PAPER C150=MC150

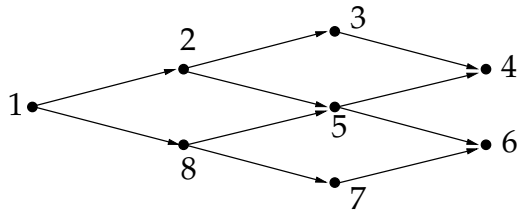
GRAPHS AND ALGORITHMS

Thursday 7 May 2020, 11:00
Duration: 80 minutes
Post-processing time: 30 minutes
Answer TWO questions

While this time-limited remote assessment has not been designed to be open book, in the present circumstances it is being run as an open-book examination. We have worked hard to create exams that assesses synthesis of knowledge rather than factual recall. Thus, access to the internet, notes or other sources of factual information in the time provided will not be helpful and may well limit your time to successfully synthesise the answers required.
Where individual questions rely more on factual recall and may therefore be less discriminatory in an open book context, we may compare the performance on these questions to similar style questions in previous years and we may scale or ignore the marks associated with such questions or parts of the questions. In all examinations we will analyse exam performance against previous performance and against data from previous years and use an evidence-based approach to maintain a fair and robust examination. As with all exams, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Paper contains 2 questions

1 a i) Use depth-first search (DFS) to perform a topological sort on the following directed acyclic graph (DAG). Assume that the adjacency lists are given in ascending numerical order.



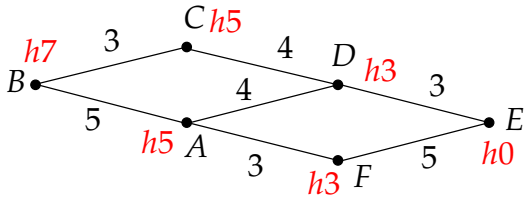
State the ordering of the nodes in the topological sort, as well as the order in which nodes are entered and the order in which they are exited. Also draw the DFS tree.

ii) What is the maximum number of arcs in a DAG with n nodes ($n \geq 0$)? State your answer as a function of n. Justify your answer.

Give an example of a DAG with nodes $1, 2, 3, 4, 5$ that has the maximum number of arcs.

b Show that in any finite simple undirected graph with $n \geq 2$ nodes there is at least one pair of nodes that have the same degree.

c The weighted graph G is as in the diagram below. The values of the heuristic function h as used by the A* algorithm are shown in red as $h0$, etc.



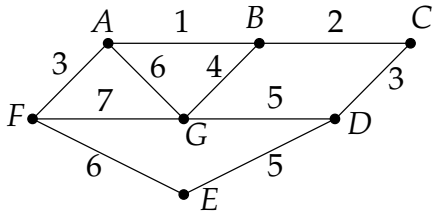
i) Use Dijkstra's algorithm to find the shortest path from A to E in G. Draw the corresponding tree, and also state the order in which nodes are added to the tree.

ii) What does it mean for the heuristic function h to be *consistent*?

iii) Use the A* algorithm to find the shortest path from A to E in G. Draw the corresponding tree, and also state the order in which nodes are added to the tree.

The three parts carry, respectively, 40%, 20%, and 40% of the marks.

2a i) Use Prim's algorithm starting from node A to find a minimum spanning tree (MST) for the following weighted graph. Give the MST as a diagram and also state the order in which the nodes are added.



ii) Does the graph from part (i) have a *unique* MST? Explain your answer.

b i) What is the minimum number of swaps needed to sort the list $[2, 0, 1]$?

ii) What is the minimum number of swaps needed to sort the list $[n-1, 0, 1, \ldots, n-2]$ (any $n \geq 2$)?

iii) Explain why your answer in (ii) is correct.

c i) Let D and D' be decision problems. What does it mean for D to *reduce* to D' (also written as $D \leq D'$)?

ii) The problem HPA is defined as follows: given an undirected (simple) graph G and an arc a joining nodes x and y of G, is there a Hamiltonian path in G which uses a?

Explain why HPA belongs to the complexity class NP.

iii) Show that HPA is NP-complete.

You may assume that the following problem HAMPATH is NP-complete: given a graph G, does G have a Hamiltonian path?

The three parts carry, respectively, 30%, 30%, and 40% of the marks.