Numerical Analysis for Iterative Filtering with New Efficient Implementations Based on FFT

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Hyunsoo, Yu

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BACKGROUND
TREND OF ANALYZING TIME SERIES DATA

- FFT & Wavelet - Conventional methods for analyzing Time series data
  - : linear decompositions
  - Not accurate on nonlinear & nonstationary time series data. (*Heisenberg uncertainty principle*)
  - **Alternate** – STFT(Short Time Fourier Transform), Synchrosqueezed Wavelet Transform, and ConceFT method…

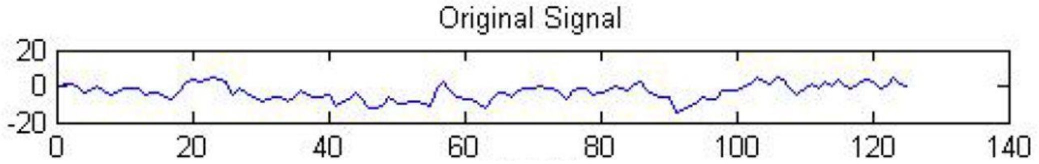
- EMD – local and adaptive data-driven method
  - : suitable nonlinear and nonstationary data analysis.
  - Unstable to perturbations
  - Susceptible to mode splitting and mode mixing
  - **Alternate** – EEMD(Ensemble EMD), NA-MEMD(Noise assisted Multivariate EMD), FMEMD(Fast MEMD)
    - Solve mode mixing problem
    - Can’t solve mode splitting problem
    - **Alternate** – IF(Iterative Filtering)

BACKGROUND
TREND OF ANALYZING TIME SERIES DATA

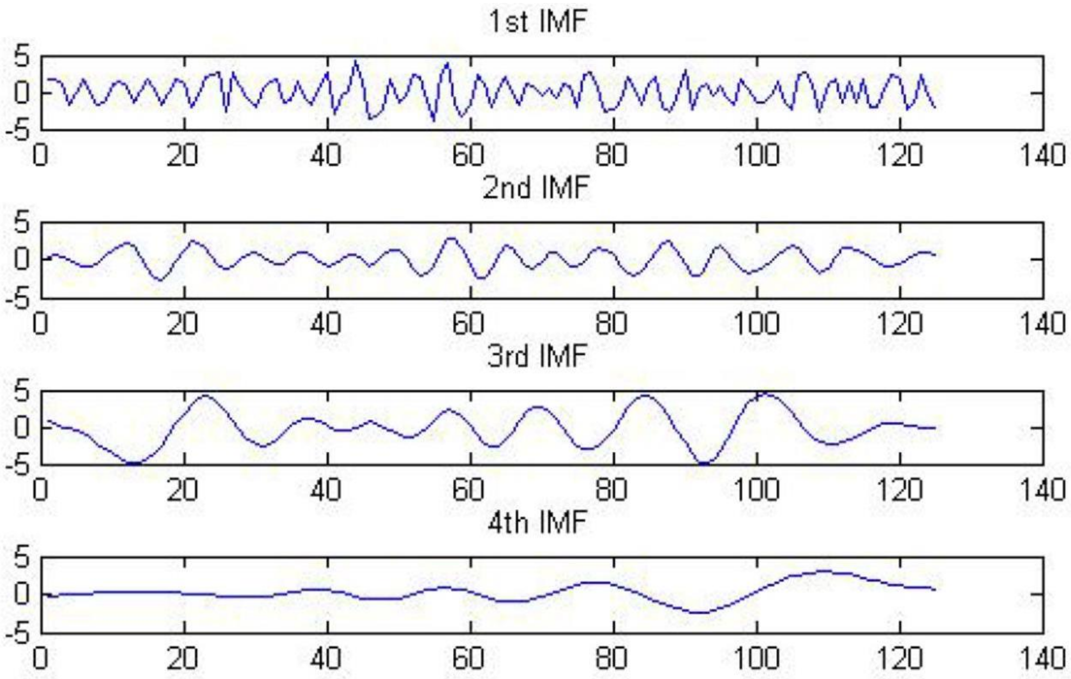
- IF(Iterative Filtering) – Produce results similar to EMD-based algorithm
  - : guarantee a priori its convergence and stability
  - Using FFT leads to faster computation. (Fast Iterative Filtering)
  - Mode splitting can be avoided by tuning the value of the stopping criterion parameter.

BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Overview



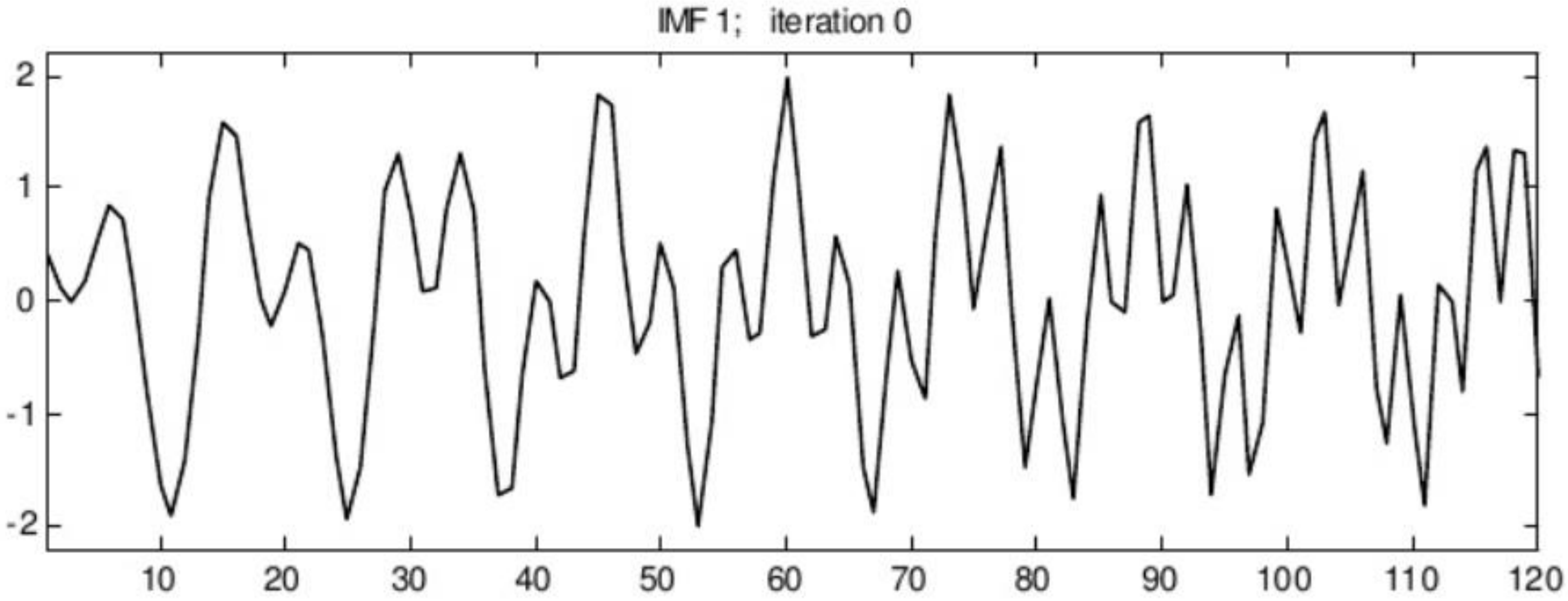
Decomposition



IMF(Intrinsic Mode Function)

