Diversion Works

Design of Hydraulic Structures (CVL381)

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Theory of Seepage Flow

From the **Continuity Equation**:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

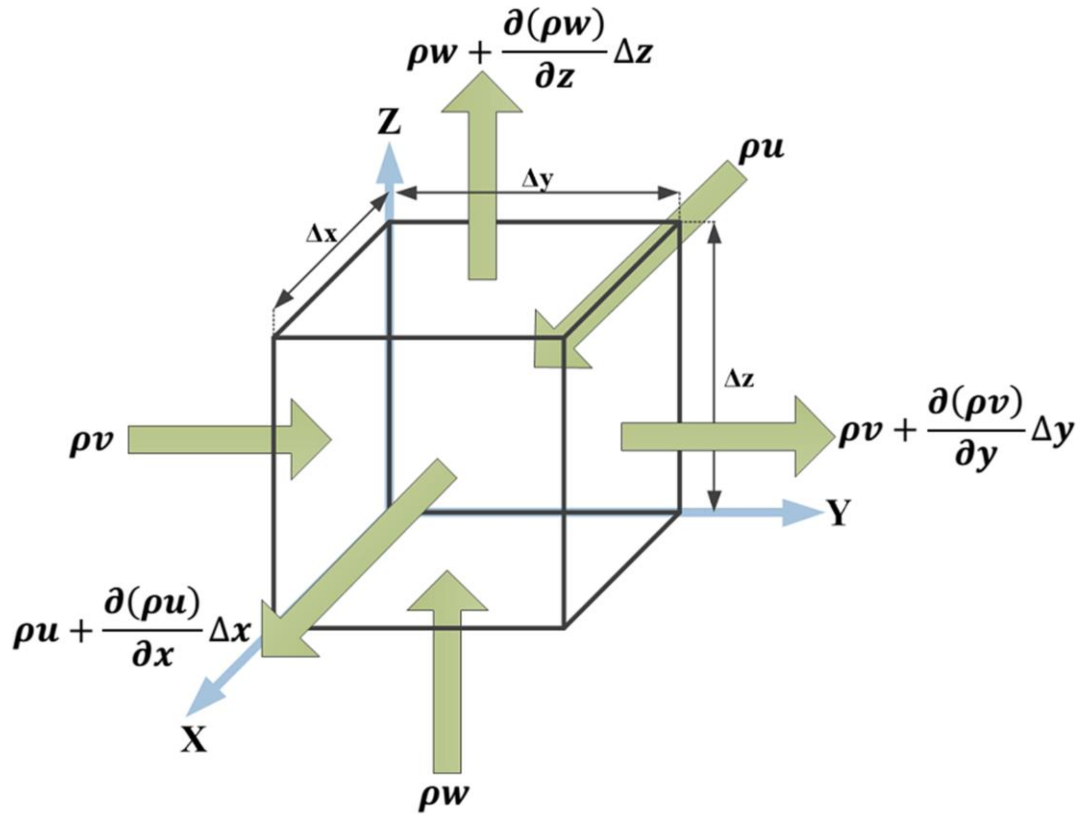
From **Darcy's Law**:

$$u = -K\frac{\partial h}{\partial x}, \quad v = -K\frac{\partial h}{\partial y}, \quad w = -K\frac{\partial h}{\partial z}$$

Putting the values of u, v and w in the continuity equation, we get:

$$K\frac{\partial^2 h}{\partial x^2} + K\frac{\partial^2 h}{\partial y^2} + K\frac{\partial^2 h}{\partial z^2} = 0$$

