AI and Deep Learning

Logistic Regression & Classification (1)

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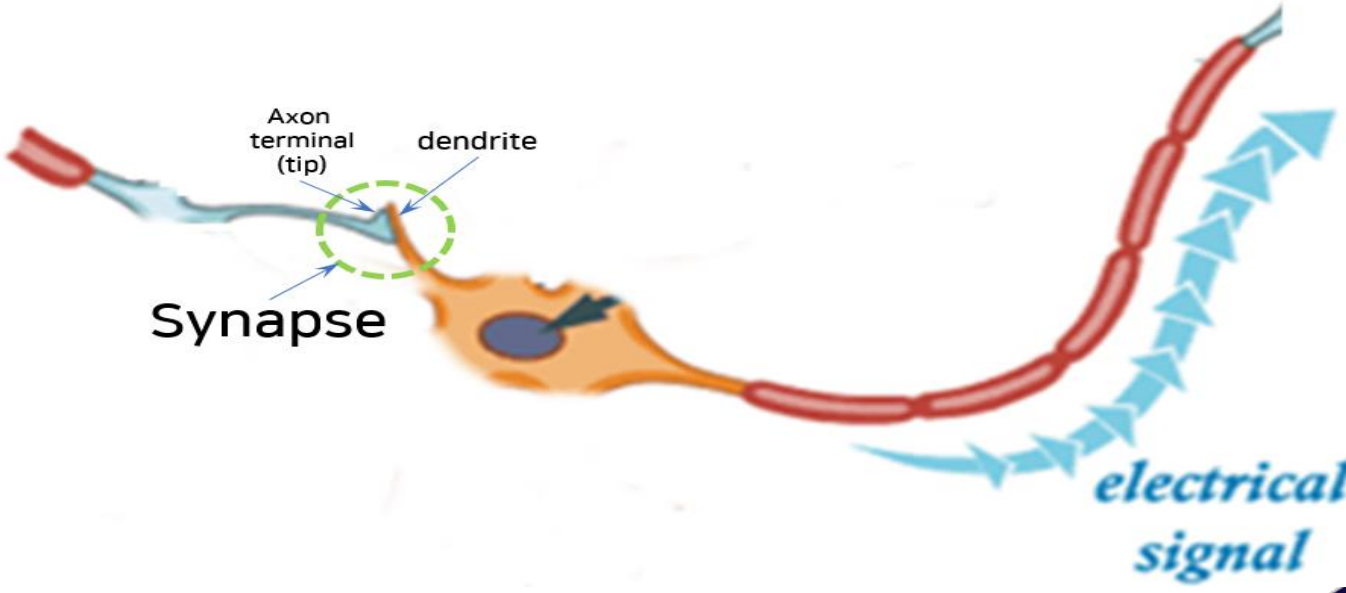
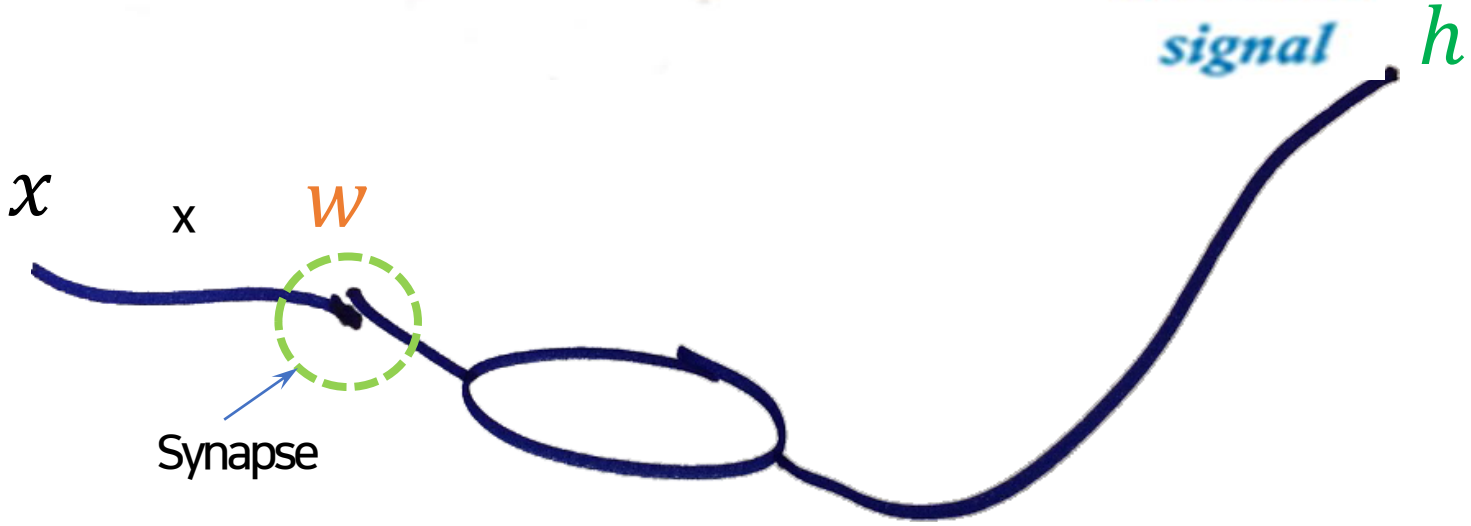
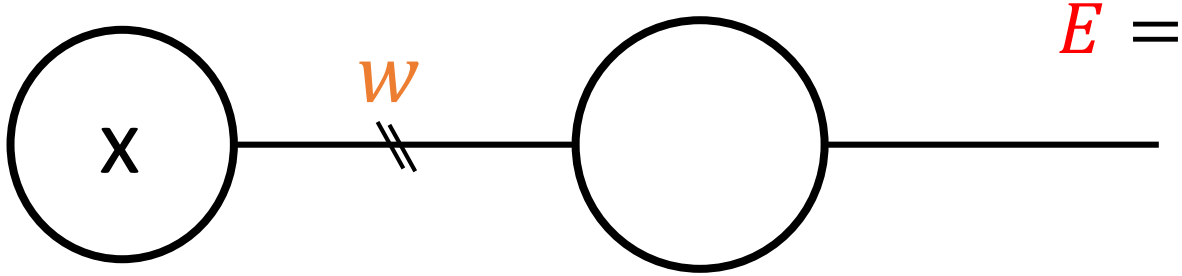
Agenda

- Artificial Intelligence
- Brain, Neurons
- Learning
- Regression
- Deep Neural Networks
- CNN


Axon terminal (tip)
dendrite
Synapse
electrical signal


x
x
w
Synapse
h


x
w
h

$$E = (h - y)^2$$

Logistic Regression

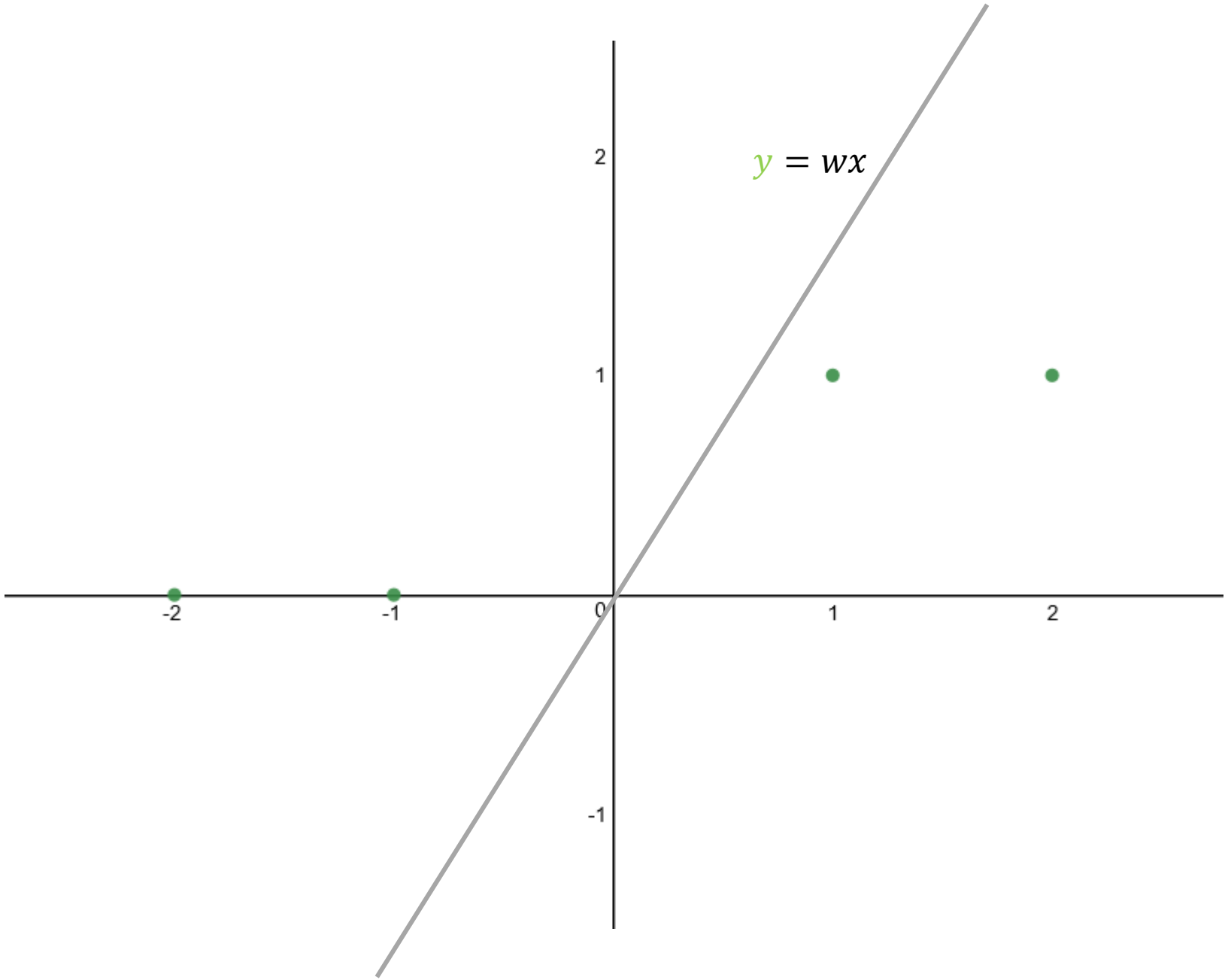
The shape of regression is not linear but logistic.