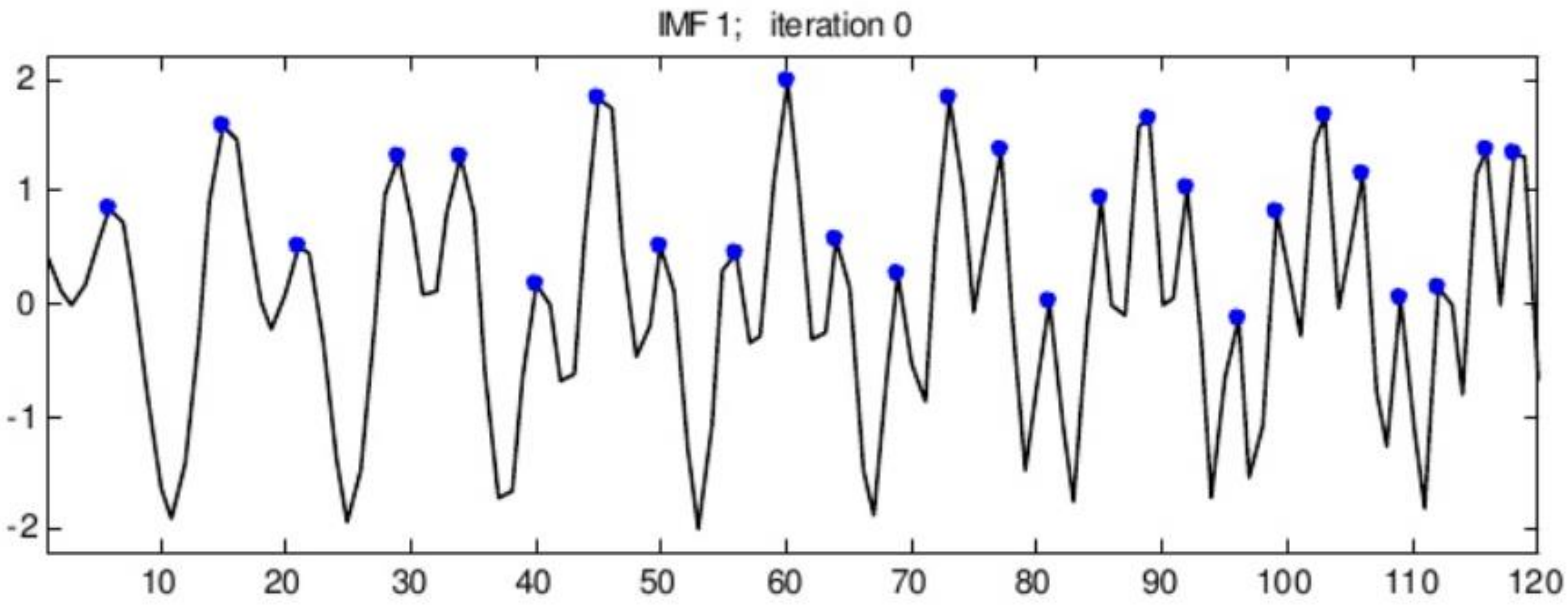
BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Method : Sifting Process
  - Signal x(t)



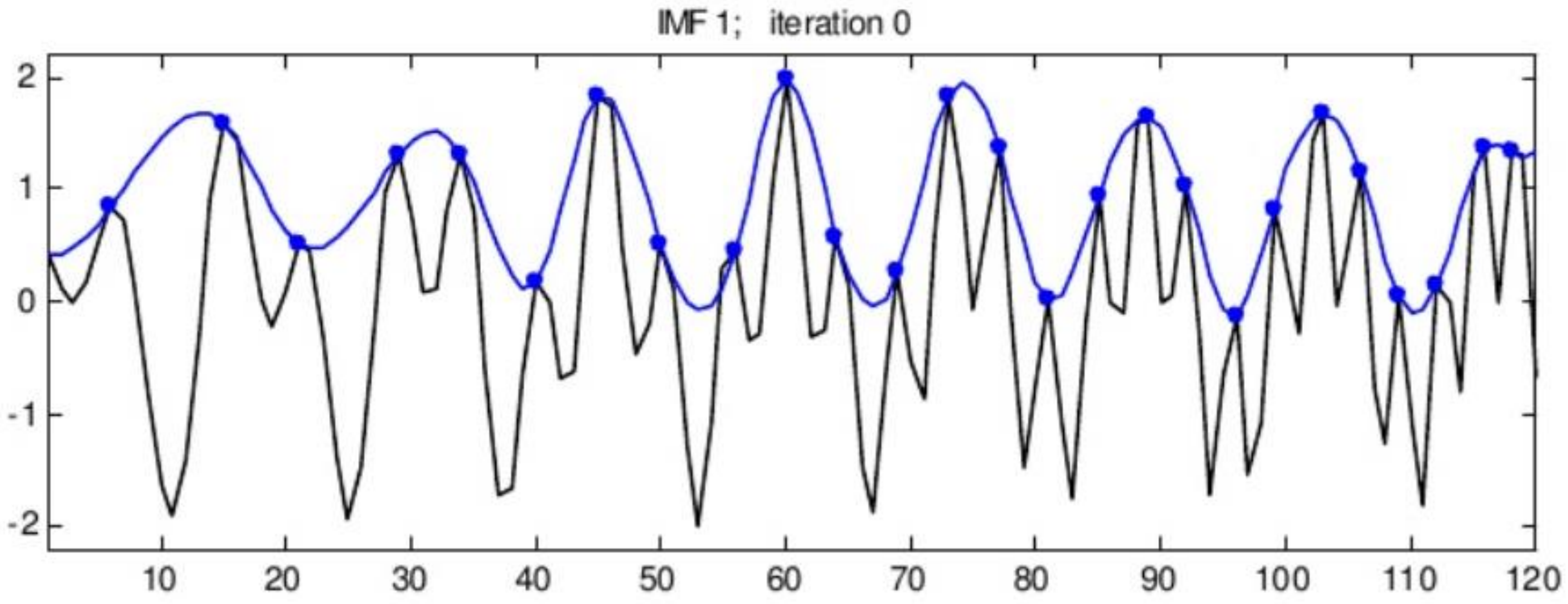
BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Method : Sifting Process
  - Get upper extrema of x(t)



