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## **MD IFTAKHAR KABIR SAKUR**

25<sup>th</sup> BATCH

**COMPUTER AND COMMUNICATION ENGINEERING**

**International Islamic University Chittagong**

**COURSE CODE: PHY-1101**

**COURSE TITLE: Physics I**

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**Lecturer in Physics**

Computer & Communication  
Engineering

Physics - Note - 01

1st Semester:-

Midterm + Final

  
Tuesday  
16.07.2020

Date: 18. 07. 20

Saturday

## Lecture - 01 Lesson - 1

### Gravity and Gravitation

(\*) Aristotle believed & said that, "a heavier body reaches the ground faster than a lighter body dropped from the same height." In 1590, Galileo disproved the idea. He did a public experiment in "pisa" tower with a stone and a paper. He said that due to a resistance offered by air on the paper, the paper reached after the stone. If that experiment was done in vacume then both stone and paper would reach on the floor at the same time.

## 2) Kepler's law of planetary motion:-

~~Orbit around sun - (fixed)~~

1] Every planet moves in an elliptical

orbit with the sun being one of  
its foci (Focus is plural)

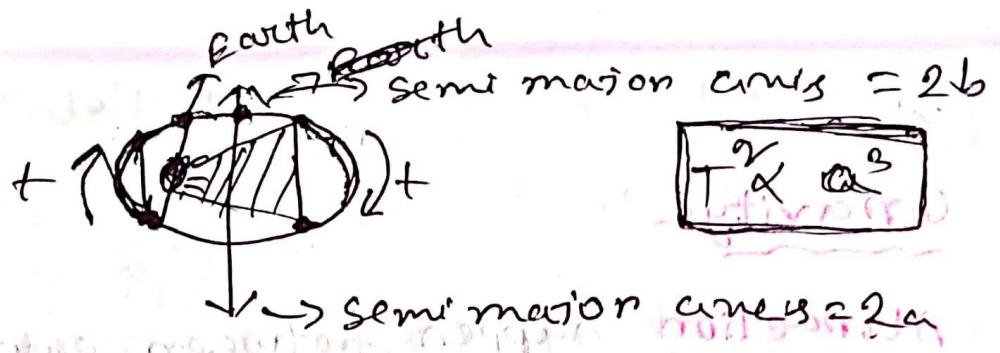
2] The radius vector drawn from the  
sun to the planet sweeps out equal  
areas in equal interval of time.

[হ্যান সময় এমন দৃষ্টি করতে  
অতিক্রম করে]  $\Rightarrow$  [Laws of areas]

3] [The periods of planets]:-

The square of the periods of revolution  
of a planet around the sun is  
proportional to the cube of the semi  
major axis of the orbit.

[মধ্যে কোনো সম্পর্ক নাই দিয়ে  
সময় অন্ত দৃষ্টি মাঝে, আবার মধ্যে গুরু  
দৃষ্টি মাঝে উভয় আপন শামন করে]

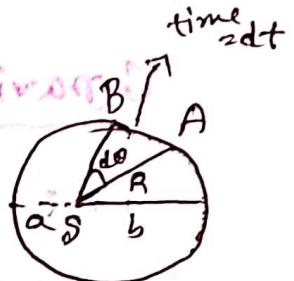


$$\text{Q} \boxed{\text{Q}}$$

Formula of Kepler's law no. 2 :-

2. Area swept in time  $\Delta t$

$$\therefore \text{Area ABS} = \frac{1}{2} R \times R d\theta = \boxed{\frac{1}{2} R^2 d\theta}$$



$$\begin{matrix} r^2 \\ r \cdot r \\ r \cdot b \end{matrix}$$

Area velocity,  $r \omega$  will be

$$\frac{\frac{1}{2} R^2 d\theta}{dt} = \frac{1}{2} R^2 \left( \frac{d\theta}{dt} \right) = \frac{1}{2} R^2 \omega = \text{Constant}$$

From  $\therefore$  so,  $\boxed{\frac{1}{2} R^2 \omega}$  is constant

As a result,  $\boxed{\frac{1}{2} M R^2 \omega}$  is also a constant

Total area of ellipse

$\pi ab$  [a is semi minor axes]  
[b is semi major axes]

सूर्योत्तरानुक्रम कोना गुणज घटना प्रमाण,

$$T = \frac{\text{Area}}{\text{Area velocity}} = \frac{\pi ab}{\frac{1}{2} R^2 \omega} = \boxed{\frac{2 \pi ab}{R^2 \omega}}$$

3

## Gravity:-

Attraction happen between earth & many other objects.