$\nabla^2 h = 0$ **(Laplace Equation)**

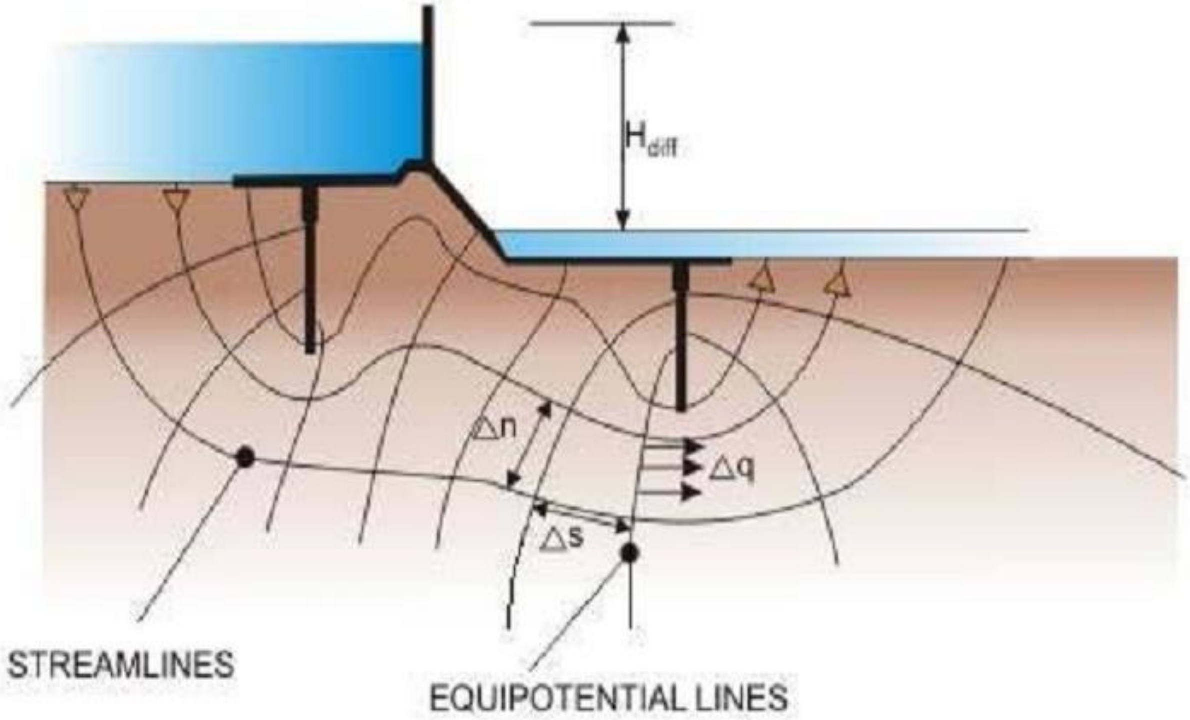


__**Assumptions:**__

- *The soil is homogeneous and isotropic.*
- *The voids are completely filled with water.*
- *No consolidation or expansion of the soil takes place.*
- *The soil and water are incompressible.*
- *The flow is steady and obeys Darcy's law.*

Flownet Diagram

When the subsurface equipotential lines and the seepage flowlines (streamlines) are plotted, a **flownet** is constructed.



Let there be $\boldsymbol{N_f}$ flowlines underneath the structure. Then,

$$q = N_f \Delta q$$

From Darcy's law,

$$\Delta q = K \frac{\Delta h}{\Delta s} \Delta n$$

Let $\boldsymbol{N_d}$ be the number of potential drops between the u/s and d/s water levels. Then,

$$\Delta h = \frac{H_{diff}}{N_d}$$

Assuming $\Delta n \approx \Delta s$,

Seepage discharge underneath the structure:

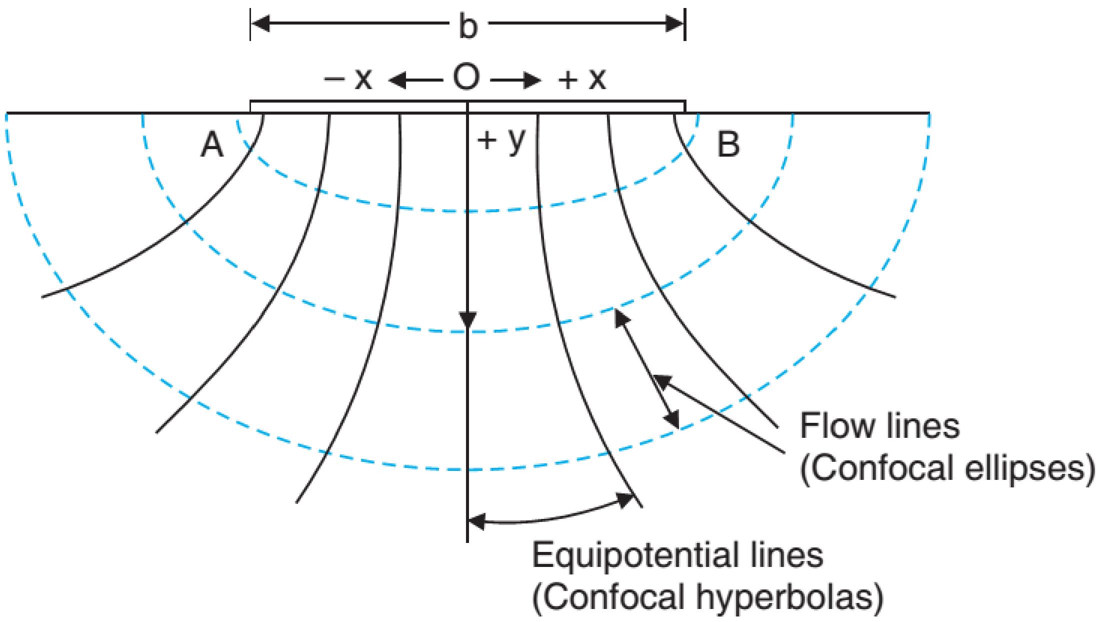
$$q = K H_{diff} \frac{N_f}{N_d}$$

Determination of Seepage Pressure underneath the structure

1. *Trial and Error OR Graphical method*
2. *Mathematical solution of the Laplace Equation*
3. *Khosla's method of independent variables*
4. *Method of electrical analogy*
5. *Method of relaxation*

Weaver's Solution of the Laplace Equation

For 2D seepage flow occurring under a straight horizontal floor:



