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hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                                        Page 1/32
   [dshin@acacia letrec] $ letrec -t /home/pl/hw07/tests/fact-of-5
2 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
4 % fact. (value=120)
5
6 letrec fact(n)
     = if zero?(n)
       then 1
       else *(n, (fact -(n, 1)))
10 in (fact 5)
13 \mid + exp=letrec fact(n)=if zero?(n) then 1 else *(n,(fact -(n,1))) in (fact 5)
14 | | env=[x=10, v=5, i=1]
15 ||+ exp=(fact 5)
16 | | | env=[fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
17 |||+ exp=fact
18 | | | | env=[fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
19 |\cdot| val=proc(n) if zero?(n) then 1 else *(n, (fact -(n,1))) [fact(n)=if zero?(n) t
   hen 1 else * (n, (fact - (n, 1))), x=10, v=5, i=1]
20 |||+ exp=5
21 | | | | env=[fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
22 |||- val=5
23 |||+ exp=if zero?(n) then 1 else *(n, (fact -(n, 1)))
| | | | | env = [n=5, fact(n) = if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
25
   |\cdot|\cdot|\cdot| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
27 | | | | | + exp=n
   ||||| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
28
29 ||||- val=5
30 ||||- val=#f
31 | | | | + \exp=*(n, (fact - (n, 1)))
|\cdot|\cdot| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
| | | | | | | env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
35 ||||- val=5
  | | | | | + \exp(fact - (n, 1))
37
   ||||| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
38
   |||||+ exp=fact
   |||||| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1
39
   |\cdot|\cdot|\cdot| val=proc(n) if zero?(n) then 1 else *(n,(fact -(n,1))) [fact(n)=if zero?(n)]
   ) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1]
  | | | | | | + \exp = -(n, 1)
   |||||| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1
42
43
   ||||||| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=
44
   11
45 ||||||- val=5
46 ||||||+ exp=1
47 ||||||| env=[n=5, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=
   1]
48 ||||||- val=1
49 |||||- val=4
50 |||||+ \exp = if zero?(n) then 1 else *(n, (fact -(n,1)))
   || || || ||  env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=1)
51
52
   |||||+ exp=zero?(n)
   ||||||| env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=
53
   11
  ||||||+ exp=n
=11
56 |||||||- val=4
57 |||||- val=#f
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hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                           Page 2/32
   || || || || + \exp=* (n, (fact - (n, 1)))
   ||||||| env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i=
   11
60 ||||||+ exp=n
| | | | | | | | | | = nv = [n=4, fact(n) = if zero?(n) then 1 else *(n, (fact -(n,1))), x=10.v=5.i
   =11
62 ||||||- val=4
63 | | | | | | | | + \exp(fact - (n, 1))
64 | | | | | | | | | env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5, i
65 | | | | | | | | | + exp=fact.
66 ||||||||| env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5,
   i = 11
   |||||||| val=proc(n) if zero?(n) then 1 else *(n,(fact -(n,1)))[fact(n)=if zero
   (n) then 1 else *(n, (fact -(n, 1))), x=10, v=5, i=1]
   | | | | | | | | | | + \exp(-(n, 1))
   || || || || || || env = [n=4, fact(n) = if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5,
   i=11
   |||||||+ exp=n
   |||||||||| env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5
   i=1
   ||||||||| val=4
73 | | | | | | | | | + exp=1
74 |||||||| env=[n=4, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5
   ,i=11
   |||||||||| val=1
   ||||||||- val=3
   ||\cdot||\cdot||\cdot| exp=if zero?(n) then 1 else *(n,(fact -(n,1)))
   |||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5,
   i=11
79 | | | | | | | | + exp=zero?(n)
80 |||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5]
   , i=1]
81 ||||||+ exp=n
   ||||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=
   5, i=1
   |||||||||| val=3
   |||||||||- val=#f
   || || || || || + \exp=*(n, (fact -(n, 1)))
   ||\cdot||\cdot||\cdot|| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=5)
   , i=1
   ||||||||+ exp=n
   |||||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=
   5, i=11
   |||||| val=3
   || || || || || || + \exp(fact - (n, 1))
   ||||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v=
   |||||||||+ exp=fact
93 ||||||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v
   =5, i=1]
94 ||||||||| val=proc(n) if zero?(n) then 1 else *(n, (fact -(n,1))) [fact(n)=if z
   ero?(n) then 1 else *(n, (fact -(n, 1))), x=10, v=5, i=1]
95 |||||||+ exp=-(n,1)
96 |||||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10, v
   =5, i=1
97 |||||||||+ exp=n
   |||||||||||| env=[n=3, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10,
   v=5, i=1
99 |||||||||- val=3
100 |||||||||||+ exp=1
v=5, i=11
102 |||||||||- val=1
103 |||||||||- val=2
104 || | | | | | | | | | + \exp if zero?(n) then 1 else *(n,(fact -(n,1)))
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hw07-LETREC-output-trace
11ìM-^[M-^T 10, 16 10:41
                                                                       Page 3/32
   |\cdot|\cdot|\cdot|\cdot|\cdot|\cdot| env=[n=2,fact(n)=if zero?(n) then 1 else *(n,(fact -(n,1))),x=10,v
   =5.i=11
106 ||||||||||+ exp=zero?(n)
107 ||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10,
   v=5.i=11
108 ||||||||||+ exp=n
109 |||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10]
   , v=5, i=1
110 |||||||||| val=2
111 |||||||| val=#f
112 | | | | | | | | | | | | | + \exp=*(n, (fact -(n, 1)))
113 |||||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10,
114 |||||||||+ exp=n
115 ||||||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10
   v=5.i=11
   117 | | | | | | | | | | + exp=(fact -(n,1))
118 | | | | | | | | | | | | | | env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=10
   , v=5, i=1]
119 ||||||||||+ exp=fact
120 ||||||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=1
   0, v=5, i=1
f zero?(n) then 1 else *(n, (fact -(n, 1))), x=10, v=5, i=1]
122 | | | | | | | | | | | | | | + \exp - (n, 1)
   |||||||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=1
   0, v=5, i=1
124 | | | | | | | | | | | | + exp=n
125 | | | | | | | | | | | | | | env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=
   10.v=5.i=11
126 ||||||||||||| val=2
127 | | | | | | | | | | | + exp=1
128 ||||||||||||||| env=[n=2, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=
   10, v=5, i=1
129 ||||||||||||- val=1
130 ||||||||||||- val=1
   |||||||||||||+ exp=if zero?(n) then 1 else *(n,(fact -(n,1)))
   |||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=1
132
   0, v=5, i=1
133 ||||||||||||+ exp=zero?(n)
   ||||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=
   10, v=5, i=11
135 ||||||||||||+ exp=n
136 |||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x
   =10, v=5, i=1
137 |||||||||||||- val=1
   ||||| val=#f
   | | | | | | | | | | | | | | + \exp^{*}(n, (fact - (n, 1)))
139
140 ||||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x=
   10.v=5.i=11
141 |||||||||+ exp=n
142 ||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x
   =10, v=5, i=1
143 |||||||||||||| val=1
144 ||||||||||||+ exp=(fact -(n,1))
145 |||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))), x
   =10, v=5, i=1
146 ||||||||||||||+ exp=fact
x=10, v=5, i=1
148 |||||||||||||- val=proc(n)if zero?(n) then 1 else *(n, (fact -(n,1)))[fact(n
   )=if zero?(n) then 1 else *(n, (fact -(n, 1))), x=10, v=5, i=1]
149 |||||||||||||+ exp=-(n,1)
150 ||||||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))),
   x=10, v=5, i=1
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hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                        Page 4/32
  ||||||+ exp=n
152 ||||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1)))
   , x=10, v=5, i=11
153 ||||||||||||||||| val=1
154 |||||||||||||||||+ exp=1
155 ||||||||||||||||| env=[n=1, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1)))
   , x=10, v=5, i=1
156 |||||||||||||||| val=1
157 |||||||||||||| val=0
158 ||||||||||||+ exp=if zero?(n) then 1 else *(n,(fact -(n,1)))
159 |||||||||||||||| env=[n=0, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1))),
160 |||||||||||||||+ exp=zero?(n)
161 ||||||||||||||||| env=[n=0, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1)))
   , x=10, v=5, i=1]
162 |||||||||||||||||||||||+ exp=n
), x=10, v=5, i=1]
164 |||||||||||||||||| val=0
166 |||||||||||||||||||+ exp=1
167 ||||||||||||||||| env=[n=0, fact(n)=if zero?(n) then 1 else *(n, (fact -(n,1)))
   ,x=10,v=5,i=1
168 |||||||||||||||||- val=1
169 |||||||||||||||| val=1
170 |||||||||||||- val=1
171 ||||||||||| val=1
172 | | | | | | | | | | | | | - val=1
173 | | | | | | | | | | | | - val=1
174 ||||||||- val=2
175 ||||||||| val=2
176 ||||||||| val=2
178 |||||||- val=6
179 |||||| val=6
180 |||||- val=24
181 | | | | | | - val=24
182 | | | | | - val=24
183 | | | | - val=120
184 |||- val=120
185 | | - val=120
   |- val=120
187 120
188 >
   [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/letrec-1
190 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
191 > % =======================
192 % letrecs. (value=32)
194 letrec f(x)
195 = -(x, 1)
196 in (f 33)
199 |+ exp=letrec f(x) = -(x, 1) in (f 33)
200 || env=[x=10, v=5, i=1]
201 | | + \exp(f 33) |
202 | | | env=[f(x) = -(x, 1), x=10, v=5, i=1]
203 |||+ exp=f
204 | | | | env=[f(x)=-(x,1), x=10, v=5, i=1]
205 | | | - val=proc(x) - (x, 1) [f(x) = -(x, 1), x=10, v=5, i=1]
206 |||+ exp=33
207 | | | | env=[f(x)=-(x,1),x=10,v=5,i=1]
208 | | | - val=33
209 | | | + \exp = -(x, 1)
210 | | | | env=[x=33, f(x)=-(x,1), x=10, v=5, i=1]
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hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                                                Page 5/32
   ||||+ exp=x
212 | | | | | env=[x=33, f(x)=-(x,1), x=10, v=5, i=1]
213 | | | | - val = 33
214 | | | | + exn=1
   |\cdot|\cdot|\cdot| env=[x=33, f(x)=-(x,1), x=10, v=5, i=1]
215
216
    || | | | - val = 1
   || |- val=32
217
218 ||- val=32
219 I- val=32
220 32
221 >
222 [dshin@acacia letrec] $ letrec -t /home/pl/hw07/tests/letrec-2
223 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
225 % letrecs. (value=8)
226
227 letrec f(x)
     = if zero?(x)
228
        then O
229
230
        else -((f - (x, 1)), -2)
231 in (f 4)
   I+ exp=letrec f(x)=if zero?(x) then 0 else -((f-(x.1)).-2) in (f 4)
