

Task 1: Gradient Calculation Assignment

SYSC 4415/5415

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Problem Statement

Calculate the gradient of the function:

$$f(x, y, z) = e^{x^2} + y^y + e^{xy} + z \cos(x)$$

at point $(0, -1, 1)$

Helpful Derivative Rules

The following derivative rules may be useful:

Basic Function Derivatives

$$\begin{aligned}\frac{d}{dx}(a^x) &= a^x \ln(a) \\ \frac{d}{dx}(\cos x) &= -\sin x \\ \frac{d}{dx}(\sin x) &= \cos x\end{aligned}$$

Composite Function Derivatives

$$\begin{aligned}\frac{d}{dx}(a^{u(x)}) &= a^{u(x)} \ln(a) \cdot \frac{d}{dx}u(x) \\ \frac{d}{dx}(x^x) &= x^x(\ln x + 1) \\ \frac{d}{dx}(a^{xy}) &= a^{xy} \ln(a) \cdot y \text{ (when differentiating with respect to } x) \\ \frac{d}{dy}(a^{xy}) &= a^{xy} \ln(a) \cdot x \text{ (when differentiating with respect to } y)\end{aligned}$$

Chain Rule

For a composite function $f(g(x))$:

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

Product Rule

For a product of functions $f(x)g(x)$:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Hint for Negative Values

When dealing with expressions like x^x where x is negative:

- Consider writing $(-1)^{-1}$ instead of $(-1)^{(-1)}$;
- Pay attention to signs and simplify carefully;
- You might want to use $|x|$ in that case.