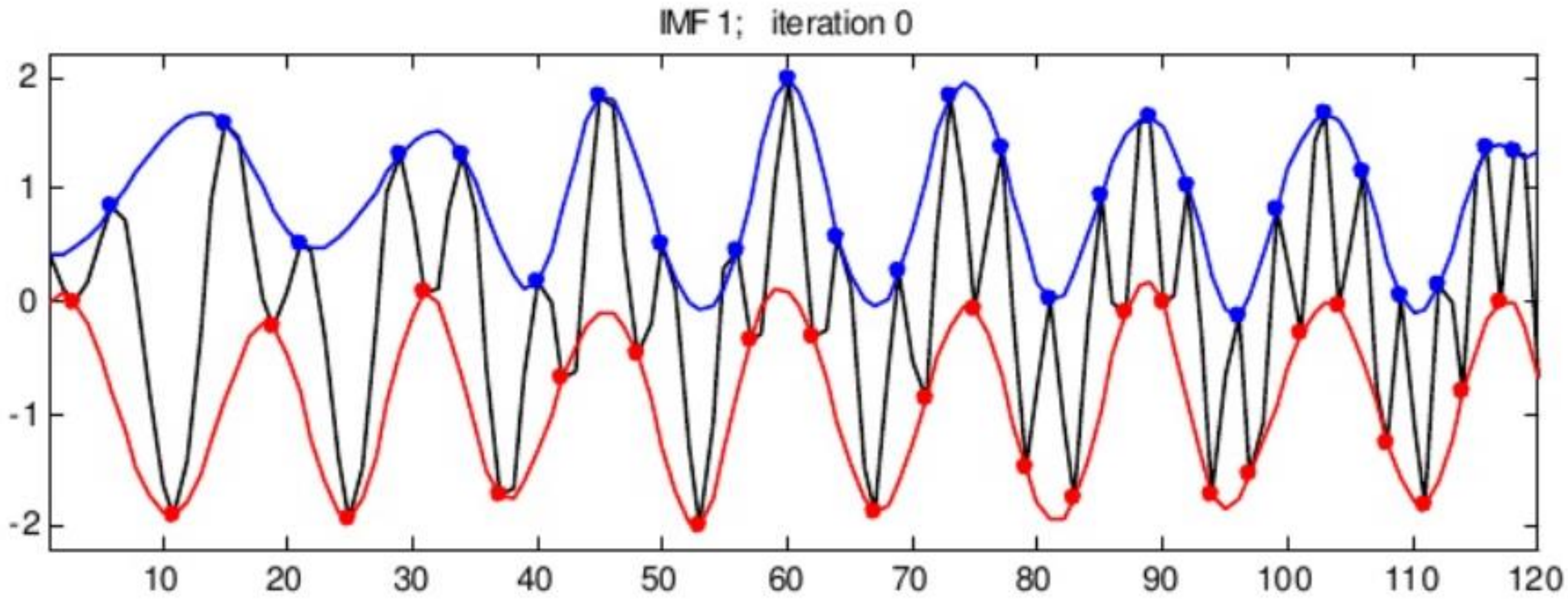
BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Method : Sifting Process
  - Interpolate(spline interpolation) the local maxima to form an upper envelope u(x)