234
    | | env=[x=10, v=5, i=1]
235
    | | + \exp(f 4)
236
    |\cdot| env=[f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
237
238
    |\cdot|\cdot| env=[f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
240 | | | - val=proc(x) if zero?(x) then 0 else -((f - (x, 1)), -2)[f(x) = if zero?(x) then 0
     else -((f - (x, 1)), -2), x=10, v=5, i=11
242 | | | | env=[f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
243 |||- val=4
   | | | + \exp = if zero?(x) then 0 else - ((f - (x, 1)), -2)
245
   |\cdot|\cdot| env=[x=4, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2), x=10, v=5, i=1]
    ||||+ \exp=zero?(x)
    |\cdot|\cdot|\cdot| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
247
248
    |\cdot|\cdot|\cdot| env=[x=4, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
249
   ||||- val=4
250
   ||||- val=#f
   | | | | + \exp(-((f - (x, 1)), -2))
   |\cdot|\cdot|\cdot| env=[x=4, f(x)=if zero?(x) then 0 else -((f -(x,1)), -2), x=10, v=5, i=1]
   | | | | | + \exp(f - (x, 1))
254
    |\cdot|\cdot|\cdot| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
255
   |||||| env=[x=4, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
258 | | | | | | | val=proc(x) if zero?(x) then 0 else -((f-(x,1)),-2)[f(x)=if zero?(x) the
   n 0 else -((f -(x,1)), -2), x=10, v=5, i=1]
259 | | | | | | | + \exp(-(x, 1))
260 | | | | | | | | env=[x=4, f(x)=if zero?(x) then 0 else -((f-(x,1)),-2),x=10,y=5, i=1]
262 |||||||| env=[x=4, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
264
   |||||| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
265
   |||||- val=1
266
   ||||| val=3
267
   || || || + \exp = if zero?(x) then 0 else - ((f - (x, 1)), -2)
268
   |\cdot|\cdot|\cdot|\cdot| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
   273 | | | | | | | | env=[x=3,f(x)=if zero?(x) then 0 else -((f-(x,1)),-2),x=10,v=5,i=1]
274 ||||||- val=3
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hw07-LETREC-output-trace
11ìM-^ſM-^T 10. 16 10:41
                                                                                                                                    Page 6/32
      |||||||| val=#f
276 | | | | | | | + \exp(-((f - (x, 1)), -2))
     |\cdot|\cdot|\cdot|\cdot| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
     || || || || || + \exp(f - (x, 1))
      |\cdot|\cdot|\cdot|\cdot| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
281 | | | | | | | | | env=[x=3, f(x) = if zero?(x) then 0 else -((f -(x,1)), -2), x=10, v=5, i=1]
282 | | | | | | | | | - val=proc(x) if zero?(x) then 0 else - ((f - (x, 1)), -2)) [f(x) = if zero?(x)
      then 0 else -((f -(x,1)), -2), x=10, v=5, i=1)
283 | | | | | | | | | + \exp = -(x, 1)
     |||||||| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10.v=5.i=11
     ||||||||| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
      ||\cdot||\cdot||\cdot|| = \text{ny} = [x=3, f(x)=\text{if zero}?(x) \text{ then } 0 \text{ else } -((f-(x,1)), -2), x=10, y=5, i=1]
     ||||||||||||||| val=1
     |||||||||| val=2
     || || || || + \exp = if zero?(x) then 0 else - ((f - (x, 1)), -2)
     |||||||| env=[x=2,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
     | | | | | | | | | | + \exp=zero?(x)
     ||||||||| env=[x=2,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1]
     |||||||+ exp=x
      ||\cdot||\cdot||\cdot|| = \text{nv} = [x=2, f(x)=\text{if zero}](x) then 0 else -((f-(x,1)), -2), x=10, v=5, i=1)
      |||||| val=2
      |||||||||- val=#f
     | | | | | | | | | | + \exp = -((f - (x, 1)), -2)
      || || || || || || env=[x=2, f(x)=if zero?(x) then 0 else -((f-(x,1)),-2),x=10,v=5,i=1]
     | | | | | | | | | | | + \exp(f - (x, 1))
      ||\cdot||\cdot||\cdot|| = nv = [x=2, f(x)=if zero?(x) then 0 else -((f-(x,1)), -2), x=10, v=5, i=1
     ||||||||||+ exp=f
305 | | | | | | | | | | | | env=[x=2, f(x)=if zero?(x) then 0 else -((f-(x,1)), -2), x=10, v=5, i=10
     |||||||||| val=proc(x) if zero?(x) then 0 else -((f -(x,1)),-2)[f(x)=if zero?(
      x) then 0 else -((f - (x,1)), -2), x=10, v=5, i=1)
     | | | | | | | | | | | + \exp = -(x, 1)
      310 ||||||||||| env=[x=2,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i
      =11
311 ||||||||| val=2
312 | | | | | | | | | | + exp=1
313 ||||||||||| env=[x=2,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i
314 | | | | | | | | | | | - val=1
315 ||||||||| val=1
316 | | | | | | | | | | | + exp=if zero?(x) then 0 else -((f - (x, 1)), -2)
317 | | | | | | | | | | | | env=[x=1, f(x)=if zero?(x) then 0 else -((f-(x,1)), -2), x=10, y=5, i=10, y=10, y=10
318 |||||||||||+ exp=zero?(x)
319 ||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i
     =1]
     ||||||||+ exp=x
     ||||||||||||| env=[x=1, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,
      i=1]
     |||||| val=1
     |||||||||||| val=#f
     | | | | | | | | | | | | | + \exp = -((f - (x, 1)), -2)
325 |||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i
      =11
326 | | | | | | | | | | | | | | + \exp(f - (x, 1))
```

```
hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                                                                                Page 7/32
     |||||+ exp=f
, i=1
330 | | | | | | | | | | | | | | | val=proc(x) if zero?(x) then 0 else -((f - (x, 1)), -2) | f(x) = if zer
     o?(x) then 0 else -((f - (x, 1)), -2), x=10, v=5, i=1)
     | | | | | | | | | | | | | | + \exp(-(x, 1))
x = 1, x
      , i=1
     ||||||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x.1)).-2).x=10.v=
335 |||||||||||||| val=1
     ||||||+ exp=1
     5.i = 11
     |||||| val=1
338
     ||||| val=0
339
     || || || || || || + \exp = if zero?(x) then 0 else - ((f - (x,1)), -2)
340
341 ||||||||||| env=[x=0, f(x)=if zero?(x) then 0 else -((f-(x,1)),-2),x=10,y=5
      i=11
     |||||||||||||||||||||||+ exp=zero?(x)
     |||||||||||||||| env=[x=0, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=
     5, i=11
344 | | | | | | | | | | | | | + exp=x
345 |||||||||||||||| env=[x=0, f(x)=if zero?(x) then 0 else -((f-(x,1)),-2).x=10.v
     |||||| val=0
     ||||| val=#t
347
     348
     5.i = 11
350 ||||||||||||| val=0
351 ||||||||||||| val=0
352 ||||||||||||- val=0
353 | | | | | | | | | | | | | | + \exp = -2
     |||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,
     i=11
     |||||| val=-2
     ||||| val=2
356
     357
     |||||||||| val=2
358
     ||||||||+ exp=-2
     ||||||||| env=[x=2, f(x)=if zero?(x) then 0 else -((f -(x,1)),-2),x=10,v=5,i=1
     |||||||||| val=-2
361
     ||||| val=4
362
363
     ||||||||- val=4
     |||||||- val=4
364
     || || || || + \exp = -2
365
366 | | | | | | | | | | env=[x=3, f(x)=if zero?(x) then 0 else -((f-(x,1)),-2),x=10,y=5,i=1]
368 ||||||- val=6
369 |||||- val=6
370 ||||- val=6
371 | | | | | + exp=-2
||||- val=-2
373
     ||||- val=8
374
     |||- val=8
375
     ||- val=8
376
377
     |- val=8
378
    8
379
     [dshin@acacia letrec] $ letrec -t /home/pl/hw07/tests/letrec-3
380
381 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
```

```
hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                                                                                              Page 8/32
      % letrecs. (value=20)
384
385 let m=-5
386 in letrec f(x)
387
               = if zero?(x)
388
                    then 0
389
                    else -((f -(x,1)),m)
            in (f 4)
300
     |+ exp=let m=-5 in letrec f(x)=if zero?(x) then 0 else -((f-(x,1)),m) in (f 4)
     | | env=[x=10,v=5,i=1]
395 | | + exp = -5
396 | | | env=[x=10, v=5, i=1]
397 ||- val=-5
398 | | + exp=letrec f(x)=if zero?(x) then 0 else -((f-(x,1)),m) in (f 4)
      | | | env=[m=-5, x=10, v=5, i=1]
     | | | + exp = (f 4)
401 | | | e^{-x} = f(x) = if(x) = if(x) = if(x) = if(x) = 0 then 0 = if(x) = (if(x), if(x), if(x
402 ||||+ exp=f
403 | | | | | env=[f(x)=if zero?(x) then 0 else -((f(x,1)),m),m=-5,x=10,v=5,i=1]
     |\cdot|\cdot| val=proc(x) if zero?(x) then 0 else -((f -(x,1)),m)[f(x)=if zero?(x) then 0
         else -((f_{-}(x,1)),m),m=-5,x=10,v=5,i=1)
      || || + \exp = 4
      |\cdot|\cdot|\cdot| env=[f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=1]
      || | | | - va| = 4
      |\cdot|\cdot| + \exp=if zero?(x) then 0 else -((f -(x,1)),m)
      |\cdot|\cdot|\cdot| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=1]
      | | | | | | + exp=zero?(x)
411 | | | | | | env=[x=4, f(x)=if zero?(x) then 0 else -((f-(x,1)), m), m=-5, x=10, v=5, i=1]
413 |||||| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=1]
414 | | | | | - val=4
415 ||||- val=#f
416 | | | | | + \exp = -((f - (x, 1)), m)
417 | | | | | | env=[x=4, f(x)=if zero?(x) then 0 else -((f-(x,1)), f(x)=-1, f(x)=-1
      | | | | | | | + \exp(f - (x, 1))
      |||||| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=1]
420
      ||||||+ exp=f
421 ||||||| env=[x=4, f(x)=if(x)=if(x)=0 then 0 else -((f-(x,1)),m),m=-5,x=10,v=5,i=1
     |\cdot|\cdot|\cdot| val=proc(x) if zero?(x) then 0 else -((f -(x,1)),m)[f(x)=if zero?(x) the
       n 0 else -((f-(x,1)),m),m=-5,x=10,v=5,i=1)
423 | | | | | | | + \exp = -(x, 1)
      ||||||| env=[x=4, f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=1
425
      ||||||| env=[x=4,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=
      11
427 ||||||- val=4
428 ||||||+ exp=1
 429 | | | | | | | | | | env=[x=4, f(x)=if zero?(x) then 0 else -((f-(x,1)), m), m=-5, x=10, v=5, i=
      1]
430 ||||||- val=1
431 |||||- val=3
432 | | | | | | + exp=if zero?(x) then 0 else - ((f - (x, 1)), m)
      ||||||| env=[x=3, f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=1
       | | | | | | | | | + exp=zero?(x)
      |||||||| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i=
435
       11
437 |||||||| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i
       =11
438 |||||||- val=3
439 | | | | | | | | - val=#f
```

11ì	M_^[M_^T 10, 16 10:41 <b>h</b>	w07-LETREC-output-trace	Page 9/32
440 441		$f(x) = \frac{1}{1} \int_{0}^{x} dx dx$ for $f(x) = \frac{1}{1} \int_{0}^{x} dx$	,x=10,v=5,i=
442 443	1]          exp=(f -(x,1))          env=[x=3,f(x)=if ze	ero?(x) then 0 else $-((f -(x,1)),m),m=-$	5,x=10,v=5,i
444	=1]         exp=f	zero?(x) then 0 else $-((f -(x,1)),m),m=$	
445 446	i=1]        val=proc(x)if zero	o?(x) then 0 else $-((f -(x,1)),m)[f(x)=$	
447 448	then 0 else $-((f - (x, 1)), m), m = $ $           + \exp(-(x, 1))$ $            + \exp(-(x, 1))$	x=-5, x=10, v=5, i=1 x=-5, x=10, v=5, i=1 x=-5, x=10, v=5, i=1 x=-5, x=10, v=5, i=1 x=-5, x=10, v=5, i=1	-5.x=10.v=5.
