AI and Deep Learning

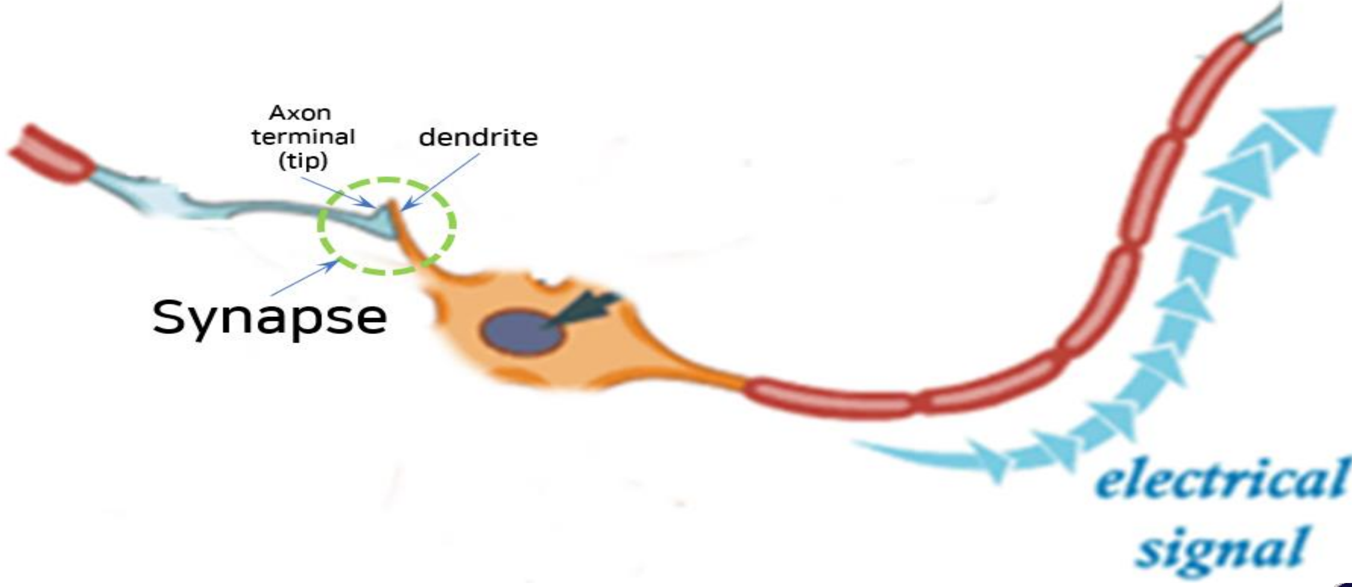
로지스틱 회귀와 분류(1)

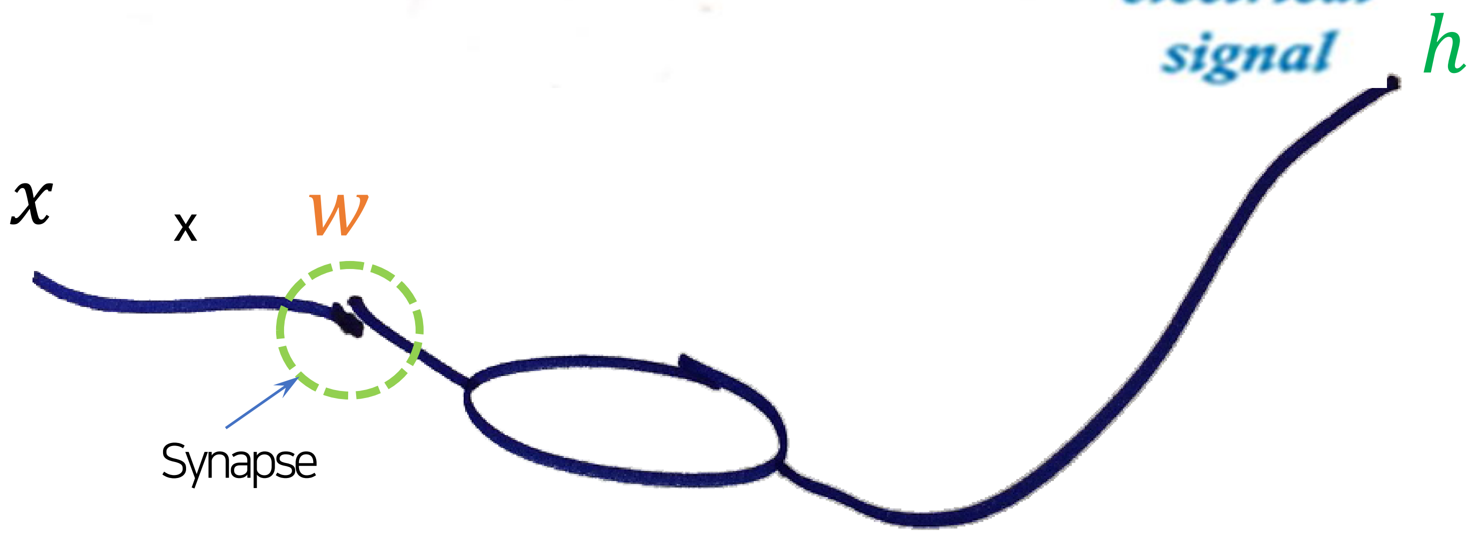
- 결정 경계 -

제주대학교

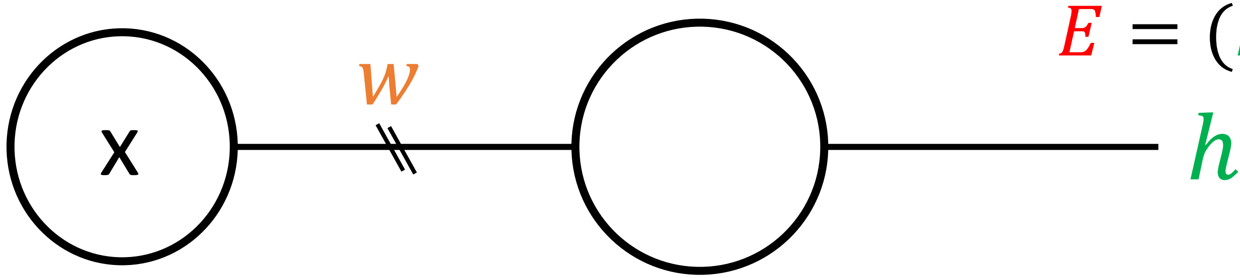
변 영 철

http://github.com/yungbyun/mllecture


Axon
terminal
(tip)
dendrite
Synapse
electrical
signal


h
x
x
w
Synapse

$$E = (h - y)^2$$


X
w
h

Logistic Regression

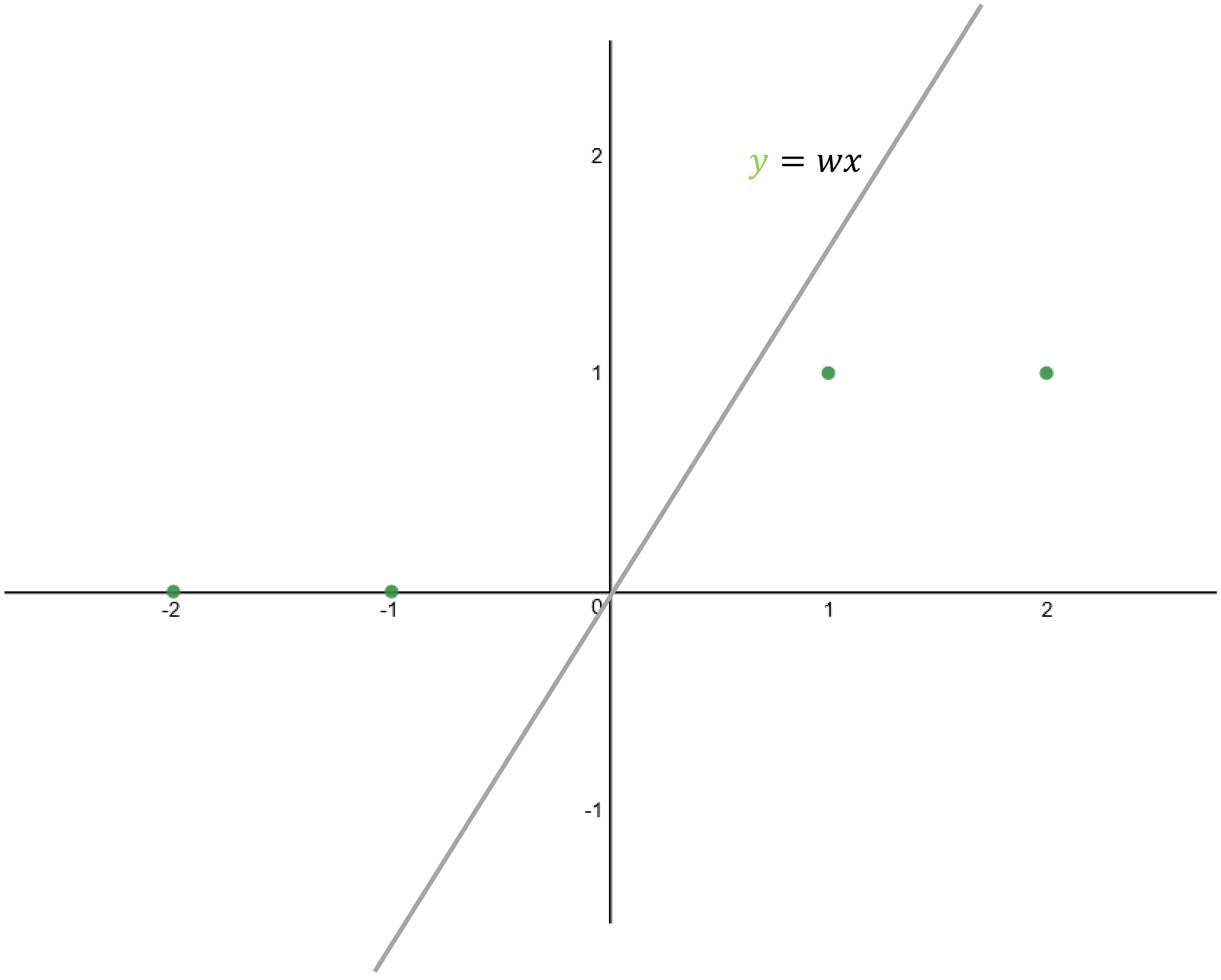
The shape of regression is not linear but logistic.