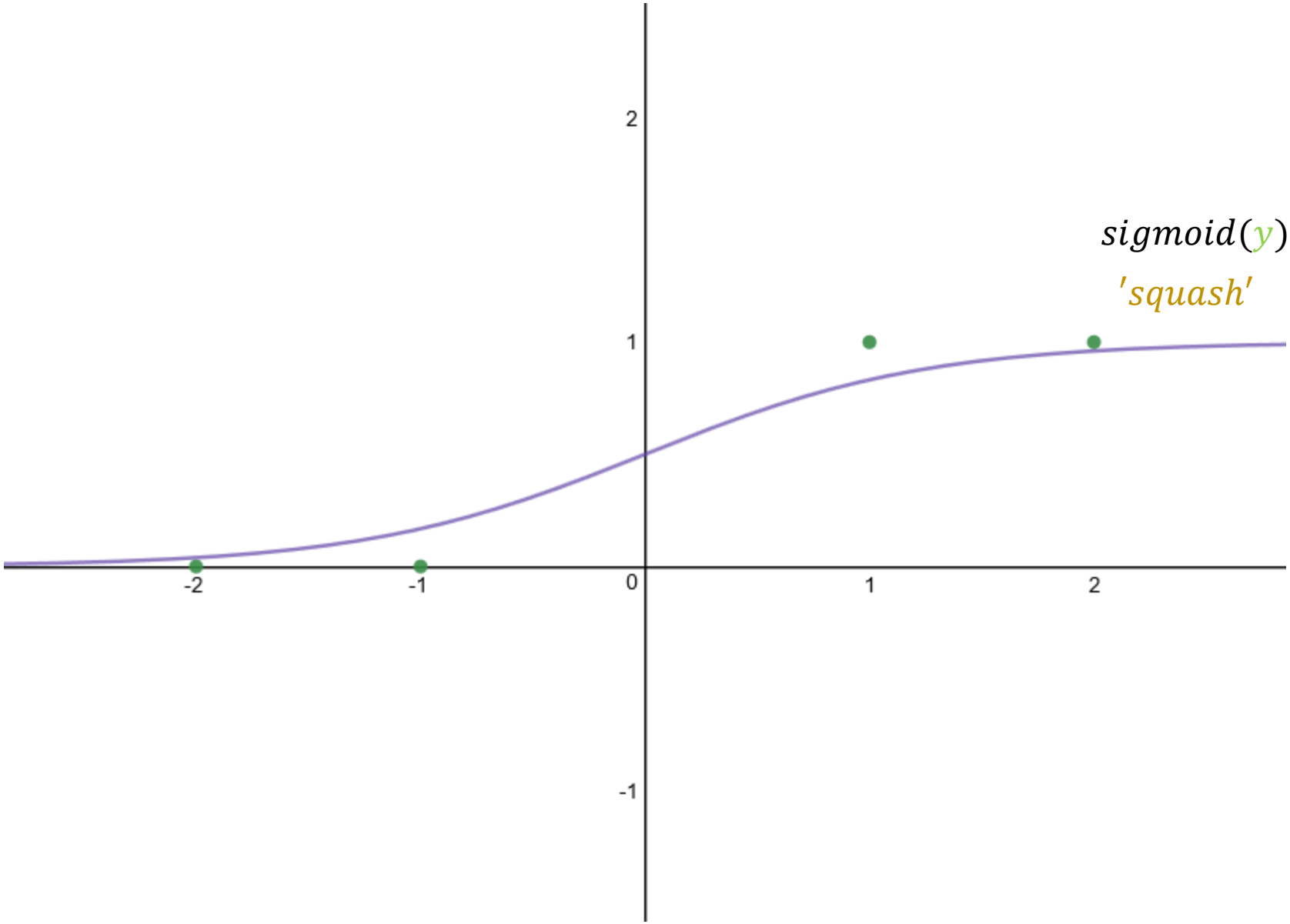
What does that mean?

