Let $f : A \to B$ be a function. Let $A_1 \subseteq A, B_1 \subseteq B$

Problem 1. Show $f(f^{-1}(B_1)) \subseteq B_1$.

Proof.

$$\exists b \in B_1 \,|\, \forall a \in A,\ f(a) \neq b$$

□

Problem 2. Show $f^{-1}(f(A_1)) \supseteq A_1$.

Proof.

$$\exists a_1, a_2 \in A_1 \,|\, a_1 \neq a_2 \,\text{and}\, \exists b \in B \,|\, f(a_1) = f(a_2) = b$$

□

Problem 3. Counter-example for $f(f^{-1}(B_1)) = B_1$ and $f^{-1}(f(A_1)) = A_1$.

Proof.

$$f : \mathbb{R} \to \mathbb{R} \text{ is defined by} f(x) = x^2$$

□