## Gravitation:-

The attraction force between any two bodies in the universe is called gravitation.

## Derivation of law of gravitation:-

(माध्यकरण आदि आविष्कार)

$$F_1 = M_1 R_1 \omega_1^2 = M_1 R_1 \left[ \frac{2\pi}{T_1} \right]^2 \rightarrow \text{Force of Planet A}$$

— i

$$F_2 = M_2 R_2 \omega_2^2 = M_2 R_2 \left[ \frac{2\pi}{T_2} \right]^2 \rightarrow \text{Force of Planet B}$$

— ii

i : ii for normal forces

$$\frac{F_1}{F_2} = \frac{M_1 R_1 \left[ \frac{2\pi}{T_1} \right]^2}{M_2 R_2 \left[ \frac{2\pi}{T_2} \right]^2} = \left( \frac{M_1}{M_2} \right) \left( \frac{R_1}{R_2} \right) \left( \frac{T_2}{T_1} \right)^2$$

— iii

Kepler's third law, a check against ellipticity.

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

Substituting this in equation (iii)

$$\text{Now } \frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right) \left(\frac{R_2}{R_1}\right)^3$$

or  $\frac{F_1 R_1^3}{M_1} = \frac{F_2 R_2^3}{M_2}$  Constant

$$\therefore F \propto \frac{m}{R^2} \text{ or, (i) } F \propto m \text{ & (ii) } F \propto \frac{1}{R^2}$$

According to the Newton:-

The Force of attraction must be mutual, a force exerted by the earth on the body, body on the earth must be equal and opposite to the force exerted by the earth on the body. Newton reduced the

Kepler's laws into a single law of gravitation.

पृथिवी नियन्त्र मात्र, बल आकर्षन द्वारा दिक रखे समान होते हैं। पृथिवी के ऊपर आकर्षन बल (मान), पृथिवी होते होते बस्तु ऊपर दूषित विपरीत बल भी समान होते हैं।

### Newton's Universal law of gravitation

(नियन्त्र मरकर्ष अध्य) :-

This law is true not only for the heavenly bodies but also for any two bodies of the universe.

Law :-

The attraction force between any two bodies in the universe is directly proportional to the square of the product of their masses & inversely proportional to the square of the distance between

Kepler's laws into a single law of gravitation.

पृथिवी - निउटन द्वारा मात्र, बल पर आकर्षण युक्ति के लिए समान होते हैं। पृथिवी के लिए ग्रavitational force आकर्षण बल (मान), पृथिवी होते होते असुन्दर ऊपर दूरज्ञ विपरीत बल व समान होते होते।

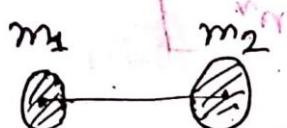
Newton's Universal law of gravitation  
(निउटन ग्राविटेशन लॉज़):-

This law is true not only for the heavenly bodies but also for any two bodies of the universe.

Law :-

The attraction force between any two bodies in the universe is directly proportional to the square of the product of their masses & inversely proportional to the square of the distance between

then



$R \rightarrow$  अंतर्वर्ती दूरी

ext to body which ext its sp to solve

Fd M.m

$$\text{Or, } Fd = \frac{1}{R^2} (Mm) \quad \text{--- (1)} \quad \therefore \text{values}$$

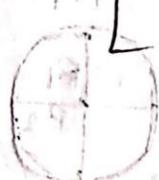
$$F = G \frac{Mm}{R^2} \quad [G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2]$$

# According to the Newton's second law

OF motion  $F = ma$ .