Streamlines: **Confocal ellipses** with AB as the major axis and the foci at A and B.

$$\frac{x^2}{\left(\frac{b}{2}\cosh u\right)^2}+\frac{y^2}{\left(\frac{b}{2}\sinh u\right)^2}=1$$

u: Stream function

$$\cosh u=\frac{1}{2}\left(e^u+e^{-u}\right)$$

$$\sinh u=\frac{1}{2}\left(e^u-e^{-u}\right)$$

Equipotential lines: **Confocal hyperbolas**

$$\frac{x^2}{\left(\frac{b}{2}\cos v\right)^2}-\frac{y^2}{\left(\frac{b}{2}\sin v\right)^2}=1$$

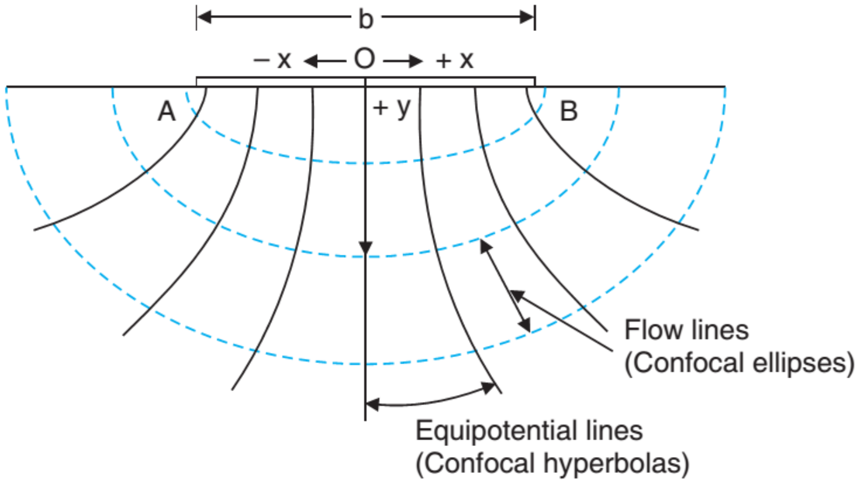
v: Pressure function - $v=\pi\Phi=\pi\frac{P}{H}$

Φ: Pressure head ratio = Pressure at any point expressed as a ratio of the total head

Weaver's Solution of the Laplace Equation

Let **P** be any point within the 2D seepage flow domain.

In order to satisfy the above equations for streamlines and equipotential lines, the following relationship is defined:



$$z = \frac{b}{2}\cosh w$$

$$z = x + iy$$

$$w = u + iv$$

$$x + iy = \frac{b}{2}\cosh(u + iv)$$

$$x + iy = \frac{b}{2}(\cosh u \cos v + i \sinh u \sin v)$$

$$x = \frac{b}{2}\cosh u \cos v$$

$$y = \frac{b}{2}\sinh u \sin v$$

Limiting Case: The first streamline is just beneath the outline of the floor AB.

Hence, we can consider: $u = 0$ at $y = 0$.