What does that mean?

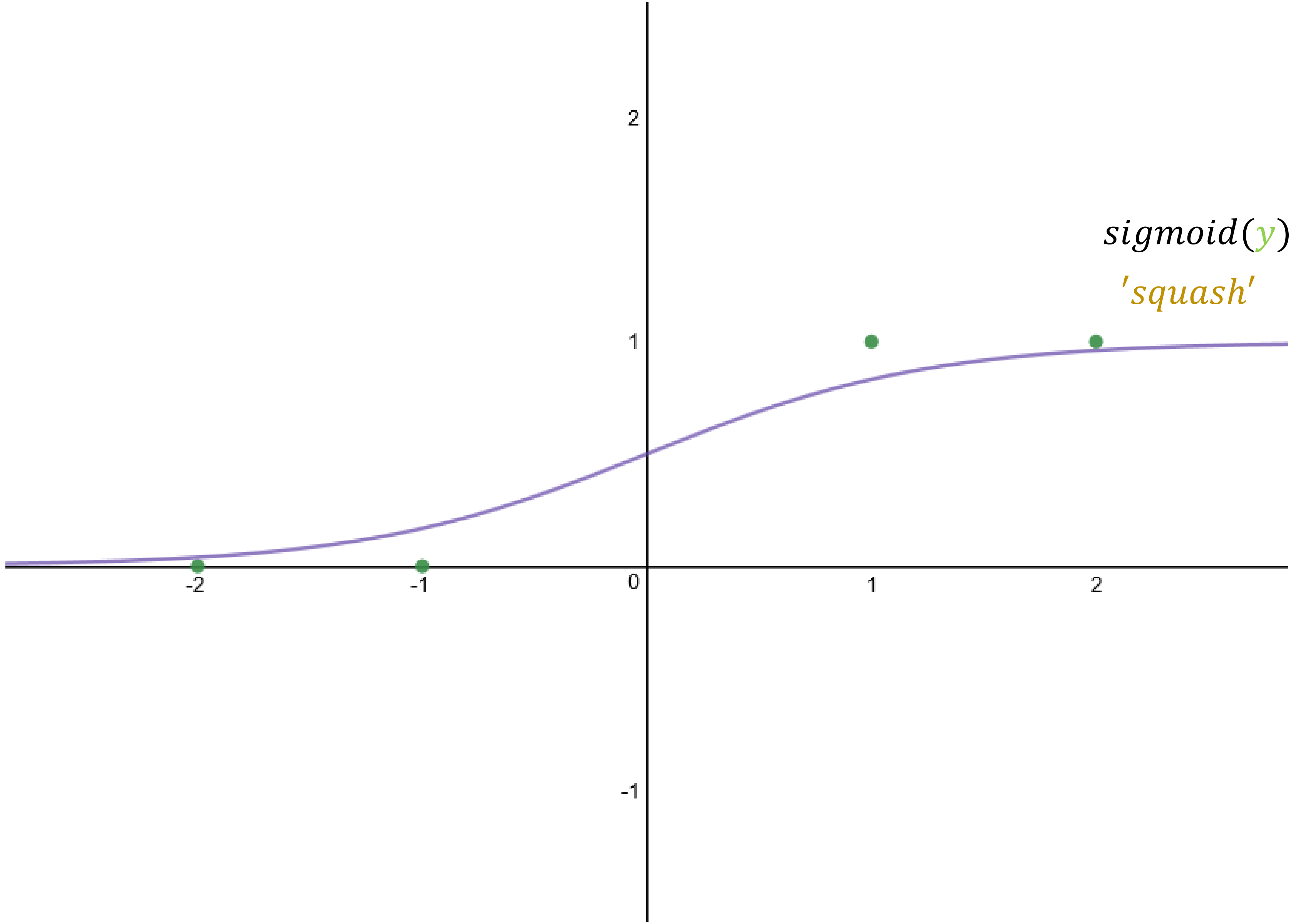
점 (-2, 0), (-1, 0), (1, 1), (2, 1) 표시

$$y = wx$$

$$y = \frac{1}{1 + e^{-wx}}$$

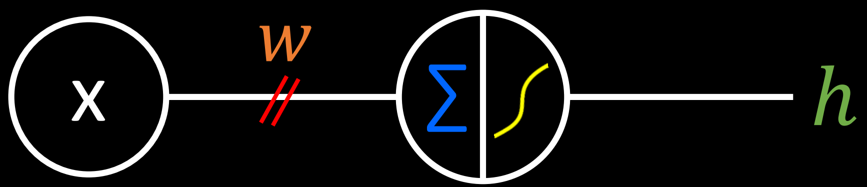


y = wx
2
1
-1
-2
-1
0
1
2



2
1
-1
-2
-1
0
1
2
$sigmoid(y)$
$'squash'$

신경 세포 기능



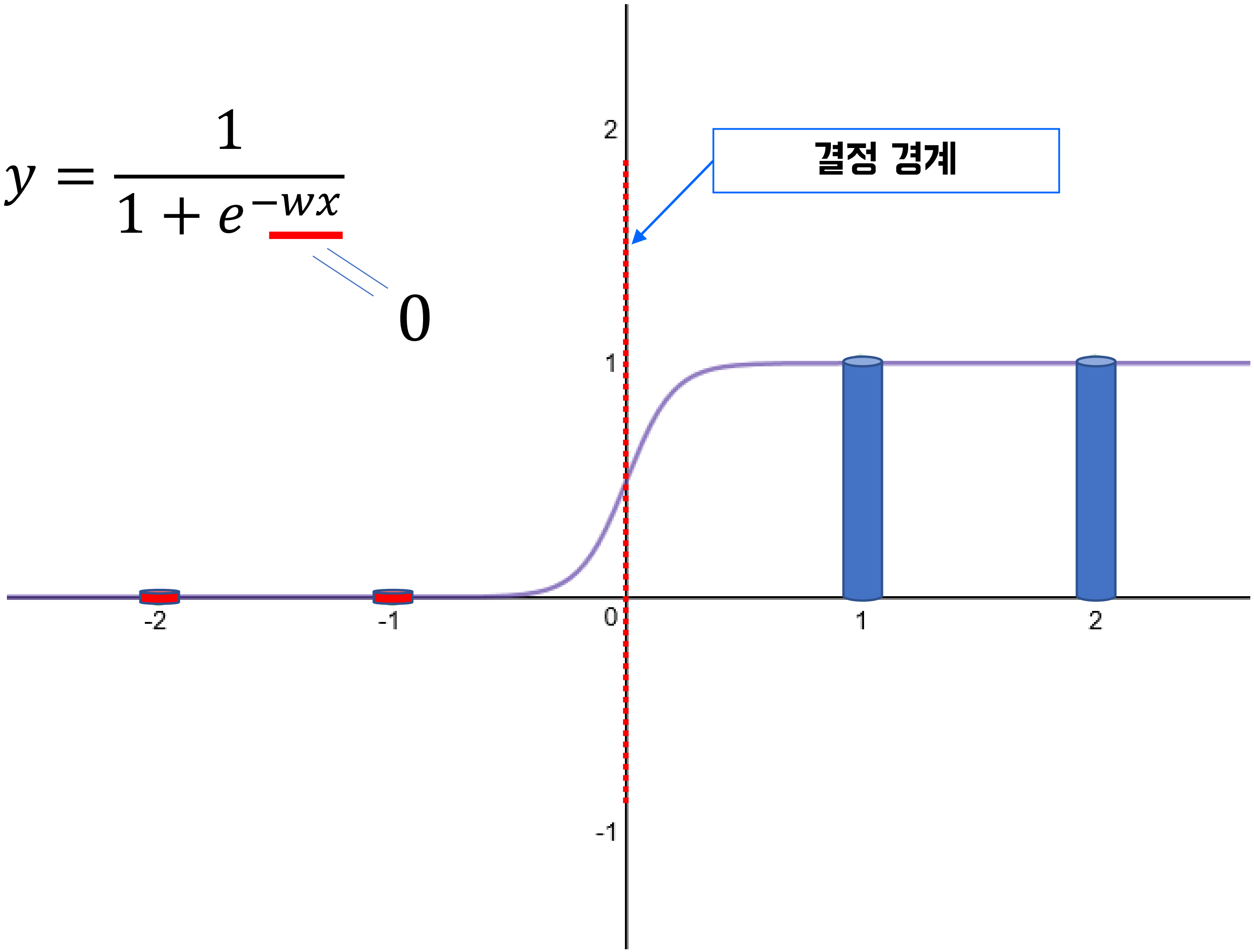
- 신경세포 1개가 할 수 있는 것은?
- 입력 x에 따라 0, 혹은 1(fire)을 출력함.
- 0 혹은 1을 결정하는 기준은?

