

AI and Deep Learning

Logistic Regression (1)

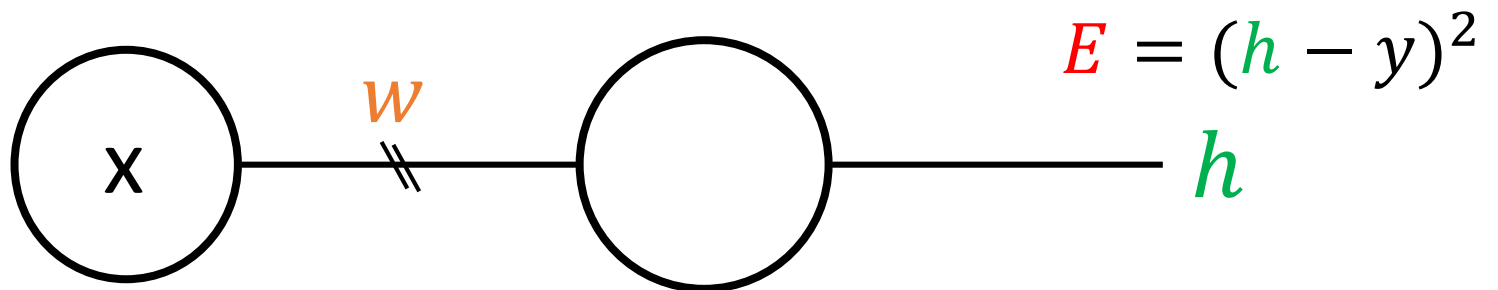
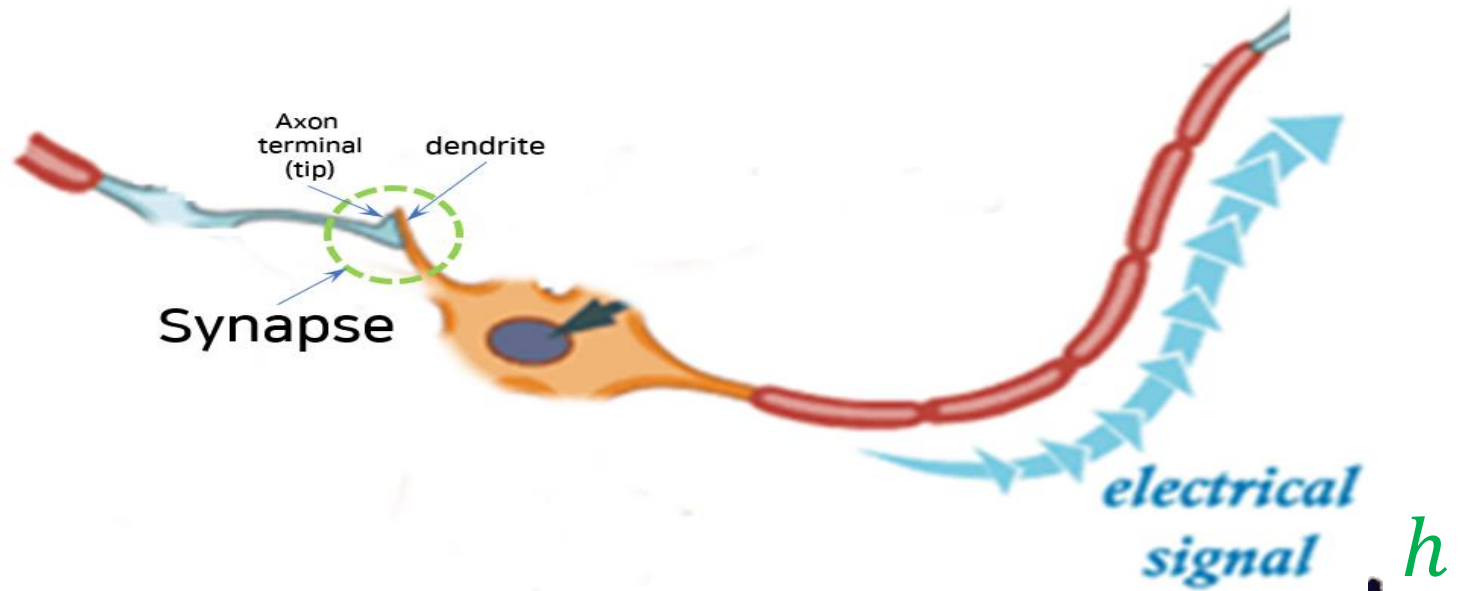
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Agenda

- Artificial Intelligence
- Brain, Neurons
- Learning
- Regression
- Deep Neural Networks
- CNN

Supervised
Learning



Logistic Regression

The shape of regression is **not linear but logistic**.

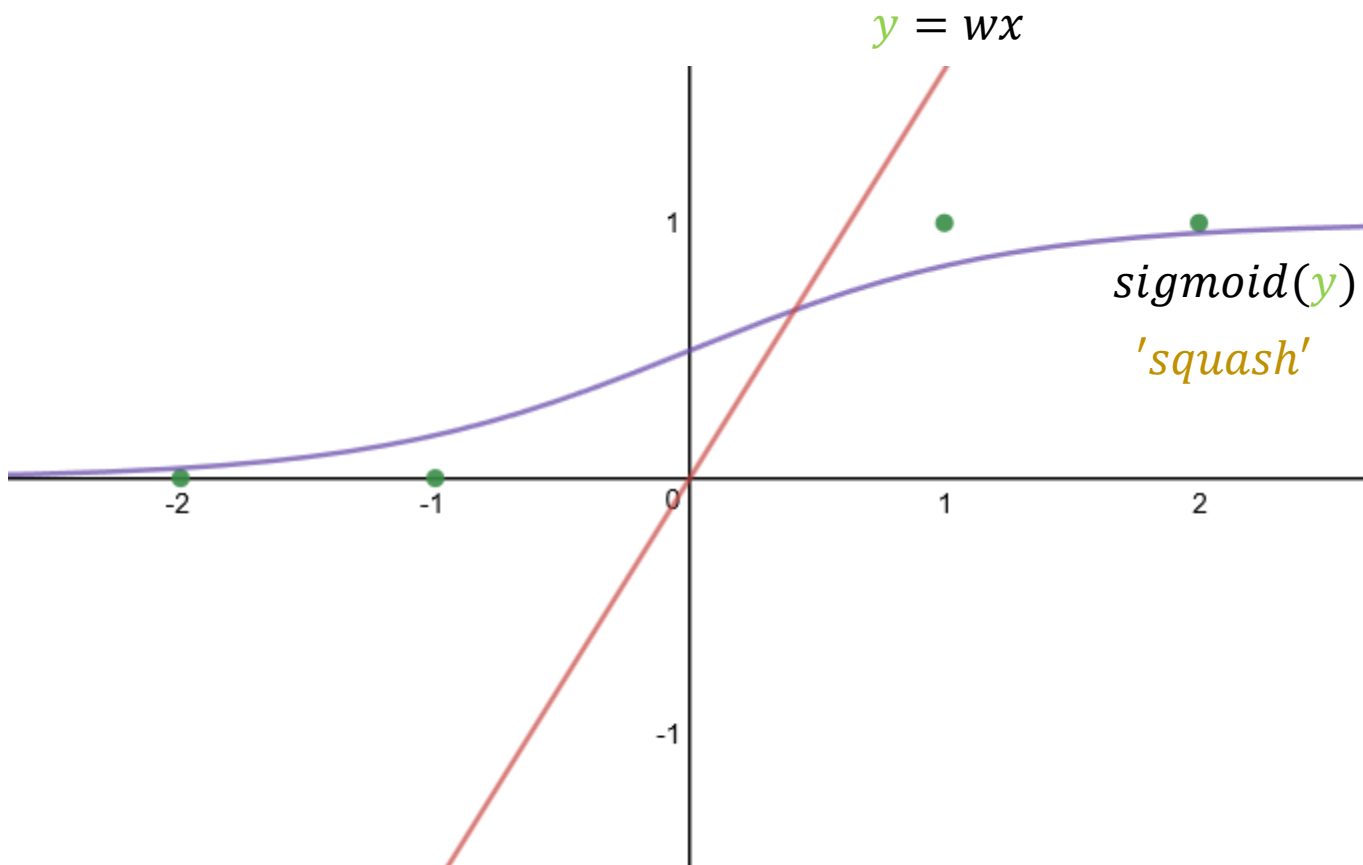
What does that mean?