Therefore, $x = \frac{b}{2}\cosh 0 \cos v = \frac{b}{2}\cos v$

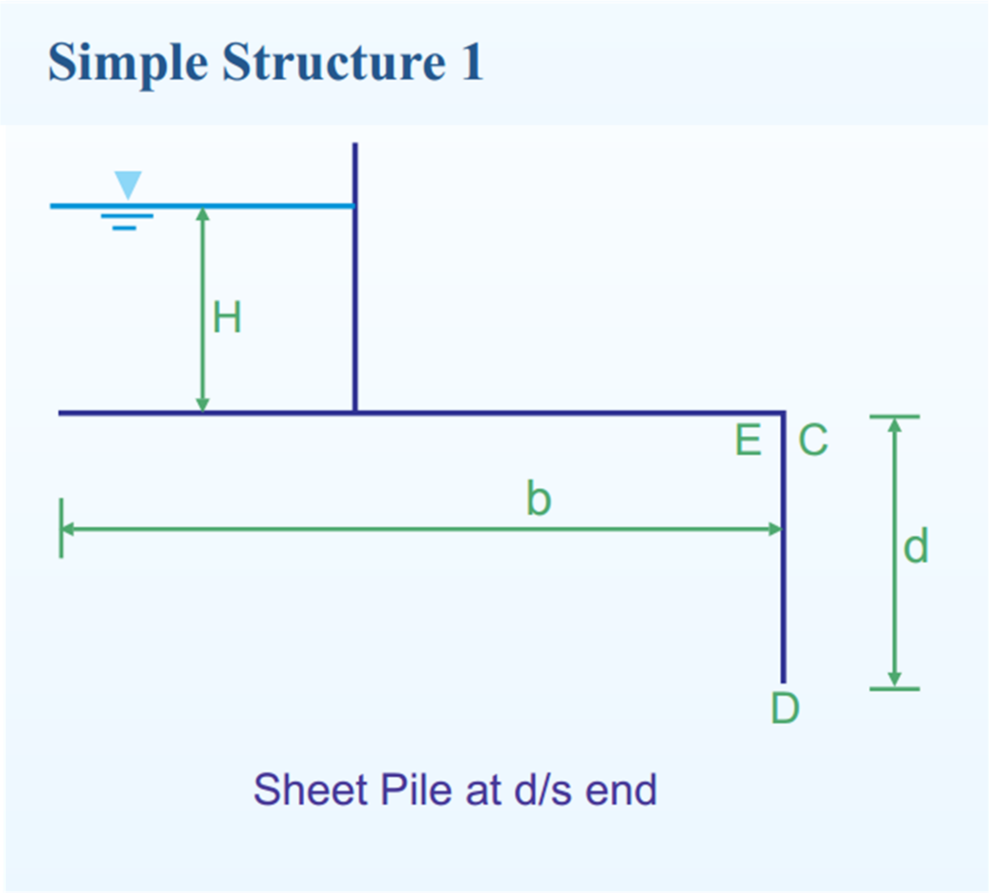
$$v = \pi\Phi = \cos^{-1}\frac{2x}{b}$$

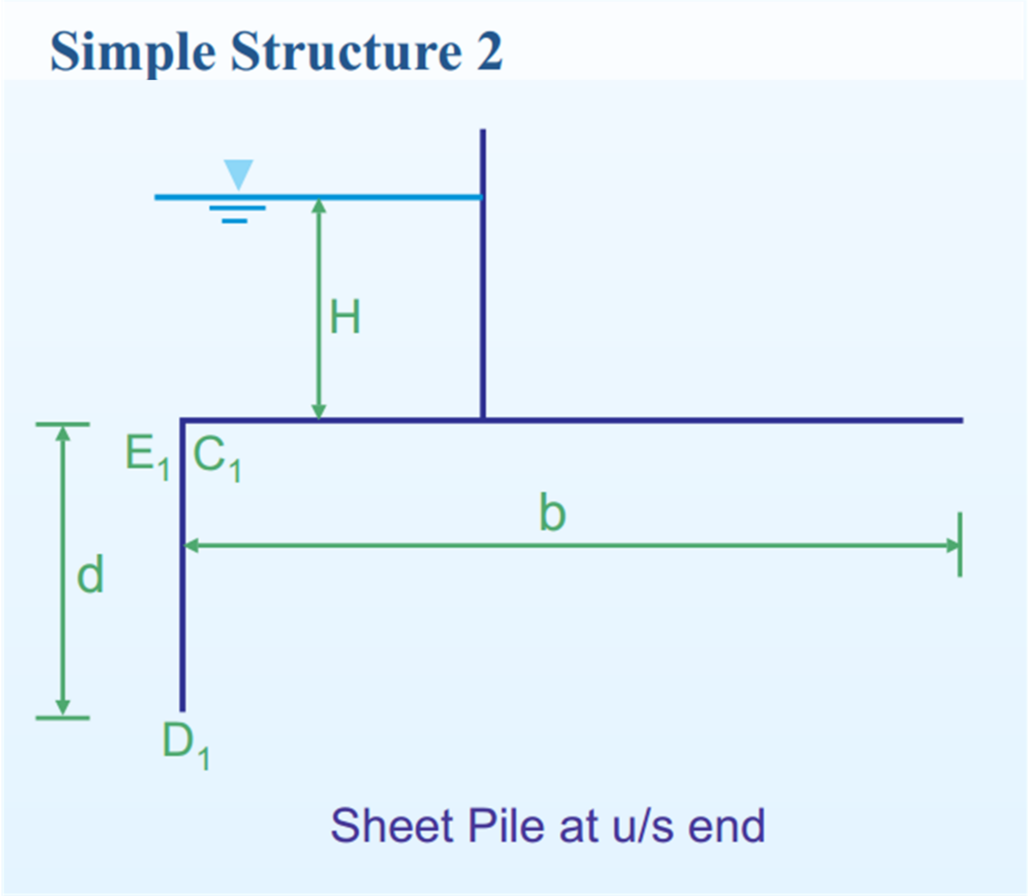
$$\Phi = \frac{1}{\pi}\cos^{-1}\frac{2x}{b}$$

Khosla's method of independent variables

- Solution of the Laplace equation for seepage flow is feasible only for a simple elementary profile of a weir section subjected to simple boundary conditions.
- Practically, barrage and weir sections conform to a composite profile with complex boundary conditions.
- Khosla along with other investigators obtained solutions of the Laplace equation for a number of **simple profiles**, using the **method of independent variables**.
- Solutions for these simple profiles are obtained in terms of the **pressure head ratio Φ** at the **key points**.
- The **key points** are the junctions of the sheet piles with the floor.
- The values of **Φ** calculated for the actual structure after breaking it down to these simple profiles is valid for design purpose after corrections for the following are made:
  1. *Floor thickness*
  2. *Mutual interference of the piles.*
  3. *Slope of the floor.*