449	i=1]        + exp=x		
450 451	,i=1]         val=3	zero?(x) then 0 else $-((f -(x,1)),m),m$	J, X-10, V-J
452 453	+ exp=1            env=[x=3,f(x)=if ,i=1]	zero?(x) then 0 else $-((f -(x,1)),m),m$	=-5, x=10, v=5
454 455 456	/          - val=1          - val=2          + exp=if zero?(x) th	nen ( else - (/f - (v 1)) m)	
457		zero?(x) then 0 else $-((f -(x,1)),m),m=$	-5, x=10, v=5,
458 459	+ exp=zero?(x)            env=[x=2,f(x)=if ,i=1]	zero?(x) then 0 else $-((f -(x,1)),m),m$	=-5, x=10, v=5
460 461		f zero?(x) then 0 else $-((f -(x,1)),m)$ ,	m=-5, x=10, v=
462 463 464		ml	
465	$  \cdot  \cdot  \cdot  \cdot $ env=[x=2, f(x)=if, i=1]	zero?(x) then 0 else $-((f -(x,1)),m),m$	=-5, x=10, v=5
466 467	exp=(f -(x,1))             env=[x=2,f(x)=if 5,i=1]	f zero?(x) then 0 else $-((f -(x,1)),m)$ ,	m=-5, x=10, v=
468 469		if zero?(x) then 0 else $-((f -(x,1)),m)$	,m=-5,x=10,v
470 471		zero?(x) then 0 else $-((f -(x,1)),m)[f(x,m-5,x=10,v=5,i=1)]$	x)=if zero?(
472		if zero?(x) then 0 else $-((f -(x,1)),m)$	, m=-5, x=10, v
473 474	v=5, i=1]	=if zero?(x) then 0 else -((f -(x,1)),m	), m=-5, x=10,
475 476 477	- val=2              + exp=1                 env=[x=2,f(x)=	=if zero?(x) then 0 else $-((f -(x,1)),m$	),m=-5,x=10.
478	v=5,i=1]                val=1	, ,	
479 480 481		then 0 else $-((f -(x,1)),m)$ if zero?(x) then 0 else $-((f -(x,1)),m)$	,m=-5,x=10,v
482 483	=5,i=1]           + exp=zero?(x)              env=[x=1,f(x)=	=if zero?(x) then 0 else $-((f -(x,1)),m$	),m=-5,x=10,
484 485	v=5, i=1]                exp=x 	=if zero?(x) then 0 else $-((f -(x,1)),$	m).m=-5.x=10
403	, v=5, i=1]	11 2010. (A, CHON 0 0130 ((1 (A, 1)))	, j.ii. 3, A-10

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hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                                                                                     Page 10/32
 486 ||||||||||||||- val=1
487 |||||||||||| val=#f
488 | | | | | | | | | | | + \exp=-((f - (x, 1)), m)
489 ||||||||| env=[x=1, f(x) = if zero?(x) then 0 else -((f-(x,1)), m), m=-5, x=10,
      v=5, i=11
490 | | | | | | | | | | | | + \exp(f - (x, 1))
491 | | | | | | | | | | | | | | env=[x=1, f(x) = if zero?(x) then 0 else -((f(x,1)), m), m=-5, x=10
      , v=5, i=1
492 | | | | | | | | | | | + exp=f
493 ||||||||||||||| env=[x=1, f(x)=if zero?(x) then 0 else -((f-(x,1)),m),m=-5,x=1
      0, v=5, i=11
494 |||||||||||| val=proc(x) if zero?(x) then 0 else -((f - (x, 1)), m) [f(x) = if zer
      o?(x) then 0 else -((f -(x,1)), m), m=-5, x=10, v=5, i=1]
495 | | | | | | | | | | | | | | | + \exp = -(x, 1)
496 |||||||||||| env=[x=1, f(x) = if zero?(x) then 0 else -((f(x,1)), m), m=-5, x=1
      0.v=5.i=11
497 ||||||||||||+ exp=x
     ||||||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=
      10, v=5, i=1]
499 |||||||||||||| val=1
500 ||||||||||||||+ exp=1
501 |||||||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=
      10, v=5, i=11
502 ||||||||||||||||- val=1
503 ||||||||||||||- val=0
504 | | | | | | | | | | | | | + exp=if zero?(x) then 0 else -((f -(x,1)),m)
      ||||||||||||||| env=[x=0, f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5, x=1
      0, v=5, i=1
506 |||||||||||||+ exp=zero?(x)
507 ||||||||||||| env=[x=0, f(x)=if zero?(x) then 0 else -((f-(x,1)), m), m=-5, x=0
      10.v=5.i=11
508 |||||||||||||||+ exp=x
=10, v=5, i=1
510 ||||||||||||||- val=0
511 |||||||||||| val=#t
512 | | | | | | | | | | | | + exp=0
      ||||||||||||||||| env=[x=0,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=
      10, v=5, i=1
514 |||||||||||||- val=0
515 |||||||||||| val=0
516 |||||||||||| val=0
517 |||||||||||+ exp=m
518 ||||||||||||||| env=[x=1,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10
      , v=5, i=1]
519 |||||||||| val=-5
520 ||||||||||||- val=5
521 ||||||||||- val=5
522 ||||||||||- val=5
523 ||||||||+ exp=m
5.i = 11
525 |||||||||- val=-5
526 |||||||||- val=10
527 ||||||||- val=10
528 | | | | | | | | - val=10
529 |||||||+ exp=m
530 ||||||||| env=[x=3,f(x)=if zero?(x) then 0 else -((f -(x,1)),m),m=-5,x=10,v=5,i
     =11
531 |||||||- val=-5
532 ||||||- val=15
533 |||||- val=15
534 ||||| val=15
535 | | | | | + exp=m
\frac{1}{1} sign in \frac{1}{1} enverges \frac{1}{1} env
537 |||||- val=-5
```

```
hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                                           Page 11/32
   ||||| val=20
539 | | | | - val=20
540 | | | | - val = 2.0
541 ||- val=20
542
   1 - val = 2.0
543 2.0
544
545
   [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/letrec-double
546 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
   % recursive double. (value=12)
550
   letrec double(x)
     = if zero?(x)
551
552
       then 0
553
       else -((double -(x,1)), -(0, 2))
554 in (double 6)
555
557 | + exp=letrec double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)) in (do
   | | env=[x=10, v=5, i=1]
   | | + \exp = (double 6)
   |\cdot| env=[double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10, v=5, i=
   11
   | \cdot | \cdot | \cdot | env=[double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10,v=5,i
   =11
563 | | | - val=proc(x) if zero?(x) then 0 else - ((double - (x, 1)), -(0, 2)) [double(x) = if z
   ero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10, v=5, i=1]
565 | | | | env=[double(x)=if zero?(x) then 0 else - ((double - (x, 1)), -(0, 2)), x=10, v=5, i
   =1]
566 |||- val=6
567 | | | + exp=if zero?(x) then 0 else - ((double - (x,1)), -(0,2))
568 | | | | env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)), x=10, y
   =5.i=11
   ||||+ exp=zero?(x)
570 ||||| env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10,
   v=5, i=1
571 | | | | + exp=x
, v=5, i=11
573 ||||- val=6
574 ||||- val=#f
575 | | | | + exp=-((double -(x,1)), -(0,2))
576 | | | | | | env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10,
   v=5, i=1
577 | | | | | + \exp(\text{double} - (x, 1))
578 |||||| env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10
   v=5, i=11
579 | | | | | | + exp=double
580 | | | | | | | env= [x=6, double(x) = if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=1
   0, v=5, i=11
581 |\cdot|\cdot|\cdot| val=proc(x)if zero?(x) then 0 else -((double -(x,1)),-(0,2))[double(x)=i
   f zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10, v=5, i=1]
582 | | | | | | + \exp = -(x, 1)
583 |||||| env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=1
   0, v=5, i=1
584 | | | | | | + exp=x
585 | | | | | | | | | env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)), x=
   10, v=5, i=1
   |||||| val=6
587 | | | | | | | + exp=1
588 | | | | | | | | env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)), x=
   10, v=5, i=1
```

```
hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                              Page 12/32
   ||||||||| val=1
590 ||||| val=5
591 | | | | | | + exp=if zero?(x) then 0 else - ((double - (x, 1)), -(0, 2))
592 |||||| env=[x=5,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=1
   0.v=5.i=11
593 | | | | | | + exp=zero?(x)
   |\cdot|\cdot|\cdot|\cdot| env=[x=5,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=
   10, v=5, i=1
595 | | | | | | | + exp=x
=10, v=5, i=11
597 ||||||- val=5
598 |||||- val=#f
599 | | | | | | | + \exp = -((double - (x, 1)), -(0, 2))
600 | | | | | | | | | env=[x=5, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=
   10.v=5.i=11
   || || || || + \exp(\text{double} - (x, 1))
\frac{1}{2} 602 | | | | | | | | | | env=[x=5, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x
   =10, v=5, i=1
603 |||||||+ exp=double
\frac{1}{1} 604 | | | | | | | | | env = [x=5, double(x) = if zero?(x) then 0 else - ((double - (x,1)), - (0,2)),
   x=10, v=5, i=11
   |\cdot|\cdot|\cdot|\cdot| val=proc(x) if zero?(x) then 0 else -((double -(x,1)),-(0,2))[double(x
   )=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10,v=5,i=1]
606 | | | | | | | | | + \exp = -(x, 1)
   ||||||||| env=[x=5,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),
   x=10, v=5, i=1
   |||||||+ exp=x
609 ||||||||| env=[x=5, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2))
   x=10.v=5.i=11
610 |||||||| val=5
611 |||||||+ exp=1
612 ||||||||| env=[x=5,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2))
   , x=10, v=5, i=1]
613 ||||||||- val=1
614 | | | | | | | | - val=4
615 ||||||||+ \exp=if zero?(x) then 0 else -((double -(x,1)), -(0,2))
616 |||||||| env=[x=4,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),
   x=10, v=5, i=1
617 | | | | | | | + exp=zero?(x)
618 ||||||||| env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2))
   ,x=10,v=5,i=1
619 | | | | | | | | | + exp=x
620 ||||||||||| env=[x=4,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)
   ), x=10, v=5, i=1]
621 ||||||||| val=4
622 ||||||||- val=#f
623 | | | | | | | | | + \exp = ((double - (x, 1)), -(0, 2))
624 | | | | | | | | | | | env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2))
   , x=10, v=5, i=1]
625 | | | | | | | | | | | + \exp(\text{double} - (x, 1))
626 |||||||||| env=[x=4,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)
   ), x=10, v=5, i=1]
627 ||||||||+ exp=double
628 |||||||||||| env=[x=4,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)
   )), x=10, v=5, i=1
629 ||||||||| = val = proc(x) if zero?(x) then 0 else -((double - (x, 1)), -(0, 2)) [double -(x, 1)), -(0, 2)
   e(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10, v=5, i=1]
   || || || || || || + \exp - (x, 1)
631 |||||||||| env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)
   )), x=10, v=5, i=1]
632 | | | | | | | | | | | + exp=x
633 ||||||||||| env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,
   2)), x=10, v=5, i=1]
634 ||||||||||- val=4
635 ||||||||||||+ exp=1
```

11ì	M-^[M-^T 10, 16 10:41 <b>hw07-LETREC-output-trace</b> Page 13/32
636	$  \cdot  \cdot  \cdot  \cdot $ env=[x=4,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,1))
	2)), x=10, v=5, i=1]
637 638	- val=1          - val=3
639	$ \cdot \cdot \cdot \cdot \cdot $ + exp=if zero?(x) then 0 else -((double -(x,1)),-(0,2))
640	env=[x=3, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2))
641	)), x=10, v=5, i=1]         + exp=zero?(x)