점 $(-2, 0)$, $(-1, 0)$, $(1, 1)$, $(2, 1)$ 표시

$$y = wx$$

$$y = \frac{1}{1 + e^{-wx}}$$



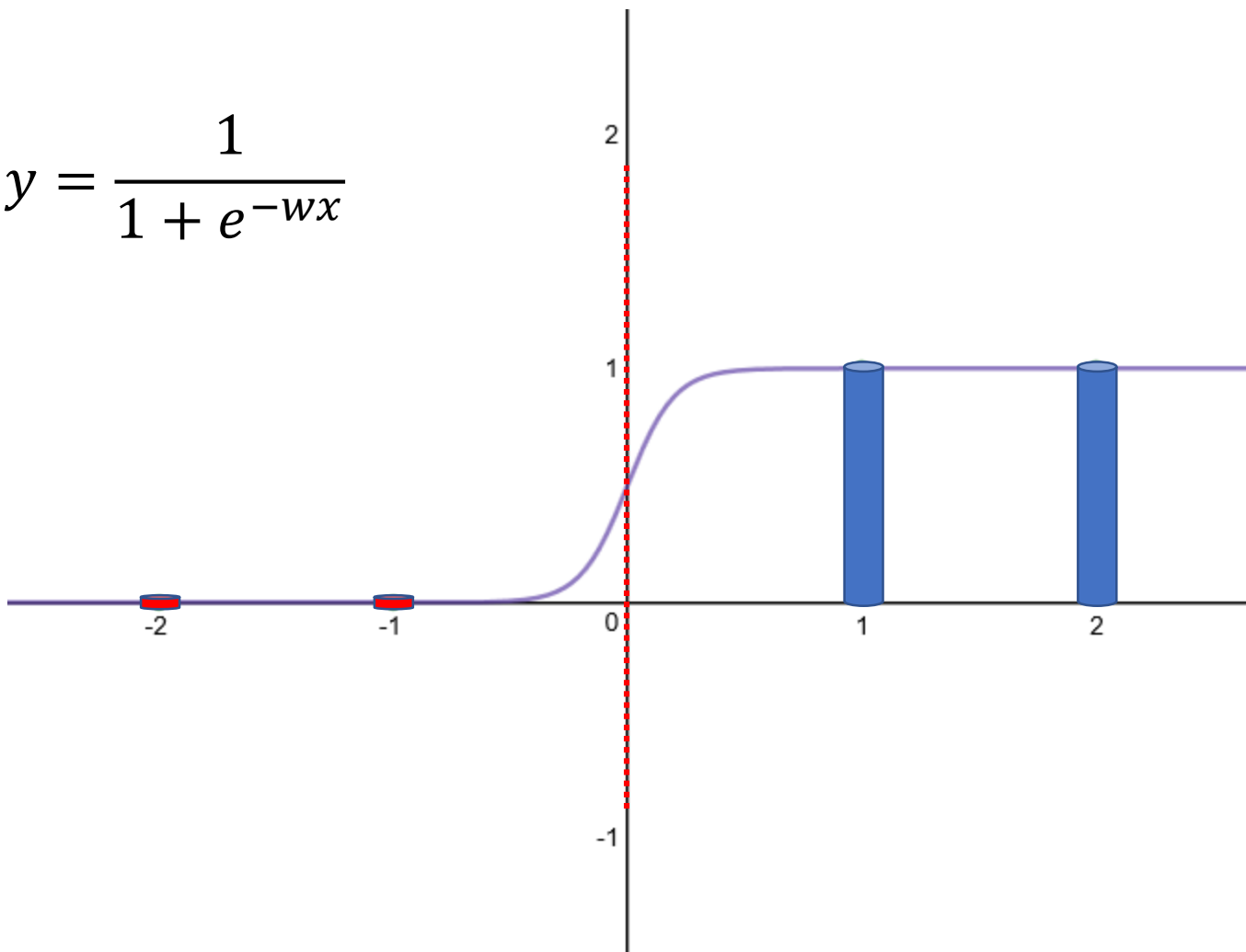
x	 y
-2	0
-1	0
1	1
2	1

 y	0	0	1	1
x	-2	-1	1	2

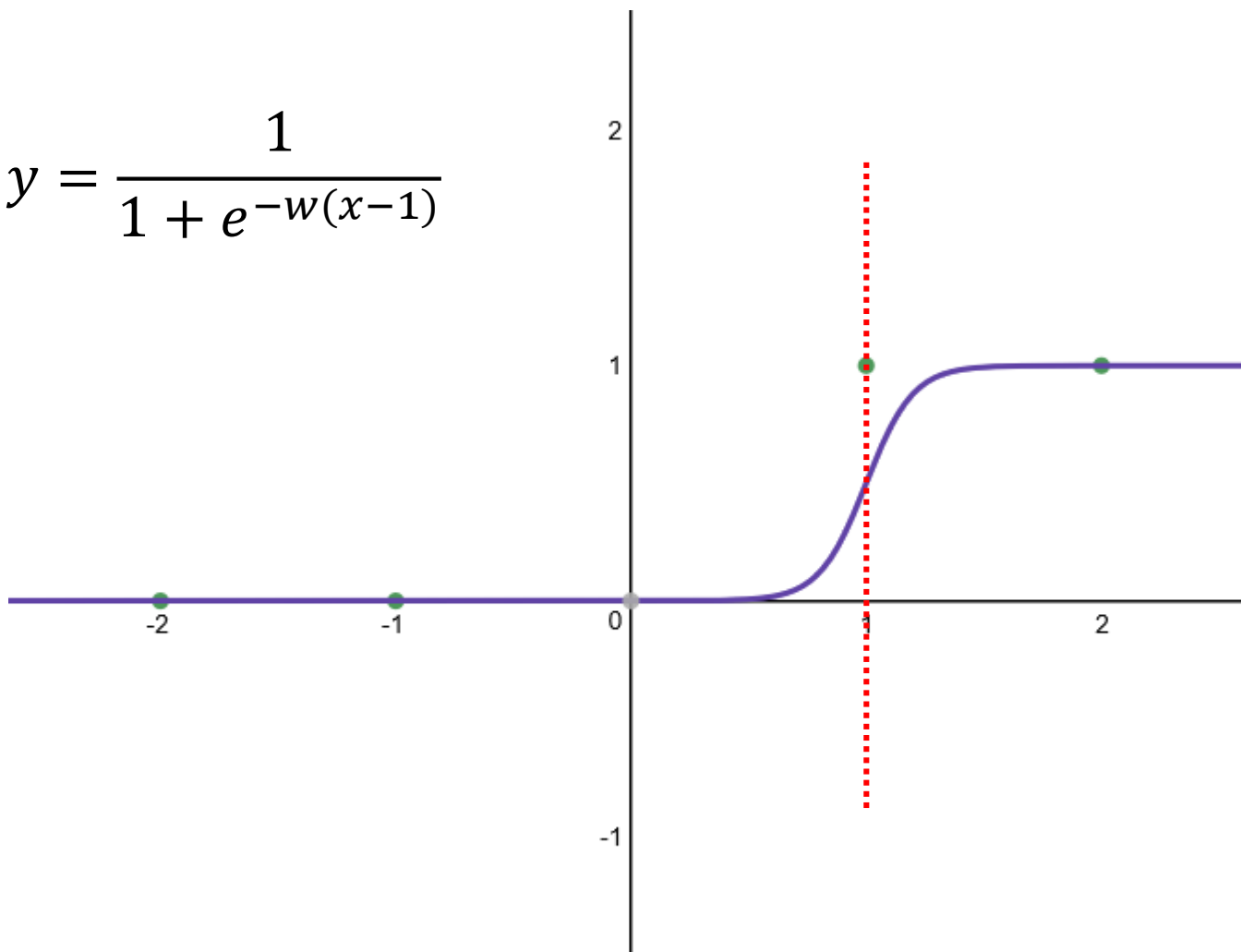


0, 1을 결정(decision)하는
경계(boundary)

