Group-Based Sparse Representation for Image Denoising

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1 Introduction

1.1 Image Denoising

As a classical problem in low level vision, image denoising aims to estimate the clean image X from its noisy observation Y = X + V[1], where V is usually assumed to be additive white Gaussian noise. Image denoising has been widely studied not only because of the evident applications it serves. Being the simplest possible inverse problem, it also provides a convenient platform over which image processing ideas and techniques can be assessed.

In the past decades, extensive studies have been conducted on developing various methods for image denoising. Statistical estimators of all sorts, spatial adaptive filters[2], stochastic analysis, partial differential equations, transform-domain methods, splines and other approximation theory methods, morphological analysis, order statistics, and more, are some of the many directions explored in studying this problem.

Among these method, sparse representation based prior is more prevalent due to its simplicity and stability. Thus, many image denoising works are based on sparse coding. In sparse based method, image is firstly sliced into many patches and then sparse algorithm is applied into each patch. For example, in K-SVD[3] method, the learned dictionary is obtained from and sparse representation is computed for each patch iteratively.

1.2 Motivation

Patch is the unit of sparse representation, the patch-based method is pretty robust by representing image with an ensemble of patches. However, each patch is considered independently in dictionary learning and sparse coding, which ignores the relationship among similar patches in essence, resulting in inaccurate sparse coding coefficients. Actually, different patches have similar information. Image patches that have similar patterns can be spatially far from each other and thus can be gathered in the whole image, which are of great significance in image restoration.

Based on the NSS property of an image, it is revealed that structured or group sparsity can provide more powerful reconstruction performance for noise removal.

In group-based method, we need to find nonlocal self-similar patches and stack them to form the group. And then we can follow the standard formulation[2] to process images:

$$\hat{\alpha_G} = arg \min_{\alpha_G} ||y - D_G\alpha_G||_2^2 + \lambda||\alpha_G||_p \tag{1}$$

where p=0 or 1, D_G is the dictionary.

There are lots of applications via group-based method, such as image denoising, debluring, inpainting, etc. However, we will mainly focus on image denoising [3] in this project.

1.3 Structure and Contribution

This project is mainly designed a new algorithm based on group-based sparse representation, combining patch, NSS and group base. And my report is made up of five with part 1 focus on the background and motivation of our project. In part 2, some prior work of this topic will be introduced. The detailed procedure of our method is shown in part 3 and our final experimental results and subjective comparisons are presented in part 4. Lastly, part 5 consists of our conclusion and future outlook.

Firstly, we mainly read, analyze algorithms and run related code in two papers, [1] and [4]. Wiejie and Xiangyu is responsible for[1], and Zhengzhao and I mainly focus on [4].Through analyzing specific methods and algorithms used in these two papers, we decided to combine methods used in these two articles and finally propose our basic approach to use Group-Based Method and ADMM optimization.

Our team is divided into two groups with one group dealing with algorithm part and achieving algorithms with MATLAB and the other analyzing and comparing experiments' outcome of four methods, including K-VSD, BM3D[5], TV[6] and Wiener Filter[7]. I was mainly responsible for obtaining outcomes(running time, PSNR) of K-VSD and Wiener Filter, while zhengzhao mainly focus on BM3D and TV.

2 Prior Work

In the paper Group-based Sparse Representation for Image Restoration[8], authors exploited the concept of group as the basic unit of sparse representation, and establish a novel sparse representation modeling of natural images, called group-based sparse representation (GSR) in order to solve a large-scale optimization problem with high computational complexity in dictionary learning.

In the paper Patch Group Based Nonlocal Self-Similarity Prior Learning for Image Denoising[4], it is proposed that only the NSS of input degraded image is exploited in most existing methods, while how to utilize the NSS of clean natural images is still an open problem awaits solving. Thus, writers proposed a patch group (PG) based NSS prior learning scheme to learn explicit NSS models from natural images for high performance denoising. PGs are extracted from training images by putting nonlocal similar patches into groups, and a PG based Gaussian Mixture Model (PG-GMM) learning algorithm is developed to learn the NSS prior. We demonstrate that, owe to the learned PG-GMM, a simple weighted sparse coding model, which has a closed-form solution, can be used to perform image denoising effectively. These two paper are the main resources to motivate us to discover and design a new model with the knowledge of patch group and nonlocal self-similarity.

In the paper Image Denoising via Group Sparsity Residual Constraint[1], authors had proposed a new prior model for image denoising via group sparsity residual constraint (GSRC). This concept of group sparsity residual is proposed in order to enhance the performance of group sparse-based image denoising. Experimental results shows that the proposed method not only outperforms many state-of-the-art denoising methods, but also results in a faster speed. This paper gives more clear instruction about how to design and implement our model.

3 Proposed Method

3.1 Flowchart

This part mainly talks about the whole process and algorithm of our method.