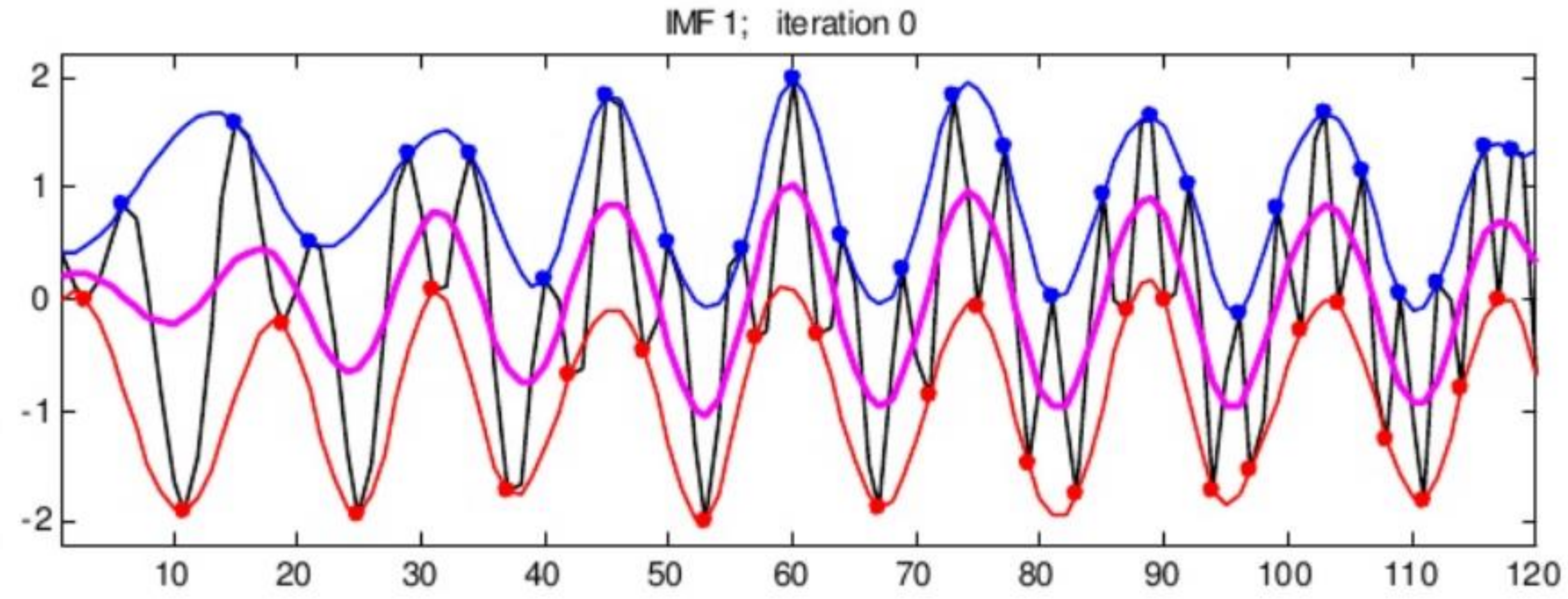
BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Method : Sifting Process
  - Get a lower envelope(l(x) with same way.



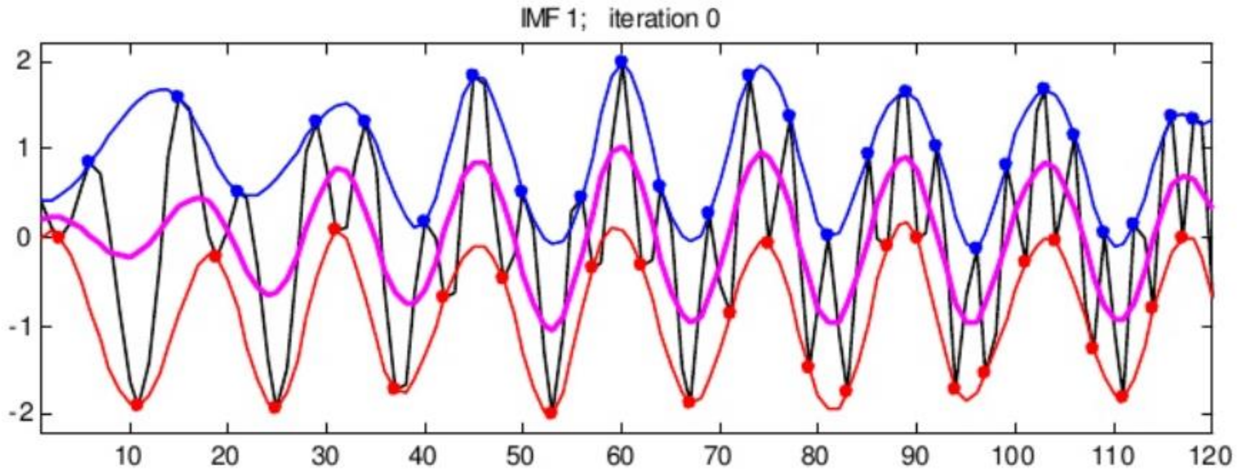
BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Method : Sifting Process
  - Calculate the mean envelope m(t) = [u(t) + l(t)]/2



BACKGROUND
WHAT IS EMD(**E**MPIRICAL **M**ODE **D**ECOMPOSITION)?

