记作$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$.

$a_{ij}(i = 1, 2; j = 1, 2)$称为行列式得元素，i,j分别叫做元素得行标和列标

$b_1 a_{22} - a_{12} b_2 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \equiv D_1$, $a_{11} b_2 - b_1 a_{21} = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} \equiv D_2$; $a_{11}a_{22} - a_{12}a_{21} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \equiv D$

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, x_2 = \frac{D_2}{D} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

例题:求解二元线性方程组$\begin{cases} 3x_1 - 2x_2 = & 12 \\ 2x_1 + x_2 = & 1 \end{cases}$,$\begin{cases} x_1 + 2x_2 = & 12 \\ 2x_1 + 4x_2 = & 1 \end{cases}$,$\begin{cases} x_1 + 2x_2 = & 12 \\ 2x_1 + x_2 = & 1 \end{cases}$

$$D = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 7, D_1 = \begin{vmatrix} 12 & -2 \\ 1 & 1 \end{vmatrix} = 14, D_2 = \begin{vmatrix} 3 & 12 \\ 2 & 1 \end{vmatrix} = -21, x_1 = \frac{D_1}{D} = 2, x_2 = \frac{D_2}{D} = -3;$$

$$D = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0, D_1 = \begin{vmatrix} 12 & 2 \\ 1 & 4 \end{vmatrix} = 46, D_2 = \begin{vmatrix} 1 & 12 \\ 2 & 1 \end{vmatrix} = -23, x_1 = \frac{D_1}{D} = ?, x_2 = \frac{D_2}{D} = ?;$$

$$D = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3, D_1 = \begin{vmatrix} 12 & 2 \\ 1 & 1 \end{vmatrix} = 10, D_2 = \begin{vmatrix} 1 & 12 \\ 2 & 1 \end{vmatrix} = -23, x_1 = \frac{D_1}{D} = -\frac{10}{3}, x_2 = \frac{D_2}{D} = \frac{23}{3};$$

三阶行列式

3行3列数表:$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$

记 $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\, a_{22}\, a_{33} + a_{12}\, a_{23}\, a_{31} + a_{13}\, a_{21}\, a_{32} - a_{11}\, a_{23}\, a_{32} - a_{12}\, a_{21}\, a_{33} - a_{13}\, a_{22}\, a_{31}$

例题：计算三阶行列式 $D = \begin{vmatrix} 1 & 2 & -4 \\ -2 & 2 & 1 \\ -3 & 4 & -2 \end{vmatrix}$

$$D = 1 * 2 * (-2) + 2 * 1 * (-3) + (-4) * (-2) * 4 - 1 * 1 * 4 - 2 * (-2) * (-2) - (-4) * 2 * (-3) = -4 - 6 + 32 - 4 - 8 - 24 = -14.$$