$$y = \frac{1}{1 + e^{-wx}}$$



$$y = \frac{1}{1 + e^{-w(x-1)}}$$



결정 경계

$$h = \frac{1}{1 + e^{-w(\underline{x})}}$$

0

결정 경계

or,

$$h = \text{sigmoid}(wx)$$

결정 경계

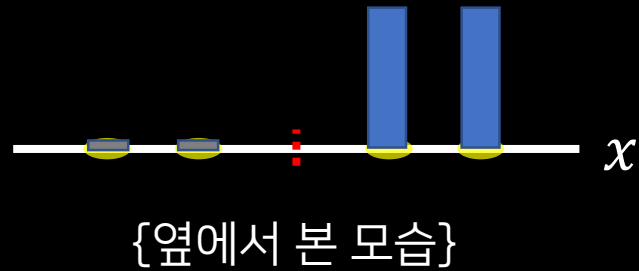
$$wx = 0$$

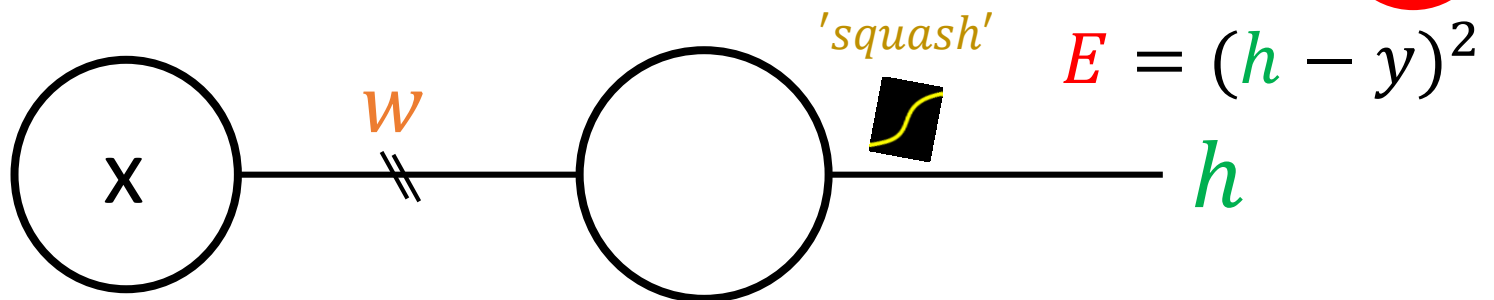
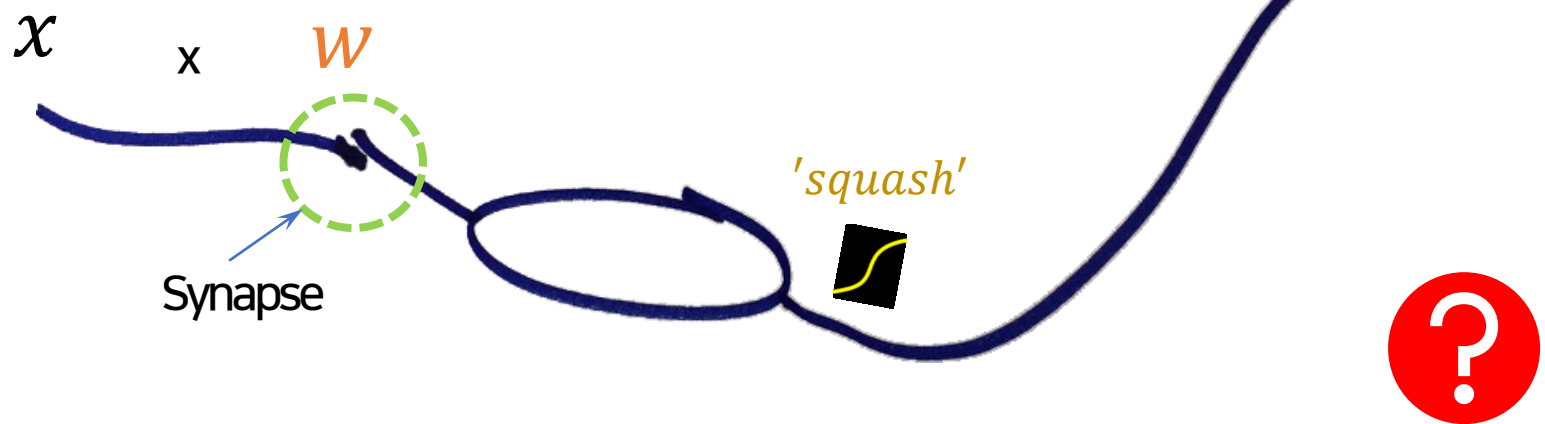
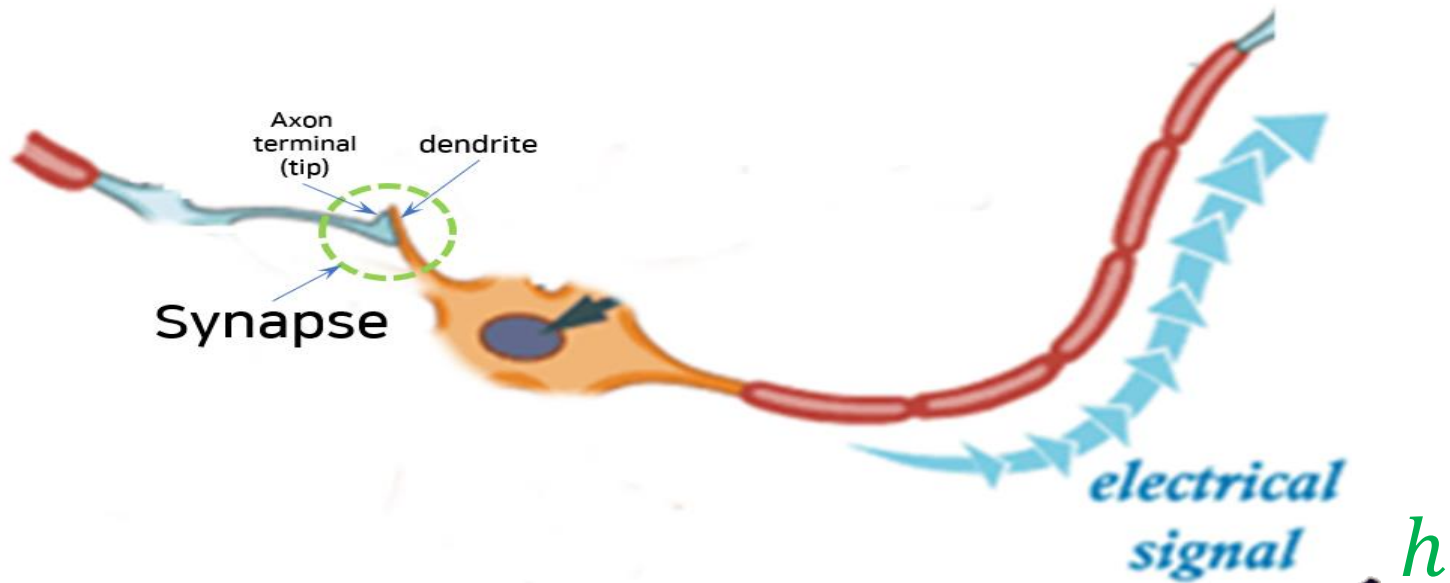
$$x = 0$$

