Program Reasoning

8. Automated Program Verification

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Towards Fully Automated Verification

- The assumption so far: a user provides inductive invariants
- Fully automated verification: combined with automated invariant generation methods
- Example:
  - Program analysis [CS524]: automatic, terminating, but may not be exact
  - Machine learning: automatic? terminating? exact?
- This lecture: verification via constraint solving
  - With Constrained Horn Clause (CHC)
  - Using SMT solvers

Horn Clause

A. Horn

- Clause: a disjunction of literals
  - E.g., $p \vee \neg q \vee \neg r$
- Horn clause: a clause with at most one positive literal
  - E.g., $\neg p \vee \neg q \vee r$ which is equivalent to $p \wedge q \rightarrow r$
- Horn clause logic: the basis of logic programming languages such as Prolog and Datalog
- Why Horn clause? Efficiency
  - Propositional Horn clause (HORNSAT): linear time
  - First-order Horn clause (e.g., Prolog): undecidable but efficient
- More details: CS402 (Introduction to Logic for Computer Science)

Constrained Horn Clause (CHC)

- A fragment of first-order logic

$$\forall x.\underbrace{\varphi}_{\textbf{Constraint}} \wedge \underbrace{p_1(X_1) \wedge \cdots \wedge p_n(X_n) \rightarrow h(X)}_{\textbf{Predicates}}$$

- φ: a constraint in a background theory T
  - E.g., $x + 1 = 2$ (Peano arithmetic)

Example

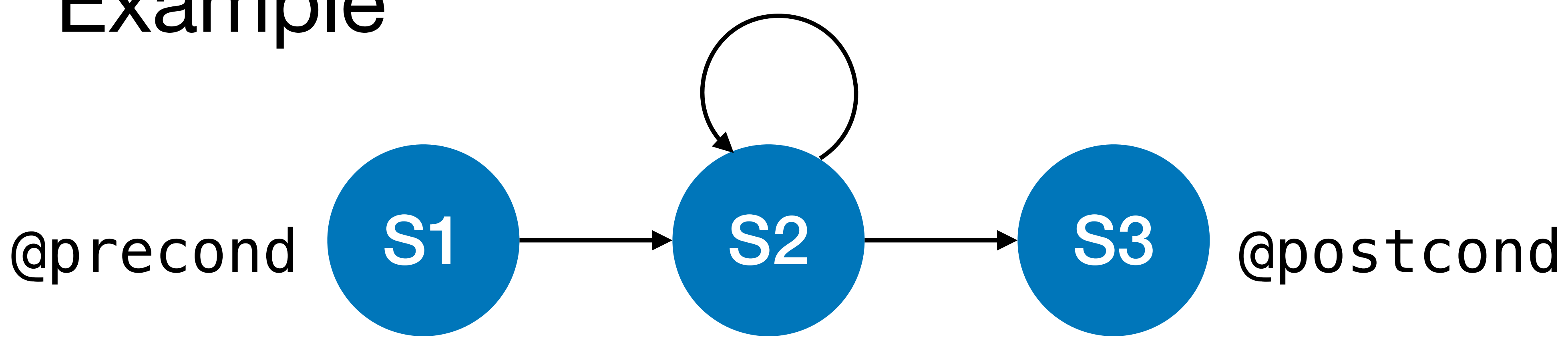
- Are the CHC formulas satisfiable? If so, what is P ?