x	y
-2	0
-1	0
1	1
2	1

x	y
-2	0
-1	0
1	1
2	1

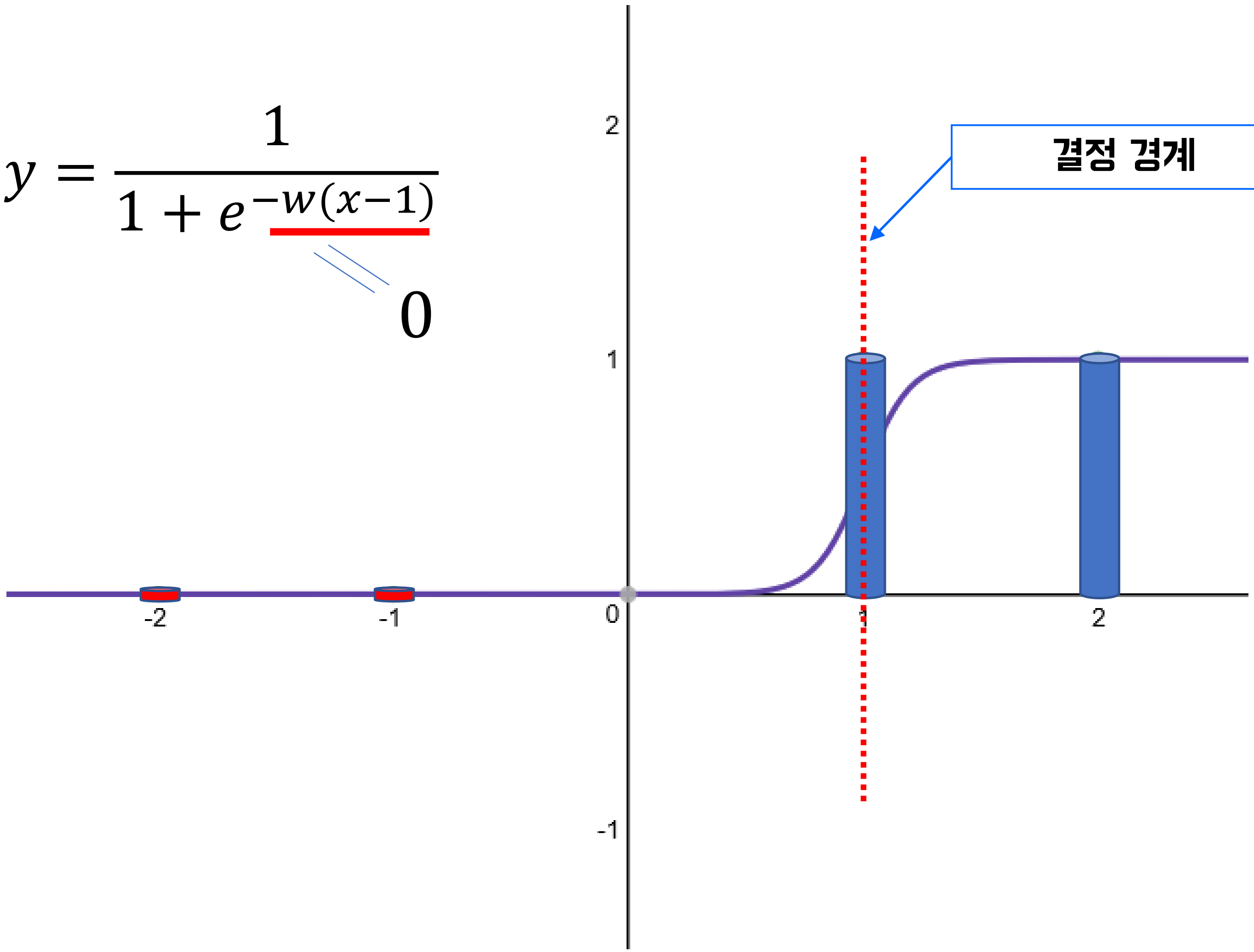
0, 1을 결정(decision)하는
경계(boundary)

$$y = \frac{1}{1+e^{\underbrace{-wx}_{=0}}}$$



$$y = \frac{1}{1 + e^{-w(x-1)}}$$

0



결정 경계

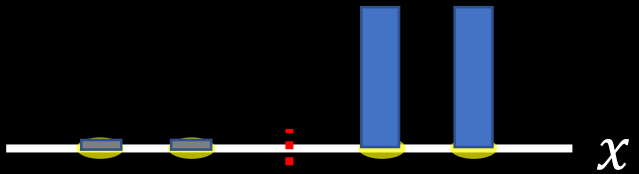
$$wx = 0$$

$$x = 0$$

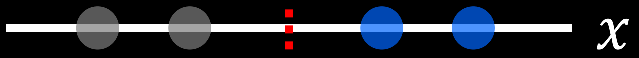
$$wx + b = 0$$

$$x + 1 = 0$$

$$2x + 3 = 0$$



{옆에서 본 모습}



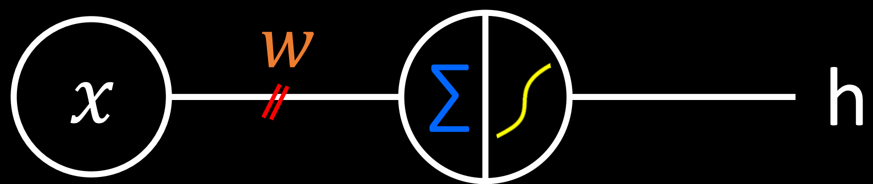
{위에서 본 모습}

Classification

- Pass(1) or Fail(0)
- Spam(1) or Ham(0)
- Scam(fraud, 1) or not(0)
- Safe(1) or Dangerous(0)
- Intrusion/virus(1) or not(0)
- Cancer(1) or not(0)
- Binary classification -> Multiple classification

뉴런 그림만 보고 결정 경계를 알 수 있을까?

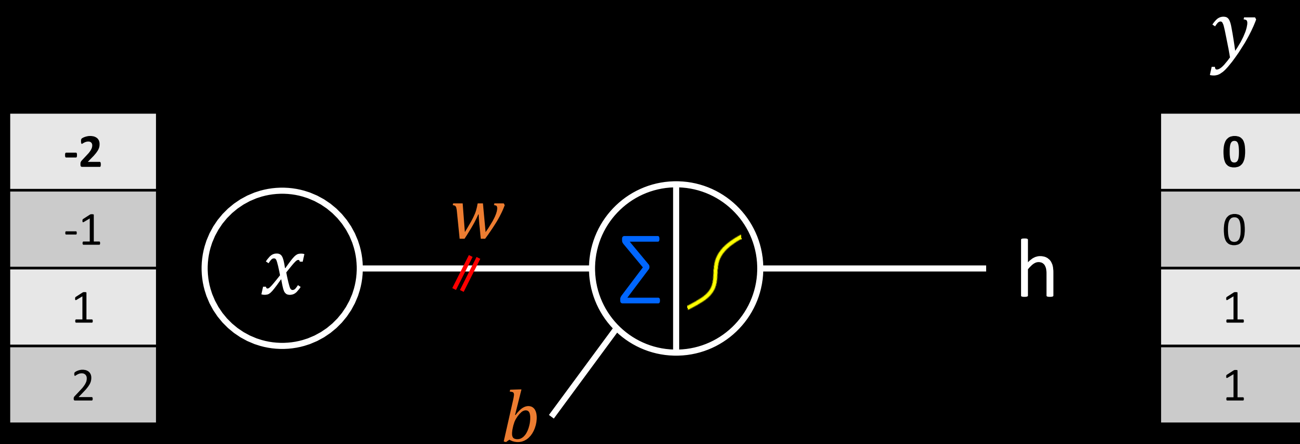
x
-2
-1
1
2



y
0
0
1
1

$$h = \begin{cases} 1 & if\ wx \geq 0 \\ 0 & otherwise \end{cases}$$

뉴런 그림만 보고 결정 경계를 알 수 있을까?



$$h = \begin{cases} 1 & if\ wx + b \geq 0 \\ 0 & otherwise \end{cases}$$

가설

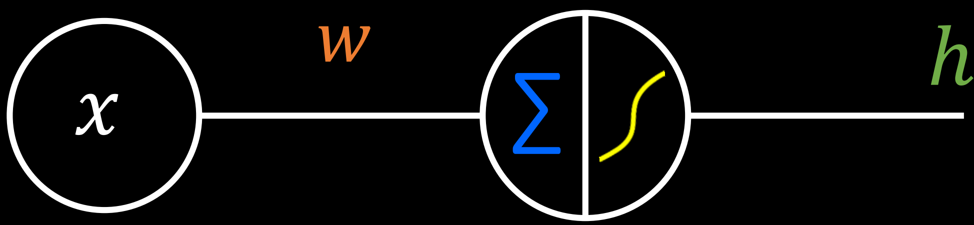
$$hypo = \frac{1}{1 + e^{-wx}}$$

가설은 뭐다?

가설 안에 결정 경계

선형 회귀 오류 함수

$$E = (h - y)^2$$



선형 회귀 오류 함수

Prediction by a neuron

$$E = \frac{1}{m}\sum_{i=1}^{m}(h_i - y_i)^2$$

Correct answer

“로지스틱 리그레션에도 동작할까?”