$$wx + b = 0$$

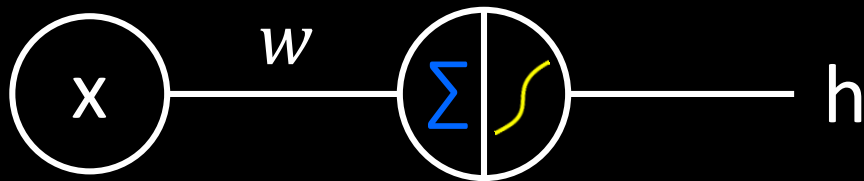
$$x + 1 = 0$$

$$2x + 3 = 0$$





신경 세포 기능

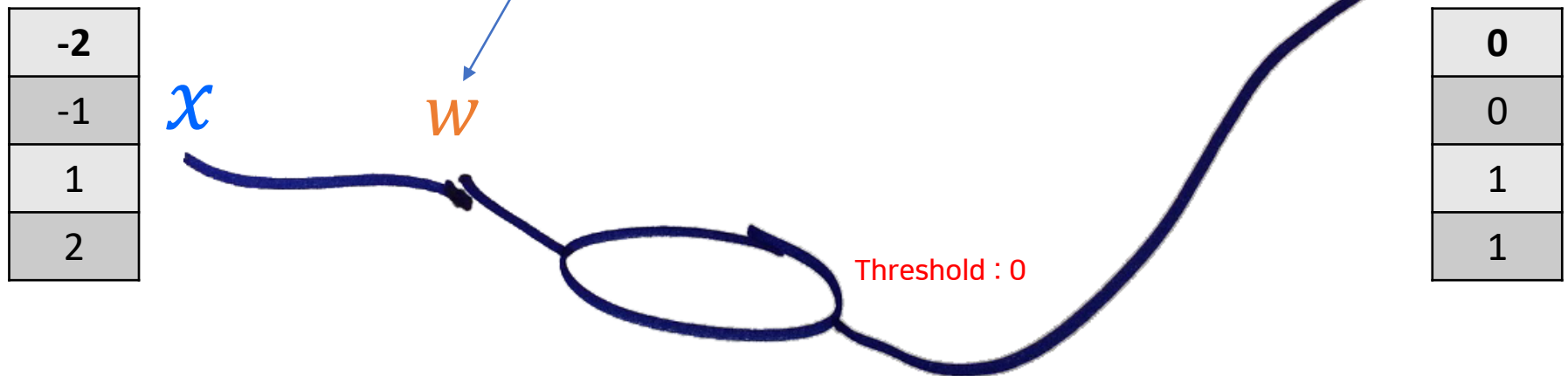


- 신경세포 1개가 할 수 있는 것은?
- 입력 x 에 따라 0, 혹은 1(fire)을 출력함.

Classification

- Pass(1) or Fail(0)
- Spam(1) or Ham(0)
- Scam(fraud, 1) or not(0)
- Safe(1) or Dangerous(0)
- Intrusion/virus(1) or not(0)
- Cancer(1) or not(0)
- Binary classification ->
Multiple classification

the parameter we tune *hypo* y



$$h = \begin{cases} 1 & \text{if } wx \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

the parameters we tune *hypo* y

-2
-1
1
2

x

w

b

$T = -\frac{b}{w}$
Threshold : T

0
0
1
1

$$y = \begin{cases} 1 & \text{if } wx + b \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

가설

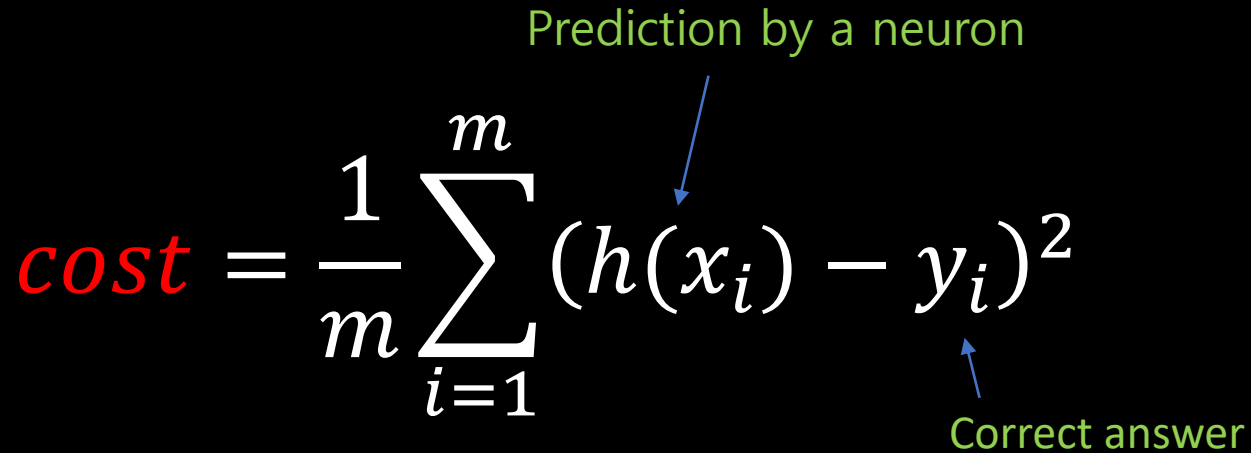
$$hypo = \frac{1}{1 + e^{-wx}}$$

오류 함수

Prediction by a neuron

$$\text{cost} = \frac{1}{m} \sum_{i=1}^m (h(x_i) - y_i)^2$$

Correct answer



“로지스틱 리그레션에도 동작할까?”



점 $(-2, 0)$, $(-1, 0)$, $(1, 1)$, $(2, 1)$ 표시

$$y = wx$$

$$y = \frac{1}{1 + e^{-wx}}$$

점 $(1, 1)$ 만 표시

$$E = \left(\frac{1}{1 + e^{-w \cdot 1}} - 1 \right)^2$$

$$(w, E)$$



점 $(-1, 0)$, $(1, 1)$ 표시

$$h = \frac{1}{1 + e^{-wx}}$$

$$E = \left(\frac{1}{1 + e^{-w \cdot -1}} - 0 \right)^2 + \left(\frac{1}{1 + e^{-w \cdot 1}} - 1 \right)^2$$

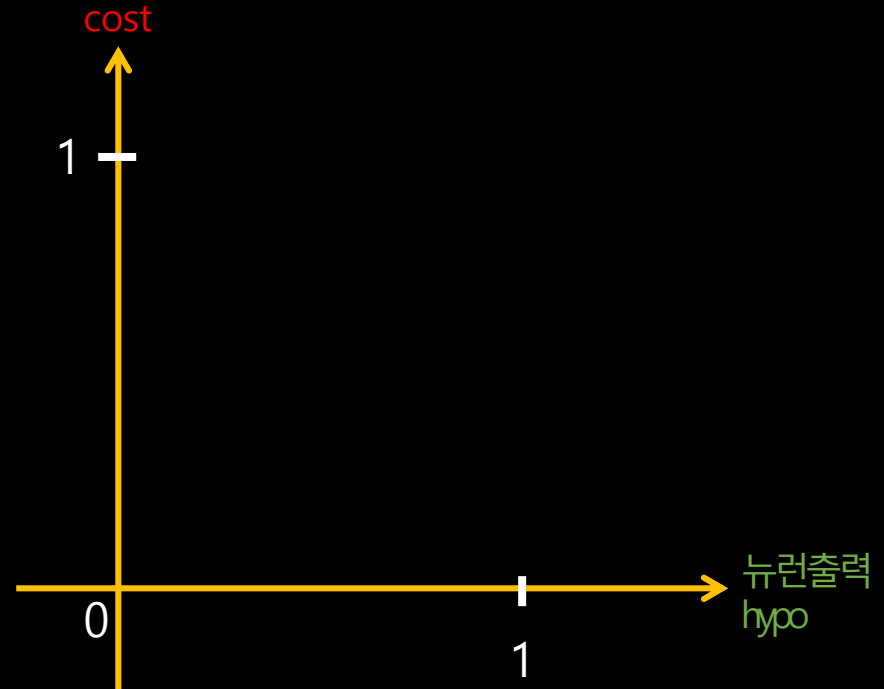
(w, E)