First, we find a patch with n*n, and set an observation window based on this patch(the target patch is the center of this searching window), then we can find some other patches witch are similar to the target patch In this window, take out both the target patch and the similar patches and write down their coordinates. Next, we take out take out these self-similar patches and write their coordinates down vectorize these patches as colunms to form a matrix, thus, a group forms. The next step is using our method to sparse represent and obtain the dictionary. Finally, we devectorize each colunm of the dictionary, reform each patches and put these patches into a blank image by coodinates (take average if there exist duplicate patches).

The flowchart below shows the process of one iteration in our algorithm, when one iteration is end, we need to pick another target patch(this new patch is usually parallel with the previous one) and set a new searching window to begin the next iteration, the distance between this new target patch and the previous target patch is called step size.

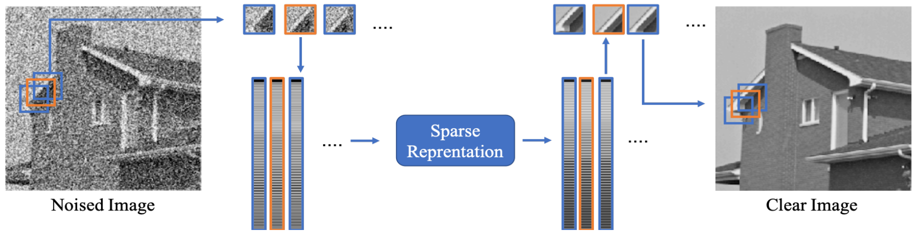

Figure 1: Flowchart of Our Algorithm

3.2 Algorithm

3.2.1 Dictionary Learning

The learning of the dictionary is mainly based on the covariance matrix of the group. Firstly, from the the flowchart presents, we can obtain the group matrix and then compute the covariance matrix $\Omega = PDP^{\mathrm{T}}$ of the group matrix. After that, we apply SVD method to the covariance matrix Ω, and take the first matrix P with orthogonal columns as the dictionary.

3.2.2 Self-Adaptive λ

In our optimization problem, the value of λ is uncertain, it depends on the noise situation of both the current group and the whole image. So in the process of optimization, the value of λ changes and it follows this equation:

$$\lambda = std(WholeImage) * std(CurrentGroup)/25 \tag{2}$$

Among the equation, std() means the standard deviation, and $\frac{1}{25}$ is a empirical parameter, it is based on the experiments we did.

3.2.3 ADMM Algorithm

The optimization problem of our algorithm is:

$$\hat{x} = arg \min_{x} ||Ax - y||_2^2 + \lambda ||x||_1 \tag{3}$$

Where A is the SVD Dictionary, y is the group.

So it's a pure LASSO problem for each group, and we can use ADMM[9] Algorithm to optimize our problem. Alternating Direction Method of Multipliers (ADMM) combines the decomposability of the dual ascending method with the upper bound convergence property of multiplier method, which can be used to solve the problem of decomposable convex optimization, especially in solving large-scale problem.

In order to apply ADMM to this problem we can rewrite (3) as:

$$\begin{aligned} \hat{x} &= \arg\min_{x} ||Ax - y||_2^2 + \lambda ||z||_1 \\ & subject\ to\ x - z = 0. \end{aligned} \tag{4}$$

And then we can use the augmented Lagrangian with penalty parameter $\tau > 0$ with equation:

$$L_{\frac{1}{\tau}}(x, y, z) = ||Ax - y||_2^2 + \lambda ||x||_1 + \frac{1}{\tau}\langle y, x - z\rangle + \frac{1}{2\tau}||x - z||_2^2 \tag{5}$$

The specific ADMM steps for solving Lasso problems are showed below. Via this algorithm, we can get updated patches, which finally contribute to the denoised image.

Algorithm 1: ADMM for LASSO

Initialization: $k = 1$, y, z and $\tau > 0$;

while *convergence criterion is not satisfied* **do**

$x_k \leftarrow (D^T D + \frac{1}{\tau} I)^{-1} (D^T G_i + \frac{1}{\tau}(z_{k-1} - y_{k-1}))$;

$z_k \leftarrow S_{\lambda\tau}(x_k + y_{k-1})$;

$y_k \leftarrow y_{k-1} + \frac{1}{\tau}(x_k - z_k)$;

$k \leftarrow k + 1$;

end

Figure 2: ADMM for Solving LASSO Problem

4 Experimental Results

In this section, our experiments' results will be shown in the form of comparisons between our algorithm and other denoising algorithms.

We use TV, BM3D, K-SVD and Wiener Filter as comparisons, and take Peak Signal to Noise Ratio (PSNR) as a kind of objective measurement of image quality, which also reflects the effects of different denoising algorithms from another side.

4.1 Results

	Lena($\sigma = 0.01$)	House($\sigma = 0.01$)	Lena($\sigma = 0.1$)	House($\sigma = 0.1$)
Group-Based	27.524	29.962	21.167	21.733
TV	27.06	28.46	20.56	21.03
BM3D	30.31	32.80	12.471	12.48
K-VSD	27.92	30.43	23.12	23.85
Wiener	26.73	27.05	18.51	18.65

Table 1: PSNR of Various Algorithm with Different Variance

From left to right, it shows original, denoised images, and images after total variation, BM3D, K-SVD and proposed method.