Khosla's method of independent variables





$$\alpha = \frac{b}{d}$$

$$\lambda = \frac{1}{2}\left[1 + \sqrt{1 + \alpha^2}\right]$$

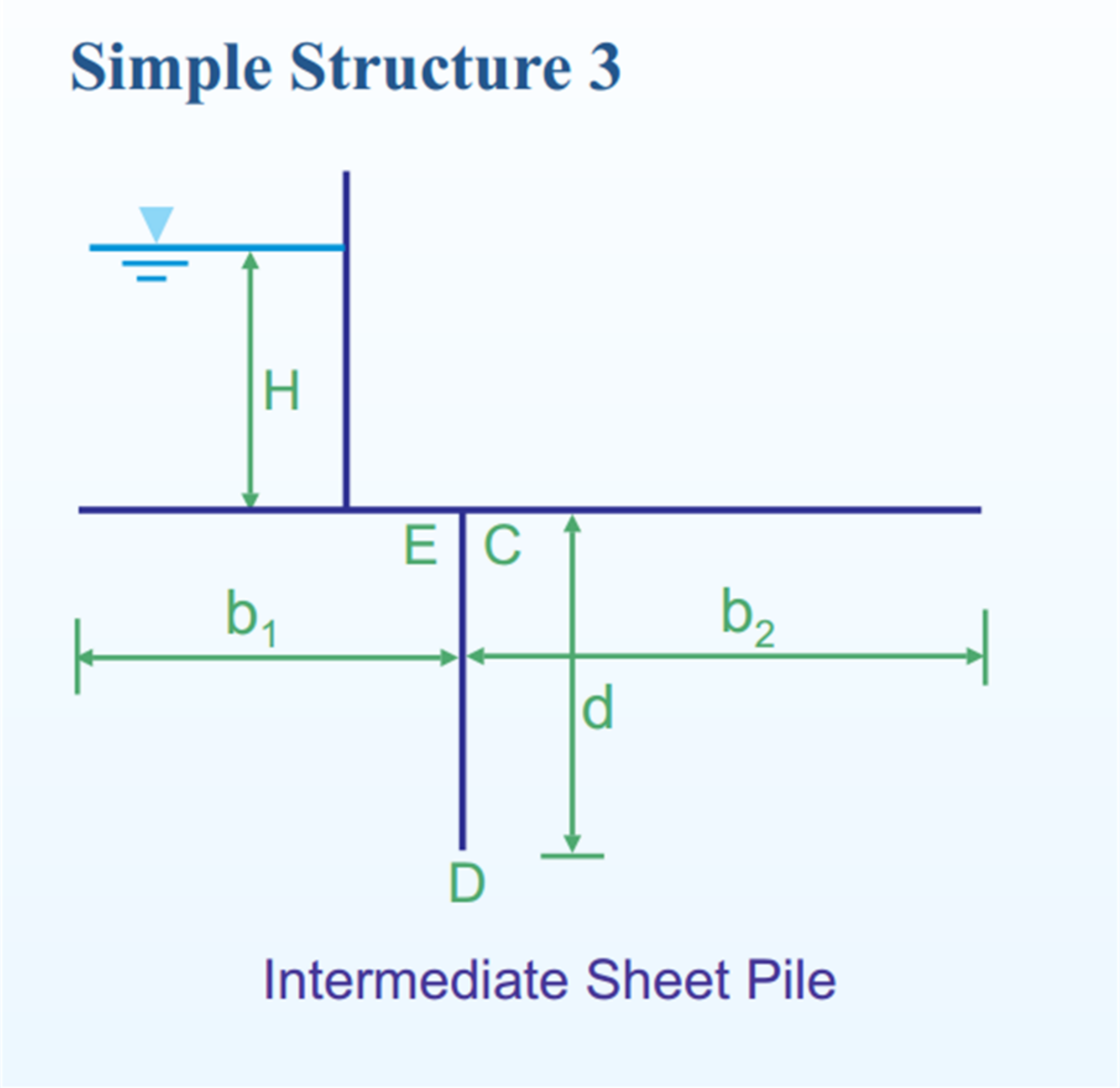
$$\Phi_E = \frac{100}{\pi}\cos^{-1}\left[\frac{\lambda - 2}{\lambda}\right]$$

$$\Phi_D = \frac{100}{\pi}\cos^{-1}\left[\frac{\lambda - 1}{\lambda}\right]$$

$$\Phi_{C_1} = 100 - \Phi_E$$

$$\Phi_{D_1} = 100 - \Phi_D$$

Khosla's method of independent variables



$$\alpha_1 = \frac{b_1}{d}$$

$$\alpha_2 = \frac{b_2}{d}$$

$$\lambda_1 = \frac{1}{2}\left[\sqrt{1+{\alpha_1}^2} - \sqrt{1+{\alpha_2}^2}\right]$$