점 (-2, 0), (-1, 0), (1, 1), (2, 1) 표시

$$y = wx$$

$$y = \frac{1}{1 + e^{-wx}}$$

점 (1, 1)만 표시

$$E = \left(\frac{1}{1 + e^{-w \cdot 1}} - 1 \right)^2$$

$$(w, E)$$

점 (-1, 0), (1, 1) 표시

$$h = \frac{1}{1 + e^{-wx}}$$

$$E = \left(\frac{1}{1 + e^{-w\cdot -1}} - 0\right)^2 + \left(\frac{1}{1 + e^{-w\cdot 1}} - 1\right)^2$$

$$(w, E)$$

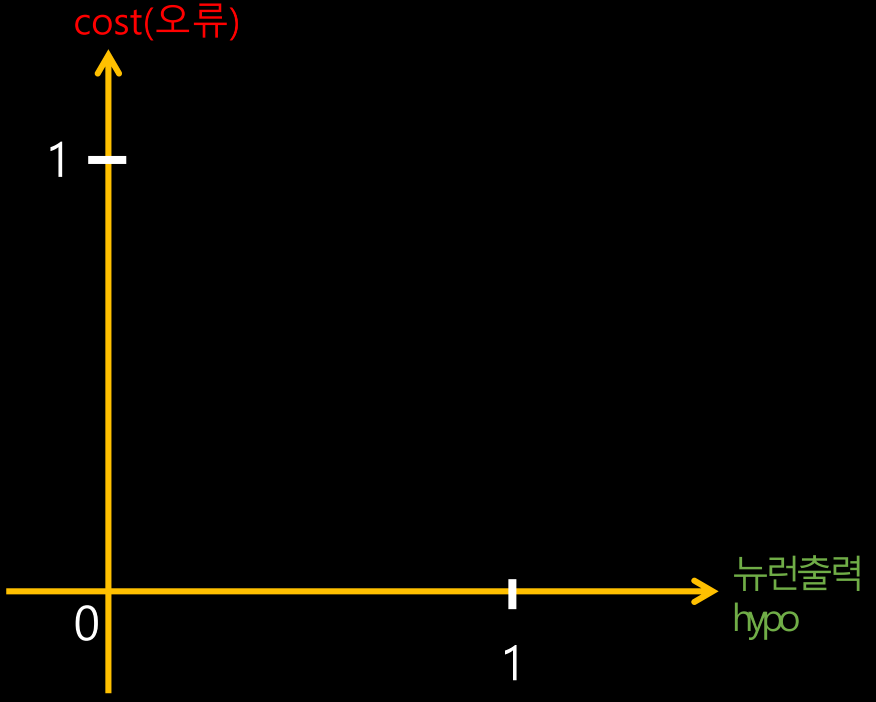
에러 그래프 모양 vs. 바람직한 모양은???

동작하지 않는다.

경사하강이 불가능

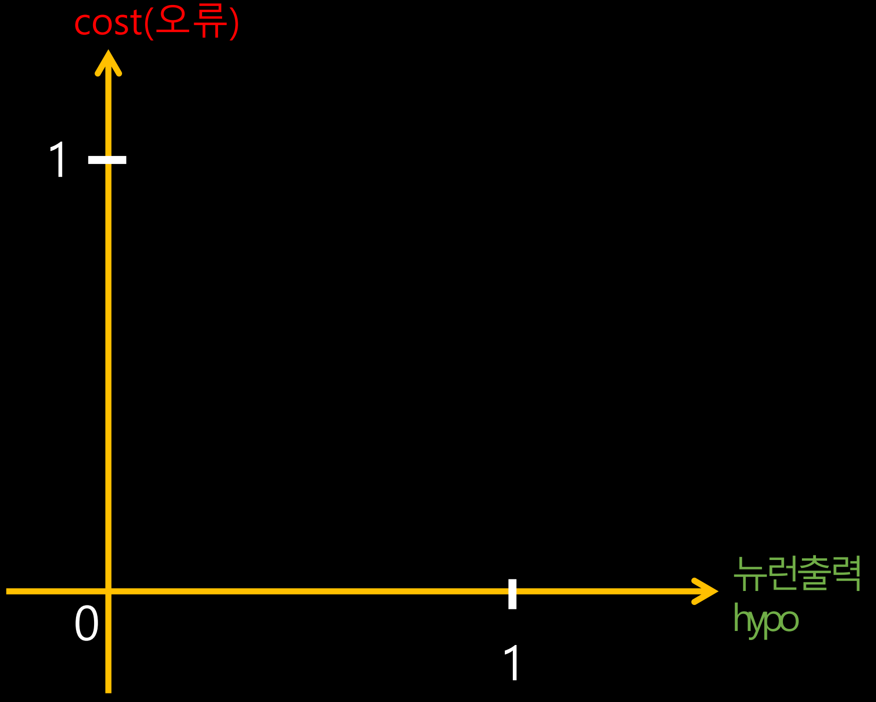
새로운 오류 함수

- 정답이 1일 때
  - 뉴런이(hypo) 정답을 맞추면(1이면) 오류는 없다고(0) 알려주고,
  - 뉴런이(hypo) 정답과는 정반대로 대답하면(0이면) 오류는 크다고(∞) 알려주자.



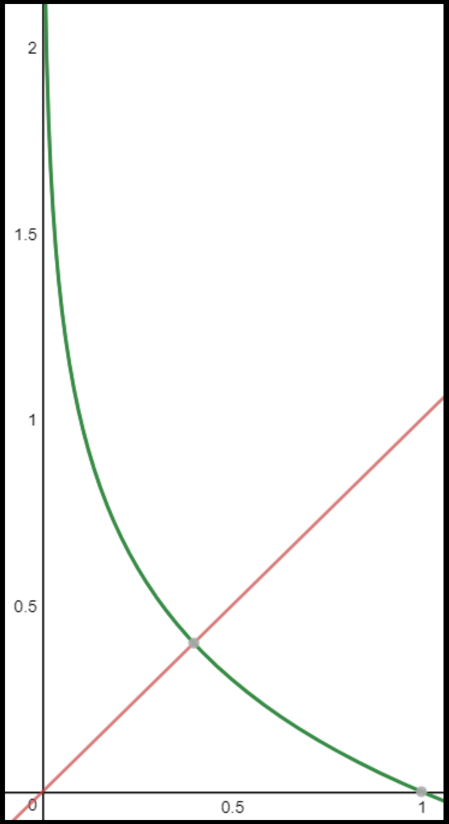
새로운 오류 함수

- 정답이 0일 때
  - 뉴런이(hypo) 정답을 맞추면(0이면) 오류는 0
  - 뉴런이(hypo) 정답과는 정반대로 대답하면(1이면) 오류는 ∞가 되게

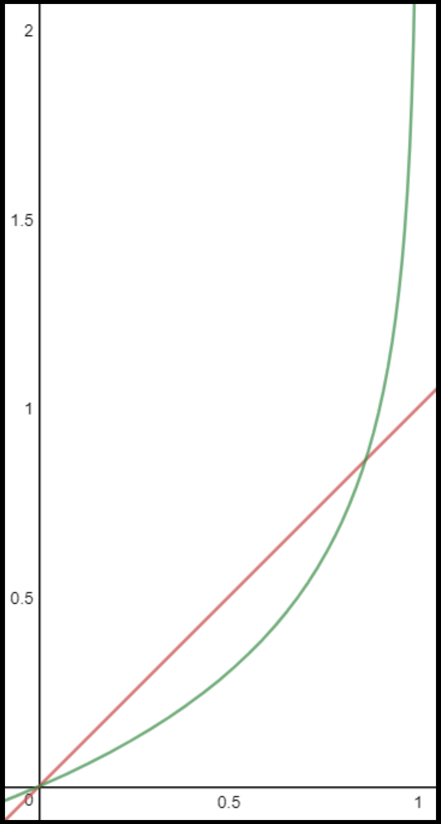


h : Prediction by a neuron : wx

$y = -\log(h)$



$y = -\log(1-h)$

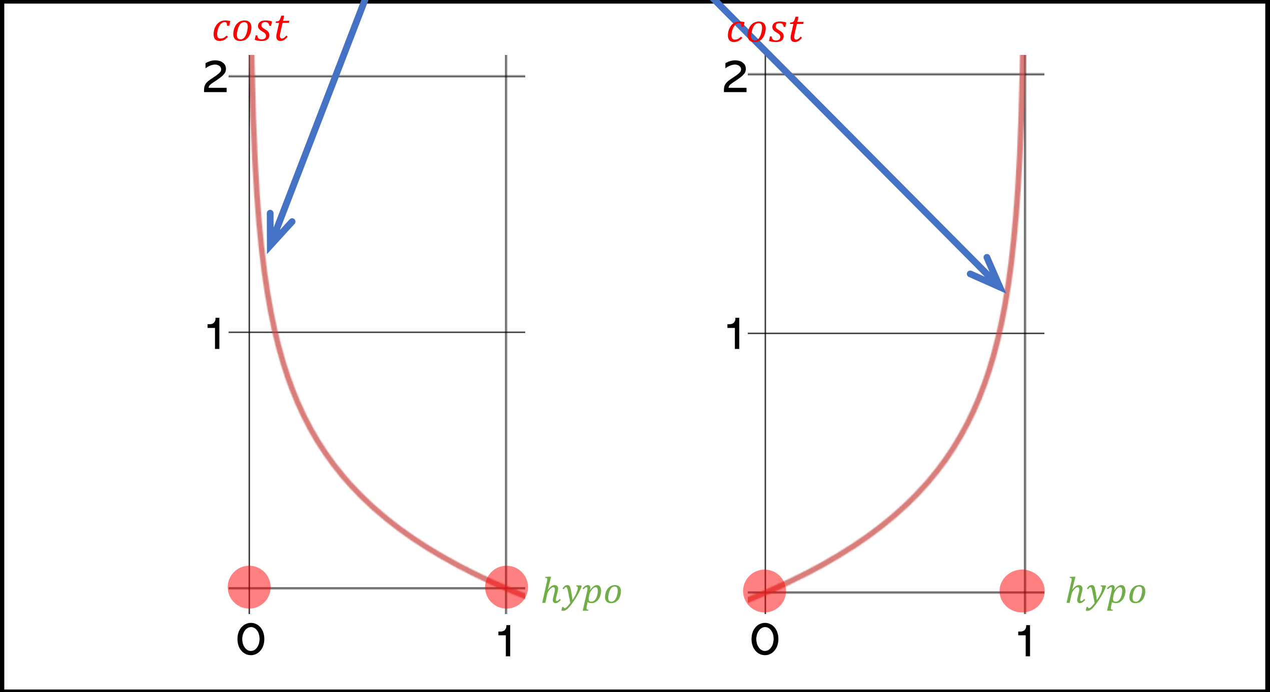


Error Function

Prediction by a neuron

Correct answer

$$cost = \begin{cases} -\log(h) & : y = 1 \\ -\log(1-h) & : y = 0 \end{cases}$$



Error Function

$$cost = \begin{cases} -\log\left(H(X)\right) & : y = 1 \\ -\log(1 - H(X)) & : y = 0 \end{cases}$$