- Method : Sifting Process
  - Subtract the mean from the signal : h(t) = x(t) – m(t)

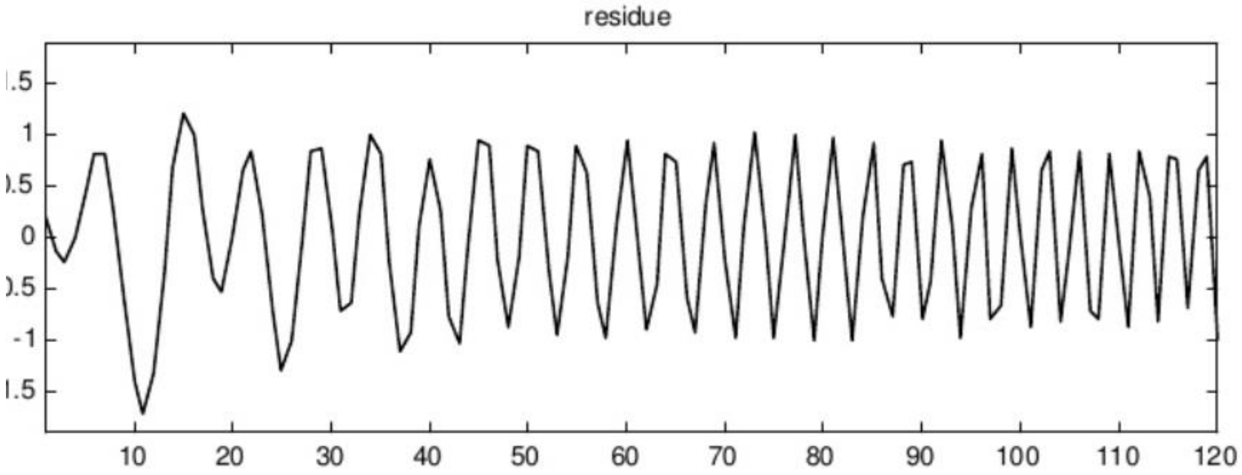


- Repeat until h(t) satisfy the IMF condition

  (Inner loop)
- Get All IMF components

  (Outer loop)

$$X(t) = \sum_{i=1}^{N} \mathrm{IMF}_i(t) + r_N(t)$$



INTRODUCTION

- Most real-life signals are **non-stationary** and **non-linear**.

  : Standard techniques(FFT, Wavelet) can't capture their **hidden features** properly.
- EMD allows to unravel the hidden features of a non-stationary signals by iteratively decomposing it into a finite sequence of simple components(IMF).
- But EMD is difficult to analyze **mathematically**.
- **Alternate** : many methods based of **IF**(Iterative Filtering)

IF ALGORITHM IN THE CONTINUOUS SETTING

- To get IMF, IF algorithm approximate the **moving average** of *s* (signal) and subtract it from s itself.
  - The approximated moving average is computed by **convolution** of *s* with a filter function *w*.
- w : filter/window

$$\int_{\mathbb{R}} w(z)dz = \int_{-L}^{L} w(z)dz = 1$$

- ℓ_m: filter length
-

$$\mathcal{M}_m(s_m) = s_m - \mathcal{L}_m(s_m) = s_{m+1}$$

$$IMF_1 = \lim_{m\to\infty} \mathcal{M}^m(s)(x) = \int_{-\infty}^{\infty} \hat{s}(\xi)\chi_{\{\hat{w}(\xi)=0\}} e^{2\pi i\xi x} d\xi$$

Algorithm 1 **Iterative Filtering** IMF $=$ IF(s)

IMF $= \{\}$
while the number of extrema of $s \geqslant 2$ **do**
  $s_1 = s$
  while the stopping criterion is not satisfied **do**
    compute the filter length l_m for $s_m(x)$
    $s_{m+1}(x) = s_m(x) - \boxed{\int_{-l_m}^{l_m} s_m(x+t)w_m(t)\mathrm{d}t}$
    $m = m + 1$
  end while
  IMF $=$ IMF $\cup \{s_m\}$
  $s = s - s_m$
end while
IMF $=$ IMF $\cup \{s\}$

Moving average
$\mathcal{L}_m(s_m)$

IF ALGORITHM IN THE DISCRETE SETTING

- Signals which we are dealing with are **discrete**.
- w : filter

$$\int_{\mathbb{R}} w(z)\,dz = \int_{-L}^{L} w(z)\,dz = 1 \xrightarrow{\text{discrete}} \sum_{p=1}^{n} w_p = 1$$