642	exp=2e10.(x) $              env=[x=3,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,x))$
	2)), x=10, v=5, i=1]
643 644	$          + \exp x$ $              = \exp x$ $               = \exp x$ $                = \exp x$
07-7	,2)),x=10,v=5,i=1]
645	- val=3
646 647	- val=#f          + exp=-((double -(x,1)),-(0,2))
648	(x,1), $(x,2)             env=[x=3,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,$
	2)), x=10, v=5, i=1]
649 650	$          + \exp(double - (x, 1))$              env=[x=3, double(x)=if zero?(x) then 0 else -((double - (x, 1)), -(0))
000	(double -(x,1)),-(x,2)),x=10,v=5,i=1]
651	+ exp=double
652	env=[x=3,double(x)=if zero?(x) then 0 else -((double -(x,1)), -0,2)),x=10,v=5,i=1]
653	-val=proc(x) if zero?(x) then 0 else $-((double -(x,1)), -(0,2))$ [double -(x,1)), $-(0,2)$
	uble(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10,v=5,i=1]
654 655	$            + \exp(x,1)$ $               = \exp(x,1)$ $               = \exp(x,1)$ ,
000	0,2)),x=10,v=5,i=1]
656	+ exp=x
657	env=[x=3,double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)),x=10,v=5,i=1]
658	
659	+ exp=1
660	env=[x=3,double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)),x=10,v=5,i=1]
661	
662	- val=2
663 664	$            + \exp(x)$ then 0 else -((double -(x,1)),-(0,2)) $               = \exp(x)$ , then 0 else -((double -(x,1)),-
004	0,2)),x=10,v=5,i=1]
665	
666	env=[x=2,double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10, v=5, i=1]
667	
668	env=[x=2, double(x)=if zero?(x) then 0 else -((double -(x,1)),
669	-(0,2)),x=10,v=5,i=1]               - val=2
670	
671	$            + \exp(-((\text{double } -(x,1)), -(0,2))$
672	env=[x=2,double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)), x=10, v=5, i=1]
673	$             + \exp(double - (x, 1))$
674	env=[x=2, double(x)=if zero?(x) then 0 else -((double -(x,1)),
675	-(0,2)),x=10,v=5,i=1]              + exp=double
	,-(0,2)), x=10, v=5, i=1]
677	val=proc(x) if zero?(x) then 0 else -((double -(x,1)),-(0,2)) [double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10,v=5,i=1]
678	exp=-(x,1)
679	$\label{eq:condition}               env=[x=2,double(x)=if zero?(x) then 0 else -((double -(x,1)))                                   $
600	,-(0,2)),x=10,v=5,i=1]
680 681	$               + \exp x$ $                 + \exp x = x = 2, double(x) = if zero?(x) then 0 else -((double -(x, 1)))$
	),-(0,2)),x=10,v=5,i=1]

11ìI	M_^[M_^T 10, 16 10:41	hw07-LETREC-output-trace	Page 14/32
682	val=2		
683 684		=2, $double(x)=if zero?(x) then 0 else$	-((double -(x.1)
001	), -(0,2)), x=10, v=5, i=1]	2, adds (m, 11 2010; (m, end) 0 0100	((000020 (11)1)
685 686	val=1       val=1		
687		zero?(x) then 0 else -((double -(x,1))	),-(0,2))
688		, double(x)=if zero?(x) then 0 else -	((double -(x,1))
689	+ exp=zer		
690		=1, $double(x) = if zero?(x) then 0 else$	-((double -(x,1)
691			
692		x=1, double(x)=if zero?(x) then 0 else	-((double -(x,1
693			
694	val=#f        exp=-((	(double - (x 1)) - (0 2)	
695 696		=1, double(x)=if zero?(x) then 0 else	-((double -(x,1)
697	), $-(0,2)$ ), $x=10$ , $v=5$ , $i=1$ ]	double -(v 1))	
698		x=1, double(x)=if zero?(x) then 0 else	-((double -(x,1
699	)),-(0,2)),x=10,v=5,i=1]	double	
700		[x=1,double(x)=if zero?(x) then 0 else	e -((double -(x,
701	1)),-(0,2)),x=10,v=5,i=1]	proc(x)if zero?(x) then 0 else -((doub	$nle = (x \ 1) ) = (0$
	2))[double(x)=if zero?(x) th	nen 0 else -((double - $(x,1)$ ),- $(0,2)$ ),:	
702 703	+ exp=-	-(x,1) [x=1,double(x)=if zero?(x) then 0 else	e - ((double - (x.
	1)), $-(0,2)$ ), $x=10$ , $v=5$ , $i=1$ ]		(((((((((((((((((((((((((((((((((((((((
704 705	+ exp=	=x =[x=1,double(x)=if zero?(x) then 0 el:	se -((double -(x
	(1)), -(0,2)), x=10, v=5, i=1]		,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
706 707	val=        + exp=		
708	env=	=[ $x=1$ , double( $x$ )=if zero?( $x$ ) then 0 els	se -((double -(x
709	,1)),-(0,2)),x=10,v=5,i=1]          val=	=1	
710	val=0		1)) (0 2))
711 712		If zero?(x) then 0 else -((double -(x) $(x=0, double(x)=if zero?(x))$ then 0 else	
	1)),-(0,2)),x=10,v=5,i=1]		
713 714	+ exp=         env=	=[x=0,double(x)=if zero?(x) then 0 el:	se -((double -(x
	,1)),-(0,2)), $x=10,v=5,i=1$ ]		
715 716		e=x =[x=0,double(x)=if zero?(x) then 0 e:	lse -((double -(
	x,1)),-(0,2)),x=10,v=5,i=1]		`\
717 718	val        val=		
719			aa ((daybla (y
720	(1,1), $(0,2)$ , $(1,1)$ , $($	=[ $x=0$ , double( $x$ )=if zero?( $x$ ) then 0 el:	se - ((double - (x
721	val=		
722 723	val=0		
724 725	+ exp=-(	(0,2) $(x=1,double(x)=if zero?(x) then 0 else$	-((double -(x 1
125	)), $-(0,2)$ ), $x=10$ , $v=5$ , $i=1$ ]		(/doddie (x,1
726 727		) [x=1,double(x)=if zero?(x) then 0 else	e - ((double - (x.
	1)),-(0,2)),x=10,v=5,i=1]		(A)
728 729	val=0		
730		x=1, double(x)=if zero?(x) then 0 else	e -((double -(x,

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11ìM-^[M-^T 10, 16 10:41
                               hw07-LETREC-output-trace
                                                                         Page 15/32
    1)), -(0,2)), x=10, v=5, i=1]
732 |||||||||||||||| val=-2
733 |||||||||||||||| val=2
734 |||||||||||||||| val=2
735
   |||||| val=2
   | | | | | | | | | | | | | | | + \exp(-(0, 2))
737 ||||||||||||||| env=[x=2,double(x)=if zero?(x) then 0 else -((double -(x,1)),
    -(0,2)), x=10, v=5, i=1]
738 | | | | | | | | | | | | | | + exp=0
739 | | | | | | | | | | | | | | | | env=[x=2, double(x)=if zero?(x) then 0 else -((double -(x,1))
    (0,2), x=10, v=5, i=1]
740 |||||||||||||||- val=0
741 | | | | | | | | | | | | | + exp=2
742 ||||||||||||||| env=[x=2, double(x)=if zero?(x) then 0 else -((double -(x,1))
    (0,2), x=10, v=5, i=1]
   |||||| val=2
744 ||||||||||||||- val=-2
   ||||| val=4
745
746 |||||||||||| val=4
747 ||||||||||||- val=4
748 | | | | | | | | | | | | | | + \exp = -(0, 2)
749 | | | | | | | | | | | | | env=[x=3, double(x)=if zero?(x) then 0 else -((double -(x, 1)), -(0
    (2)), x=10, v=5, i=1
   ||||||+ exp=0
   0,2)), x=10, v=5, i=1]
752 | | | | | | | | | | | | - val=0
   753
754 | | | | | | | | | | | | env=[x=3, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(
   0.2), x=10, v=5, i=11
755 | | | | | | | | | | | | | | | val=2
756 |||||||||||- val=-2
757 |||||||||||- val=6
758 |||||||||| val=6
759
   |||||||||| val=6
   | | | | | | | | | | | + \exp = -(0, 2)
   ||||||||| env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)
   ), x=10, v=5, i=1]
   ||||||||||+ exp=0
763 | | | | | | | | | | | | env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)
   )), x=10, v=5, i=11
764 |||||||||| val=0
765 ||||||||||+ exp=2
766 | | | | | | | | | | | | env=[x=4, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)
   )), x=10, v=5, i=1]
   ||||||||||- val=2
   ||||||||| val=-2
768
   |||||||||- val=8
769
770 ||||||||- val=8
771 ||||||- val=8
773 ||||||||| env=[x=5, double(x)=if zero?(x) then 0 else -((double -(x,1)), -(0,2)),x
   =10, v=5, i=11
774 | | | | | | | | + exp=0
775 | | | | | | | | | | env = [x=5, double(x) = if zero?(x) then 0 else - ((double - (x,1)), -(0,2)),
   x=10, v=5, i=11
   ||||||||- val=0
777 |||||||+ exp=2
778 |||||||| env=[x=5,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),
   x=10, v=5, i=11
   ||||||||- val=2
   ||||||||| val=-2
781 ||||||- val=10
782 | | | | | | - val=10
783 | | | | | - val=10
```

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hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                            Page 16/32
   |||||+ \exp(0.2)
785 | | | | | | | env=[x=6,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=10
   , v=5, i=1
786 | | | | | | + exp=0
   | | | | | | | | env=[x=6, double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=1
   0, v=5, i=11
   |||||- val=0
   |||||+ exp=2
   |\cdot|\cdot|\cdot| env=[x=6,double(x)=if zero?(x) then 0 else -((double -(x,1)),-(0,2)),x=1
   0, v=5, i=1
791 |||||- val=2
792 | | | | | - val=-2
793 | | | | - val=12
794 |||- val=12
795 ||- val=12
796
   I- val=12
   12
797
798
   [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/letrec-sumto
800 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
   % recursive sumto. (value=55)
804 letrec sumto(n)
     = if zero?(n)
805
806
        then 0
        else -((sumto -(n,1)), -(0,n))
   in (sumto 10)
811 \mid + exp=letrec sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)) in (sumt
   0 10)
812 | | env=[x=10, v=5, i=1]
813 | | + exp=(sumto 10)
814 | | | env=[sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=5, i=1]
815 |||+ exp=sumto
816 | | | | env=[sumto(n)=if zero?(n) then 0 else - ((sumto-(n,1)), -(0,n)), x=10, v=5, i=1)
   |\cdot| val=proc(n)if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))[sumto(n)=if zer
   o?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=5, i=1]
819 | | | | env=[sumto(n)=if zero?(n) then 0 else - ((sumto-(n,1)), -(0,n)), x=10, v=5, i=1)
   |||- val=10
821 | | | + exp=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
822 | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=
   ||||+ exp=zero?(n)
824 | | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v
   =5, i=1]
825 |||||+ exp=n
826 |||||| env=[n=10,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=10,
   v=5, i=1
827 | | | | | - val=10
828 ||||- val=#f
829 | | | | + \exp = ((sumto - (n, 1)), -(0, n))
830 | | | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v
   =5, i=1
   | | | | | + exp = (sumto - (n, 1))
832 |||||| env=[n=10,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=10,
   v=5, i=11
833 |||||+ exp=sumto
834 |||||| env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10
   , v=5, i=11
835 |\cdot|\cdot|\cdot|\cdot| val=proc(n) if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))[sumto(n)=if
    zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=5, i=1]
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                                               Page 17/32
   | | | | | | + \exp = -(n, 1)
837 | | | | | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10
    , v=5, i=11
838 | | | | | | | + exp=n
   ||||||| env=[n=10,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=1
839
    0, v=5, i=1
   |||||| val=10
841 |||||+ exp=1