오류 함수

- 뉴런이 잘 맞추면 에러=0
- 뉴런이 정반대로 맞추면 에러= ∞

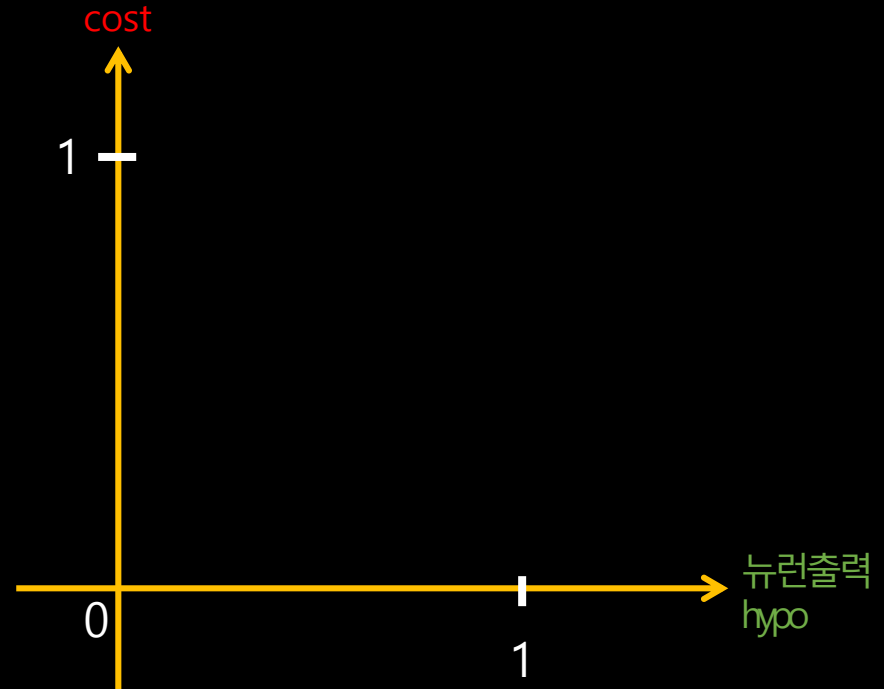
오류 함수

- 정답이 1일 때
 - 뉴런 출력(*hypo*)이 1이면 오류는 0이 되게
 - 뉴런 출력(*hypo*)이 0이면 오류는 ∞ 가 되게



오류 함수

- 정답이 0일 때
 - 뉴런 출력(*hypo*)이 0이면 오류는 0이 되게
 - 뉴런 출력(*hypo*)이 1이면 오류는 ∞ 가 되게





$$y = -\log(x)$$

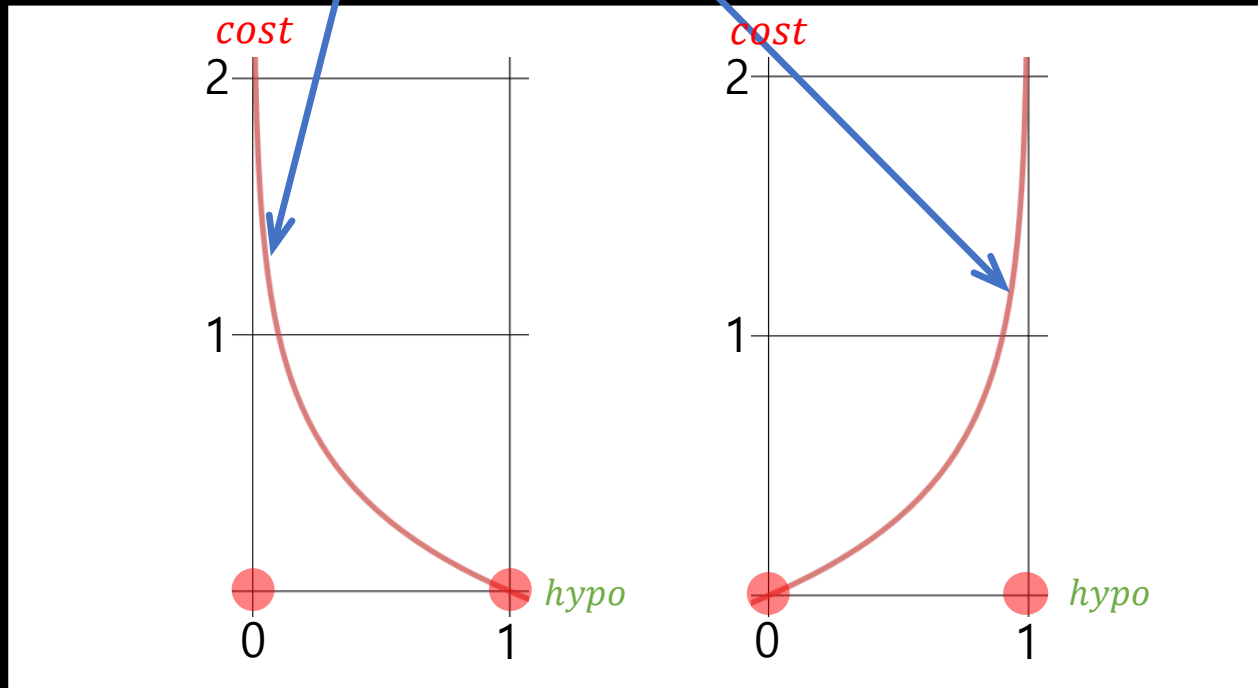
$$y = -\log(1 - x)$$

오류 함수

Prediction by computer

Correct answer

$$\text{cost} = \begin{cases} -\log(\text{hypo}) & : y = 1 \\ -\log(1 - \text{hypo}) & : y = 0 \end{cases}$$



오류 함수

$$cost = \begin{cases} -\log(H(X)) & : y = 1 \\ -\log(1 - H(X)) & : y = 0 \end{cases}$$



$$cost = -y \log(H(X)) - (1 - y) \log(1 - H(X))$$

$$cost = -(y \log(H(X)) + (1 - y) \log(1 - H(X)))$$

$$W = W - \alpha \frac{\partial}{\partial W} cost(W)$$

(실습) 11.py

음수는 0으로 양수는 1로 분류

$$cost = -(y \log(H(X)) + (1 - y) \log(1 - H(X)))$$

```
x_data = [-2., -1, 1, 2]
y_data = [0., 0, 1, 1]
```

```
#----- a neuron
w = tf.Variable(tf.random_normal([1]))
hypo = tf.sigmoid(x_data * w)
```

```
#----- learning
cost = -tf.reduce_mean(y_data * tf.log(hypo) +
                       tf.subtract(1., y_data) * tf.log(tf.subtract(1., hypo)))

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

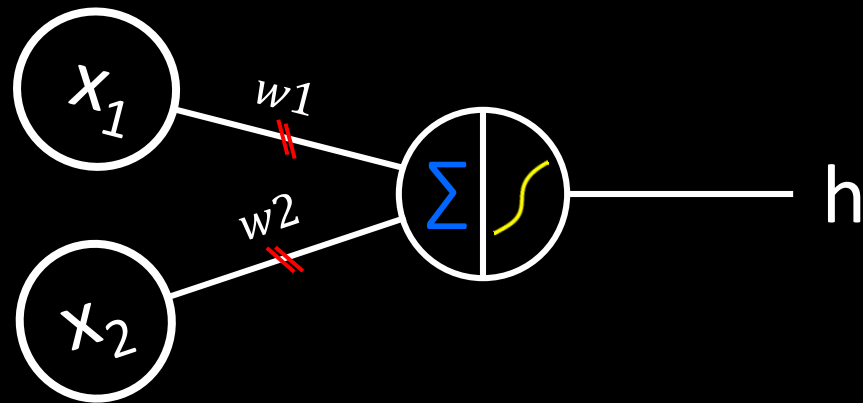
for step in range(5001):
    sess.run(train)
```

```
#----- testing(classification)
predicted = tf.cast(hypo > 0.5, dtype=tf.float32)
p = sess.run(predicted)
print("Predicted: ", p)
```

(실습) 12.py

바이어스를 갖는 뉴런

신경 세포 (2 입력)



$$h = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2)}}$$

신경 세포 (2 입력)

- 결정 경계는?