점 (-2, 0), (-1, 0), (1, 1), (2, 1) 표시

$$y = wx$$

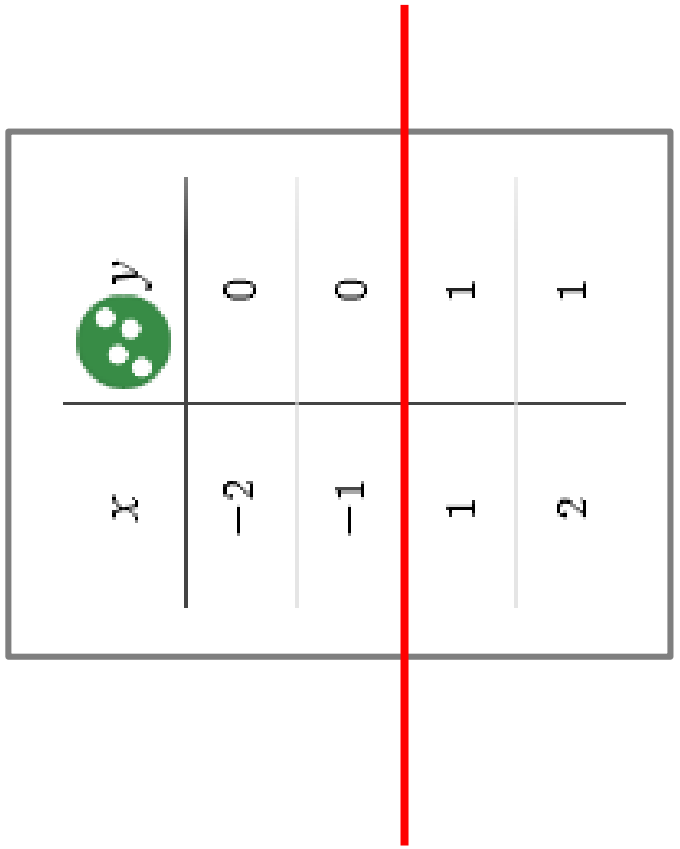
$$y = \frac{1}{1 + e^{-wx}}$$


$y = wx$
2
1
-1
-2
-1
0
1
2


2
1
-1
-2
-1
0
1
2
$sigmoid(y)$
$'squash'$

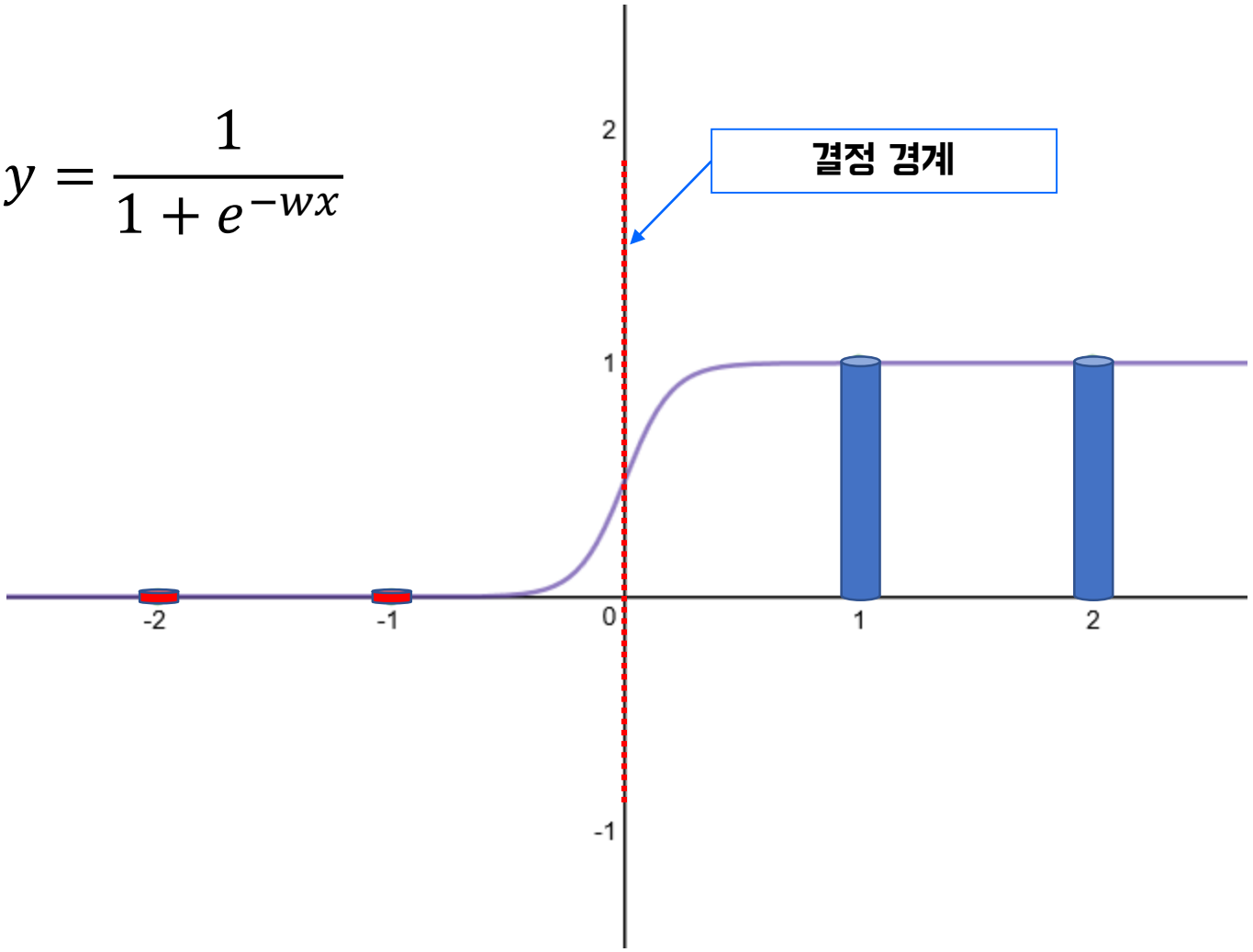
x	y
-2	0
-1	0
1	1
2	1

y	0	0	1	1
x	-2	-1	1	2

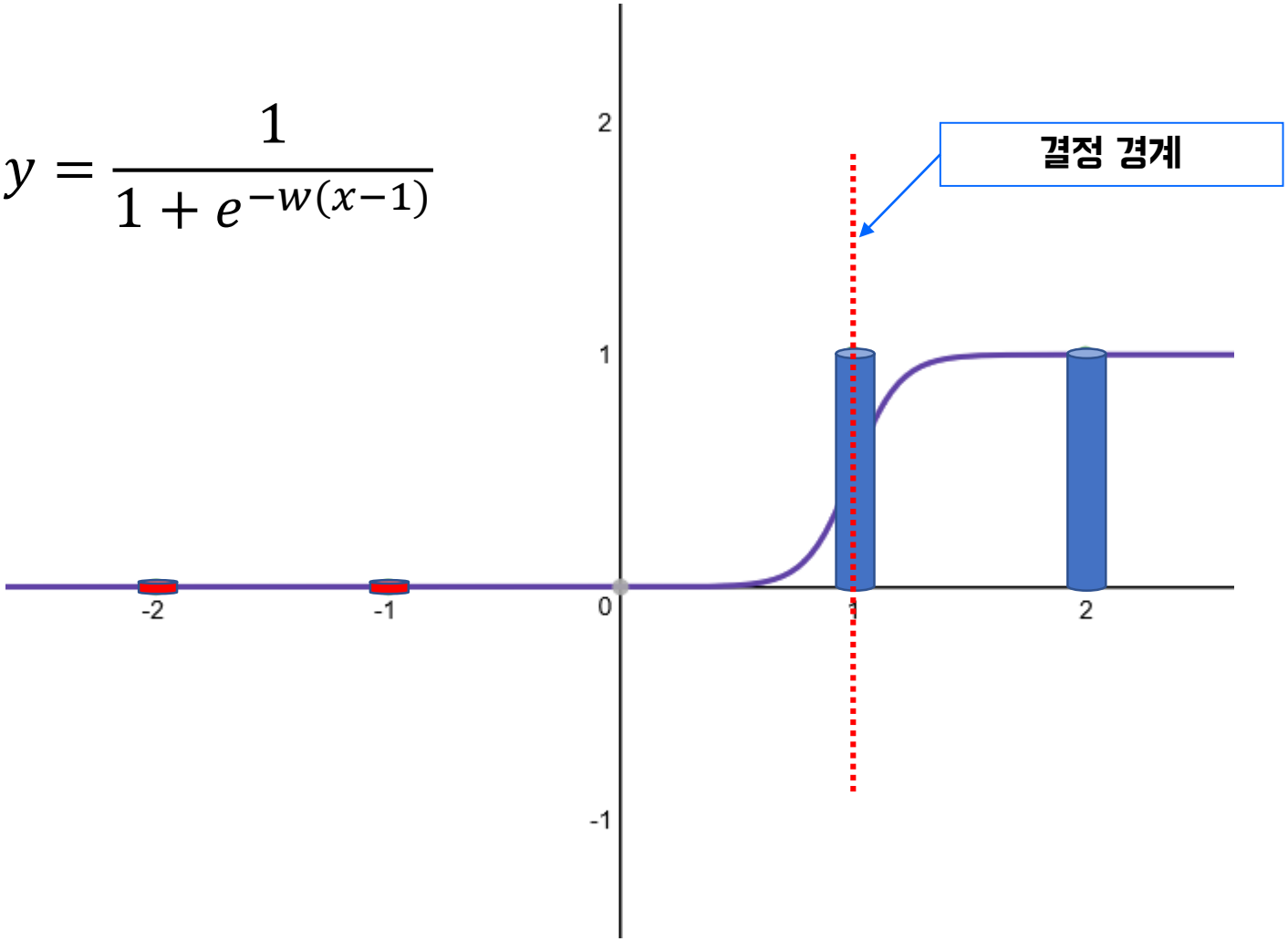


0, 1을 결정(decision)하는
경계(boundary)

