

3.1. This problem is simply a binomial distribution. To represent the number 'n' we would need  $2^n$  bags. For example:

- Number 0  $\Rightarrow 2^0$  bags = 1 bag
- Number 1  $\Rightarrow 2^1$  bags = 2 bag
- Number 2  $\Rightarrow 2^2$  bags = 4 bag
- Number 3  $\Rightarrow 2^3$  bags = 8 bag
- Number 4  $\Rightarrow 2^4$  bags = 16 bag
- Number 5  $\Rightarrow 2^5$  bags = 32 bag
- Number 6  $\Rightarrow 2^6$  bags = 64 bag
- Number 7  $\Rightarrow 2^7$  bags = 128 bag
- Number 8  $\Rightarrow 2^8$  bags = 256 bag

3.2. The professor will need to carry  $2^{64}$  bags in his vehicle.

- If each bags weights 4 grams, then he will need to carry  $2^{64} * 4g / 1000kg / 1000 = \text{tons}$
- This is equal to  $18446744073709551616 * 4g / 1000kg / 1000 = \text{tons}$
- The total number of tons is:  $7.3786976294838206464 \times 10^{13} \text{ tons}$

3.3. To prove the number of bags needed to represent the number 'n' the formula I have derived is:

- *For  $b = \text{number of bags needed to represent 'n'}$*   

$$b = 2^n$$

- The base case is:

$$\text{For } n = 0, \quad b = 2^0 = 1$$

- *For  $n = 1$ ,*

$$2^0 + 2^{n-1} = 2^0 + 2^0 = 2$$

- *For  $n = 2$ ,*

$$2^0 + 2^{n-1} + 2^{n-2} \\ 2^0 + (2^0 + 2^0) + 2^0 = 4$$

- *For  $n = 3$ ,*

$$2^0 + 2^{n-1} + 2^{n-2} + 2^{n-3} \\ (2^0) + (2^0 + 2^0) + (2^0 + 2^0 + 2^0 + 2^0) + 2^0 = 8$$

We can see that this is simply the sum of the previous sets.

I have grouped the sets using parenthesis to denote each set starting with  $n=0$ . Finally we add a final  $2^0$ .

- As can be seen from the pattern above the formula is equal to:

$$b = \sum_{i=0}^{n-1} 2^i + 2^0$$

- As we can see, because this hold true for the base case ( $n=0$ ), then it must be true for when  $n=n+1$ . As we see above, this continues to hold throughout.