842 | | | | | | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=1
    0, v=5, i=1
843 |||||- val=1
844 |||||- val=9
845 |||||+ \exp = if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))
846 |||||| env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10,
   v = 5 \cdot i = 11
   |||||||+ exp=zero?(n)
   ||||||| env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10
    , v=5, i=1]
    |||||||+ exp=n
   |\cdot|\cdot|\cdot|\cdot| env=[n=9,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=1
850
    0, v=5, i=1
851 ||||||- val=9
852 |||||- val=#f
853 || || || || + \exp = -((sumto - (n, 1)), -(0, n))
854 | | | | | | | | | env=[n=9, \text{sumto}(n) = \text{if zero}(n) \text{ then } 0 \text{ else } -((\text{sumto} -(n,1)), -(0,n)), x=10
    v=5, i=1
    || || || || + \exp = (sumto - (n, 1))
   |||||||| env=[n=9,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=1
856
    0.v=5.i=11
857 | | | | | | | + exp=sumto
858 | | | | | | | | | | env=[n=9.sumto(n)=if zero?(n) then 0 else - ((sumto - (n,1)), - (0,n)), x=
   |\cdot|\cdot|\cdot|\cdot| val=proc(n) if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))[sumto(n)=
    if zero?(n) then 0 else - ((sumto - (n,1)), - (0,n)), x=10, v=5, i=1]
   | | | | | | | | | | + \exp(-(n, 1))
   |||||||| env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=
    10, v=5, i=1
   ||||||||+ exp=n
863 ||||||||||| env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x
    =10, v=5, i=1
864 ||||||||- val=9
865 ||||||||+ exp=1
866 | | | | | | | | | | | env= [n=9, sumto(n) = if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x
    =10, v=5, i=11
867 ||||||||- val=1
868
   || || || || + \exp = if zero?(n)  then 0 else -((sumto -(n,1)), -(0,n))
   |||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=
    10, v=5, i=1
871 |||||||+ exp=zero?(n)
872 | | | | | | | | | | | env=[n=8.sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x
    =10.v=5.i=11
873 | | | | | | | | | + exp=n
874 |||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),
   x=10, v=5, i=1
875 ||||||||- val=8
876 ||||||||- val=#f
877 || | | | | | | + \exp -((sumto -(n,1)), -(0,n))
878 |||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x
    =10, v=5, i=1
879 ||||||||+ exp=(sumto -(n,1))
880 |||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)),
    x=10.v=5.i=11
881 ||||||||+ exp=sumto
882 |||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
    , x=10, v=5, i=1]
```

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hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                     Page 18/32
   ||||||||| val=proc(n) if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))[sumto(
   n) = if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=5, i=11
884 | | | | | | | | | | | + \exp = -(n, 1)
885 |||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
   ,x=10,v=5,i=11
   ||||||||||+ exp=n
   |||||||||||| env=[n=8,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)
   ), x=10, v=5, i=1]
888 ||||||||||| val=8
889 ||||||||||+ exp=1
890 |||||||||||| env=[n=8,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)
   ), x=10, v=5, i=1]
891 |||||||||||- val=1
892 ||||||||||- val=7
893 |||||||||+ \exp = if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
   , x=10, v=5, i=1]
   ||||||||||+ exp=zero?(n)
896 | | | | | | | | | | | | env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
   ), x=10, v=5, i=1]
897 |||||||||||+ exp=n
  ||||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))
   )), x=10, v=5, i=1]
899 ||||||||||||- val=7
900 ||||||||||- val=#f
901 | | | | | | | | | | | | + \exp -((sumto -(n,1)), -(0,n))
   ||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)
   ), x=10, v=5, i=1]
903 | | | | | | | | | | + exp=(sumto -(n,1))
  |\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|\cdot| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
   )), x=10, v=5, i=1]
905 |||||||||||+ exp=sumto
906 ||||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,
   n)), x=10, v=5, i=1]
  || || || || || || || - val = proc(n) if zero?(n) then 0 else - ((sumto -(n,1)), -(0,n)) [sum
   to(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=10,v=5,i=11
   | | | | | | | | | | | | | | + \exp(-(n, 1))
   |||||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,
   n)), x=10, v=5, i=1]
   ||||||+ exp=n
(n)), x=10, v=5, i=1
912 |||||||||||||- val=7
913 |||||||||||||+ exp=1
914 |||||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0))]
   (n)), x=10, v=5, i=1]
   |||||| val=1
   |||||| val=6
917 | | | | | | | | | | | | + exp=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
n)), x=10, v=5, i=1]
919 ||||||||||+ exp=zero?(n)
920 ||||||||||||||| env=[n=6, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0)
   (n)), x=10, v=5, i=1]
921 ||||||||||||+ exp=n
922 |||||||||||||||| env=[n=6, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(
   0, n)), x=10, v=5, i=1]
   |||||| val=6
  |||||| val=#f
  | | | | | | | | | | | | | | + \exp -((sumto -(n,1)), -(0,n))
926 ||||||||||||||| env=[n=6, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0))]
   (n)), x=10, v=5, i=1]
  | | | | | | | | | | | | | | | | + \exp = (sumto - (n, 1))
  ||||||||||||||| env=[n=6,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(
   (0,n), x=10, v=5, i=1
929 ||||||||||||+ exp=sumto
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                         Page 19/32
  ||||||||||||||||| env=[n=6, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),
  (0,n), x=10, v=5, i=11
931 ||||||||||||| val=proc(n)if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))[
  sumto(n) = if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=5, i=1]
  | | | | | | | | | | | | | | | | + \exp(-(n, 1))
  (0,n)), x=10, v=5, i=1
  ||||||+ exp=n
  -(0,n), x=10, v=5, i=1
936 |||||||||||||| val=6
 ||||||+ exp=1
  -(0,n), x=10, v=5, i=1]
 |||||| val=1
  |||||| val=5
  ||||||||||||||+ \exp if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))
  ||||||||||||||||| env=[n=5, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-
  (0,n), x=10, v=5, i=1]
  -(0,n), x=10, v=5, i=11
 ||||||+ exp=n
 (0,n), x=10, v=5, i=1
  |||||| val=5
  || || || || || || || + \exp(-((sumto - (n, 1)), -(0, n)))
  -(0,n)), x=10, v=5, i=1]
 || || || || || || || || + \exp(\operatorname{sumto} - (n, 1))
952 ||||||||||||||||| env=[n=5, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1))
  (0,n), x=10, v=5, i=1]
 ||||||||||||||||||||| env=[n=5,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)))
  ), -(0,n)), x=10, v=5, i=1]
  )) [sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10, v=5, i=1]
   || || || || || || || || + \exp(-(n, 1))
  ), -(0,n)), x=10, v=5, i=1
  ||||||+ exp=n
  )),-(0,n)),x=10,v=5,i=1]
  |||||| val=5
  ||||||+ exp=1
  )), -(0,n)), x=10, v=5, i=1]
  |||||| val=4
  |\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|\cdot|+ \exp=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
  ), -(0, n)), x=10, v=5, i=1]
 ||||||| exp=zero?(n)
  )), -(0,n)), x=10, v=5, i=1]
  ||||||+ exp=n
  1)), -(0, n), x=10, v=5, i=1]
  |||||| val=#f
  || || || || || || || || || + \exp - ((sumto - (n, 1)), - (0, n))
  |||||||||||||||||||| env=[n=4, sumto(n)=if zero?(n) then 0 else -((sumto -(n, 1)))
  )), -(0,n)), x=10, v=5, i=1]
```

11ì	M-^[M-^T 10, 16 10:41	hw07-LETREC-output-trace	Page 20/32
	1)),-(0,n)),x=10,v=5,i=1]	•	
977			
978	(1), -(0,n), x=10, v=5, i=1	env= $[n=4$ , sumto $(n)=if$ zero? $(n)$ then 0 else	-((sumto -(n
979		val=proc(n)if zero?(n) then 0 else -((sumt) then 0 else -((sumto -(n,1)),-(0,n)), $x=10$	
980 981		exp=-(n,1) env=[n=4,sumto(n)=if zero?(n) then 0 else	- ( (sumto - (n
	,1)),-(0,n)), $x=10,v=5,i=1$	]	((Sameo (ii
982 983	n,1)),-(0,n)),x=10,v=5,i=	env=[n=4, sumto(n)=if zero?(n) then 0 else	e -((sumto -(
984		- val=4	
985			
986	n,1)),-(0,n)),x=10,v=5,i=		e -((sumto -(
987			
988			1)) (0))
989		exp=if zero?(n) then 0 else -((sumto -(n, $\frac{1}{2}$ )) then 0 else	
990	(0,n), x=10, v=5, i=1	env= $[n=3, sumto(n)=if zero?(n) then 0 else$	-((sumto -(n
991			
992		env=[n=3, sumto(n)=if zero?(n) then 0 else	e -((sumto -(
	n,1)),-(0,n)),x=10,v=5,i=		
993		+ exp=n	
994		$\mid \mid$ env=[n=3, sumto(n)=if zero?(n) then 0 els	se -((sumto -
	(n,1), $-(0,n)$ , $x=10$ , $v=5$ , i		
995 996			
996		- vai=#1 + exp=-((sumto -(n,1)),-(0,n))	
998		env=[n=3, sumto(n)=if zero?(n) then 0 else	e -((sumto -(
	(n,1), -(0,n), x=10, v=5, i=		
999 1000	(n,1)),-(0,n)),x=10,v=5,i	env=[n=3, sumto(n)=if zero?(n) then 0 else	se -((sumto -
1001			
1002	-(n,1)),-(0,n)),x=10,v=5,		
1003	,-(0,n)) [sumto(n)=if zero	- val = proc(n) if zero?(n) then 0 else -((:?(n) then 0 else -((sumto -(n,1)),-(0,n)),:	
1004 1005		$    + \exp(-(n, 1))  $       = nv = [n = 3, sumto(n) = if zero?(n) then 0 expressions	lse - ((sumto
	-(n,1)),-(0,n)),x=10,v=5,	i=1]	( ( ) dame 0
1006 1007		+ exp=n $     env=[n=3, sumto(n)=if zero?(n) then 0 \in$	else -((sumto
	-(n,1)),-(0,n)),x=10,v=5	, i=1]	
1008			
1009			alaa -//a
1010	-(n,1)),-(0,n)),x=10,v=5	$ \cdot \cdot $ env=[n=3,sumto(n)=if zero?(n) then 0 $\epsilon$	eise - ((Sumco
1011			
1012			
1013		+ exp=if zero?(n) then 0 else -((sumto -	(n,1)),-(0,n)
1014	-(n,1)),-(0,n)),x=10,v=5,	$ \cdot $ env=[n=2,sumto(n)=if zero?(n) then 0 exist $ \cdot $	lse -((sumto
1015			
1016		$ \cdot \cdot $ env=[n=2, sumto(n)=if zero?(n) then 0 e	else -((sumto
	-(n,1)),-(0,n)),x=10,v=5	, i=1]	
1017			-1 // :
1018	(0, 1) = (	$ \cdot \cdot \cdot $ env=[n=2, sumto(n)=if zero?(n) then 0	eise - ((sumt
1019	0 - (n, 1)), - (0, n)), x=10, v=		
1019			
1021		+ exp=-((sumto -(n,1)),-(0,n))	
1022		$ \cdot \cdot $ env=[n=2, sumto(n)=if zero?(n) then 0	else -((sumto

11ì	M_^[M_^T 10, 16 10:41	hw07-LETREC-output-trace	Page 21/32
	-(n,1)),-(0,n)),x=10,v=5	-	-
1023 1024		+ exp=(sumto -(n,1))        env=[n=2,sumto(n)=if zero?(n) then	0 else -((sumt
1025 1026		+ exp=sumto         env=[n=2,sumto(n)=if zero?(n) then	0 else -((sum
1027		val=proc(n)if zero?(n) then 0 else zero?(n) then 0 else -((sumto -(n,1)),-(0,	
1028 1029		$ \cdot \cdot \cdot \cdot $ env=[n=2,sumto(n)=if zero?(n) then	0 else -((sum
1030 1031		+ exp=n         env=[n=2,sumto(n)=if zero?(n) the	en O else -((su