Here  $a = g$ , where  $g$  is the acceleration due  
to gravity.

$$\boxed{\frac{g'}{g}} = \frac{r^2}{R^2}$$



$$\therefore g' = \frac{gr^2}{R^2}$$

$g'$  = अंतर्वर्ती अक्षर्त घूमन

$g$  = पृथिवी का अक्षर्त घूमन

$r$  = पृथिवी का व्यास

$R$  = ग्रह का व्यास

$$\therefore g' = \frac{v^2}{R^2}$$

$$\left( \frac{2\pi R}{T} \right)^2 = \frac{4\pi^2 R^2}{T^2}$$

$$\therefore \frac{gr^2}{R^2} = \frac{4\pi^2 R}{T^2}$$

$$\text{Ans}, R^3 = \frac{gp^2 T^2}{4\pi^2} \quad \text{Ans}, R = \left[ \frac{gp^2 T^2}{4\pi^2} \right]^{1/3}$$

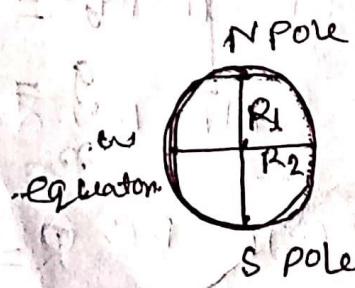
Q]

value of 'g' at the poles and at the equator :-

(মৃত্তি ও অক্ষরেখায় 'g' এর মান)

⇒ The shape of the earth is slightly

ellipsoidal (অক্ষিপৃষ্ঠা). It is bulging (অপুর্ণ) at the equator (অক্ষরেখায়) and flattened (চোটি) at the poles (মৃত্তি). Its equatorial (নিয়ন্ত্রিত) radius is more than the pole polar radius.



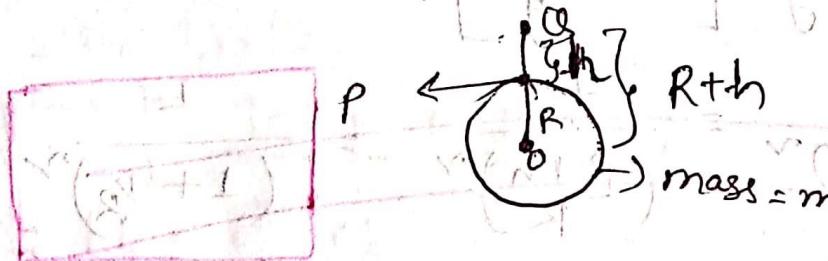
$$\therefore g = \frac{GM}{R^2}$$

$$\therefore g \propto \frac{1}{R^2}$$

variation of g with altitude :-

(सूर्योदय और सूर्यास्त के दौरान g का विवरण)

प्रति मात्रा (मेला वे परिवर्तन हैं) :-



A body of mass ( $m$ ) when placed at the point  $P$  experiences a Force ( $mg$ ) towards the centre of the earth.

$$mg = \frac{GMm}{R^2} \quad \text{--- (i)} \quad \therefore F = \frac{GMm}{R^2}$$

$$F = mg$$

When the body is at  $Q$ , let the acceleration due to gravity be  $g'$ ,

$$\therefore mg' = \frac{GMm}{(R+h)^2} \quad \text{--- (ii)}$$

$$(ii) \div (i)$$

$$\frac{mg'}{mg} = \frac{(R+h)^2}{R^2}$$

~~প্রশ্ন~~, পৃষ্ঠা সমতল হলে গুরুত্ব কি?

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}} = \frac{1}{\left(\frac{R+h}{R}\right)^2} = \boxed{\frac{1}{\left(1 + \frac{h}{R}\right)^2}}$$

→ ১ম

⇒ পৃষ্ঠা সমতল হলে কেনে যতকে মাত্র উপরে  
বেগ কমতে,  $h$  এর মান উত্তর কমতে যাকে।

কমতে কমতে ০ এর মতো চলে আসে।

$$\therefore \left(\frac{h}{R}\right)^2 = 0$$

$$\therefore g' = g \left(1 - \frac{2h}{R}\right)$$

→ ২য়

Therefore, the acceleration due to gravity decreases with altitude.