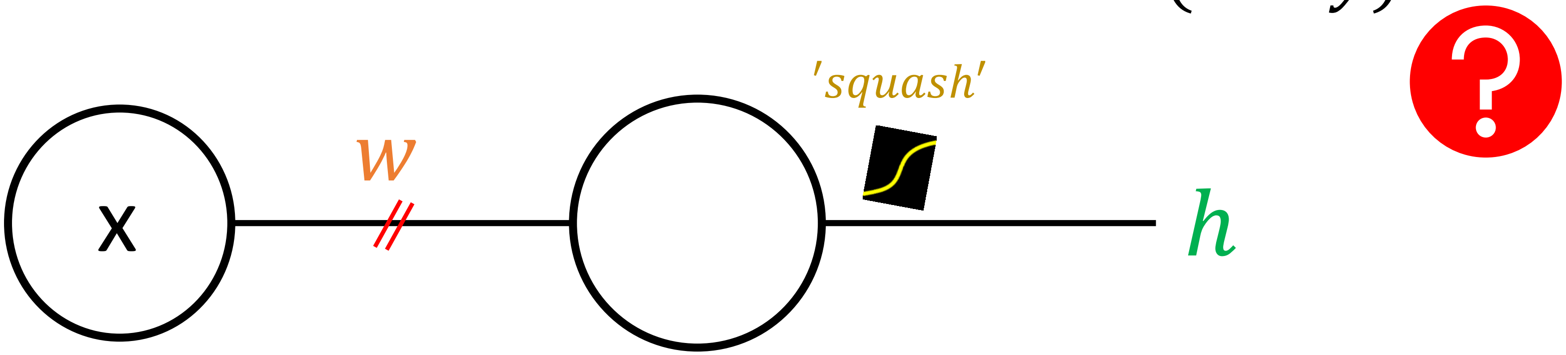
$$E = -y\log\left(H(X)\right) - (1 - y)\log(1 - H(X))$$

$$E = -\left(y\log\left(H(X)\right) + (1 - y)\log(1 - H(X))\right)$$

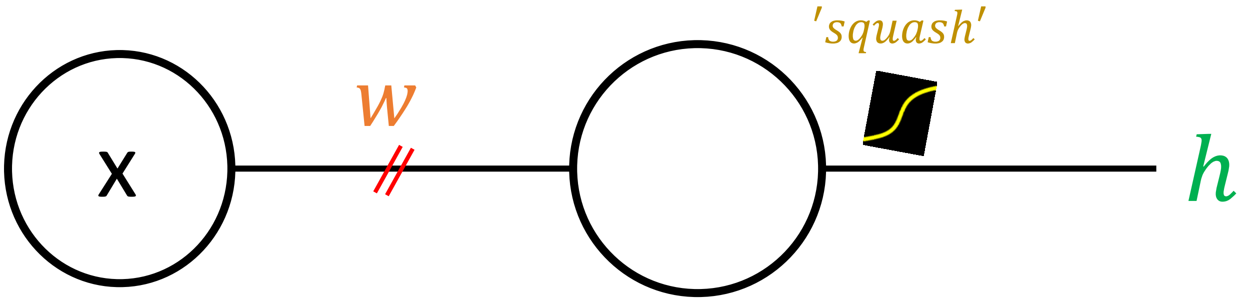
$$w = w - \alpha \cdot Gradient$$

$$\frac{\partial E}{\partial w}$$

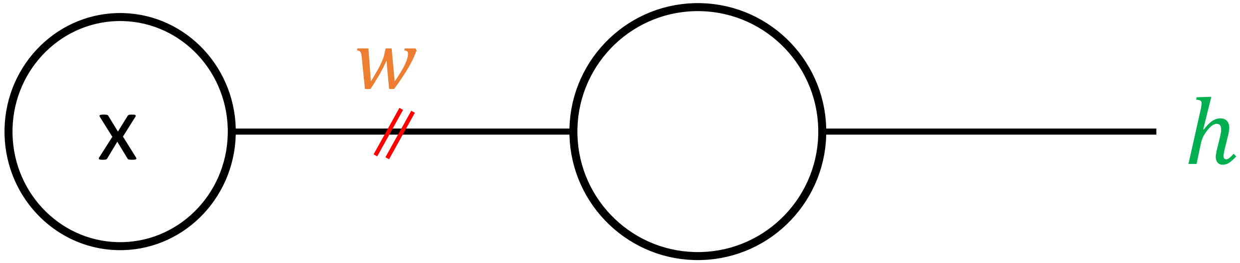
$$E = (h - y)^2$$



$$E = -(y\log(h) + (1-y)\log(1-h))$$



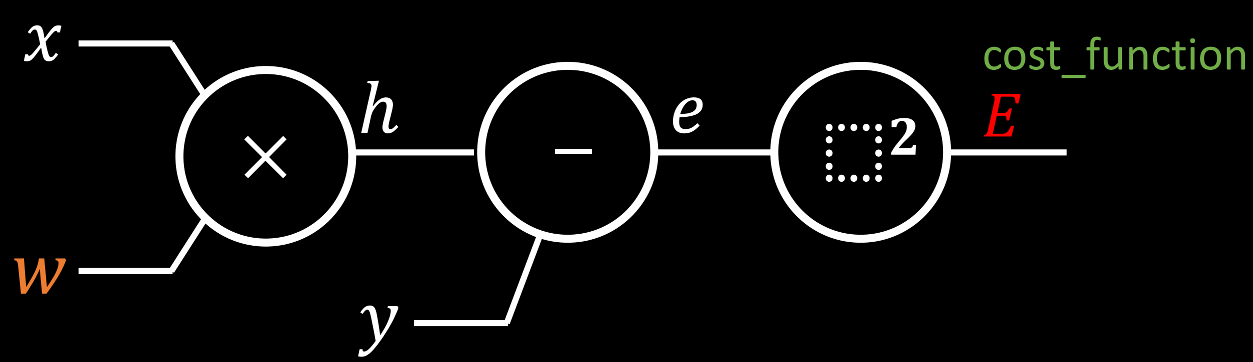
$$E = (h - y)^2$$



Computational Graph

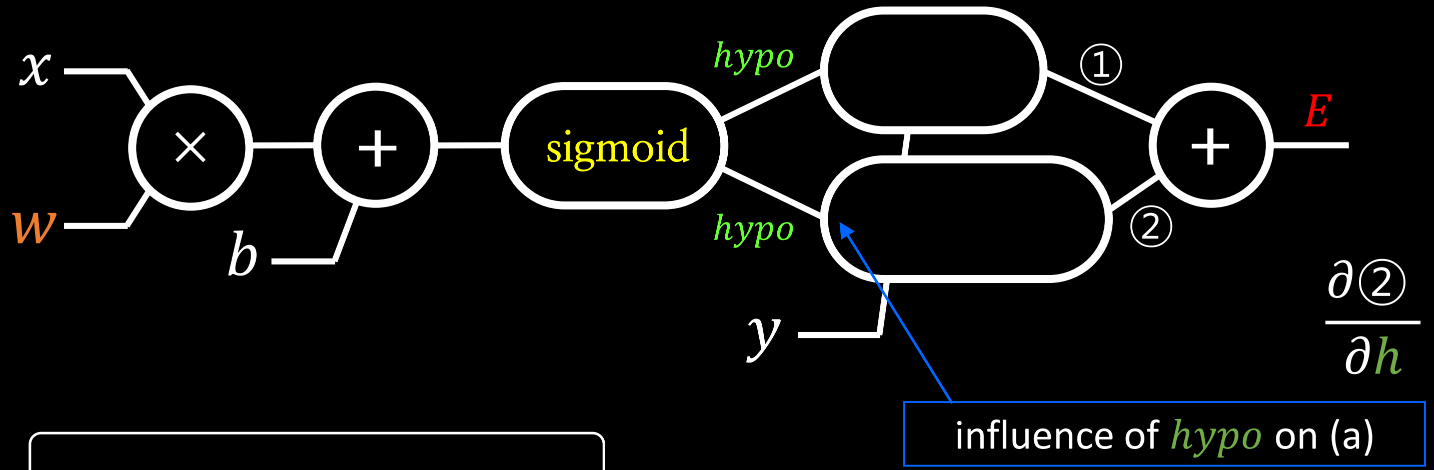
$$E = (wx - y)^2$$

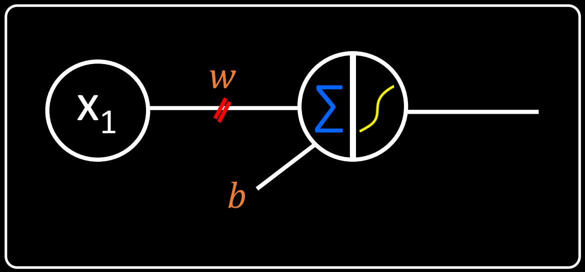
```
hypo = w * x
cost_function(E) = (hypo - y) ** 2)
```