1032 1033 1034		- val=2	en 0 else -((su
1035 1036	mto -(n,1)),-(0,n)),x=10,	v=5,i=1]       - val=1	
1037		$ \cdot \cdot \cdot $ + exp=if zero?(n) then 0 else -((sum	nto -(n,1)),-(0
1038	to -(n,1)),-(0,n)),x=10,v		0 else -((sum
1039 1040	mto -(n,1)),-(0,n)),x=10,	env=[n=1, sumto(n)=if zero?(n) the v=5, i=1]	en O else -((su
1041 1042	umto $-(n,1)),-(0,n)),x=10$	env=[n=1,sumto(n)=if zero?(n) th  ,v=5,i=1]	en 0 else -((s
1043 1044			
1045 1046		+ exp=-((sumto -(n,1)),-(0,n))          env=[n=1,sumto(n)=if zero?(n) the   v=5,i=1	en O else -((su
1047 1048		$ \begin{array}{lll}        + \exp(sumto -(n,1)) \\          & env=[n=1,sumto(n)=if zero?(n) th \end{array} $	en 0 else -((s
1049 1050		+ exp=sumto          env=[n=1,sumto(n)=if zero?(n) t	hen 0 else -((
1051		val=proc(n)if zero?(n) then 0 eig zero?(n) then 0 else -((sumto -(n,1)),-	
1052 1053		$ \cdot \cdot \cdot \cdot \cdot $ env=[n=1,sumto(n)=if zero?(n) t	hen 0 else -((
1054 1055		+ exp=n            env=[n=1,sumto(n)=if zero?(n)	then 0 else -(
1056 1057		- val=1          + exp=1	
1058	(sumto -(n,1)),-(0,n)),x=	env=[n=1, sumto(n)=if zero?(n)	then 0 else -(
1059		- val=1	
1060 1061		val=0           + exp=if zero?(n) then 0 else -((	sumto -(n,1)),
1062	sumto $-(n,1)),-(0,n)), x=1$		hen 0 else -((
1063 1064	(sumto -(n,1)),-(0,n)),x=	env=[n=0, sumto(n)=if zero?(n)	then 0 else -(
1065	4 AZNA AT`NA ANA ANA ANA		

11ì	M_^[M_^T 10, 16 10:41	hw07-LETREC-output-trace Page 22/32
1066	· '	
	((sumto -(n,1)),-(0,n)),	
1067		
1068 1069		- val=#t
1070		$ \cdot \cdot \cdot \cdot \cdot \cdot $ env=[n=0, sumto(n)=if zero?(n) then 0 else -(
1071	(sumto -(n,1)),-(0,n)),x	
1071		
1073		
1074		
1075	umto -(n,1)),-(0,n)),x=1	env=[n=1,sumto(n)=if zero?(n) then 0 else -((s 0.v=5. $i=1$ )
1076		
1077		$ \cdot \cdot \cdot \cdot \cdot $ env=[n=1, sumto(n)=if zero?(n) then 0 else -((
1078		
1079		
1080		env=[n=1, sumto(n)=if zero?(n) then 0 else -((
1081	sumto -(n,1)),-(0,n)),x=	
1082		
1083		
1084		
1085		
1086		+ exp=-(0,n)        env=[n=2,sumto(n)=if zero?(n) then 0 else -((sumt
1087	o -(n,1)),-(0,n)),x=10,v	=5, i=1]
1088		exp=0         env=[n=2,sumto(n)=if zero?(n) then 0 else -((sum
1089	to $-(n,1)),-(0,n)),x=10,$	v=5, i=1]
1090		
1091 1092		$ \cdot \cdot \cdot \cdot $ env=[n=2,sumto(n)=if zero?(n) then 0 else -((sum
1093	to -(n,1)),-(0,n)),x=10,	
1094		- val=-2
1095		- val=3
1096		
1097		
1098		
1099	(n,1)),-(0,n)),x=10,v=5,	$ \cdot $ env=[n=3,sumto(n)=if zero?(n) then 0 else -((sumto - i=1)
1100		
1101		$ \cdot \cdot $ env=[n=3,sumto(n)=if zero?(n) then 0 else -((sumto
	-(n,1)),-(0,n)),x=10,v=5	, i=1]
1102		
1103		
1104	-(n,1),-(0,n),x=10,v=5	$ \cdot \cdot $ env=[n=3,sumto(n)=if zero?(n) then 0 else -((sumto $\cdot$ i=1)
1105		
1106		
1107		
1108		- val=6
1109		val=6
1110		
1111	1)),-(0,n)),x=10,v=5,i=1	
1112		
1113	,1)),-(0,n)), $x=10$ , $v=5$ , $i=$	
1114		
1115		
1116	,1)),-(0,n)), $x=10,v=5,i=$	
1117	111111111111111111111111111111111111111	- val=4

```
hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                         Page 23/32
  |||||| val=10
1120 ||||||||||||||||||||| val=10
1121 |||||||||||||||| val=10
1122 | | | | | | | | | | | | | | | + exp=-(0, n)
(0,n), x=10, v=5, i=11
1124 | | | | | | | | | | | | | | | + exp=0
  ||||||||||||||||||| env=[n=5, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1))
1125
   ), -(0,n)), x=10, v=5, i=1]
1126 ||||||||||||||||||| val=0
  ), -(0,n)), x=10, v=5, i=1]
|||||| val=15
   |||||| val=15
1132
   |||||| val=15
1133
1134 | | | | | | | | | | | | | | + \exp=-(0,n)
0, n), x=10, v=5, i=1
1136 ||||||||||||||+ exp=0
1137 ||||||||||||||||| env=[n=6, sumto (n)=if zero? (n) then 0 else -((sumto -(n,1)), -(n,1))
   (0,n), x=10, v=5, i=1]
  |||||| val=0
   ||||||+ exp=n
  |||||||||||||||||||| env=[n=6, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-
1140
   (0,n)), x=10, v=5, i=1]
1141 |||||||||||||| val=6
1142 ||||||||||||| val=-6
1143 |||||||||||| val=21
1144 ||||||||||||- val=21
1145 |||||||||||- val=21
1146 | | | | | | | | | | | + \exp=-(0, n)
)), x=10, v=5, i=1]
  1149 |||||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,
  n)), x=10, v=5, i=1]
1150 ||||||||||||| val=0
1151 |||||||||||+ exp=n
1152 |||||||||||||| env=[n=7, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,
  n)),x=10,v=5,i=1
1153 |||||||||||| val=7
  ||||| val=-7
1154
1155
   |||||| val=28
   |||||| val=28
1156
  |||||||||| val=28
1157
1158 ||||||||+ exp=-(0,n)
x=10, v=5, i=1
1160 | | | | | | | | | + exp=0
1161 |||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n))
   , x=10, v=5, i=11
1162 ||||||||- val=0
1163 |||||||||+ exp=n
  ||||||||||| env=[n=8, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n))
   , x=10, v=5, i=1]
   ||||| val=8
  |||||| val=-8
1166
  |||||||||| val=36
  ||||||||||| val=36
1169
  |||||||| val=36
  || || || || + \exp = -(0, n)
1171 | | | | | | | | | env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=1
```

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hw07-LETREC-output-trace
11ìM-^[M-^T 10, 16 10:41
                                                                                 Page 24/32
    0, v=5, i=1
1172 | | | | | | | | | + \exp=0
1173 | | | | | | | | | | | env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=
    10, v=5, i=11
1174 | | | | | | | | - val=0
1175 | | | | | | | + exp=n
    |\cdot|\cdot|\cdot|\cdot|\cdot| env=[n=9, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=
    10, v=5, i=1
1177 |||||||- val=9
1178 ||||||- val=-9
1179 |||||- val=45
1180 ||||| val=45
1181 ||||- val=45
1182 | | | | + exp=-(0, n)
1183 | | | | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10,
    v=5, i=11
1184 | | | | | + exp=0
1185 ||||||| env=[n=10,sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)),-(0,n)),x=10
    , v=5, i=1]
1186 ||||| val=0
1187 | | | | | + exp=n
1188 | | | | | | | | env=[n=10, sumto(n)=if zero?(n) then 0 else -((sumto -(n,1)), -(0,n)), x=10
    , v=5, i=11
1189 |||||- val=10
   ||||| val=-10
1190
    ||||- val=55
    |||- val=55
1193 ||- val=55
    |- val=55
1194
1195 55
1196 >
1197 [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-apply-1
1198 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
1200 % proc application. (value=29)
1201
    (proc(x) - (x, 1) 30)
1202
1204
    |+ \exp = (\operatorname{proc}(x) - (x, 1) \ 30)
1205
    | | env=[x=10, v=5, i=1]
1207 | | + exp=proc(x) - (x, 1)
    | | | env=[x=10, v=5, i=1]
    | - val = proc(x) - (x, 1) [x = 10, v = 5, i = 1]
1210 | | + exp=30
1211 | | | env=[x=10, v=5, i=1]
1212 | | - val=30
1213 | | + \exp = -(x, 1)
1214 | | | env=[x=30, x=10, v=5, i=1]
1215 |||+ exp=x
1216 | | | | env=[x=30, x=10, v=5, i=1]
1217 | | | - val=30
1218 | | | + exp=1
1219 | | | | env=[x=30, x=10, v=5, i=1]
1220 |||- val=1
1221 ||- val=29
    |- val=29
1222
1223 29
1224 >
    [dshin@acacia letrec] $ letrec -t /home/pl/hw07/tests/proc-apply-2
1226 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
1228 % proc application. (value=29)
1230 let f = proc(x) - (x, 1)
1231 in (f 30)
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10. 16 10:41
                                                                                  Page 25/32
1233
   2 ______
    I + \exp = let f = proc(x) - (x, 1) in (f 30)
1234
1235
   | | env=[x=10, v=5, i=1]
    | \cdot | + \exp = \operatorname{proc}(x) - (x.1)
1236
1237
    | | | env=[x=10, v=5, i=1]
1238
    | | - val = proc(x) - (x, 1) [x = 10, v = 5, i = 1]
   | | + \exp(f 30) |
1239
1240 | | | env=[f=proc(x)-(x,1)[x=10,v=5,i=1],x=10,v=5,i=1]
1241 |||+ exp=f
1242 | | | | env=[f=proc(x)-(x,1)[x=10,v=5,i=1],x=10,v=5,i=1]
   | | | - val = proc(x) - (x, 1) [x = 10, v = 5, i = 1]
1244 | | | + exp=30
1245 | | | | env=[f=proc(x)-(x,1)[x=10,v=5,i=1],x=10,v=5,i=1]
    |||- val=30
1246
1247
    | | | + \exp(-(x, 1))
    |\cdot|\cdot| env=[x=30, x=10, v=5, i=1]
1248
1249
    ||||+ \exp=x
    |\cdot|\cdot|\cdot| env=[x=30, x=10, v=5, i=1]
1250
1251 | | | | - val=30
1252 | | | | + exp=1
   | | | | | | env=[x=30, x=10, v=5, i=1]
1254 | | | | - val=1
1255 | | | - val=29
   11 - val = 29
1256
    |- val=29
1257
1258 29
1259
    [dshin@acacia letrec] $ letrec -t /home/pl/hw07/tests/proc-apply-3
1260
1261 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
   % proc can be applied twice. p75. (value=55)
   let f=proc(x) - (x,11)
1265
   in (f (f 77))
1266
1267
    %
1268
    |+ \exp = \text{let } f = \text{proc}(x) - (x, 11) \text{ in (f (f 77))}
    | | env=[x=10, v=5, i=1]
1270
1271 | | + exp=proc(x) - (x, 11)
1272 \mid \mid \mid \mid env=[x=10, v=5, i=1]
    | | - val = proc(x) - (x, 11) [x = 10, v = 5, i = 1]
1274 \mid | + exp = (f (f 77))
    |\cdot| = \text{env} = [f = \text{proc}(x) - (x, 11) [x = 10, v = 5, i = 1], x = 10, v = 5, i = 1]
1275
1276
1277
    |\cdot|\cdot| env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
1278
    | | | - val = proc(x) - (x, 11) [x = 10, v = 5, i = 1]
    | | | + \exp(f 77)
1279
    |\cdot|\cdot| env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
1280
1281
| | | | - val = proc(x) - (x, 11) [x = 10, v = 5, i = 1]
1284 | | | | + exp=77
1285 | | | | | env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
   ||||- val=77
1286
    | | | + \exp(-(x, 11))
1287
    |\cdot|\cdot|\cdot| env=[x=77, x=10, v=5, i=1]
1288
1289
    | | | | | | | env=[x=77, x=10, v=5, i=1]
1290
   |||||| val=77
1291
1292 | | | | | + exp=11
1293 | | | | | | env=[x=77, x=10, v=5, i=1]
1294 ||||- val=11
1295 ||||- val=66
1296 |||- val=66
1297 | | | + \exp = -(x, 11)
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10. 16 10:41
                                                                                Page 26/32
    | | | | env=[x=66, x=10, v=5, i=1]
   ||||+ exp=x
1300 | | | | | env=[x=66, x=10, v=5, i=1]
1301 | | | | - val=66
1302 | | | | + exp=11
1303
    | | | | | | env=[x=66, x=10, v=5, i=1]
    ||||- val=11
1305 |||- val=55
1306 | | - val=55
1307
    |- val=55
1308 5.5
    [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-currying-1
1311 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
1313
    % multiple arguments with proc that returns proc. (value=-1)
    % (This is called Currying.)