Figure 3: Experiment Results of Lena with $\sigma = 0.1$

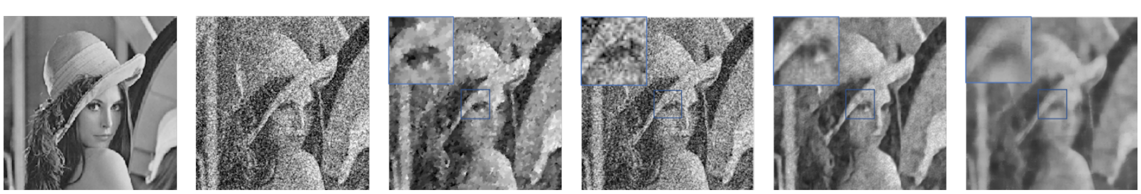

Figure 4: Experiment Results of Lena with $\sigma = 0.1$

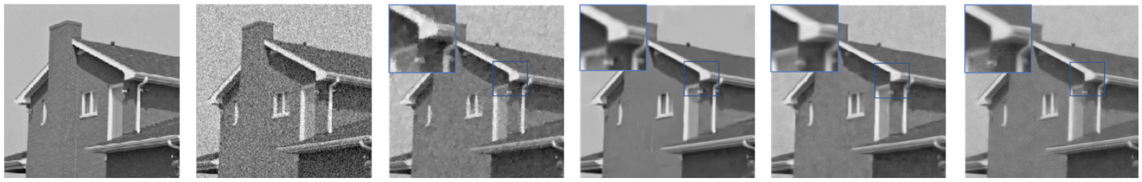

Figure 5: Experiment Results of House with $\sigma = 0.1$

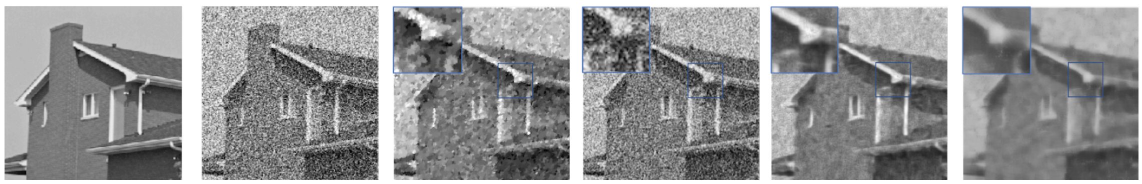

Figure 6: Experiment Results of House with $\sigma = 0.1$

4.2 Analysis

In the experiment, we add gaussian noise into ground-truth image to obtain noised image with standard deviation of noise data, $\sigma = 0.01$ and $\sigma = 0.1$. And in order to compare our method with other popular method, total variation denoising, BM3D and K-SVD are also implemented as comparisons. Through comparing the outcomes, we can see pros and cons of different denoised method.

- Group-based Method

  Pros:

  1. High information utilization via non-local similarity
  2. No need to train dictionary
  3. Good performance of processing details
  4. Good performance even in high noise environment

  Cons:

  1. Relatively slow running speed
  2. The effect is not good as expected

- TV

  Pros: Good performance in processing contour

  Cons: PSNR is pretty low which reflects that the denoised effect is not good

- BM3D

  Pros: Good performance of processing contour in high-noise environment

  Cons: PSNR is relatively low which reflects that the denoised effect is not good

- K-SVD

  Pros: PSNR is relatively high which reflects that the denoised effect is pretty good

  Cons: In low-noise environment, the details of denoised image are blurred and present in blocks; also, its running speed is relatively slow

- Wiener

  Pros: Fast running speed

  Cons: Relatively weak performance in processing details

5 Conclusion

5.1 Conclusion

In this project, our group designs a new algorithm based on dictionary and group. Our group tries to gather nonlocal self-similar patches into a group and solve a LASSO problem with ADMM using SVD dictionary.

Based on the experiments' outcomes, we can clearly see that the method we proposed perform pretty well in processing details and high noise environment. Also, there is no need for us to train extra dictionary and its information utilization is pretty high via non-local similarity. However, there are some problems that its running speed is relatively slow and final results are not so good as expected.

5.2 Future outlook

Obviously, our algorithms still need great improvement, so we come up with some future outlook.

- Improve running speed

  Instead solving each group one by one as there are many iterations in a whole process which leads to the slow speed, we could solve several groups at the same time. In other word, we can parallel compute the group solution by GPU.

- Regularization can be added to obtain better outcome

  At first, we decided to add a TV regularization, but after discussion we found we couldt do any treatment to the edge of each patch, so if we add a TV regularization, it may destroy the details rather than improve its performance. So whether to add regularization, which regularization we should add is still a problem awaits for research and discussion.

References

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