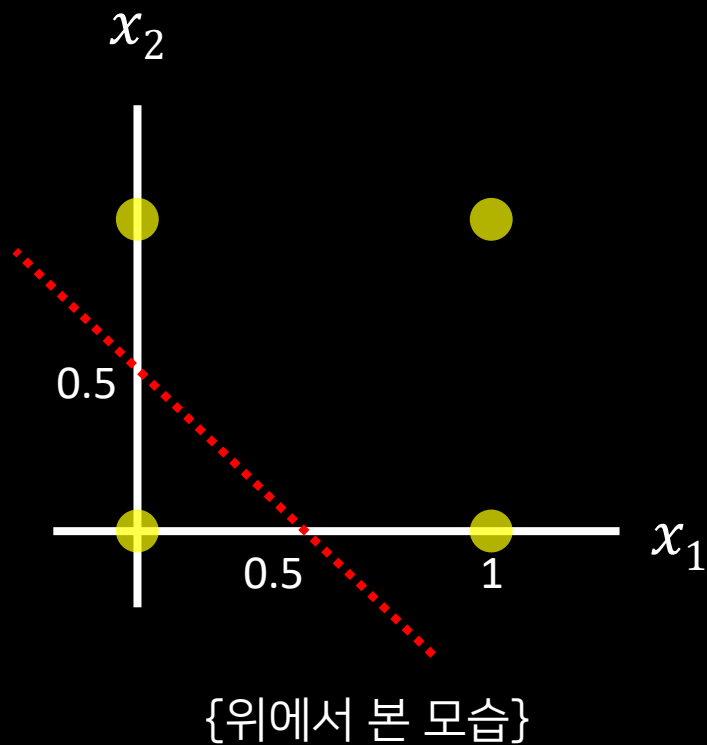
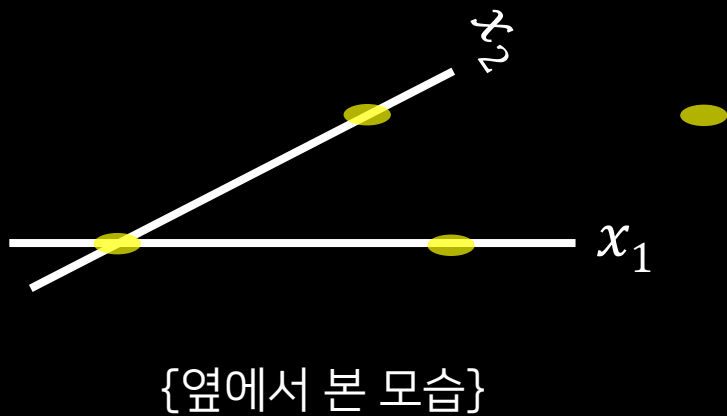
$$w_1x_1 + w_2x_2 = 0$$

$$x_1 + x_2 = 0$$

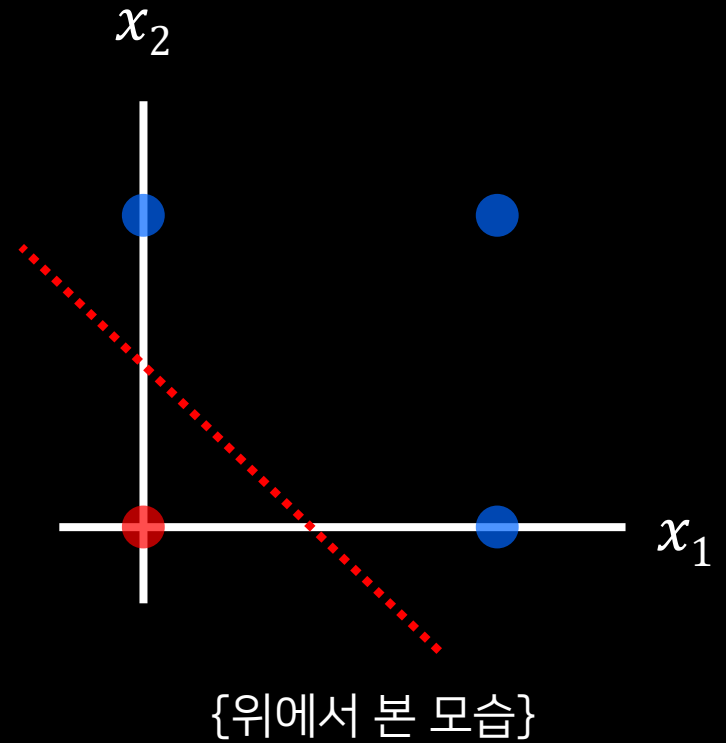
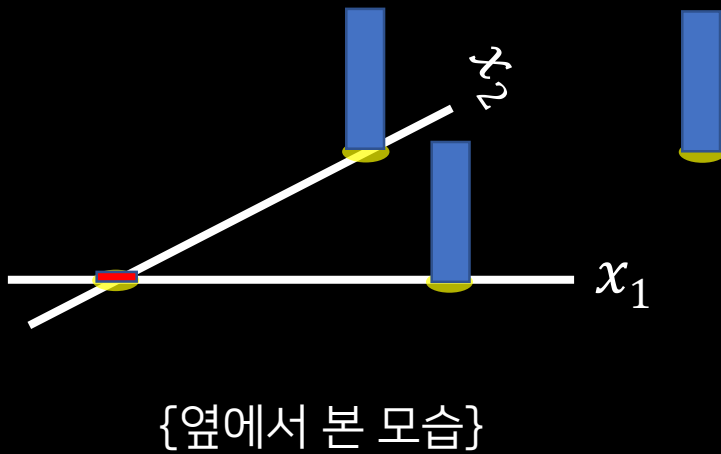
$$w_1x_1 + w_2x_2 + b = 0$$

$$x_1 + x_2 - 0.5 = 0$$

신경 세포 (2 입력)