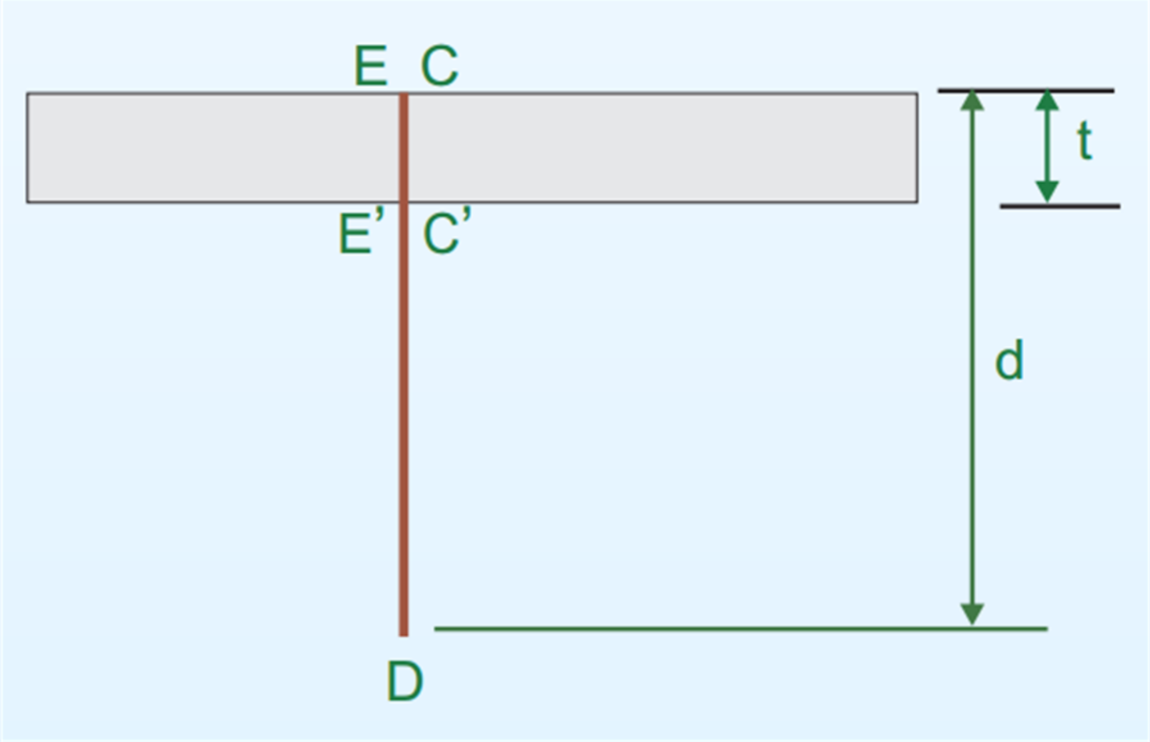
$$\lambda_2 = \frac{1}{2}\left[\sqrt{1+{\alpha_1}^2} + \sqrt{1+{\alpha_2}^2}\right]$$

$$\Phi_C = \frac{100}{\pi}\cos^{-1}\left[\frac{\lambda_1+1}{\lambda_2}\right]$$

$$\Phi_D = \frac{100}{\pi}\cos^{-1}\left[\frac{\lambda_1}{\lambda_2}\right]$$

$$\Phi_E = \frac{100}{\pi}\cos^{-1}\left[\frac{\lambda_1-1}{\lambda_2}\right]$$

Khosla's method – Correction for Floor Thickness



The key points **E/E_1** and **C/C_1** correspond to the top of the floor.

The values of the pressure head ratio at the corresponding key points **E'/E_1'** and **C'/C_1'** at the bottom of the floor is calculated using linear interpolation method.