$$y = \frac{1}{1 + e^{-wx}}$$

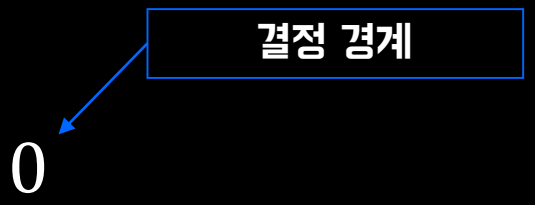


$$y = \frac{1}{1 + e^{-w(x-1)}}$$



결정 경계

$$h = \frac{1}{1 + e^{-w(x)}}$$



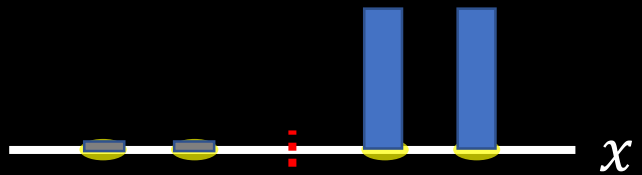
$or,$

$h = \text{sigmoid}\ (wx)$

결정 경계

$wx = 0$

$x = 0$



{옆에서 본 모습}

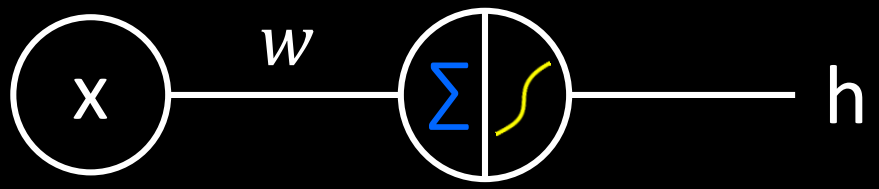
$wx + b = 0$

$x + 1 = 0$

$2x + 3 = 0$

{위에서 본 모습}

신경 세포 기능

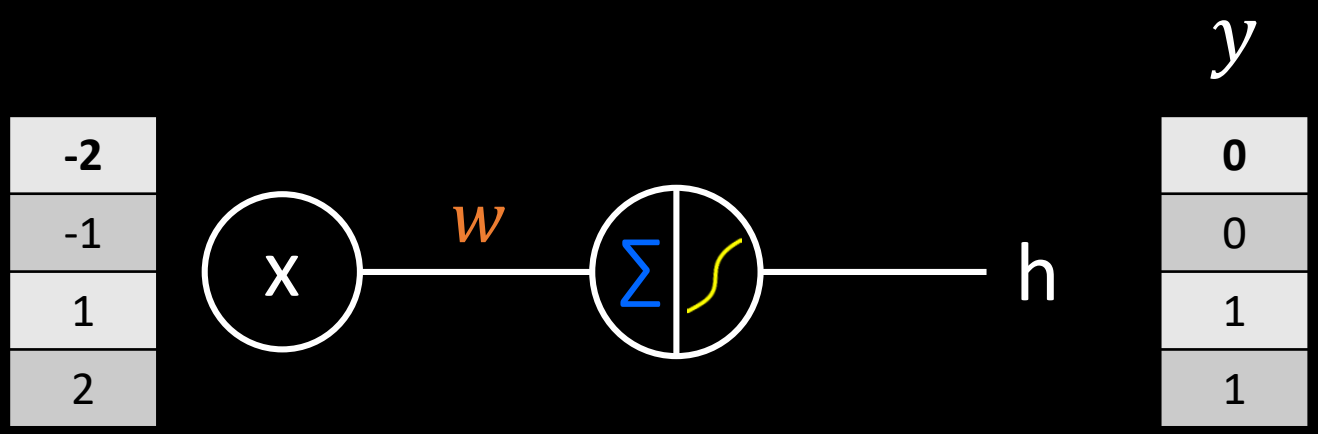


- 신경세포 1개가 할 수 있는 것은?
- 입력 x에 따라 0, 혹은 1(fire)을 출력함.
- 그림만 보고 결정 경계를 알 수 있을까?

Classification

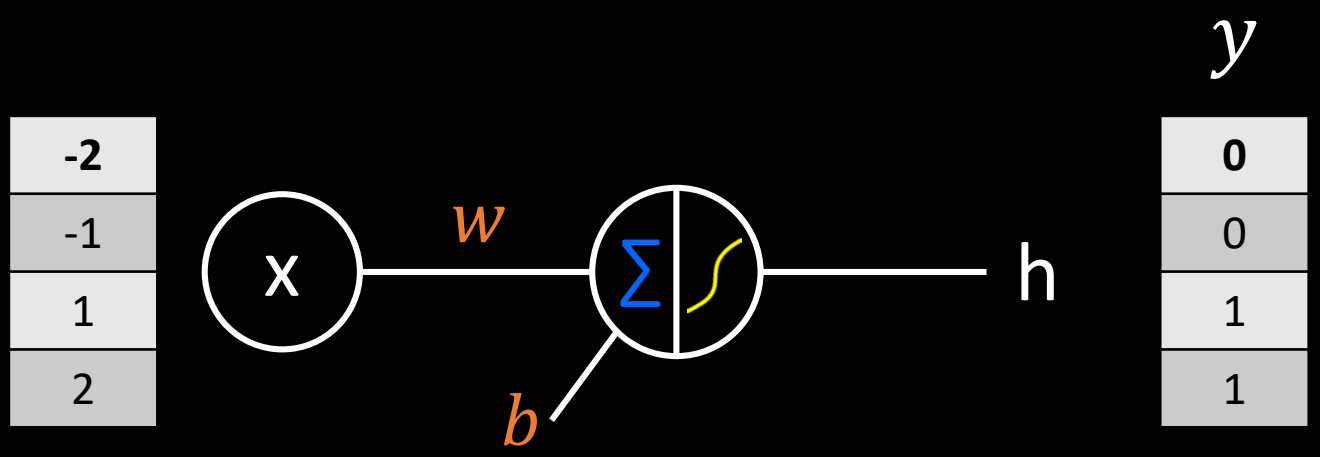
- Pass(1) or Fail(0)
- Spam(1) or Ham(0)
- Scam(fraud, 1) or not(0)
- Safe(1) or Dangerous(0)
- Intrusion/virus(1) or not(0)
- Cancer(1) or not(0)
- Binary classification -> Multiple classification

그림만 보고 결정 경계를 알 수 있을까?



$$h = \begin{cases} 1 & if\ wx \geq 0 \\ 0 & otherwise \end{cases}$$

그림만 보고 결정 경계를 알 수 있을까?



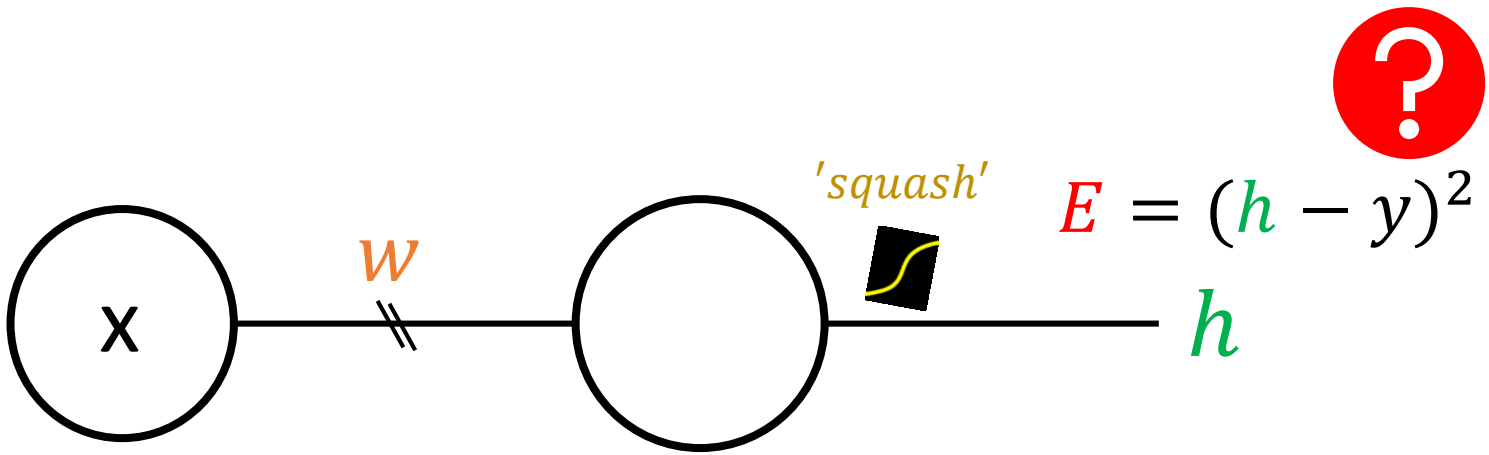
$$h = \begin{cases} 1 & if\ wx + b \geq 0 \\ 0 & otherwise \end{cases}$$

가설

$$hypo = \frac{1}{1 + e^{-wx}}$$

가설은 뭐다?

가설 안에 결정 경계


x
w
'squash'
E = (h − y)²
h

선형 회귀 오류 함수

Prediction by a neuron

$$cost = \frac{1}{m}\sum_{i=1}^{m}(hypo(x_i) - y_i)^2$$

Correct answer

"로지스틱 리그레션에도 동작할까?"

