DAA
ASSIGNMENT-4

Q)What are the differences between dynamic programming and divide and conquer approaches?

Ans) The **main difference** between divide and conquer and dynamic programming is that the **divide and conquer combines the solutions of the sub-problems to obtain the solution of the main problem while dynamic programming uses the result of the sub-problems to find the optimum solution of the main problem.**Divide and conquer and dynamic programming are two algorithms or approaches to solving problems. Divide and conquer algorithm divides the problem into subproblems and combines those solutions to find the solution to the original problem. However, dynamic programming does not solve the subproblems independently. It stores the answers of subproblems to use them for similar problems.

Q)How dynamic programming is used to solve the Knapsack problem?

Ans) In the Dynamic programming we will work considering the same cases as mentioned in the recursive approach. In a DP[][] table let's consider all the possible weights from '1' to 'W' as the columns and weights that can be kept as the rows. The state DP[i][j] will denote the maximum value of 'j-weight' considering all values from '1 to ith'. So if we consider 'wi' (weight in 'ith' row) we can fill it in all columns which have 'weight values > wi'. Now two possibilities can take place:

- Fill 'wi' in the given column.
- Do not fill 'wi' in the given column.

Now we have to take a maximum of these two possibilities, formally if we do not fill 'ith' weight in 'jth' column then DP[i][j] state will be same as DP[i-1][j] but if we fill the weight, DP[i][j] will be equal to the value of 'wi'+ value of the column weighing 'j-wi' in the previous row. So we take the maximum of these two possibilities to fill the current state.

Q3) Consider the travelling salesperson instance defined by the

following cost matrix

□ 20 30 10 11

15 □ 16 4 2

3 5 □ 2 4

19 6 18 □ 3

16 4 7 16 □

Draw the state space tree with Least Cost B&B and show the

reduced matrices corresponding to each of the nodes.

Ans3)

DAA

Travelling Salesman Problem.

$\boxed{Q_3}$ Matrix:

$$\begin{array}{c c} \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} & \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

$= 21$

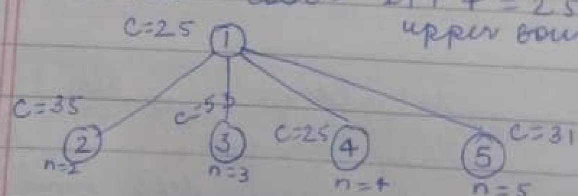
→ reduction ①

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$4 = \quad 1 \quad 0 \quad 3 \quad 0 \quad 0$

reduced cost $= 21 + 4 = 25$

upper bound $= \infty$



→ Now, for 2 node cost

$= C(1,2) + r + \hat{r}$

$= 10 + 25 + 0$

$= \underline{\underline{35}}$

②

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

(2)

→ cost for 3 rd node

$$\begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} & \begin{matrix} \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 11 & 0 & 0 & 0 & 0 \end{matrix} & = 4 \end{array}$$

$$= \quad 3 \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$c(1,3) + r + \hat{r}$

$17 + 25 + 11 = \underline{\underline{53}}$

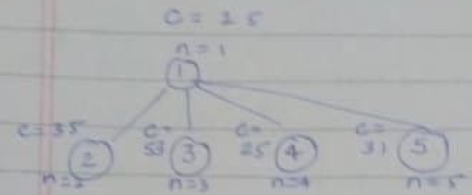
→ cost for 4 th node

$$\begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} & \end{array} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

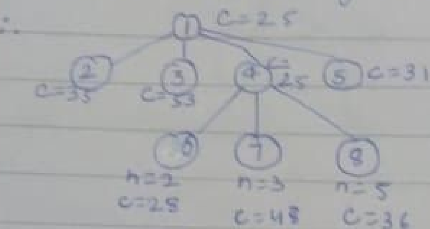
$c(1,4) + r + \hat{r}$

$0 + 25 + 0 = \underline{\underline{25}}$

→ similarly cost for 5 th node $= \underline{\underline{31}}$



∵ lowest cost is of node 4
∴



→ Now for 6, cost

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ 0 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

$= c(2\,4,2) + r + \hat{r} = 25 + 3 + 0 = 28$

→ Now for 7, cost

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \end{array} \begin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{array} = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \\ 11 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \end{array} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

reduced cost = 11 + 2 = 13

$C(4,3) + r + \hat{r}$

$= 12 + 25 + 13$

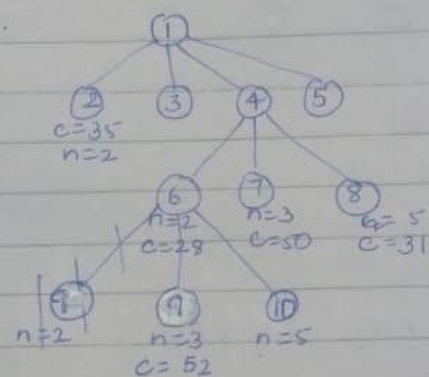
$= \cancel{77}\ \underline{\underline{50}}$