also Note:-

ज्यामिती फूटो :-

$$(1+n)^m = 1 - 2n + 3n^2 - 4n^3$$

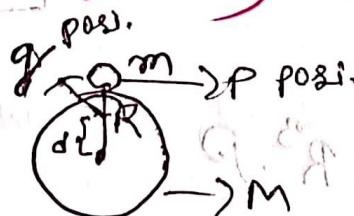
अतः

$$(1+\frac{h}{R})^m = 1 - \frac{2h}{R} + \left(\frac{3h^2}{R} - \dots\right)$$

$$\therefore (1+\frac{h}{R})^m = (1 - \frac{2h}{R}) \quad \boxed{\text{ज्यामिती अवधारणा का उपयोग}}$$

variation of g with Depth:-

(जलाभास्थ g का मान)



Let  $g$  &  $g'$  be the accelerations due to gravity at  $P$  &  $Q$  respectively. At  $P$  the whole mass of the earth attracts the body & at  $Q$  it is attracted by the

mass of the earth of radius ( $R - h$ )

$$\therefore mg = \frac{G M m}{R^2}$$

$$\text{पर), } g = \frac{GM}{R^2}$$

Same

$$mg' = \frac{GM'm}{(R-b)^2(R-d)^2}$$

$$g' = \frac{GM'}{(R-d)^2}$$

ମ' ଏବଂ କାହାନ ଶିଳ୍ପୀ,  
ପ୍ରାଚ୍ୟାନୀତି ଜ୍ଞାନୀ କୋଣେ  
ଏକାଦିକୁ ଯୁଗମାଧ' କମଳେ  
ଚାନ୍ଦିକେର୍ଣ୍ଣ କମେ ଯାଏ ।  
ଅମ୍ବାତନ ଛୋଟ ଶଳେ ଛେ  
ଓ ଛୋଟ ଶଫ୍ତେ ।

at point P,

$$\text{GST}, M' = \frac{4}{3} \pi R^3 \cdot P$$

at point Q,

$$m^r = \frac{4}{3} \pi (R-d)^3 \cdot p$$

$$\therefore g = \frac{GM}{R^2} = \frac{G \cdot \frac{4}{3}\pi R^3 \cdot \rho}{R^2} = \boxed{\frac{G \cdot 4 \cdot \pi R \cdot \rho}{3}} \quad \boxed{P = \text{যন্ত্র পৃষ্ঠিতে}}$$

$$\therefore g' = \frac{Gm'}{(R-d)^2} = \frac{\frac{4}{3}\pi(R-d)^3 \cdot \rho}{(R-d)^2} = \boxed{\frac{4\pi(R-d)\rho}{3}} \quad \text{---(4)}$$

#  $\rho$  is the Density of the earth

$$(4) \div (3)$$

$$\frac{g'}{g} = \frac{4\pi G R \rho (R-d) \cdot \rho}{3} \times \frac{3}{4\pi G R \rho}$$

$$= \frac{R-d}{R} = 1 - \frac{d}{R}$$

$$\therefore g' = g \left(1 - \frac{d}{R}\right)$$

$\because g \ll R$

$$g' = g(0.9 \dots)$$

$g' < g$  (এতে মানে হলো, পৃষ্ঠিতে যে তাঁকি আম  
মাঝের মানে এতেই  $g'$  এতে মান কামবে)

পৃষ্ঠীতে  $\boxed{g' = g(1 - \frac{d}{R})}$

□ ইঞ্জিনেরোলজি চুম্বকার্য কর্ম, কিন্তু এগুলো

অতিকর্ষ স্থুলের মান কর্ম কেনা?

$$\Rightarrow \frac{G_1 M_1}{R^2} = \frac{G_1 \frac{4}{3} \cdot \pi R^3 \cdot \rho}{R^{n+2}}$$

এখানে, অন্তর্ভুক্ত মান কর্ম  $R^3$  আবশ্যিক আবশ্যিক কুম্ভ কর্ম  $R^n$  আবশ্যিক। মেঝের মধ্যে ~~কুম্ভ~~ একটি কর্ম, তাহে অতিকর্ষ স্থুলের মান কাটা।