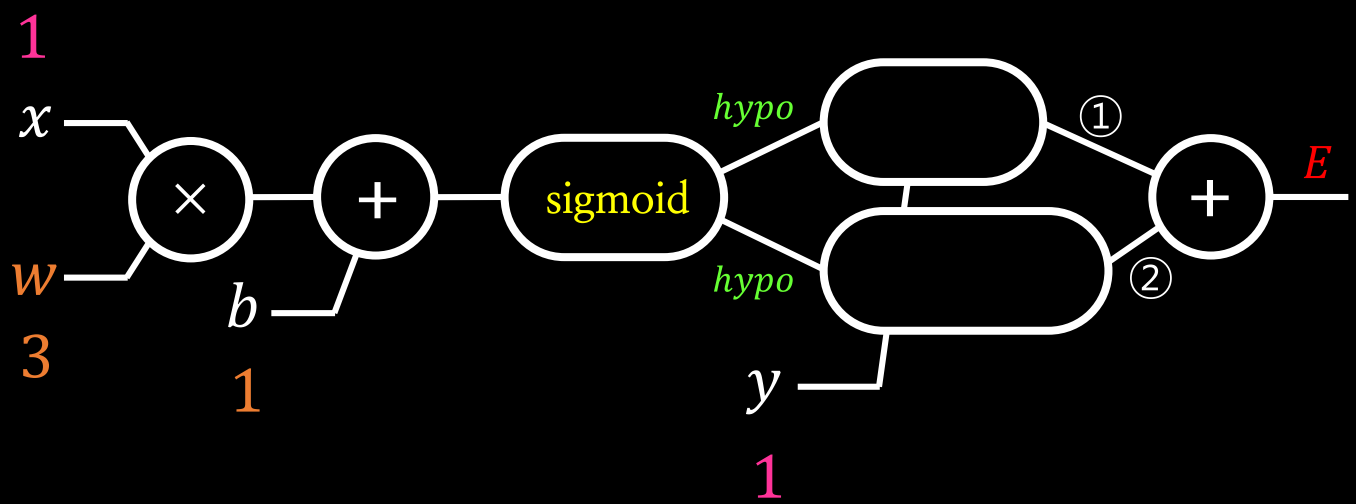
Computational Graph

$$E = \overbrace{-(y \cdot \log(hypo)}^{①} + \overbrace{(1-y) \cdot \log(1-hypo))}^{②}$$

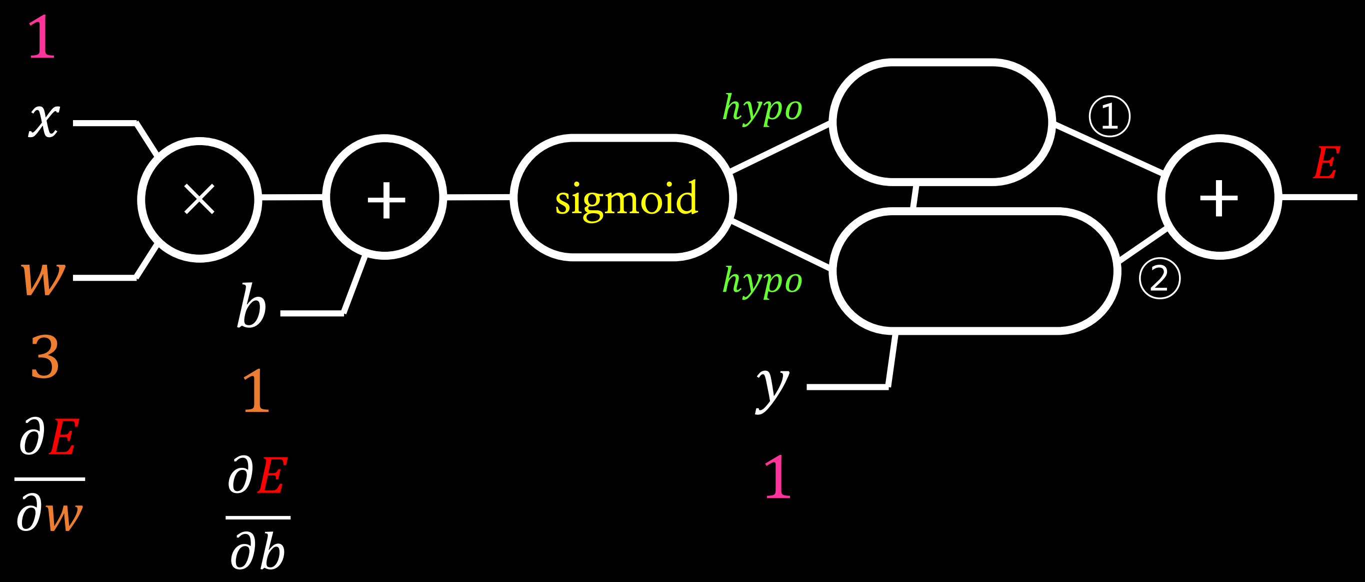




Forward propagation



Back-propagation



$$w = w - \propto \cdot \frac{\partial E}{\partial w}$$

$$b = b - \propto \cdot \frac{\partial E}{\partial b}$$

(실습) 11.py

입력이 음수면 0을,

입력이 양수면 1을 출력

$$cost = -(y \log(H(X)) + (1-y)\log(1 - H(X)))$$

```
x_data = [-2., -1, 1, 2]
y_data = [0., 0, 1, 1]
```

```
#------- a neuron
w = tf.Variable(tf.random_normal([1]))
hypo = tf.sigmoid(x_data * w)
```

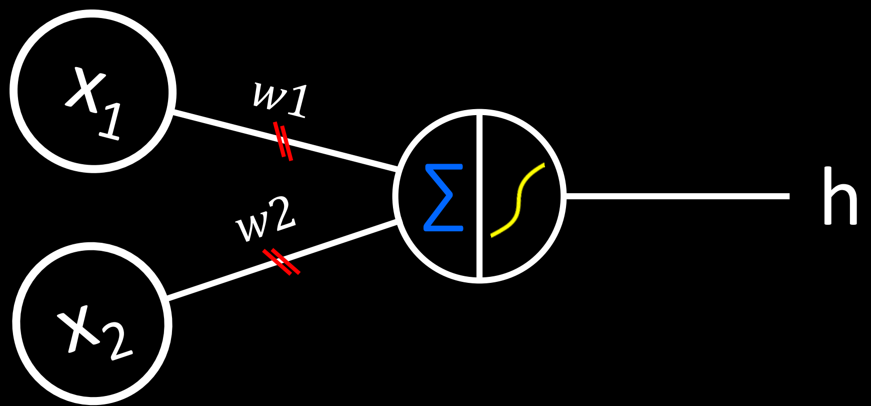
```
#----- learning
cost = -tf.reduce_mean(y_data * tf.log(hypo) +
        tf.subtract(1., y_data) * tf.log(tf.subtract(1., hypo)))

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

for step in range(5001):
    sess.run(train)
```

```
#----- testing(classification)
predicted = tf.cast(hypo > 0.5, dtype=tf.float32)
p = sess.run(predicted)
print("Predicted: ", p)
```

(실습) 12.py

바이어스를 갖는 뉴런

신경 세포 (2 입력)



$$h = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2)}}$$

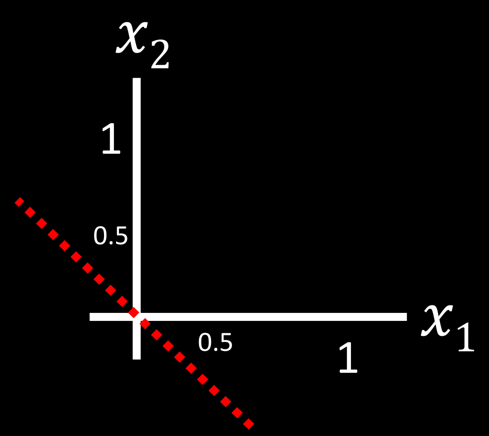
신경 세포 (2 입력)

{위에서 본 모습}

- 결정 경계는?

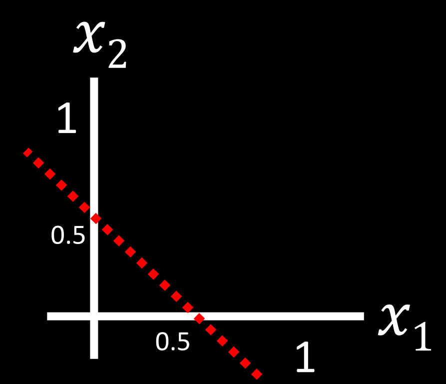
$$w_1 x_1 + w_2 x_2 = 0$$

$$x_1 + x_2 = 0$$

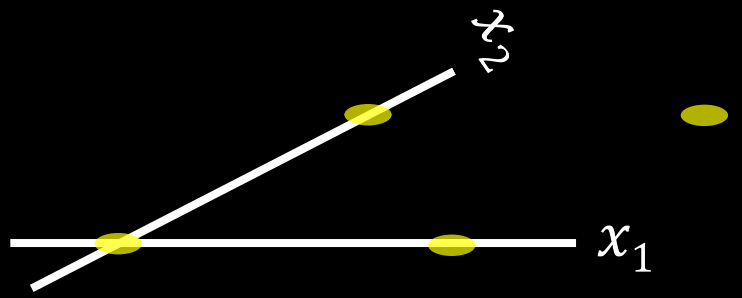


$$w_1 x_1 + w_2 x_2 = b$$

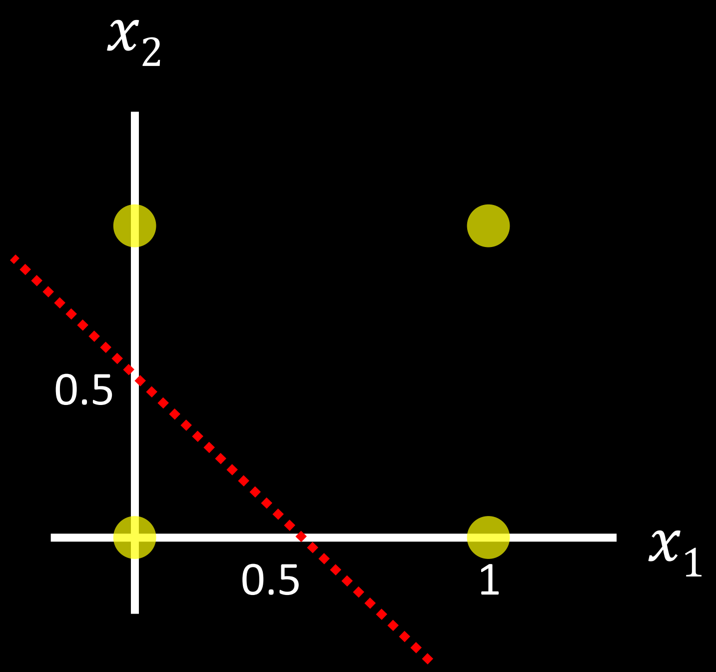
$$x_1 + x_2 = 0.5$$



신경 세포 (2 입력)

