Problem 1. Let $w = Pdy \wedge dz + Qdz \wedge dx + Rdx \wedge dy$ be a 2-form.

Solution:

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\begin{aligned}
dw &= d(Pdy \wedge dz + Qdz \wedge dx + Rdx \wedge dy) \\
&= \quad dP \wedge (dy \wedge dz) - P \wedge d(dy \wedge dz) \\
&\quad + dQ \wedge (dz \wedge dx) - Q \wedge d(dz \wedge dx) \\
&\quad + dR \wedge (dx \wedge dy) - P \wedge d(dx \wedge dy) \\
&= \quad \left(\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz\right) \wedge (dy \wedge dz) - P \wedge (\cancel{d^2y} \wedge dz + dy \wedge \cancel{d^2z}) \\
&\quad + \left(\frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy + \frac{\partial Q}{\partial z}dz\right) \wedge (dz \wedge dx) - Q \wedge (\cancel{d^2z} \wedge dx + dz \wedge \cancel{d^2x}) \\
&\quad + \left(\frac{\partial R}{\partial x}dx + \frac{\partial R}{\partial y}dy + \frac{\partial R}{\partial z}dz\right) \wedge (dx \wedge dy) - R \wedge (\cancel{d^2x} \wedge dy + dx \wedge \cancel{d^2y}) \\
&= \left\{\left(\frac{\partial P}{\partial x}dx + \cancel{\frac{\partial P}{\partial y}dy} + \frac{\partial P}{\partial z}dz\right) \wedge dy\right\} \wedge dz - \cancel{P \wedge 0} \\
&\quad \left\{\left(\frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy + \cancel{\frac{\partial Q}{\partial z}dz}\right) \wedge dz\right\} \wedge dx - \cancel{Q \wedge 0} \\
&\quad \left\{\left(\cancel{\frac{\partial R}{\partial x}dx} + \frac{\partial R}{\partial y}dy + \frac{\partial R}{\partial z}dz\right) \wedge dx\right\} \wedge dy - \cancel{R \wedge 0} \\
&= -\left\{dz \wedge \left(\frac{\partial P}{\partial x}dx + \cancel{\frac{\partial P}{\partial z}dz}\right) \wedge dy\right\} \\
&\quad - \left\{dx \wedge \left(\cancel{\frac{\partial Q}{\partial x}dx} + \frac{\partial Q}{\partial y}dy\right) \wedge dz\right\} \\
&\quad - \left\{dy \wedge \left(\cancel{\frac{\partial R}{\partial y}dy} + \frac{\partial R}{\partial z}dz\right) \wedge dx\right\} \\
&= \frac{\partial P}{\partial x}(dx \wedge dy \wedge dz) + \frac{\partial Q}{\partial y}(dy \wedge dz \wedge dx) + \frac{\partial R}{\partial z}(dz \wedge dx \wedge dy) \\
&= \frac{\partial P}{\partial x}(dx \wedge dy \wedge dz) + \frac{\partial Q}{\partial y}\left[dy \wedge \{-(dx \wedge dz)\}\right] + \frac{\partial R}{\partial z}\left[\{-(dx \wedge dz)\} \wedge dy\right] \\
&= \frac{\partial P}{\partial x}(dx \wedge dy \wedge dz) + \frac{\partial Q}{\partial y}\left\{(dx \wedge dy) \wedge dz\right\} + \frac{\partial R}{\partial z}\left\{dx \wedge (dy \wedge dz)\right\} \\
&= \frac{\partial P}{\partial x}(dx \wedge dy \wedge dz) + \frac{\partial Q}{\partial y}(dx \wedge dy \wedge dz) + \frac{\partial R}{\partial z}(dx \wedge dy \wedge dz) \\
&= \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) dx \wedge dy \wedge dz
\end{aligned}
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