□ variation of  $g$  with rotation of the earth:-

$$g' = g (1 - \omega^2 R \cos^2 \delta)$$

$\omega$  = কোণিক বেচা

$R$  = পৃষ্ঠা দূরত্ব

$\cos \delta$  = কেন্দ্রীয় কোণ

বালুটা কোণ

$g$  = অতিকর্ষ পৃষ্ঠা

$g'$  = পরিবর্তিত  $g$

বিশুদ্ধ উৎস পথাবৰ্ত,  $\delta = 90^\circ$ ;

অর্থাৎ,  $\cos \delta = \cos 90^\circ = 0$

$$\therefore g'_e = g - \omega^2 R \Rightarrow \text{বিশুদ্ধ উৎস}$$

मेसुर अस्थले,

$$\theta = 90^\circ; \text{ ताकी } \cos 90^\circ = 0$$

$$g_{90} = g$$

आर्थिक डातिकृ मले,

विश्वाया अस्थले सुन्नत एवं मान मवाचय कम  
मेसुर " (मात्रात्, स्थिरता) " विका

एष मले, आर्थिक डातिकृ मले वसुत्तु उक्त विश्वाय  
अस्थल इति मेसुर अस्थलेष प्रिक अमक  
दृष्टि दाय,

■ Gravitational Field:-

(मराकर्षीय भूज)

The gravitational force of attraction is

perceived (अनुकूल) is called gravitational  
field.

If the gravitational field at a point is  
 $F$ , the force acting on a mass  $m$  is  $F$ .

$$\therefore F = mE$$

∴,  $E = \frac{F}{m}$  (i)

$= -\frac{dV}{dr}$  [also defined as the negative gradient of gravitational potential]

## Gravitational Potential

(মহাকর্ষীয় সম্মুখোত্তৃত্ব)

Consider a particle A of mass  $m$ , P is a point at a distance  $r$  from A.

The gravitational intensity at P.

$$E = \frac{F}{m} = \frac{GM}{r^2} \quad \text{--- (i)}$$

$$E = -\frac{dV}{dr} \quad \text{--- (ii)}$$

∴  $dV = -Exdr$

$$= -\left(\frac{GM}{r^2}\right) dr \quad \text{[ (ii) রেখাংশ ]}$$

Integration between the limits of infinity and  $r$ ,

$$V = -\frac{GM}{r}$$

∴ The gravitational potential at a point due to a point mass,

$$= \left( -\frac{GM}{r} \right)$$

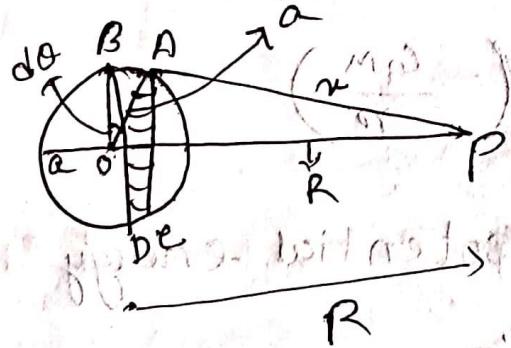
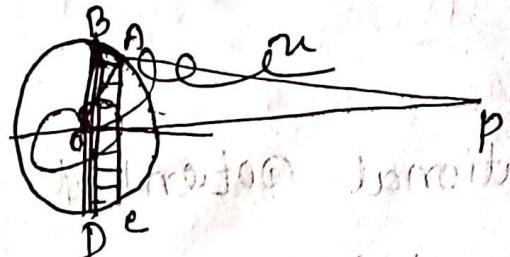
Q Gravitational potential energy of a body depends upon:-

- (i) The mass of the body at A.
- (ii) The mass of the body at BP
- (iii) the distance between two masses.