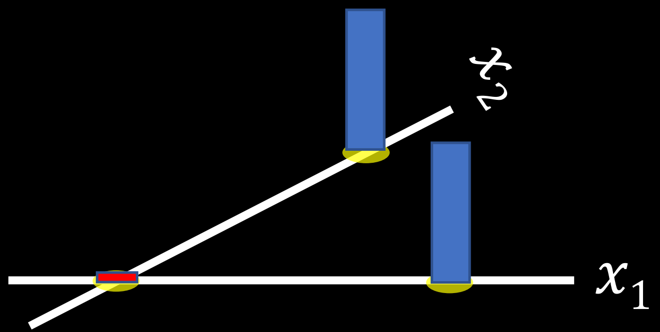


{옆에서 본 모습}

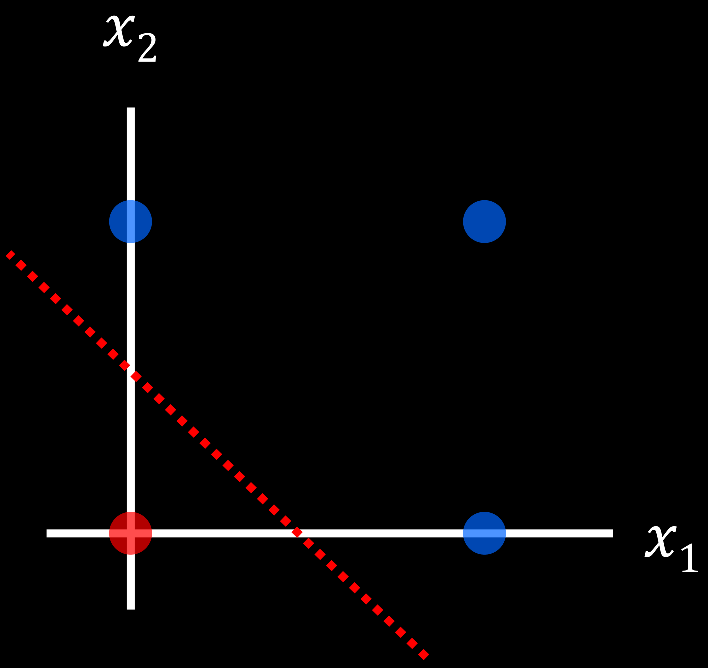


{위에서 본 모습}

신경 세포 (2 입력)