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$$s_{m+1}(x_i) = s_m(x_i) - \int_{x_i - l_m}^{x_i + l_m} s_m(y) w_m(x_i - y)\mathrm{d}y$$

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$$\int_{\mathbb{R}} w(z)\,dz = \int_{-L}^{L} w(z)\,dz = 1 \xrightarrow{\text{discrete}} \sum_{p=1}^{n} w_p = 1$$

$$s_{m+1} = (I - W_m)s_m$$

Matrix form

$$s_{m+1}(x_i) = s_m(x_i) - \boxed{\int_{x_i - l_m}^{x_i + l_m} s_m(y)\,w_m(x_i - y)\,\mathrm{d}y} \approx s_m(x_i) - \boxed{\sum_{x_j = x_i - l_m}^{x_i + l_m} s_m(x_j)\,w_m(x_i - x_j)\frac{1}{n}}, \quad j = 0, \ldots, n-1$$

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$$IMF_1 = \lim_{m\to\infty} \mathcal{M}^m(s)(x) \xrightarrow{\text{discrete}} \mathrm{IMF}_1 = \lim_{m\to\infty}(I - W_m)s_m$$

W_m = W for every m

$$\mathrm{IMF}_1 = \lim_{m\to\infty}(I - W)^m s$$

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$$s_{m+1}(x_i) = s_m(x_i) - \boxed{\int_{x_i - l_m}^{x_i + l_m} s_m(y) w_m(x_i - y)\,\mathrm{d}y} \approx s_m(x_i) - \boxed{\sum_{x_j = x_i - l_m}^{x_i + l_m} s_m(x_j) w_m(x_i - x_j) \frac{1}{n}}, \quad j = 0, \ldots, n-1$$

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W_m = W for every m

$$\boxed{\mathrm{IMF}_1 = \lim_{m \to \infty} (I - W)^m s}$$

IF ALGORITHM IN THE DISCRETE SETTING

Algorithm 2 **Discrete Iterative Filtering** IMF $=$ DIF(s)

IMF $= \{\}$
while the number of extrema of $s \geqslant 2$ **do**
  $s_1 = s$
  while the stopping criterion is not satisfied **do**
    compute the function $w_m(\xi)$, whose half support length l_m is based on the signal $[s_m(x_i)]_{i=0}^{n-1}$
    $s_{m+1}(x_i) = s_m(x_i) - \boxed{\sum_{j=0}^{n-1} s_m(x_j) w_m(|x_i - x_j|) \frac{1}{n}}, \qquad i = 0, \ldots, n-1$
    $m = m + 1$
  end while
  IMF $=$ IMF $\cup \{s_m\}$
  $s = s - s_m$
end while
IMF $=$ IMF $\cup \{s\}$

EFFICIENT IMPLEMENTATION OF THE DIF ALGORITHM

W : digonalizable

$$W = UDU^T$$

D : diagonal matrix containing in its diagonal eigenvalues of W

$$(I - W) = U(I - D)U^T$$

$$\lim_{m \to \infty} (I - W)^m = \lim_{m \to \infty} U(I - D)^m U^T = UZU^T$$

$$IMF_1 = \lim_{m \to \infty} (I - W)^m s = UZU^T s$$

Z : diagonal matrix

U : Consists of u_p as columns

$$u_p = \frac{1}{\sqrt{n}} \left[1,\ e^{-2\pi i p \frac{1}{n}}, \ldots,\ e^{-2\pi i p \frac{n-1}{n}}\right]^T$$

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$$W_m = \begin{bmatrix} c_0 & c_{n-1} & \cdots & c_1 \\ c_1 & c_0 & \cdots & c_2 \\ \vdots & \vdots & \ddots & \vdots \\ c_{n-1} & c_{n-2} & \cdots & c_0 \end{bmatrix}$$

: Discrete convolution operator
<= circulant matrix

Circulant matrix's eigenvalues

$$\lambda_j = c_0 + c_{n-1}\omega_j + \ldots + c_1\omega_j^{n-1}$$

$$\lambda_j = c_0 + 2\sum_{k=1}^{\frac{n-1}{2}} c_k \cos\left(\frac{2\pi jk}{n}\right)$$

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Approximated IMF1

$$\overline{\mathrm{IMF}}_1 = (I - W)^{N_0} s = U(I - D)^{N_0} U^T s$$

N_0 : minimum Natural number

$$W_m = \begin{bmatrix} c_0 & c_{n-1} & \cdots & c_1 \\ c_1 & c_0 & \cdots & c_2 \\ \vdots & \vdots & \ddots & \vdots \\ c_{n-1} & c_{n-2} & \cdots & c_0 \end{bmatrix}$$

