

Resampling Method - Seminar 3

Math 567: Winter 2016

February 18, 2016

The jackknife Resampling Method

The **jackknife** was a forerunner of the bootstrap, developed by Maurice Quenouille in 1949 to estimate the **bias** of an estimator $\hat{\theta}$ of a population parameter θ , and extended (and given its name) by John Tukey in 1958 to estimate the **standard error** of $\hat{\theta}$. Draper

1. Estimation

To find the **jackknife** estimate of a parameter, we estimate the parameter for each subsample omitting the i th observation to estimate the previously unknown value of parameter \bar{x}_i

$$\bar{x}_i = \frac{1}{n-1} \sum_{j \neq i}^n x_j$$

Variance Estimation

An estimate of the variance of an estimator can be calculated using the jackknife technique.

$$\text{Var}_{(\text{jack})} = \frac{n-1}{n} \sum_{i=1}^n (\bar{x}_i - \bar{x}_{(\cdot)})^2$$

\bar{x}_i is the parameter estimate based on leaving out the i th observation, and $\bar{x}_{(\cdot)}$ is the estimator based on all of the subsamples.

Bais Estimation

$$\widehat{Bias}_{jack} = (n-1)(\hat{\theta}_{(\cdot)} - \hat{\theta})$$

first order estimate of θ is

$$\widehat{Bias}_{(\theta)} = n\hat{\theta} - (n-1)\hat{\theta}_{(\cdot)}$$

The multiplicative factor $(n - 1)$ is chosen to make this work out exactly right for the case of estimating the variance. Cowles

The Relationship Between the Jackknife and the Bootstrap

The jackknife is like a bootstrap in which sampling is done without replacement instead of with, and the samples are of size $(n1)$ instead of n .

unless n is large, jackknife is less computationally intensive. But if jackknife uses fewer samples than bootstrap, then jackknife is using less information.

2. The jackknife estimate of bias of our dataset using R language

We are computing the jackknife estimate of bias of the median

First we install and load the "bootstrap" package

```
install.packages("bootstrap")  
library(bootstrap)
```

load the data

```
data <- read.csv('Seminar_2.csv', header = TRUE, sep = "")
```

we extract each vector from the dataset

```
col1 <- data[[1]]  
col2 <- data[[2]]
```

theta function

This is the theta function which will be passed to jackknife method

```
theta <- function(x){ median(x) }
```

```
result1 <- jackknife(col1, theta)  
result1$jack.bias  
result : > 0.1124775
```

```
result2 <- jackknife(col2, theta)  
result2$jack.bias  
result : > -0.04999
```

References

Kate Cowles. Computing in statistics.

David Draper. Some notes on jackknife.