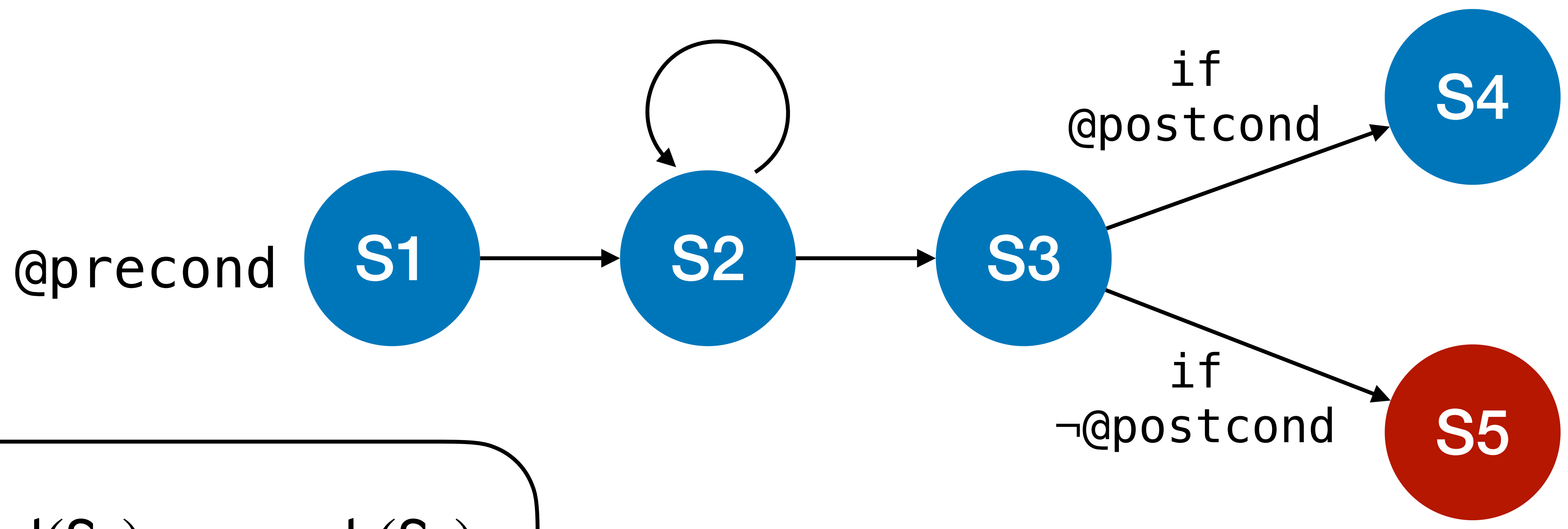
$$P(0)$$
$$\forall x, x' . P(x) \wedge x < 10 \wedge x' = x + 1 \rightarrow P(x')$$
$$\forall x . P(x) \wedge x > 11 \rightarrow \mathit{false}$$

$$\forall x . x \leq 0 \rightarrow P(x)$$
$$\forall x, x' . P(x) \wedge x < 5 \wedge x' = x + 1 \rightarrow P(x')$$
$$\forall x . P(x) \wedge x > 5 \rightarrow \mathit{false}$$

Program Verification via CHC

- Given a program and a specification, generate verification conditions using CHC
- Check the satisfiability of the CHC formula using SMT solvers
- Idea: partial correctness check as a reachability problem
- Example





$\text{pre} \rightarrow \text{reach}(S_1)$

$\text{reach}(S_1) \land \text{cmd}(S_1) \rightarrow \text{reach}(S_2)$

$\text{reach}(S_2) \land \text{cond}(S_2) \land \text{cmd}(S_2) \rightarrow \text{reach}(S_2)$

$\text{reach}(S_2) \land \neg\text{cond}(S_2) \rightarrow \text{reach}(S_3)$

$\text{reach}(S_3) \land \text{post} \rightarrow \text{reach}(S_4)$

$\text{reach}(S_3) \land \neg\text{post} \rightarrow \text{reach}(S_5)$

Query: Is S_5 reachable?
(i.e., $\text{reach}(S_5)$?)

Language

- Program = control flow graph
- Node = basic block = list of commands (end with jump)

$$
\begin{aligned}
C &\rightarrow \texttt{skip} \mid x := E \mid x := \texttt{input}() \mid \texttt{br}\ B\ l_1\ l_2 \\
&\mid\ \texttt{goto}\ l \mid \texttt{assume}(E) \mid \texttt{assert}(E) \\
E &\rightarrow n \mid x \mid E + E \mid E - E \mid E \times E \mid E\ /\ E \\
B &\rightarrow \mathsf{true} \mid \mathsf{false} \mid E < E \mid E = E \mid \neg B
\end{aligned}
$$

- Example:

```
Entry:
  x := input()
  assume(x > 1)
  assert(x == 0)
```

```
Entry:
  x := input()
  y := x - 1
  br x / 2 != 0 L1 L2
L1:
  assert(y != 0)
L2:
  skip
```

Specification

- Annotated in programs using assertions
- Checking an assertion = checking a reachability
  - Assertion is false = error state is reachable
- Example

```
Entry:
  x := input()
  y := x - 1
  assert(y != 0)
```

➡

```
Entry:
  x := input()
  y := x - 1
  br y != 0 L1 L2
L1:
  skip
L2:
  assert false
```

State

- A predicate parameterized by values of variables defined so far
  - One relation per basic block
- Example

```
Entry:
  x := input()
  y := x - 1
  br y != 0 L1 L2
L1:
  skip
L2:
  assert false
```

