

Show that followings with definition.

Definition 1. A function $f(x)$ is said to be continuous at a point a if $\lim_{x \rightarrow a} f(x) = f(a)$.

Problem 1. Show that $(0, 1)$ is open.

Solution: Fix any x in $(0, 1)$, choose $e = e(x) = \min(x, 1 - x)$. Then, $(x - e, x + e) = (x - e, x + e) = (0, 1)$ is subset of $(0, 1)$. \square

Problem 2.

Solution: With negation: 'There exists x in A s.t. for any $\epsilon > 0$, $(x - \epsilon, x + \epsilon)$ is not a subset of A ', let $x = 0$. Then, $(0 - \epsilon, 0 + \epsilon)$ is not a subset of A . Because, $(-\epsilon, 0)$ is subset of $(-\epsilon, \epsilon)$ but, not subset of A . \square