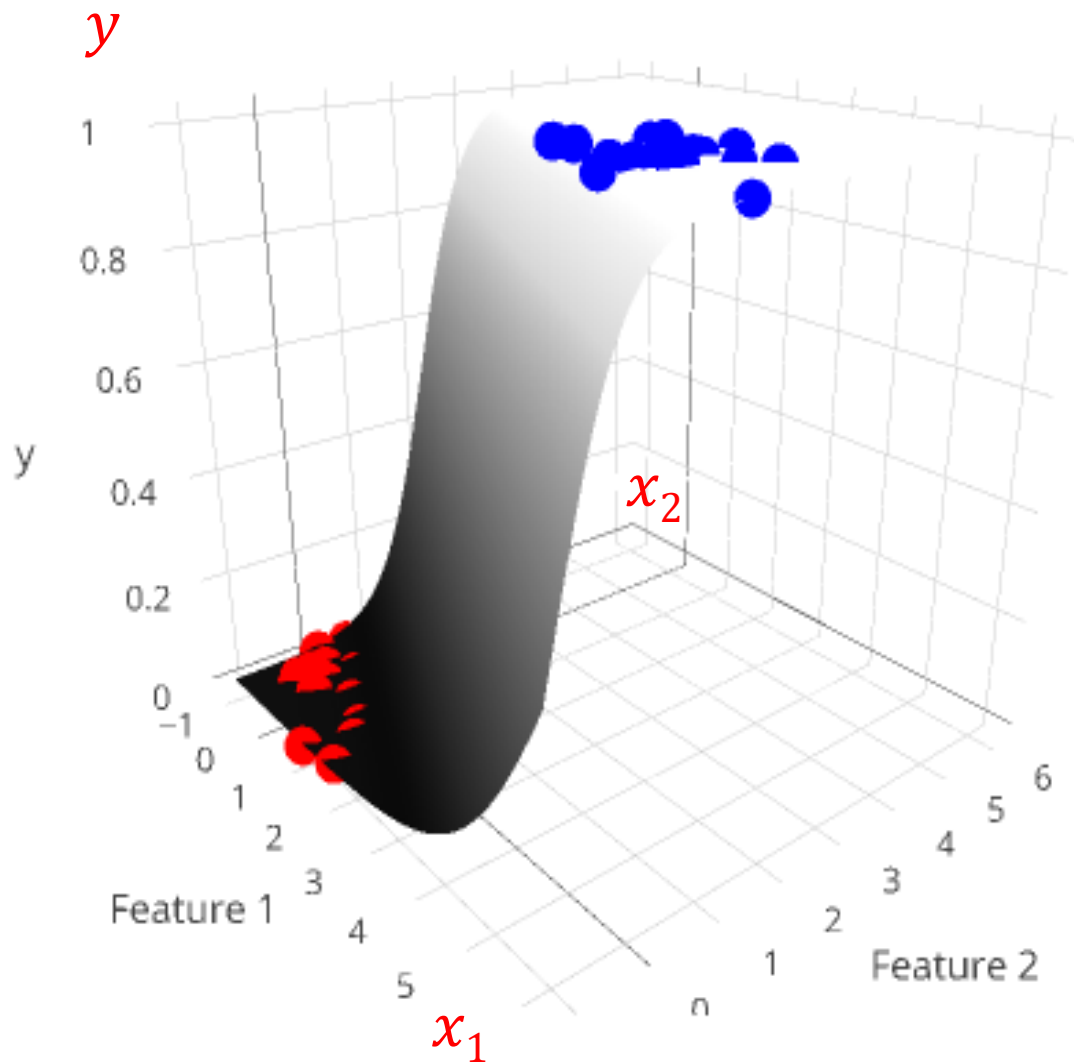
신경 세포 (2 입력)



{옆에서 본 모습}

Logistic Regression: 2 Features

미끄럼틀



● $y = 1$

● $y = 0$

$P(y=1)$

1

0.8

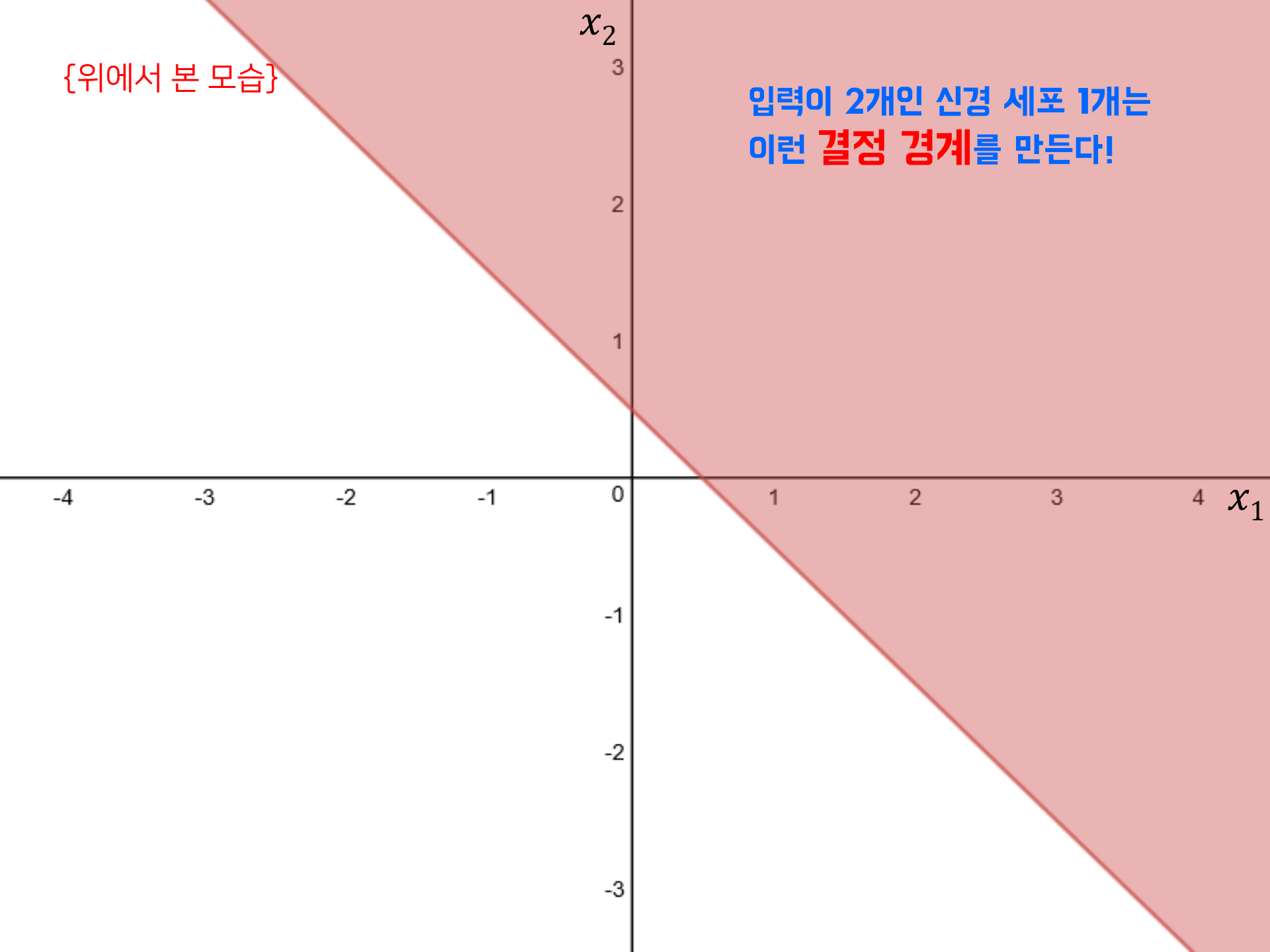
0.6

0.4

0.2

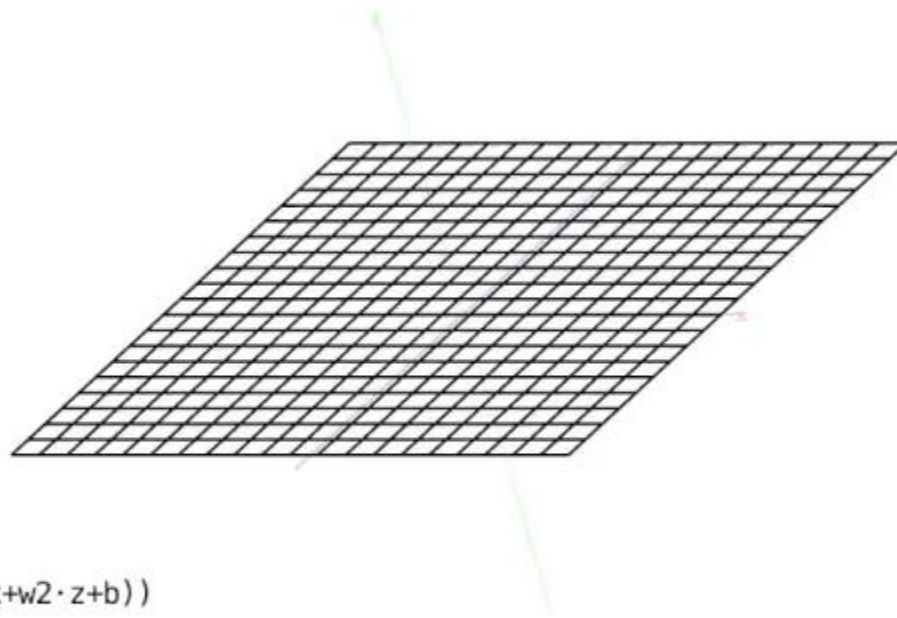
{위에서 본 모습}

입력이 2개인 신경 세포 1개는
이런 **결정 경계**를 만든다!