    ((proc(x)proc(y) - (x,y) 5) 6)
1316
1317
    | + \exp((\operatorname{proc}(x)\operatorname{proc}(y) - (x, y) + 5)) |
    | | env=[x=10, v=5, i=1]
1321 | | + exp= (proc(x)proc(y) - (x,y) 5)
1322 | | | env=[x=10, v=5, i=1]
    | \cdot | \cdot | + \exp = \operatorname{proc}(x) \operatorname{proc}(y) - (x, y)
    | | | |  env=[x=10, v=5, i=1]
1325 | | | - val=proc(x)proc(y) - (x, y) [x=10, v=5, i=1]
1326 |||+ exp=5
1327 | | | | env=[x=10, v=5, i=1]
1328 |||- val=5
1329 | | | + exp=proc(y) - (x, y)
1330 | | | | env=[x=5, x=10, v=5, i=1]
1331 | | | - val=proc(y) - (x, y) [x=5, x=10, v=5, i=1]
1332 | | - val=proc(y) - (x, y) [x=5, x=10, v=5, i=1]
1333 | | + exp=6
1334 | | | env=[x=10, v=5, i=1]
1335 ||- val=6
1336 | | + exp = -(x, y)
1337 | | | env=[y=6, x=5, x=10, v=5, i=1]
1338 | | | + exp=x
1339 | | | | env=[v=6, x=5, x=10, v=5, i=1]
1340 |||- val=5
1341 | | | + exp=v
1342 | | | | env=[y=6, x=5, x=10, v=5, i=1]
1343 |||- val=6
1344 ||- val=-1
1345 |- val=-1
1346 -1
1347 >
1348 [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-currying-2
1349 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
1351 % multiple arguments with proc that returns proc. (value=-1)
   % (This is called Currying.)
    let f=proc(x)proc(y)-(x,y)
1354
1355 in ((f 5) 6)
|+ exp=let f=proc(x)proc(y)-(x,y) in ((f 5) 6)
1359 || env=[x=10, v=5, i=1]
1360 | | + exp=proc(x)proc(y) - (x, y)
1361 | | | env=[x=10, v=5, i=1]
1362 | | - val=proc(x)proc(y) - (x, y) [x=10, v=5, i=1]
1363 | | + exp=((f 5) 6)
```

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hw07-LETREC-output-trace
11ìM-^IM-^T 10, 16 10:41
                                                                                                                                             Page 27/32
              env=[f=proc(x)proc(y)-(x,y)[x=10,v=5,i=1],x=10,v=5,i=1]
       | | | + \exp(f 5)
       |\cdot|\cdot| env=[f=proc(x)proc(y)-(x,y)[x=10,v=5,i=1],x=10,v=5,i=1]
1367
       ||\cdot|| + \exp = f
1368
        |\cdot|\cdot|\cdot| env=[f=proc(x)proc(y)-(x,y)[x=10,v=5,i=1],x=10,v=5,i=1]
1360
             |-val=proc(x)proc(y)-(x,y)[x=10,y=5,i=1]
1370
1371
        |\cdot|\cdot|\cdot| env=[f=proc(x)proc(y)-(x,y)[x=10,v=5,i=1],x=10,v=5,i=1]
        ||||- val=5
1372
        | | | | + exp=proc(y) - (x, y)
1373
       | | | | | | env=[x=5, x=10, v=5, i=1]
         | | | - val = proc(y) - (x, y) [x = 5, x = 10, v = 5, i = 1]
        | | | - val = proc(y) - (x, y) [x = 5, x = 10, v = 5, i = 1]
       || | + exp = 6
1377
1378
        |\cdot|\cdot| env=[f=proc(x)proc(y)-(x,y)[x=10,v=5,i=1],x=10,v=5,i=1]
1379
        |||- val=6
        | | | + \exp = -(x, y)
1380
                env=[y=6, x=5, x=10, v=5, i=1]
1381
1382
          |||+ exp=x
        | | | | | | env=[y=6, x=5, x=10, v=5, i=1]
1383
       ||||- val=5
1384
        | | | | | | env=[y=6, x=5, x=10, v=5, i=1]
1387
        IIII- val=6
        || - va| = -1
1388
        | | - va | = -1
1389
1390
       |- val=-1
      -1
1391
1392
       [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-currying-3
1393
1394 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
       % multiple arguments with proc that returns proc. (value=40)
      % (This is called Currying.)
1397
1398
1399
       let plus=proc(x) proc(y) -(x, -(0, y))
       in let minus=proc(x) proc(y) -(x,y)
1400
1401
             in ((minus ((plus 10) 20)) ((minus 40) 50))
1402
1403
1404
       |+ exp=let plus=proc(x)proc(y)-(x,-(0,y)) in let minus=proc(x)proc(y)-(x,y) in
        (minus ((plus 10) 20)) ((minus 40) 50))
        | | env=[x=10, v=5, i=1]
        | \ | + \exp = \operatorname{proc}(x) \operatorname{proc}(y) - (x, -(0, y))
1406
        | | | env=[x=10, v=5, i=1]
1407
1408
        | - val = proc(x) proc(y) - (x, -(0, y)) [x=10, v=5, i=1]
1409
       | + \exp = let minus=proc(x)proc(y)-(x,y) in ((minus ((plus 10) 20)) ((minus 40) 50
       ))
       |\cdot| env=[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
1410
       | | | + \exp=\operatorname{proc}(x)\operatorname{proc}(y) - (x, y)
1412 | | | | env=[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
1413 | | | - val=proc(x)proc(y) - (x, y) [plus=proc(x)proc(y) - (x, -(0, y)) [x=10, v=5, i=1], x=10, v=10, 
       v=5, i=1
1414 |||+ exp=((minus ((plus 10) 20)) ((minus 40) 50))
       | | | | | env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1]
       [x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10, v=5, i=1], x=10, v=5, i=1]
       | | | | + \exp = (\min us ((plus 10) 20))
        |\cdot|\cdot|\cdot| env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=
       1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
       | | | | | + exp=minus
      || || ||  env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i]
1419
       =1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1], x=10,v=5,i=1]
      || || | - val = proc(x) proc(y) - (x, y) [plus = proc(x) proc(y) - (x, -(0, y)) [x = 10, v = 5, i = 1], x = 1
       0, v=5, i=1
1421 ||||+ exp=((plus 10) 20)
1422 |||||| env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i]
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10. 16 10:41
                                                                                  Page 28/32
    =1], x=10, v=5, i=1], plus=proc(x) proc(y) - (x, -(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1]
1423 |||||+ exp=(plus 10)
   || || || || env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,v=5]
    i=1], x=10, v=5, i=1], plus=proc(x) proc(y) - (x, -(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1]
   |||||||+ exp=plus
    || | | | | | | env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5]
    i=1, x=10, y=5, i=1, plus=proc(x)proc(y)-(x,-(0,y))[x=10,y=5,i=1], x=10,y=5,i=1]
   | | | | | | | - val = proc(x) proc(y) - (x, -(0, y)) [x=10, y=5, i=1]
   || || || + \exp = 10
i=1, x=10, v=5, i=1, plus=proc(x)proc(y) - (x, -(0,y))[x=10, v=5, i=1], x=10, v=5, i=1]
1431 | | | | | | + \exp=\operatorname{proc}(y) - (x, -(0, y))
1432 | | | | | | | env=[x=10, x=10, v=5, i=1]
    | | | | | | | - val = proc(y) - (x, -(0, y)) [x=10, x=10, v=5, i=1]
    || || || - val = proc(y) - (x, -(0, y)) [x=10, x=10, v=5, i=1]
1434
    || || || || + \exp = 20
    || || || || env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,
    i=1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1], <math>x=10, v=5, i=1]
   ||||||| val=20
   | | | | | | + \exp = (x, -(0, y))
    | | | | | | | | env=[v=20, x=10, x=10, v=5, i=1]
   || || || || | env=[y=20, x=10, x=10, v=5, i=1]
   |||||| val=10
1442
    | | | | | | | + \exp = (0, y)
    ||||||| env=[y=20, x=10, x=10, v=5, i=1]
    || || || || + \exp = 0
    | | | | | | | | | | env=[y=20, x=10, x=10, v=5, i=1]
   |||||| val=0
   |||||||+ exp=y
   | | | | | | | | | | env=[y=20, x=10, x=10, v=5, i=1]
   |||||| val=20
1451 | | | | | | - val=-20
1452 | | | | | | - val=30
1453
   ||||- val=30
    ||||+ \exp=\operatorname{proc}(y) - (x, y)
    |\cdot|\cdot|\cdot| env=[x=30, plus=proc(x) proc(y)-(x,-(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1]
1456
   ||\cdot|| - \text{val=proc}(y) - (x, y) [x=30, plus=proc(x)proc(y) - (x, -(0, y)) [x=10, v=5, i=1], x=10,
    v=5, i=1
1457
   |\cdot|\cdot| val=proc(y)-(x,y)[x=30,plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
    =5, i=11
    ||||+ \exp((\min 40) 50)
    | | | | | | env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=
    1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1], x=10,v=5,i=1]
   |||||+ \exp(\min 40)
    | | | | | | | env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i]
    =1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1], x=10,v=5,i=1]
   ||||||+ exp=minus
   |||||| env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,v=5]
    i=1], x=10, v=5, i=1], plus=proc(x) proc(y) - (x, -(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1]
   |||||-val=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=
    10, v=5, i=1
1465 | | | | | | + exp=40
   | | | | | | | | env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,v=5]]
    i=1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1], x=10,v=5,i=1]
   ||||| val=40
   | | | | | | + \exp = \operatorname{proc}(y) - (x, y)
   |||||| env=[x=40,plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
1470 | | | | | | - val=proc(y) - (x, y) [x=40, plus=proc(x) proc(y) - (x, -(0, y)) [x=10, v=5, i=1], x=10
    v=5, i=1
1471 |\cdot|\cdot|\cdot| val=proc(y)-(x,y)[x=40,plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,
    v=5, i=11
1472 | | | | | + exp=50
1473 |||||| env=[minus=proc(x)proc(y)-(x,y)[plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i]
    =1], x=10, v=5, i=1], plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
```

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11ìM-^[M-^T 10, 16 10:41
                                       hw07-LETREC-output-trace
                                                                                             Page 29/32
     ||||| val=50
1475 | | | | + \exp = (x, y)
    |\cdot|\cdot|\cdot| env=[y=50, x=40, plus=proc(x)proc(y)-(x,-(0,y))[x=10, v=5, i=1], x=10, v=5, i=1]
1/177
1478
     |\cdot|\cdot|\cdot|\cdot| env=[y=50,x=40,plus=proc(x)proc(y)-(x,-(0,y))[x=10,v=5,i=1],x=10,v=5,i=1]
1479
     ||||| val=40
1480
     |||||+ exp=v
    |\cdot|\cdot|\cdot|\cdot| env=[v=50, x=40, plus=proc(x)proc(y)-(x,-(0,y))[x=10, v=5, i=1], x=10, v=5, i=1]
1481
1482 | | | | | | | - val = 50
1483 ||||| val=-10
1484 | | | | - val=-10
    | | | | + \exp = -(x, y)
1485
    |\cdot|\cdot|\cdot| env=[y=-\bar{1}0, x=30, plus=proc(x) proc(y) - (x, -(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1]
1486
1487
     |\cdot|\cdot|\cdot| env=[y=-10, x=30, plus=proc(x) proc(y) - (x, -(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1
1488
     | | | | | | | - val = 30
1489
    ||||+ exp=y
1490
    |\cdot|\cdot|\cdot| env=[y=-10, x=30, plus=proc(x) proc(y)-(x,-(0,y)) [x=10, v=5, i=1], x=10, v=5, i=1]
1491
1492
     ||||- val=-10
     | | | | | - val = 40
1/103
     || | - va | = 40
1494
     11 - val = 40
1495
1496
    |- val=40
1497 40
1498
    [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-higher-1
1499
1500 Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
1502 % proc's arg is proc. (value=29)
    % (This is called higher-order function.)