점 (-2, 0), (-1, 0), (1, 1), (2, 1) 표시

$$y = wx$$

$$y = \frac{1}{1 + e^{-wx}}$$

점 (1, 1)만 표시

$$E = \left(\frac{1}{1 + e^{-w \cdot 1}} - 1 \right)^2$$

(w, E)

점 (-1, 0), (1, 1) 표시

$$h = \frac{1}{1 + e^{-wx}}$$

$$E = \left(\frac{1}{1 + e^{-w \cdot -1}} - 0\right)^2 + \left(\frac{1}{1 + e^{-w \cdot 1}} - 1\right)^2$$

(w, E)

Cost/Loss/Stress Function

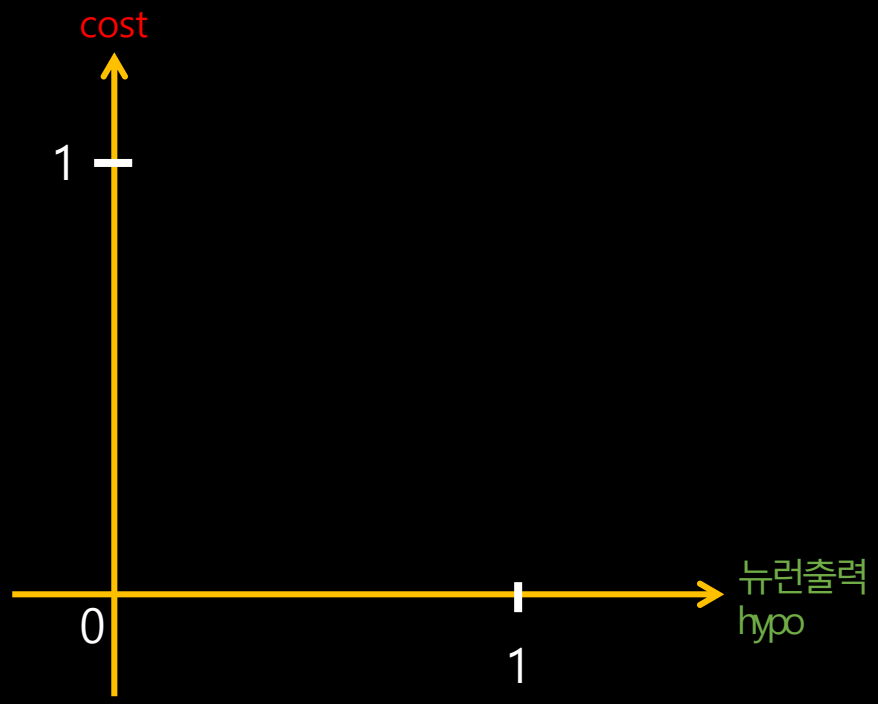
문제

오류 함수

- 뉴런의 출력 (hypo, 가설)이 정답과 일치하면 에러=0
- 뉴런의 출력 (hypo, 가설)이 정답과는 정반대 값이면 에러=∞

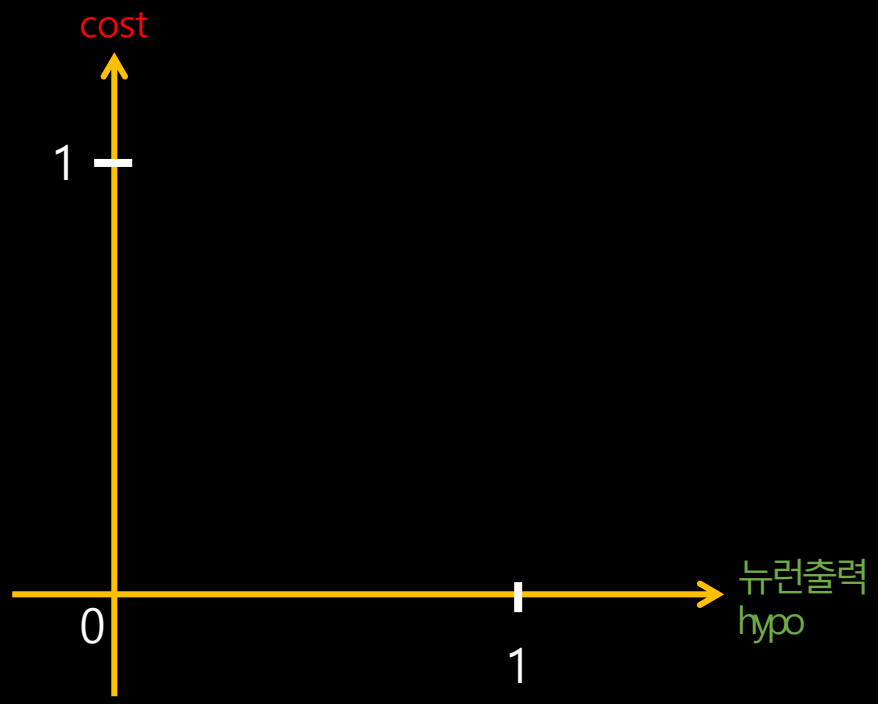
오류 함수

- 정답이 1일 때
  - 뉴런 출력(hypo)이 1이면 오류는 0이 되게
  - 뉴런 출력(hypo)이 0이면 오류는 ∞가 되게



오류 함수

- 정답이 0일 때
  - 뉴런 출력(hypo)이 0이면 오류는 0이 되게
  - 뉴런 출력(hypo)이 1이면 오류는 ∞가 되게

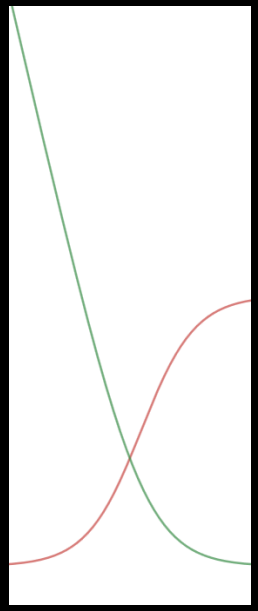


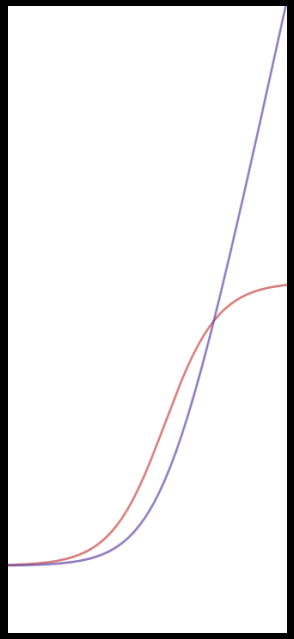
desmos

h : Prediction by a neuron

$$y = -\log(h)$$

$$y = -\log(1 - h)$$



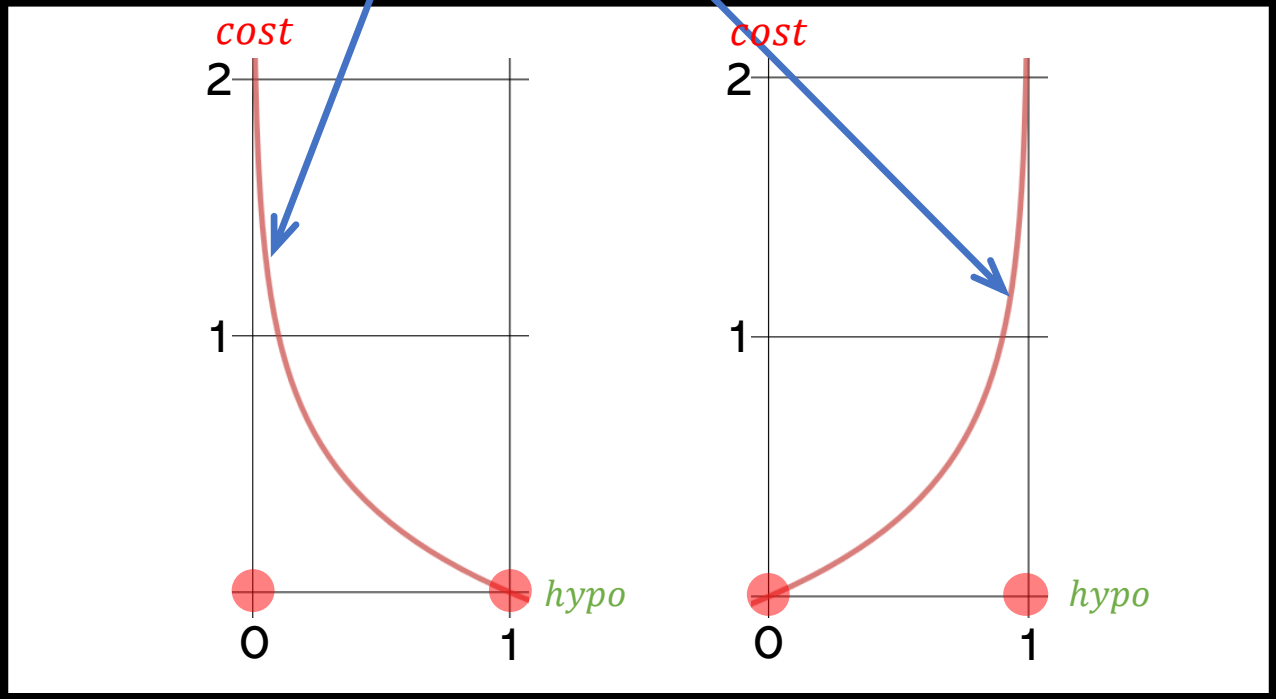


오류 함수

Prediction by a neuron

Correct answer

$$cost = \begin{cases} -\log(h) & : y = 1 \\ -\log(1-h) & : y = 0 \end{cases}$$



오류 함수

$$cost = \begin{cases} -\log\big(H(X)\big) & : y = 1 \\ -\log(1 - H(X)) & : y = 0 \end{cases}$$