Decision Boundary in 3D

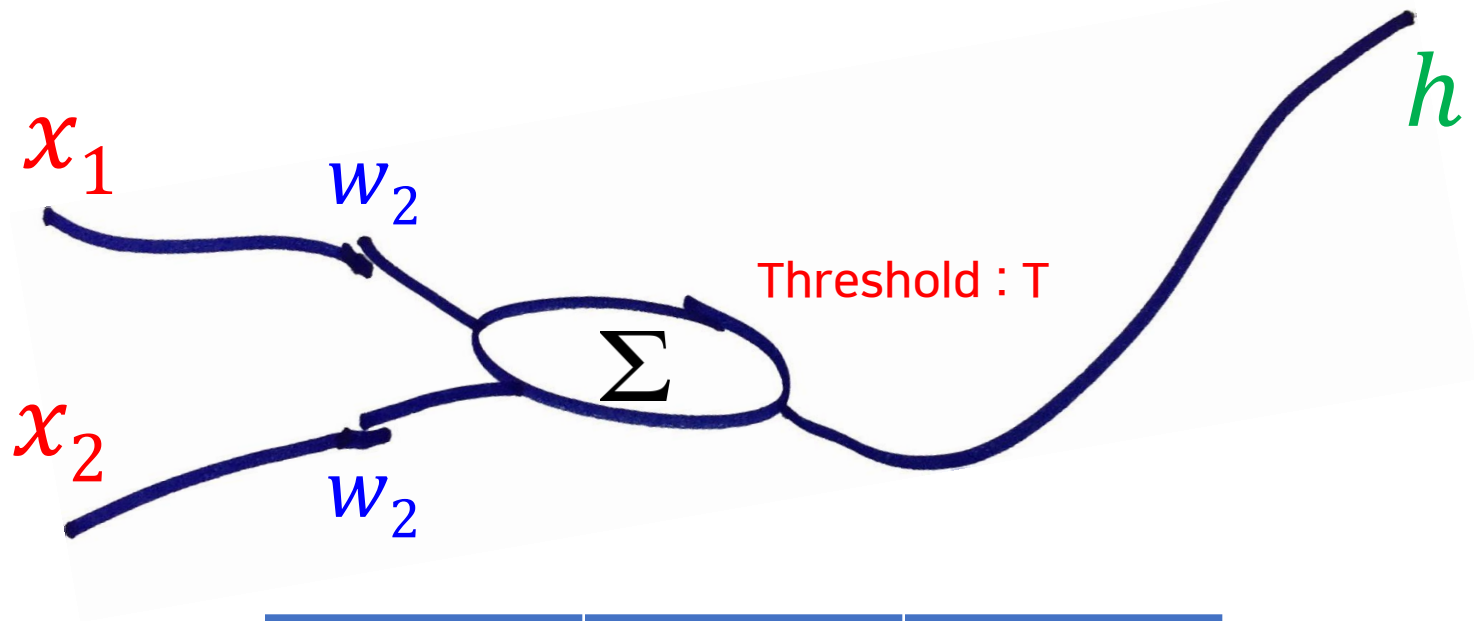
```
sigmoid(w1·length + w2·width + b)
```



```
surface(f(x,z)=sig(w1·x+w2·z+b))  
w1 = 0.00  
w2 = 0.00  
b = 0.00
```

(실습) 13.py

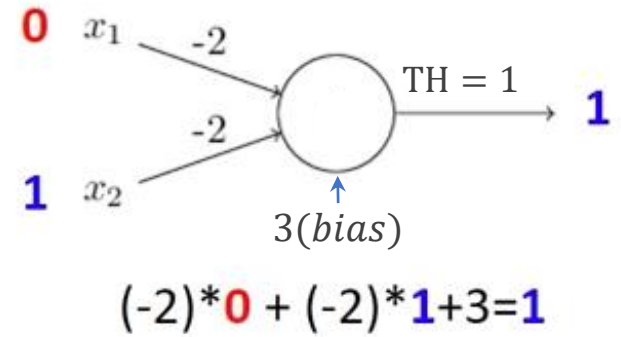
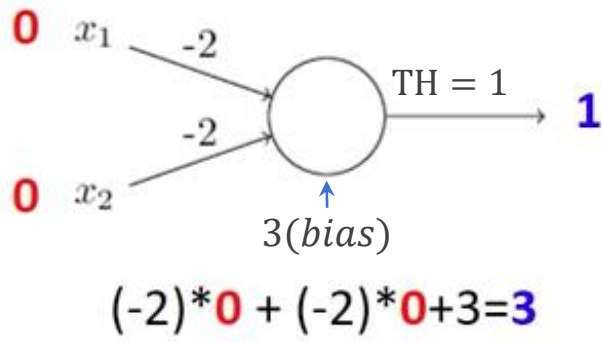
- 입력 두개(x_1, x_2)를 갖는 뉴런을 이용하여 OR 게이트를 구현함
- 한 개의 결정 경계
- $(0, 0) \rightarrow 0$, 그 외 $\rightarrow 1$



x_1	x_2	$AND(h)$
0	0	0
0	1	0
1	0	0
1	1	1

NAND

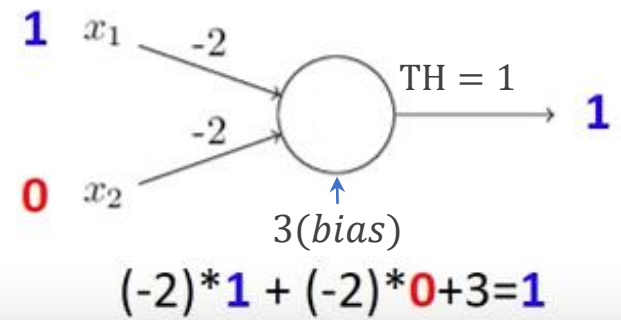
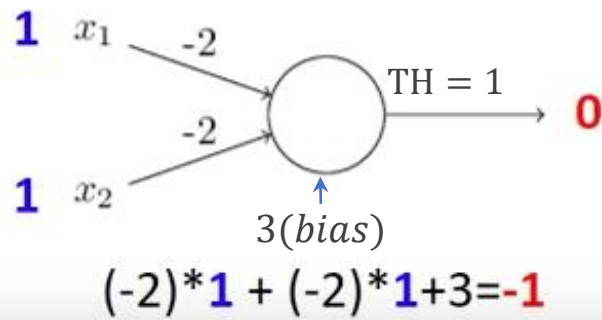
- NAND gates are functionally complete.
- We can build any logical function out of them.



NAND

Truth Table

Input A	Input B	Output Q
0	0	1
0	1	1
1	0	1
1	1	0



요약

- 로지스틱 리그레션과 분류(classification)
- 로지스틱 리그레션을 위한 cost 함수
- 한 개의 뉴런이 만들어 내는 결정 경계
- 텐서 플로우를 이용한 ML 프로그래밍