→ similarly for ~~n~~ 8, cost

$= \underline{\underline{36}}$

∵ 6th node has the lowest cost

∴



→ For node 9, cost

$$\begin{array}{c}
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 11 & \infty & 0 \\
\infty & \infty & \infty & \infty & 2 \\
\infty & \infty & \infty & \infty & \infty \\
11 & \infty & 0 & \infty & \infty
\end{bmatrix}
\begin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{array} \\
\begin{array}{ccccc} 11 & 0 & 0 & 0 & 0 \end{array}
\end{array}$$

$C(2,3) + r + \hat{r}$

$11 + 28 + 13$

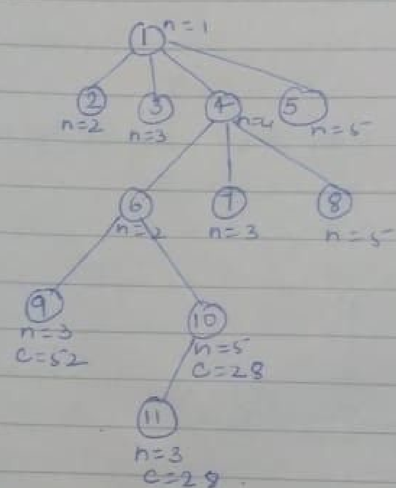
$= \underline{\underline{39}} + 13$

$= \underline{\underline{52}}$

$$\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 11 & \infty & 0 \\
\infty & \infty & \infty & \infty & 2 \\
\infty & \infty & \infty & \infty & \infty \\
0 & \infty & 0 & \infty & \infty
\end{bmatrix}$$

similarly for 10th node
c=28

∴ lowest cost is of node 10



Now for node 11, cost:

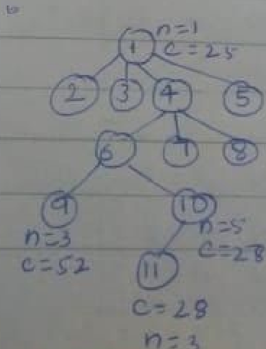
$$\begin{array}{cc} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} & \end{array}$$

$c(5,3) + r + \hat{r}$

$0 + 28 + 0$

$= 28$

∴ space tree



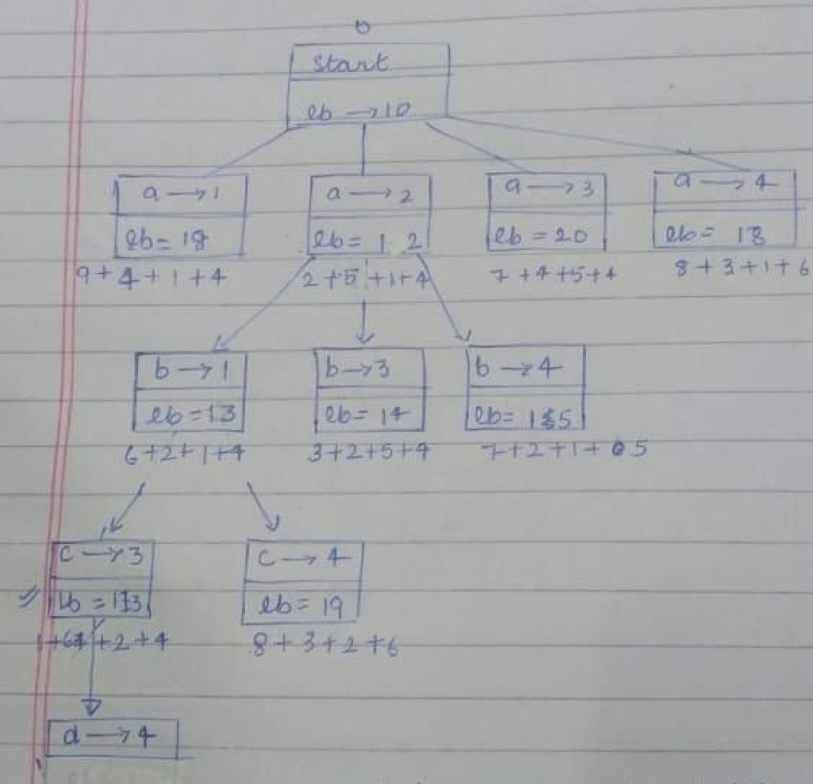
upper bound = 28.

Q-4.Find the optimal solution using Branch and Bound for the following assignment problem.

	JOB1	JOB2	JOB3	JOB4
A	9	2	7	8
B	6	4	3	7
C	5	8	1	8
D	7	6	9	4

Q4

	Jobs → 1	2	3	4	
A	9	2	7	8	2
B	6	4	3	7	3
C	5	8	1	8	1
D	7	6	9	4	4
					= 10



Total cost = 2 + 6 + 1 + 4 = 13