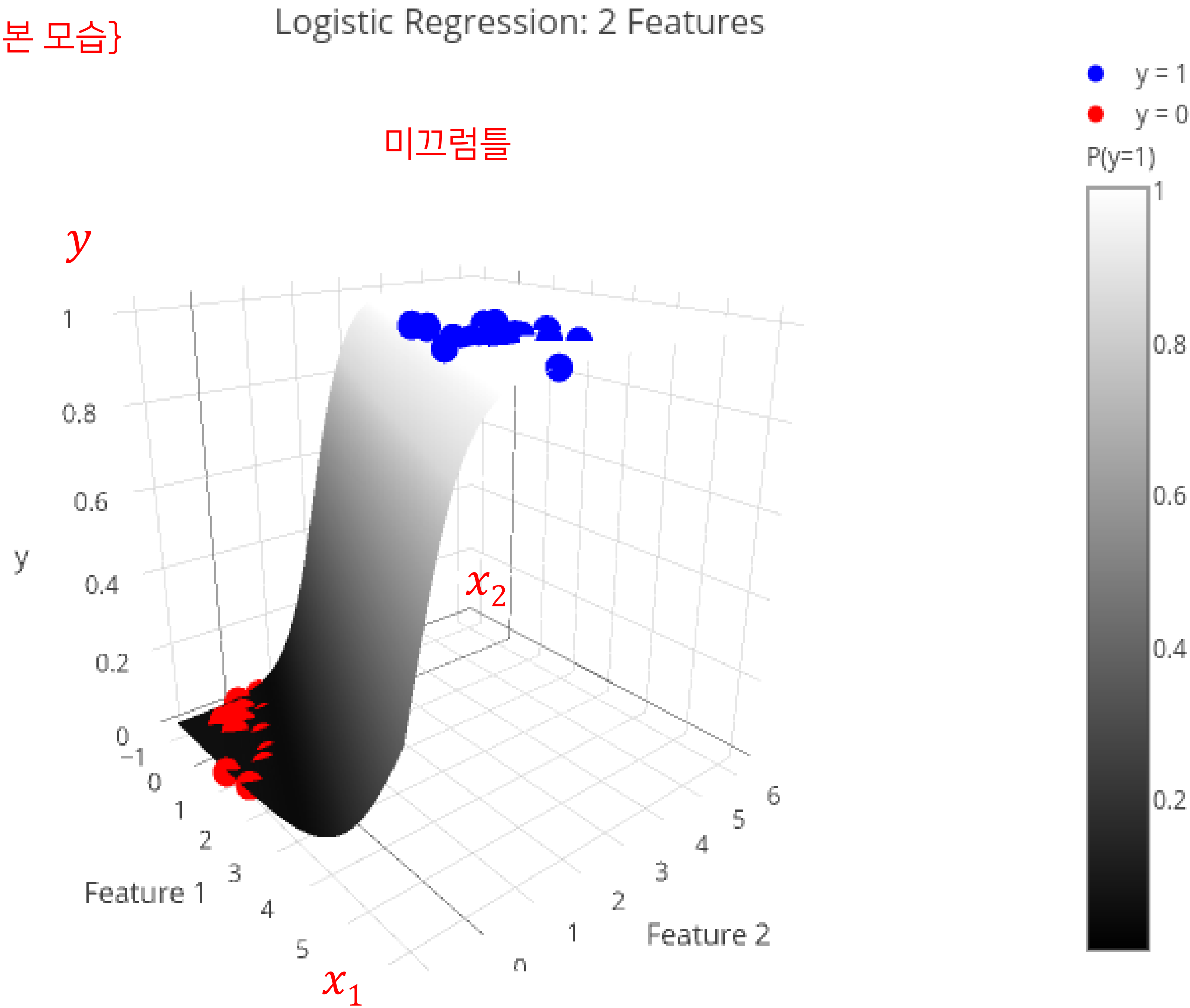


{옆에서 본 모습}

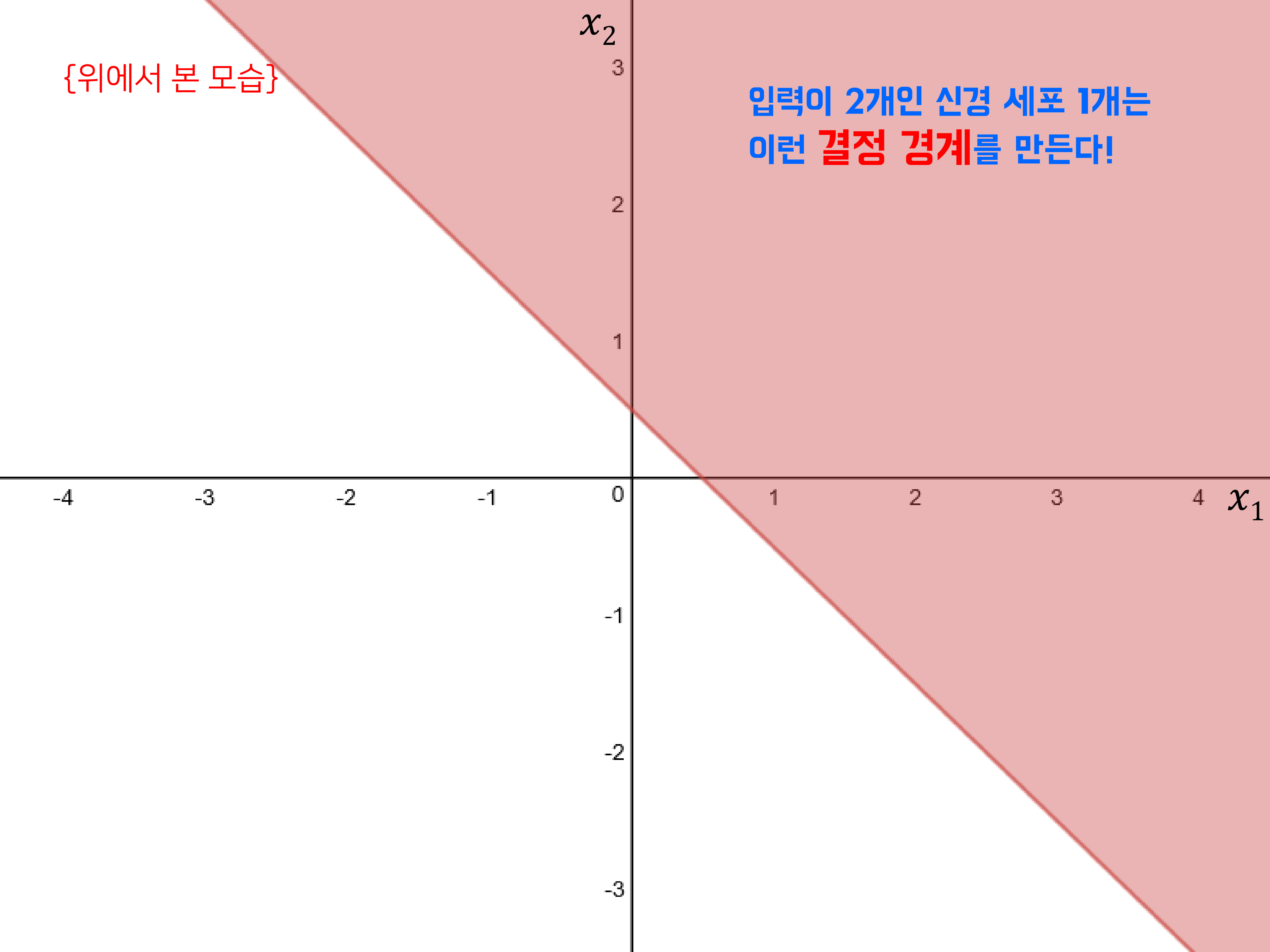


{위에서 본 모습}

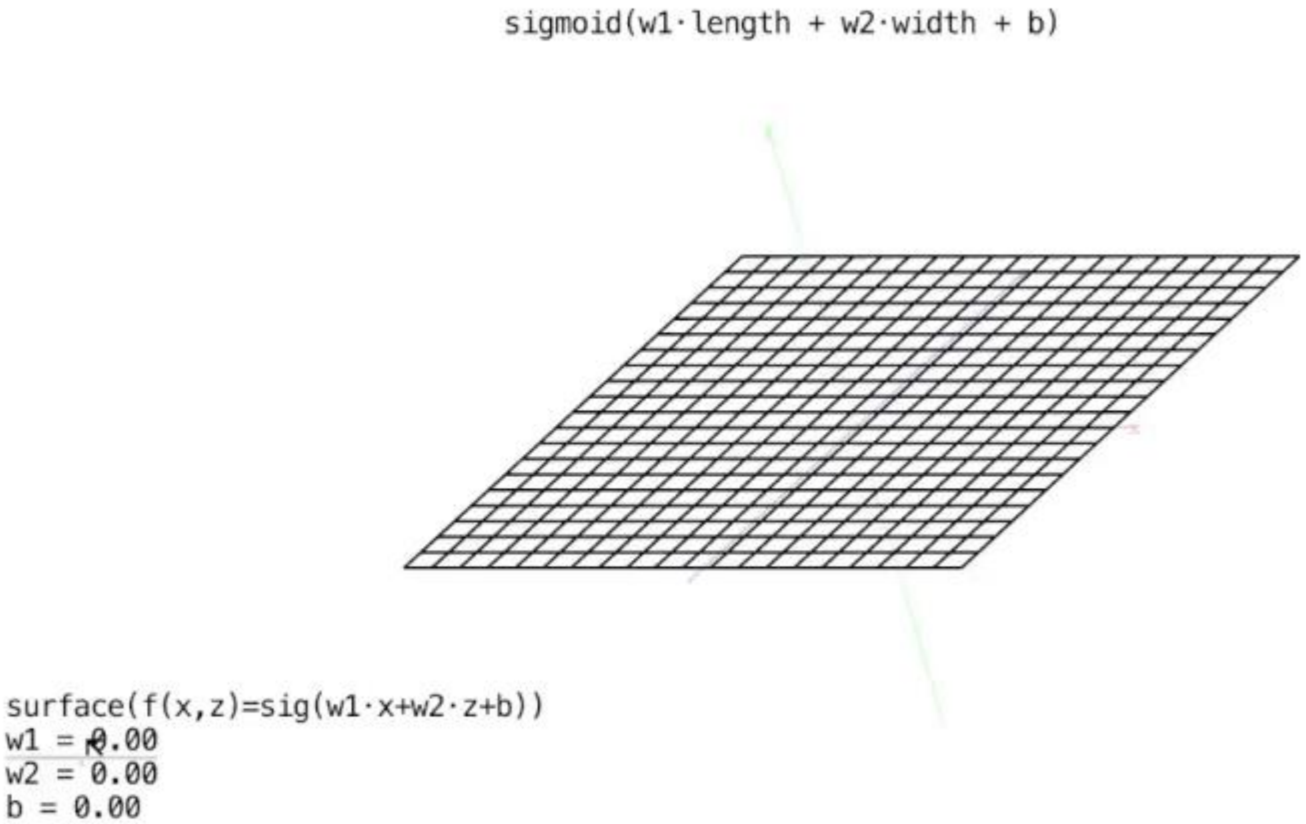
{옆에서 본 모습}



Logistic Regression: 2 Features
미끄럼틀
y
x_2
x_1
y
Feature 1
Feature 2
y = 1
y = 0
P(y=1)


{위에서 본 모습}
입력이 2개인 신경 세포 1개는
이런 결정 경계를 만든다!
x_2
x_1
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
1
2
3
4

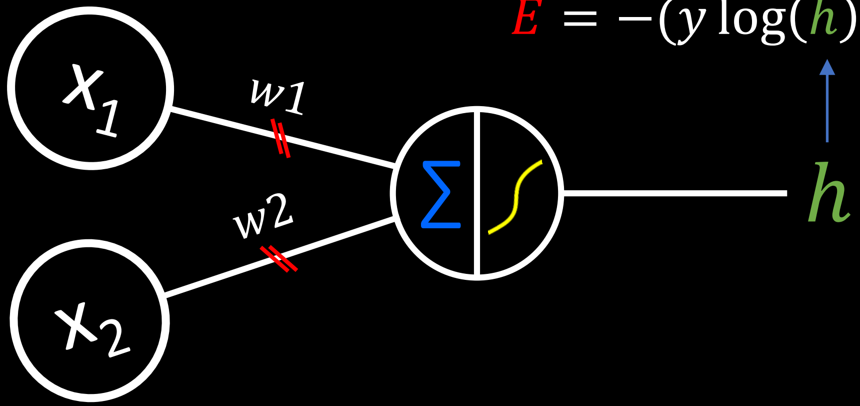
Decision Boundary in 3D



(실습) 13.py

- 입력 두개(x1, x2)를 갖는 뉴런을 이용하여 OR 맞추기
- 한 개의 결정 경계

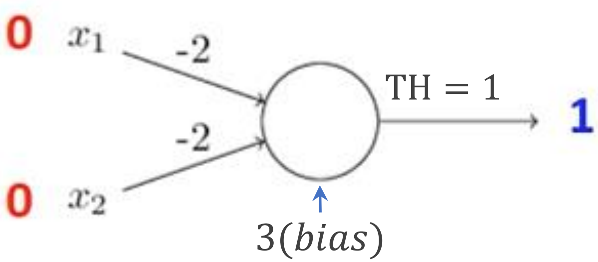
$$E = -(y\log(h) + (1-y)\log(1-h))$$



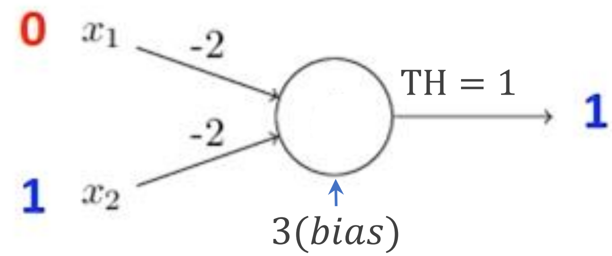
x_1	x_2	$AND(h)$
0	0	0
0	1	0
1	0	0
1	1	1

NAND

- NAND gates are functionally complete.
- We can build any logical functions out of them.



(-2)*0 + (-2)*0+3=3

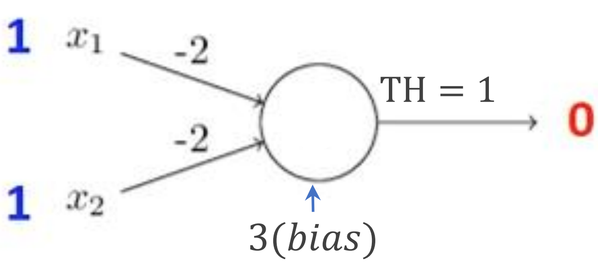


(-2)*0 + (-2)*1+3=1

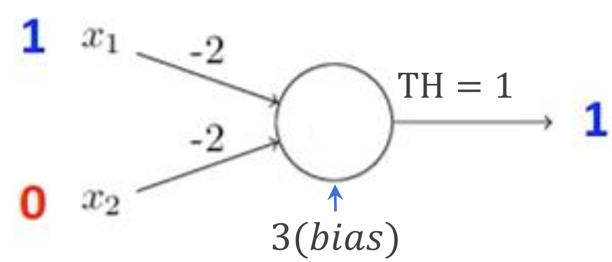
NAND

Truth Table

Input A	Input B	Output Q
0	0	1
0	1	1
1	0	1
1	1	0



(-2)*1 + (-2)*1+3=-1



(-2)*1 + (-2)*0+3=1

이번 학습에서는

- 로지스틱 리그레션을 위한 오류함수를 어떻게 디자인 하는지 이해할 수 있다.
- 한 개의 뉴런이 만들어 내는 결정 경계를 이해할 수 있다.
- 결정경계를 옆에서 본 모습, 위에서 본 모습을 이해할 수 있다.
- 신경세포의 입력의 수에 따른 결정경계를 이해할 수 있다.