Problem 1. Compute $D(g \circ f)(0,1)$ When,

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \qquad g : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

$$(x, y) \longmapsto (x^3 + y, e^{xy}, 2 + xy) \tag{1}$$

$$(u, v, w) \longmapsto (u^2 + v, uv + w^3) \tag{2}$$

Proof. From Equation 1 and 2, Jacobian of function f and g are

$$Df = \begin{bmatrix} \nabla(x^3+y) \\ \nabla(e^{xy}) \\ \nabla(2+xy) \end{bmatrix} = \begin{bmatrix} \frac{d}{dx}(x^3+y) & \frac{d}{dy}(x^3+y) \\ \frac{d}{dx}(e^{xy}) & \frac{d}{dy}(e^{xy}) \\ \frac{d}{dx}(2+xy) & \frac{d}{dy}(2+xy) \end{bmatrix} = \begin{bmatrix} 3x^2 & 1 \\ ye^{xy} & xe^{xy} \\ y & x \end{bmatrix} \tag{3}$$

$$Dg = \begin{bmatrix} \nabla(u^2+v) \\ \nabla(uv+w^3) \end{bmatrix} = \begin{bmatrix} \frac{d}{du}(u^2+v) & \frac{d}{dv}(u^2+v) & \frac{d}{dw}(u^2+v) \\ \frac{d}{du}(uv+w^3) & \frac{d}{dv}(uv+w^3) & \frac{d}{dw}(uv+w^3) \end{bmatrix} = \begin{bmatrix} 2u & 1 & 0 \\ v & u & 3w^2 \end{bmatrix} \tag{4}$$

$$D(g \circ f) = Dg(\underbrace{f(0,1)}_{(1,1,2)}) \times Df(0,1) \tag{5}$$

$$= \begin{bmatrix} 2 \times 1 & 1 & 0 \\ 1 & 1 & 3 \times 2^2 \end{bmatrix} \begin{bmatrix} 3 \times 0^2 & 1 \\ 1 \times e^{0\times1} & 0 \times e^{0\times1} \\ 1 & 0 \end{bmatrix} \tag{6}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 12 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} 1 & 2 \\ 13 & 1 \end{bmatrix} \tag{8}$$

$\square$