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$$u_p = \frac{1}{\sqrt{n}}\left[1,\ e^{-2\pi i p\frac{1}{n}}, \ldots,\ e^{-2\pi i p\frac{n-1}{n}}\right]^T$$ [Cpq]'s eigenvectors

$$W_m = \begin{bmatrix} c_0 & c_{n-1} & \cdots & c_1 \\ c_1 & c_0 & \cdots & c_2 \\ \vdots & \vdots & \ddots & \vdots \\ c_{n-1} & c_{n-2} & \cdots & c_0 \end{bmatrix}$$

: Discrete convolution operator

<= circulant matrix

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$$\lambda_j = c_0 + 2\sum_{k=1}^{\frac{n-1}{2}} c_k \cos\left(\frac{2\pi jk}{n}\right)$$

$$\lambda_p = \sum_{q=0}^{n-1} c_{1q} e^{-2\pi i p\frac{q}{n}}$$

DFT of $\{C_{1q}\}$ & [Cpq]'s eigenvalues

Approximated IMF1

$$\overline{\mathrm{IMF}}_1 = (I-W)^{N_0}s = U(I-D)^{N_0}U^T s$$

N_0 : minimum Natural number

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$$W_m = \begin{bmatrix} c_0 & c_{n-1} & \cdots & c_1 \\ c_1 & c_0 & \cdots & c_2 \\ \vdots & \vdots & \ddots & \vdots \\ c_{n-1} & c_{n-2} & \cdots & c_0 \end{bmatrix}$$

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Approximated IMF1

$$\overline{\mathrm{IMF}}_1 = (I-W)^{N_0}s = U(I-D)^{N_0}U^T s$$

N_0 : minimum Natural number

$$\mathrm{IMF} = \sum_{k=0}^{n-1} u_k(1-\lambda_k)^{N_0}\sigma_k = \mathrm{IDFT}\left((I-D)^{N_0}\mathrm{DFT}(s)\right)$$

: FIF (Fast Iterative Filtering)

$$\lambda_p = \sum_{q=0}^{n-1} c_{1q} e^{-2\pi i p\frac{q}{n}}$$

DFT of {C_{1q}} & [Cpq]'s eigenvalues

σ_k : the k-th element of the DFT of the signal s

CONCLUSIONS AND OUTLOOK

- We prove that the DIF is also convergent.
- We show that each IMF is a smart summation of eigenvectors of a circulant matrix.
- The DIF algorithm and the explicit formula for the IMFs derive FIF(Fast Iterative Filtering)
    - It increase its efficiency and reduce its computational complexity.
    - It decompose a signal by means of the FFT
    - Instantaneous analysis of non-stationary signals.

APPENDIX

[HTML] A fast iterative filtering decomposition and symmetric difference analytic energy operator for bearing fault extraction
Y Xu, F Fan, X Jiang - ISA transactions, 2021 - Elsevier
The fault vibration signals extracted from defective bearings are generally non-stationary and non-linear. Besides, such signals are extremely weak and easily buried by inevitable background noise and vibration interferences. Thus, the development of methods capable of ...
☆ 99 2회 인용 관련 학술자료 전체 4개의 버전 no code implementation

Impact factor : 4.035

[HTML] Schizophrenia detection technique using multivariate iterative filtering and multichannel EEG signals
K Das, RB Pachori - Biomedical Signal Processing and Control, 2021 - Elsevier
A new approach for extension of univariate iterative filtering (IF) for decomposing a signal into intrinsic mode functions (IMFs) or oscillatory modes is proposed for multivariate multi-component signals. Additionally the paper proposes a method to detect schizophrenia (Sz) ...
☆ 99 전체 3개의 버전 no code implementation

Impact factor : 3.137

Multidimensional iterative filtering: a new approach for investigating plasma turbulence in numerical simulations
E Papini, A Cicone, M Piersanti, L Franci... - Journal of Plasma ..., 2020 - cambridge.org
Turbulent space and astrophysical plasmas exhibit a complex dynamics, which involves nonlinear coupling across different temporal and spatial scales. There is growing evidence that impulsive events, such as magnetic reconnection instabilities, lead to a spatially ...
☆ 99 7회 인용 관련 학술자료 전체 6개의 버전 no code implementation

Impact factor : 1.91