1503
1504
1505
     (proc(f) (f 30)
      proc(x) - (x, 1)
1506
1507
1508
    |+ \exp(\operatorname{proc}(f)(f 30) \operatorname{proc}(x) - (x, 1))|
1509
    | | env=[x=10, v=5, i=1]
1510
    | | + exp=proc(f)(f 30)
     | | | env=[x=10, v=5, i=1]
     | - val=proc(f)(f 30)[x=10,v=5,i=1]
     | + \exp = \operatorname{proc}(x) - (x, 1)
1514
     | | | env=[x=10, v=5, i=1]
1515
1516
     | - val = proc(x) - (x, 1) [x = 10, v = 5, i = 1]
     | | + \exp(f 30)
1517
     |\cdot| env=[f=proc(x)-(x,1)[x=10,v=5,i=1],x=10,v=5,i=1]
1518
1519
    |\cdot|\cdot| env=[f=proc(x)-(x,1)[x=10,v=5,i=1],x=10,v=5,i=1]
1520
1521 | | | - val=proc(x) - (x, 1) [x=10, v=5, i=1]
1522 | | | + exp=30
1523 | | | | env=[f=proc(x)-(x,1)[x=10,v=5,i=1],x=10,v=5,i=1]
1524 | | | - val = 30
    | \cdot | + \exp = -(x, 1)
1525
     |\cdot|\cdot| env=[x=30, x=10, v=5, i=1]
1526
1527
     ||||| env=[x=30, x=10, v=5, i=1]
1528
    ||||- val=30
1529
    | | | | + exp=1
1530
    | | | | | | env=[x=30, x=10, v=5, i=1]
    ||||- val=1
1533 | | | - val=29
1534 ||- val=29
1535 |- val=29
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10, 16 10:41
                                                                                 Page 30/32
   29
1537
    [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-higher-2
   Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
    % proc's arg is proc. p75. (value=55)
    % (This is called higher-order function.)
    (proc(f) (f (f 77))
    proc(x) - (x, 11)
1545
   |+ \exp(\operatorname{proc}(f)(f(f.77))\operatorname{proc}(x) - (x,11))
    | | env=[x=10, v=5, i=1]
    | \cdot | + exp=proc(f)(f (f 77))
    | | | env=[x=10, v=5, i=1]
    | - val = proc(f)(f(f77))[x=10, v=5, i=1]
   | \ | + \exp = \operatorname{proc}(x) - (x, 11)
   | | | env=[x=10, v=5, i=1]
   | - val=proc(x) - (x, 11) [x=10, v=5, i=1]
   | + \exp(f (f 77))
   |\cdot| = \text{env} = [f = \text{proc}(x) - (x, 11) [x = 10, v = 5, i = 1], x = 10, v = 5, i = 1]
   |\cdot|\cdot| env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
    | | | - val = proc(x) - (x, 11) [x = 10, v = 5, i = 1]
    | | | + \exp(f 77)
    |\cdot|\cdot| env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
    |\cdot|\cdot|\cdot| env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
   | | | | - val = proc(x) - (x, 11) [x = 10, v = 5, i = 1]
1567 | | | | | env=[f=proc(x)-(x,11)[x=10,v=5,i=1],x=10,v=5,i=1]
1568 ||||- val=77
1569 | | | | + \exp = -(x, 11)
1570 | | | | | env=[x=77, x=10, v=5, i=1]
1571 | | | | + exp=x
1572 | | | | | | env=[x=77, x=10, v=5, i=1]
   ||||- val=77
   ||||+ exp=11
   | | | | | | | env=[x=77, x=10, v=5, i=1]
1576 | | | | | - val=11
1577 ||||- val=66
1578 | | | - val=66
   | | | + exp = -(x, 11)
   | | | | env=[x=66, x=10, v=5, i=1]
1580
   |||+ exp=x
1582
   | | | | | | env=[x=66, x=10, v=5, i=1]
   ||||- val=66
1584 | | | | + exp=11
1585 | | | | | | env=[x=66, x=10, v=5, i=1]
1586 | | | | | - val=11
1587 | | | - val=55
1588 ||- val=55
    1 - val = 55
1589
1590 55
1591
    [dshin@acacia letrec]$ letrec -t /home/pl/hw07/tests/proc-in-let
1592
    Welcome to MzScheme v370 [3m], Copyright (c) 2004-2007 PLT Scheme Inc.
   % procs in let is very useful. p76. (value=-100)
1595
   let x=200
1598 in let f=proc(z) - (z,x)
       in let x=100
          in let g=proc(z) - (z,x)
1600
              in - ((f 1), (q 1))
1601
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10. 16 10:41
                                                                                       Page 31/32
1603
   2 ______
    |+ exp=let x=200 in let f=proc(z)-(z,x) in let x=100 in let g=proc(z)-(z,x) in
    ((f 1), (g 1))
    | | env=[x=10, v=5, i=1]
1605
1606
    | | + \exp = 200
1607
    | | | env=[x=10, v=5, i=1]
1608
    ||- val=200
    | \cdot | + exp=let f=proc(z)-(z,x) in let x=100 in let q=proc(z)-(z,x) in -((f 1),(g 1)
1609
   | | | env=[x=200, x=10, v=5, i=1]
   | | | + exp=proc(z) - (z, x)
    |\cdot|\cdot| env=[x=200, x=10, v=5, i=1]
    | | | - val = proc(z) - (z, x) [x = 200, x = 10, v = 5, i = 1]
    |\cdot| + \exp = \text{let } x = 100 \text{ in let } q = \operatorname{proc}(z) - (z, x) \text{ in } - ((f 1), (q 1))
1614
         env = [f = proc(z) - (z, x) [x = 200, x = 10, v = 5, i = 1], x = 200, x = 10, v = 5, i = 1]
1615
1616
     |\cdot|\cdot|\cdot| env=[f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10,v=5,i=1]
1617
     ||||- val=100
1618
    | | | | + \exp = \det g = \operatorname{proc}(z) - (z, x) \text{ in } - ((f 1), (g 1))
1610
    |\cdot|\cdot|\cdot| env=[x=100, f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10,v=5,i=1]
1620
    | | | | | + \exp=proc(z) - (z, x)
    |\cdot|\cdot|\cdot| env=[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10,v=5,i=1]
1623
    ||\cdot|| - val = proc(z) - (z, x) [x=100, f=proc(z) - (z, x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
    5, i=11
1624 | | | | | + exp=-((f 1), (q 1))
    |\cdot|\cdot|\cdot| env=[g=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10]
    v=5, i=1, v=100, f=proc(z)-(z, x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
    | | | | | | | + \exp(f 1)
   |\cdot|\cdot|\cdot|\cdot| env=[q=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=1
1627
    0, v=5, i=1, x=100, f=proc(z)-(z, x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
    |\cdot|\cdot|\cdot|\cdot| env=[q=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=
    10, v=5, i=1], x=100, f=proc(z)-(z,x)[x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
    | | | | | | | - val = proc(z) - (z, x) [x = 200, x = 10, v = 5, i = 1]
1630
    ||||||+ \exp=1
1631
    |\cdot|\cdot|\cdot|\cdot| env=[q=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=
1632
    10, v=5, i=1, x=100, f=proc(z)-(z, x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
    |||||- val=1
1633
    | | | | | | | + \exp = -(z, x)
1634
    | | | | | | | | |  env=[z=1, x=200, x=10, v=5, i=1]
1635
    |||||+ exp=z
    || || || || ||  env=[z=1, x=200, x=10, v=5, i=1]
1637
1638
    |||||||| val=1
1639
    |||||+ exp=x
    | | | | | | | | | |  env=[z=1, x=200, x=10, v=5, i=1]
1640
1641
     |||||| val=200
    |||||| val=-199
1642
    ||||| val=-199
1643
1644 |||||+ exp=(g 1)
   |\cdot|\cdot|\cdot|\cdot| env=[g=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=1
    0, v=5, i=1, x=100, f=proc(z)-(z,x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
   |\cdot|\cdot|\cdot|\cdot| env=[q=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=0
    10, v=5, i=1, x=100, f=proc(z)-(z, x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
   1648
    v=5, i=11
    | | | | | | | | + \exp = 1
    |\cdot|\cdot|\cdot|\cdot| env=[g=proc(z)-(z,x)[x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10]
1650
    10, v=5, i=1, x=100, f=proc(z)-(z, x) [x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
    |||||- val=1
1651
    | | | | | | | + exp = -(z, x)
    |\cdot|\cdot|\cdot|\cdot| env=[z=1, x=100, f=proc(z)-(z, x)[x=200, x=10, v=5, i=1], x=200, x=10, v=5, i=1]
1654
    ||||||| env=[z=1,x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10,v=5,i=1]
1655
1656 |||||||- val=1
```

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hw07-LETREC-output-trace
11ìM-^ſM-^T 10. 16 10:41
                                                                         Page 32/32
   |||||||| env=[z=1,x=100,f=proc(z)-(z,x)[x=200,x=10,v=5,i=1],x=200,x=10,v=5,i=1]
   |||||| val=100
   ||||||| val=-99
1661
   |||||||| val=-99
1662
   ||||- val=-100
   ||||- val=-100
1664 |||- val=-100
1665 ||- val=-100
1666 |- val=-100
1667 -100
1669 [dshin@acacia letrec]$
```