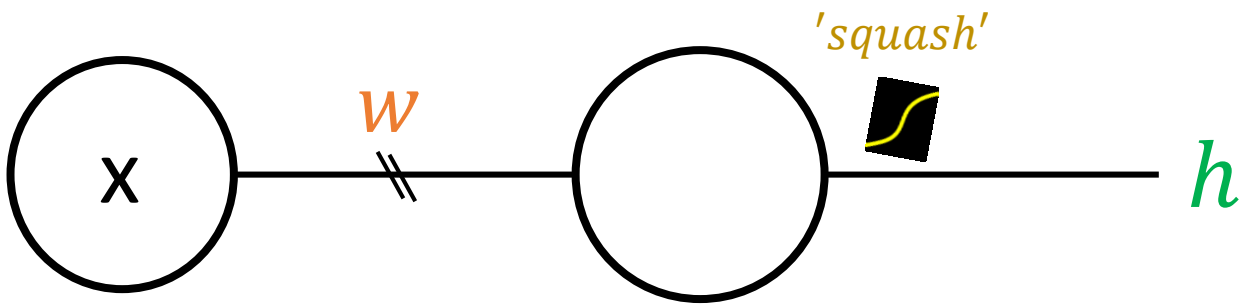
$$E = -y\log\big(H(X)\big) - (1 - y)\log(1 - H(X))$$

$$E = -(y\log\big(H(X)\big) + (1 - y)\log(1 - H(X)))$$

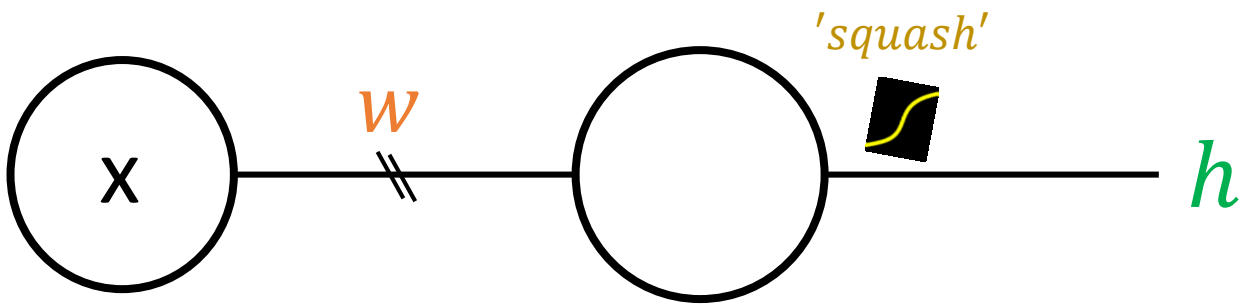
$$W = W - \alpha \cdot \text{기울기}$$

$$\frac{\partial E}{\partial W}$$

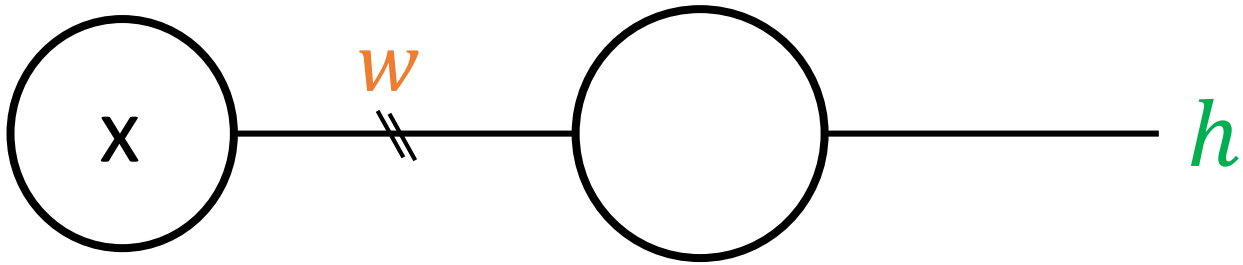
$E = (h - y)^2$



$$E = -(y \log(h) + (1 - y)\log(1 - h))$$



$$E = (h - y)^2$$



(실습) 11.py

음수는 0으로 양수는 1로 분류

$$cost = -(y \log(H(X)) + (1 - y)\log(1 - H(X)))$$

```
x_data = [-2., -1, 1, 2]
y_data = [0., 0, 1, 1]
```

```
#------- a neuron
w = tf.Variable(tf.random_normal([1]))
hypo = tf.sigmoid(x_data * w)
```

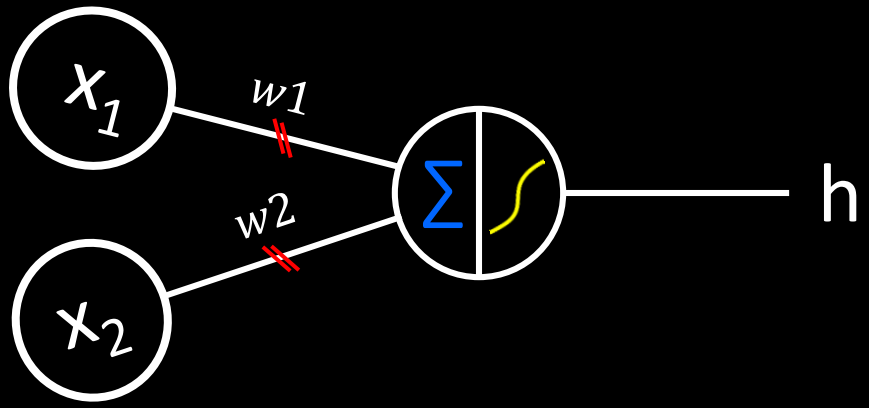
```
#----- learning
cost = -tf.reduce_mean(y_data * tf.log(hypo) +
        tf.subtract(1., y_data) * tf.log(tf.subtract(1., hypo)))

train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)

sess = tf.Session()
sess.run(tf.global_variables_initializer())

for step in range(5001):
    sess.run(train)
```

```
#----- testing(classification)
predicted = tf.cast(hypo > 0.5, dtype=tf.float32)
p = sess.run(predicted)
print("Predicted: ", p)
```

(실습) 12.py

바이어스를 갖는 뉴런

신경 세포 (2 입력)



$$h = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2)}}$$

신경 세포 (2 입력)

- 결정 경계는?

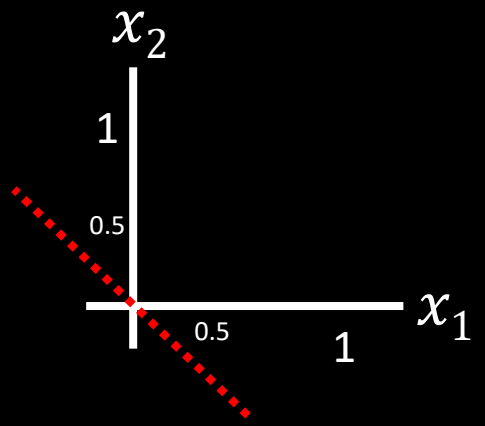
$w_1 x_1 + w_2 x_2 = 0$

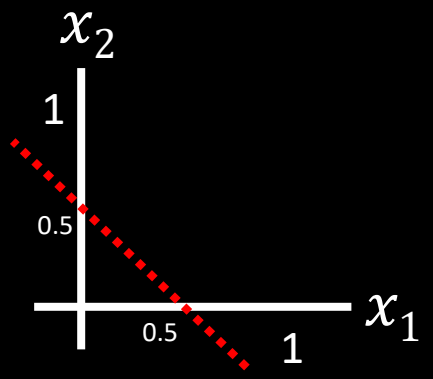
$x_1 + x_2 = 0$

$w_1 x_1 + w_2 x_2 = b$

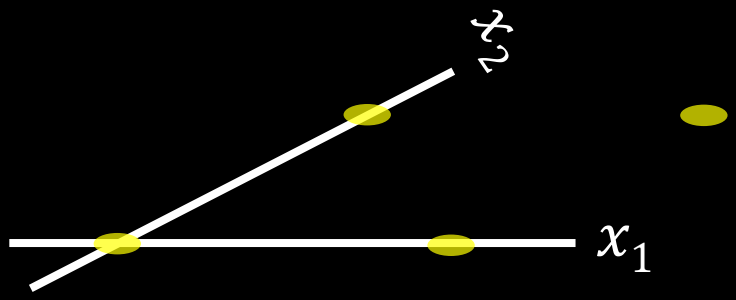
$x_1 + x_2 = 0.5$

{위에서 본 모습}

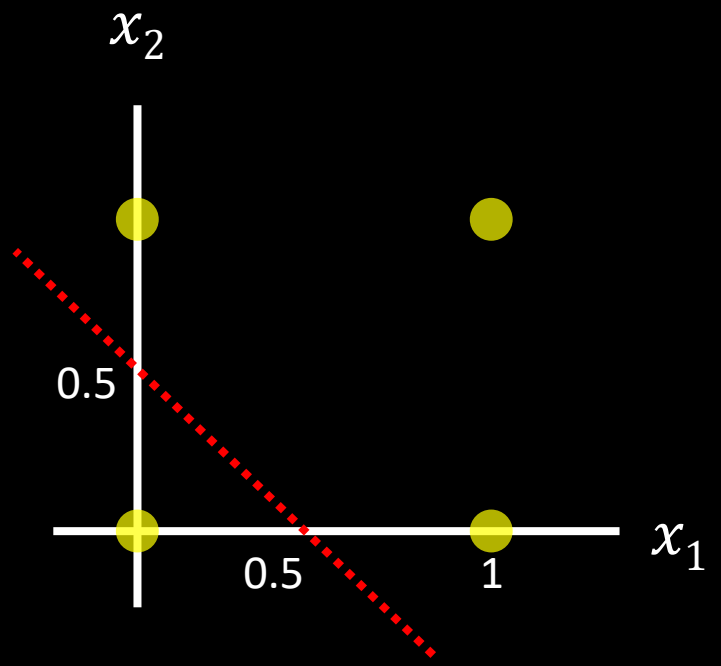




신경 세포 (2 입력)