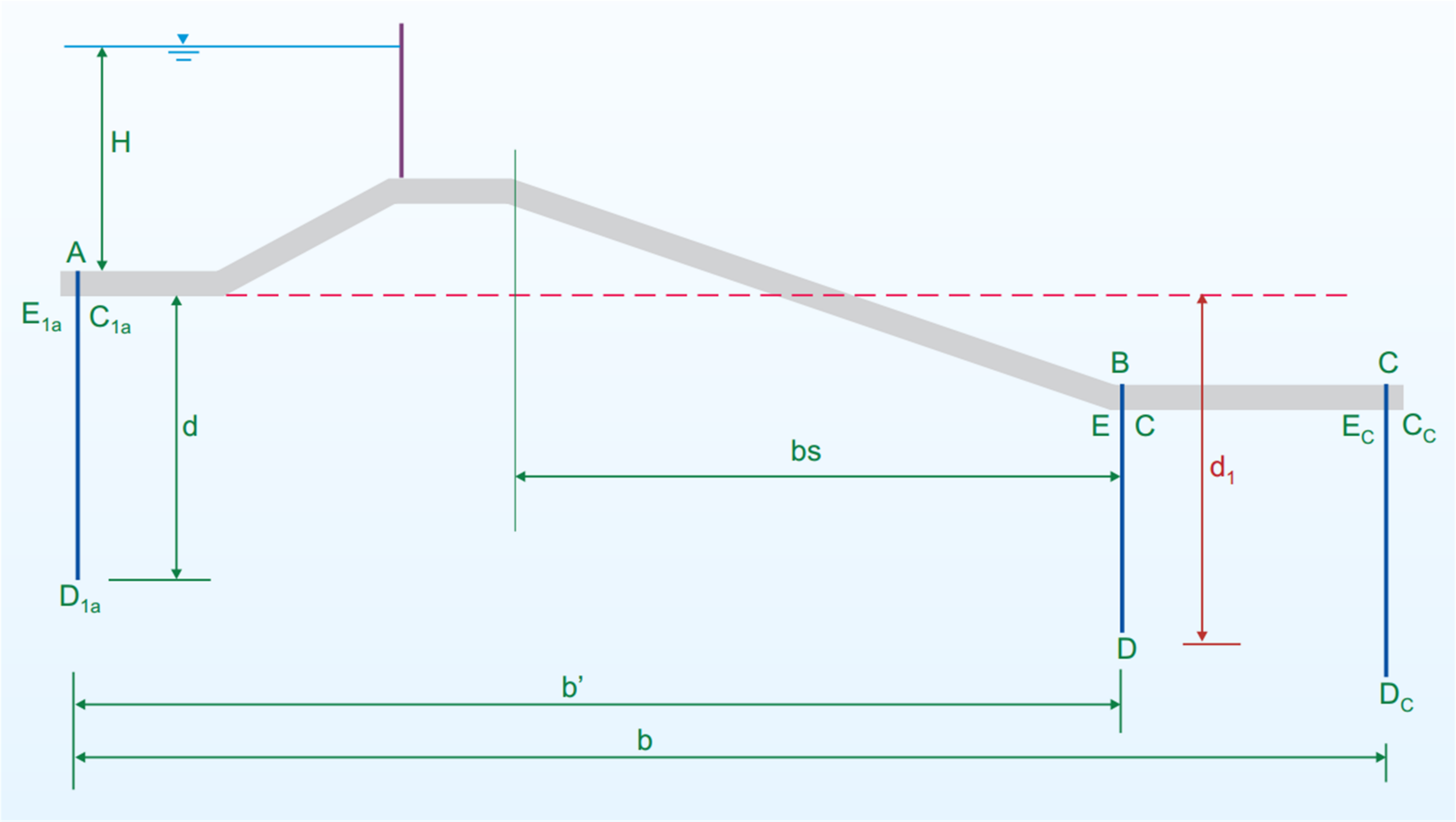
$$\Phi_E' = \Phi_E - \left(\frac{\Phi_E - \Phi_D}{d}\right) t$$

$$\Phi_C' = \Phi_C + \left(\frac{\Phi_D - \Phi_C}{d}\right) t$$

Khosla's method – Correction for Mutual Interference of Piles

Correction **C%** for the key point C_{1a} of sheet pile A for the interference of pile B:

$$C = 19\sqrt{\frac{d_1}{b'}}\left(\frac{d_1+d}{b}\right)$$



b': Distance between two pile lines A and B.

d_1: Depth of the interfering pile B (whose influence is to be determined) below the level at which the interference is desired (i.e., below the top of the neighbouring pile A).

d: Depth of the pile A.

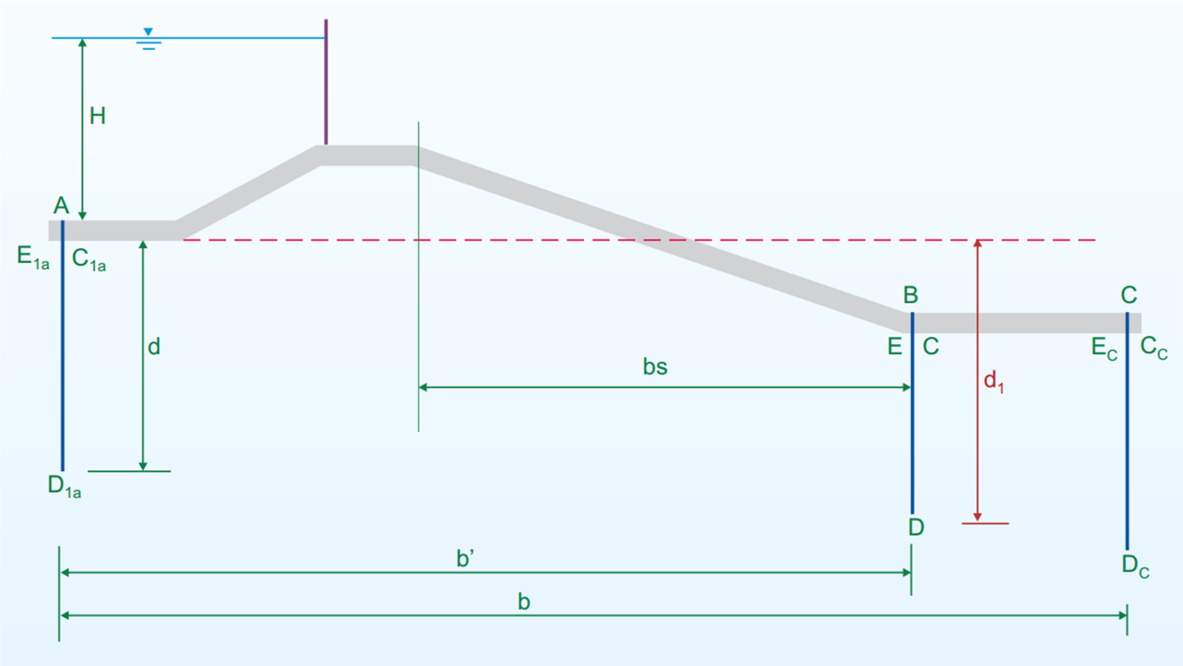
b: Floor length.