Relations: Entry, L1(x, y), L2(x, y)

Reachable states:
Entry,
L1(2, 1), L1(3,2), L1(4, 3), …
L2(1, 0)

Verification Condition

- CHC formula: the relationship among all nodes + unreachability of the error node
- Loop invariants will be computed by the underlying solver (But not always! Why?)
- Example

```
Entry:
  x := input()
  y := x - 1
  assert(y != 0)
```

```
Entry:
  x := input()
  y := x - 1
  br y != 0 L1 L2
L1:
  skip
L2:
  assert false
```

The condition is SATISFIABLE iff the program is correct

$$
\begin{aligned}
& Entry \\
& \forall x, y \,.\, Entry \land y = x - 1 \land y \neq 0 \rightarrow L_1(x, y) \\
& \forall x, y \,.\, Entry \land y = x - 1 \land y = 0 \rightarrow L_2(x, y) \\
& \forall x, y \,.\, L_2(x, y) \rightarrow false
\end{aligned}
$$

Example (1)

```
x := input();
assume(x < 10);
while(x < 10) {
  x++;
}
assert(x == 10);
```

↓

```
Entry:
  x0 := input()
  assume(x0 < 10)
  goto Cond
Cond:
  x1 := ϕ [x0, Entry] [x2, Body]
  br (x1 < 10) Body End
Body:
  x2 := x1 + 1
  goto Cond
End:
  br (x1 = 10) Then Else
Then
  skip
Else:
  assert false
```

The condition is SATISFIABLE
iff
the program is correct

$Entry$

$\forall x\,.\,Entry \wedge x < 10 \rightarrow Cond(x)$

$\forall x\,.\,Cond(x) \wedge x < 10 \rightarrow Body(x)$

$\forall x\,.\,Cond(x) \wedge x \geq 10 \rightarrow End(x)$

$\forall x, x'\,.\,Body(x) \wedge x' = x + 1 \rightarrow Cond(x')$

$\forall x\,.\,End(x) \wedge x = 10 \rightarrow Then(x)$

$\forall x\,.\,End(x) \wedge x \neq 10 \rightarrow Else(x)$

$\forall x\,.\,Else(x) \rightarrow false$

Reachable states:

Entry	: $\top$
Cond(10), Cond(9), Cond(8), Cond(7), ...,	: $x \leq 10$
Body(9), Body(8), Body(7), ...,	: $x \leq 9$
End(10)	: $x = 10$
Then(10)	: $x = 10$

{ $\top$ } Entry $\{x \leq 10\}$
$\{x \leq 10\}$ Cond; Body $\{x = 10\}$

Example (2)

- In practice, typically solve the incorrectness rather than correctness for efficiency
  - Query the reachability to an error state from the initial state
  - E.g., in your homework

The condition is SATISFIABLE iff the program is correct	The condition is SATISFIABLE iff the program is **incorrect**

$$
\begin{array}{l}
Entry \\
\forall x \,.\, Entry \land x < 10 \rightarrow Cond(x) \\
\forall x \,.\, Cond(x) \land x < 10 \rightarrow Body(x) \\
\forall x \,.\, Cond(x) \land x \geq 10 \rightarrow End(x) \\
\forall x, x' \,.\, Body(x) \land x' = x + 1 \rightarrow Cond(x') \\
\forall x \,.\, End(x) \land x = 10 \rightarrow Then(x) \\
\forall x \,.\, End(x) \land x \neq 10 \rightarrow Else(x) \\
\forall x \,.\, Else(x) \rightarrow false
\end{array}
$$

$$
\begin{array}{l}
Entry \\
\forall x \,.\, Entry \land x < 10 \rightarrow Cond(x) \\
\forall x \,.\, Cond(x) \land x < 10 \rightarrow Body(x) \\
\forall x \,.\, Cond(x) \land x \geq 10 \rightarrow End(x) \\
\forall x, x' \,.\, Body(x) \land x' = x + 1 \rightarrow Cond(x') \\
\forall x \,.\, End(x) \land x = 10 \rightarrow Then(x) \\
\forall x \,.\, End(x) \land x \neq 10 \rightarrow Else(x) \\
\textbf{Query}(Else)
\end{array}
$$

Summary

- Automated program verification for partial correctness
  - Equivalently, checking reachability of error states
- Constrained Horn clause: a fragment of FOL
- Program verification using CHC
  - Verification condition = unreachability of error states
- Automatically solved by theorem provers