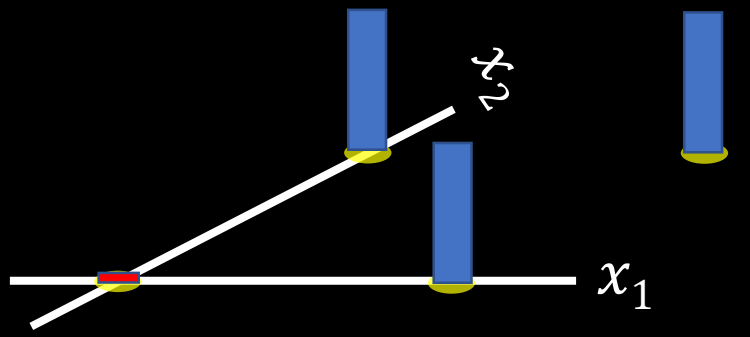


{옆에서 본 모습}

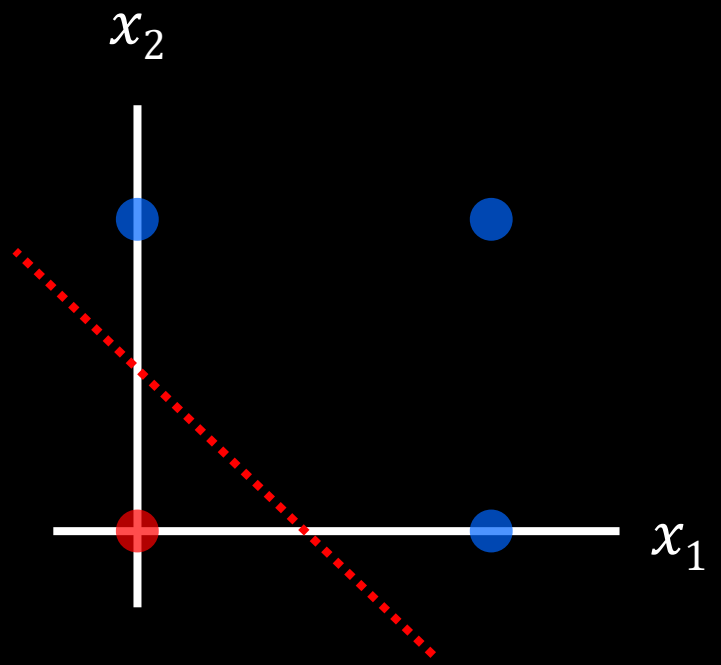


{위에서 본 모습}

신경 세포 (2 입력)