Khosla's method – Correction for Mutual Interference of Piles

The correction C% is positive for key points upstream and negative for key points downstream of the interfering pile.

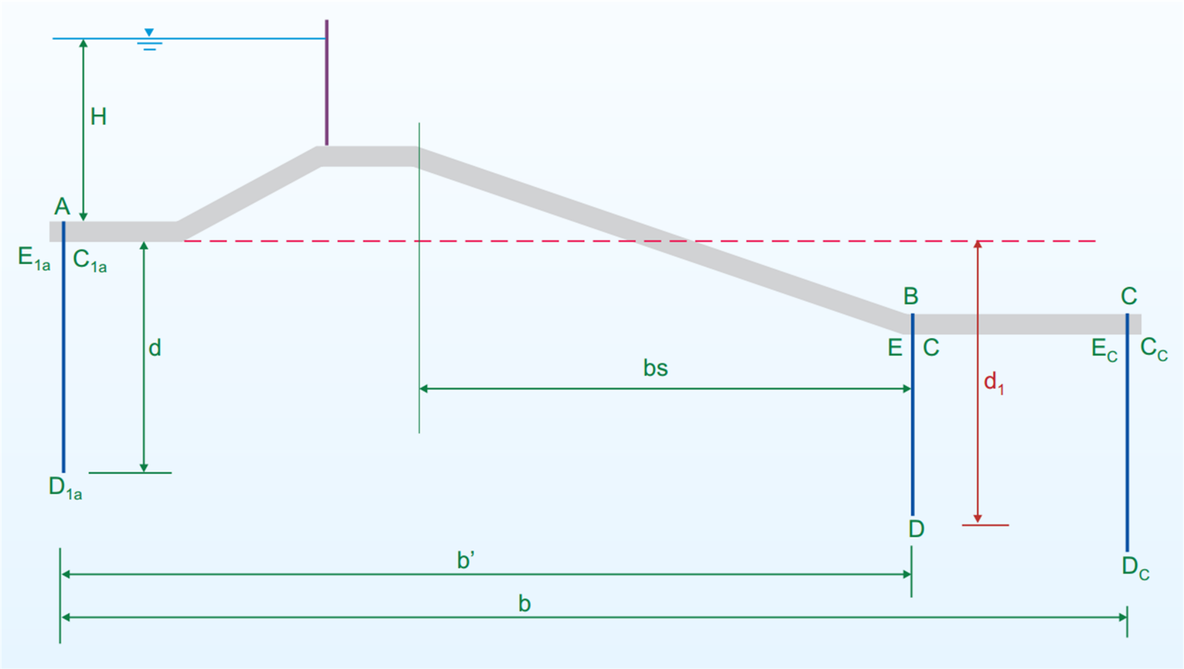
The correction is calculated only for the key points of the adjacent pile towards the interfering pile.

Key Points	Interference of Pile	Correction Type
C_{1a} (Pile A)	Pile B	+
E (Pile B)	Pile A	-
C (Pile B)	Pile C	+
E_C (Pile C)	Pile B	-



Khosla's method – Correction for Slope of the Floor

- The correction for slope of the floor is applied to the pressures of the key point of that pile which is fixed either at the beginning or at the end of the slope.
- Positive correction – positive slope.
- Negative correction – negative slope.



Correction at key point E of pile B:

$$C = C_s \frac{b_s}{b'}$$

Slope (V:H)	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8
Correction (C_s) in %	11.2	6.5	4.5	3.3	2.8	2.5	2.3	2.0

Khosla's method -Exit Gradient

Exit gradient is calculated as:

$$G_E = \frac{H}{d}\frac{1}{\pi\sqrt{\lambda}}$$

H: Effective hydraulic head

d: Depth of the downstream pile

$\alpha = \frac{b}{d}$, $\lambda = \frac{1}{2}\left[1 + \sqrt{1 + \alpha^2}\right]$

- If there is no d/s pile, $d = 0$, and therefore, $G_E = \infty$.
- **It is therefore necessary to provide a vertical sheet pile at the d/s end of the impervious floor.**
- To prevent piping failure, the exit gradient is not allowed to exceed a critical value depending on the soil type.
- The critical exit gradient for sand varies between $\frac{1}{7}$ and $\frac{1}{5}$.

Numerical Problem: Example 3

Using the Khosla's method, calculate the residual seepage pressures at the key points for the weir profile shown in the figure. Also calculate the value of the exit gradient.