

Let 2nd order linear PDE:

$$a_{11}u_{xx} + a_{12}u_{xy} + a_{21}u_{yx} + a_{22}u_{yy} + a_1u_x + a_2u_y + a_0u = 0$$

. From the PDE, can get

$$\left(\partial x + \frac{a_{12}}{a_{11}}\partial y\right)^2 u + \left(\frac{\det A}{a_{11}^2}\right)(\partial y)^2 u + l.o.t = 0. \quad (1)$$

s.t.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

**Case 1.**  $\det A > 0$ .

*Solution:*

$$\begin{aligned} & \left(\partial x + \frac{a_{12}}{a_{11}}\partial y\right)^2 u + \left(\frac{\det A}{a_{11}^2}\right)(\partial y)^2 u + l.o.t \\ & \Leftrightarrow (\partial t)^2 u + (\partial s)^2 u + l.o.t \\ & \Leftrightarrow \underbrace{u_{tt} + u_{ss}}_{\text{harmonic form}} + l.o.t = 0 \end{aligned}$$

□

**Case 2.**  $\det A = 0$ .

*Solution:*

$$\begin{aligned} & \left(\partial x + \frac{a_{12}}{a_{11}}\partial y\right)^2 u + \left(\frac{\det A}{a_{11}^2}\right)^0 (\partial y)^2 u + l.o.t \\ & \Leftrightarrow (\partial t)^2 u + l.o.t \\ & \Leftrightarrow u_{tt} + l.o.t = 0 \end{aligned}$$

□

**Case 3.**  $\det A < 0$ .

*Solution:*

$$\begin{aligned} & \left(\partial x + \frac{a_{12}}{a_{11}}\partial y\right)^2 u + \left(\frac{\det A}{a_{11}^2}\right)(\partial y)^2 u + l.o.t \\ & \Leftrightarrow (\partial t)^2 u - (\partial s)^2 u + l.o.t \\ & \Leftrightarrow u_{tt} - u_{ss} + l.o.t = 0 \end{aligned}$$

□