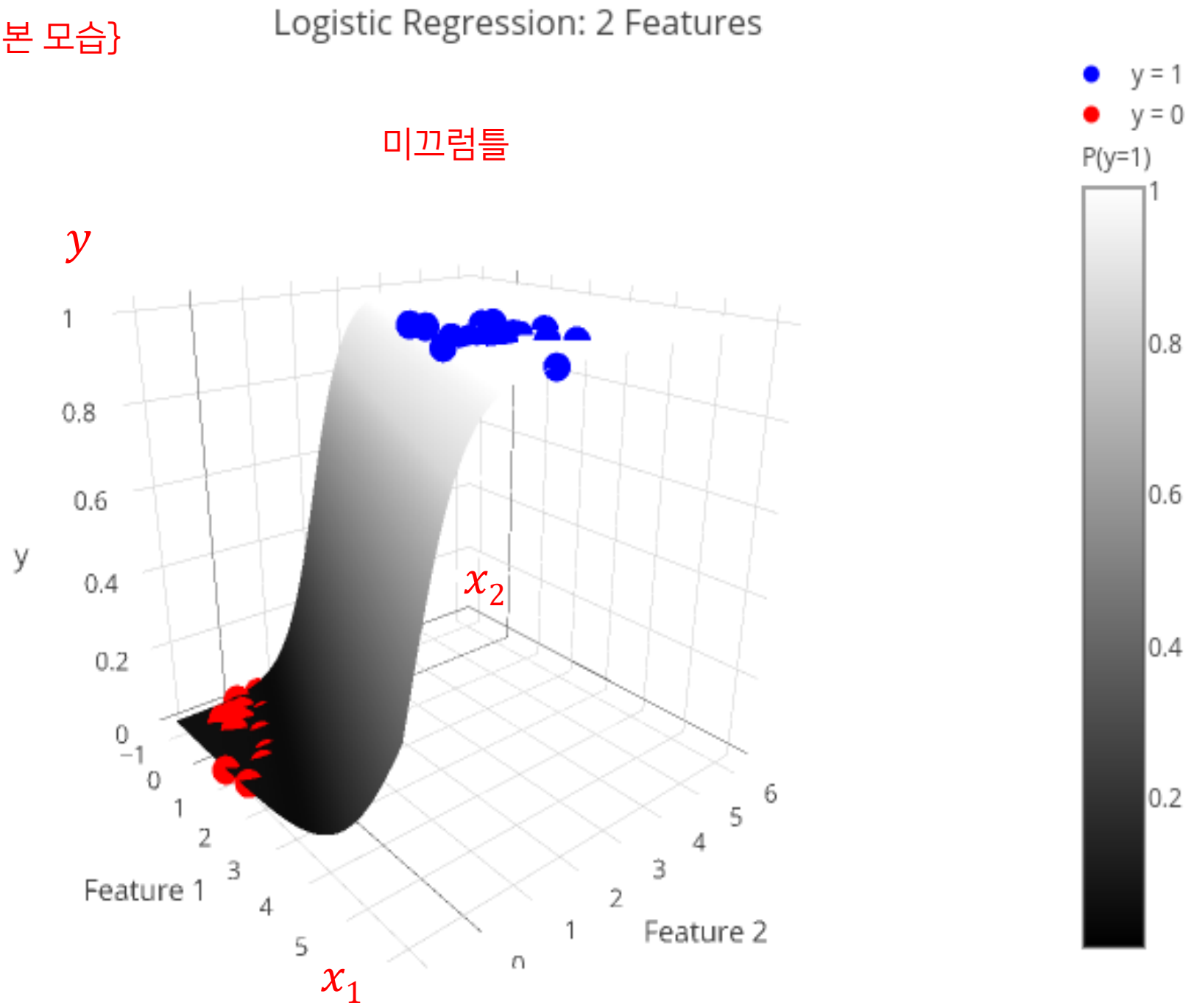


{옆에서 본 모습}



{위에서 본 모습}

{옆에서 본 모습}



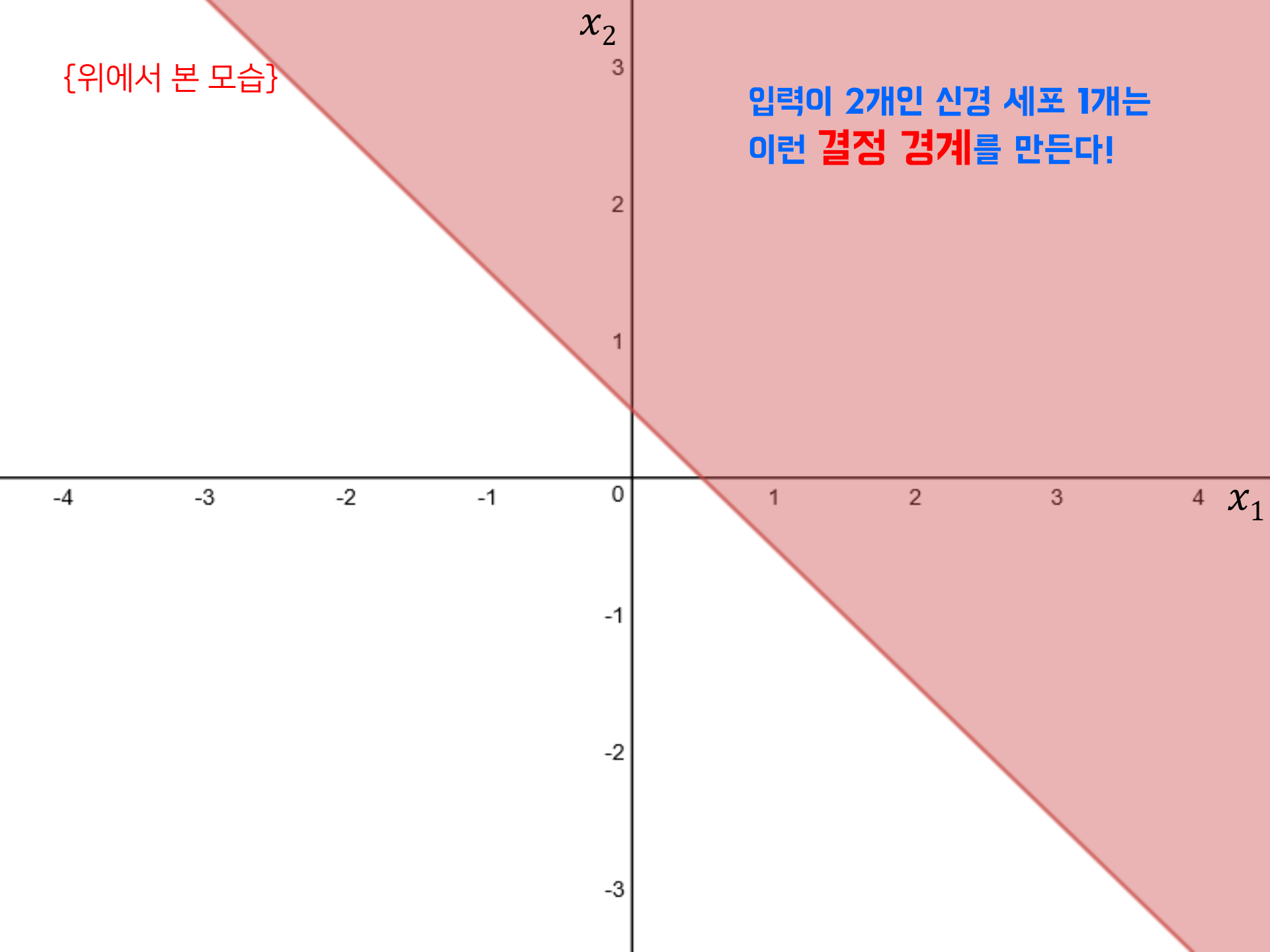
미끄럼틀

y

x_2

x_1

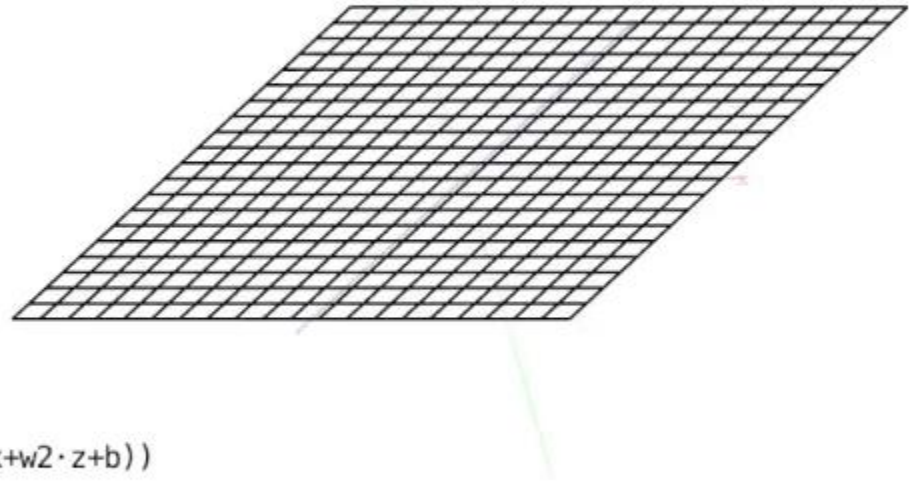
{위에서 본 모습}



입력이 2개인 신경 세포 1개는
이런 **결정 경계**를 만든다!

Decision Boundary in 3D

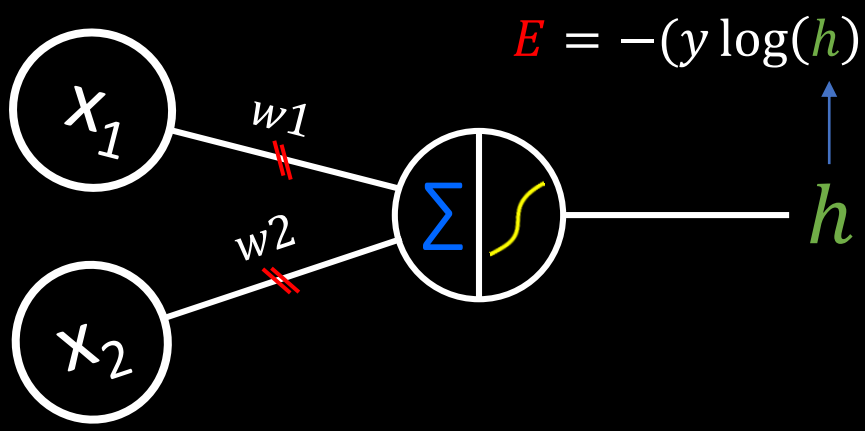
sigmoid(w1·length + w2·width + b)



surface(f(x,z)=sig(w1·x+w2·z+b))
w1 = 0.00
w2 = 0.00
b = 0.00

(실습) 13.py

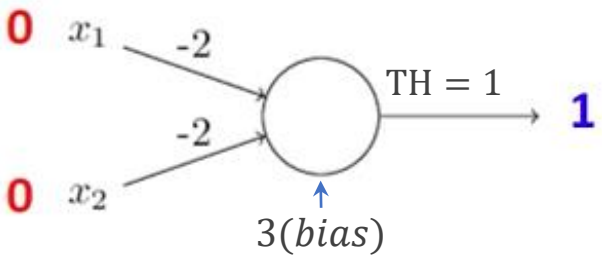
- 입력 두개(x1, x2)를 갖는 뉴런을 이용하여 OR 맞추기
- 한 개의 결정 경계



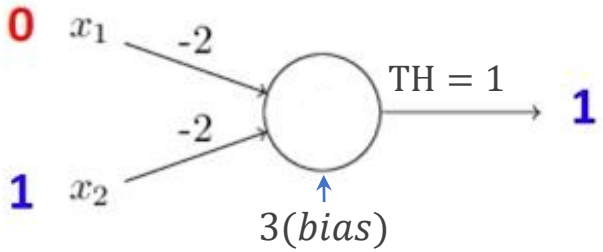
x_1	x_2	$AND(h)$
0	0	0
0	1	0
1	0	0
1	1	1

NAND

- NAND gates are functionally complete.
- We can build any logical functions out of them.



(-2)*0 + (-2)*0+3=3

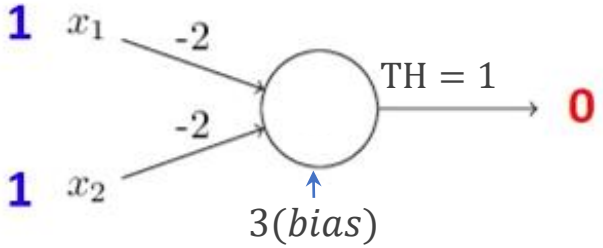


(-2)*0 + (-2)*1+3=1

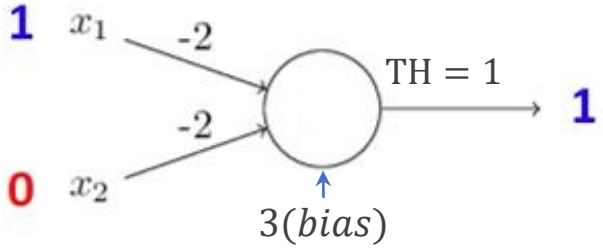
NAND

Truth Table

Input A	Input B	Output Q
0	0	1
0	1	1
1	0	1
1	1	0



(-2)*1 + (-2)*1+3=-1



(-2)*1 + (-2)*0+3=1

요 약

- 로지스틱 리그레션과 분류(classification)
- 로지스틱 리그레션을 위한 cost 함수
- 한 개의 뉴런이 만들어 내는 결정 경계
- 텐서 플로우를 이용한 ML 프로그래밍