\*\*\*  Gravitational potential & field at a point due to a spherical shell:-

⇒

P.T.O.



$$AB = a \sin \theta \quad [\because s = r\theta]$$

Surface area of element AC, BD

$$= 2\pi(a \sin \theta) a d\theta \quad [\sin \theta = \frac{r}{a}]$$

$$= 2\pi a^2 \sin \theta d\theta \quad [r = a \sin \theta] \quad (2)$$

Mass of the element,

$$m = (2\pi a^2 \sin \theta d\theta) \rho \quad (3)$$

Now,

$$\Delta P = \rho$$

P, point at gravitational potential,

$$dr = - \frac{Edr}{m}, E = \frac{dr}{dn}$$

$$\therefore E = \frac{GMMm}{rn^2} = \frac{GM}{rn}$$

$$\therefore dr = - \frac{GM}{rn}$$

$$= - G \frac{2\pi a^2 \sin \theta d\theta}{n} \quad (4)$$

$\angle AOP$  এর জন্য,

$$x^2 = a^2 + R^2 - 2aR \cos \theta$$

এখন, Differentiation করে পাই,

$$2ndr = 2aR \sin \theta d\theta$$

$$\therefore \frac{dr}{dt}, n = \frac{aR \sin \theta d\theta}{(dn) \text{ বার } (n+1)} \quad (5)$$

(5) নং equation-এ  $n$  এর মান বসাই,

$$dr = \frac{2\pi Gr a^2 \sin \theta d\theta}{aR \sin \theta d\theta} dn$$

$$\therefore dr = - \left( \frac{2\pi Gr a^2}{R} \right) dn \quad (6)$$

$$v = - \int_{R+a}^{R+a} \frac{2\pi Gr a^2}{R} dn = \frac{2\pi Gr a^2}{R} [R+a - R+a]$$

$$v = - \frac{4\pi a^3 \rho}{R} \quad \text{--- (7)}$$

आमरा कानि,

$$\therefore M = 4\pi a^3 \rho$$

$$\therefore v = - \frac{GM}{R} \quad \text{--- (8)}$$

$$F = - \frac{dv}{dR} = - \frac{d}{dR} \left[ - \frac{GM}{R} \right]$$

$$= - \frac{GM}{R^2}$$

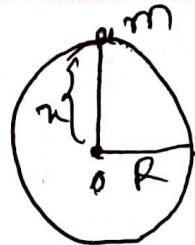
$$\therefore \vec{F} = - \frac{GM\vec{R}}{R^2}$$

मुक्तिवेत्ता

(Escape velocity)

Escape velocity is defined as the velocity with which a body has to be projected vertically upwards from the earth's gravitational field altogether.

(यदि कोना वृत्त के gravitational field  
पर वाईच्छनिय सांडर रहे)



$R$  = Radius of the earth

$M$  = mass of the earth

$m$  = mass of the object

$$F = \frac{GMm}{r^2}$$

(1)

$$dW = F \cdot dr$$

$$dW = \left( \frac{GMm}{r^2} \right) dr \quad (2)$$

To take the body from earth's surface to infinity we will get total work ( $W$ ).

$$W = \int_{R}^{\infty} F dr = \int_{R}^{\infty} \frac{GMm}{r^2} dr = \int_{R}^{\infty} \frac{GMm}{r^2} dr$$

$$= GMm \int_{R}^{\infty} \frac{1}{r^2} dr$$

$$= GMm \left[ -\frac{1}{r} \right]_{R}^{\infty}$$

$$= GMm \left[ -\frac{1}{R} + \frac{1}{\infty} \right]$$

$$= \left[ -\frac{GMm}{R} \right] = \left[ \frac{GMm}{R} \right]$$

$$\text{Ans} \quad \text{Q. K.E} = \frac{1}{2}mv^2$$



$$\text{Ans} \quad \text{Q. } \frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$\therefore v = \sqrt{\frac{2GM}{R}} \quad \text{--- (iv)}$$

Also at the surface of the earth,

$$mg = \frac{GMm}{R^2}$$

$$\therefore GM = gR^2$$

Substituting the value of  $GM$  in equation (iv)

$$v = \sqrt{2gR} \quad \text{--- (v)}$$

. The value of  $g$  at the earth's surface  $= 9.8 \text{ ms}^{-2}$ ,

$$\text{and } R = 6 \times 10^6 \text{ m}$$

$$v = \sqrt{2 \times 9.8 \times 6 \times 10^6}$$

$$\therefore v = 11.2 \times 10^6 \text{ ms}^{-2}$$

$$= 11.2 \text{ km s}^{-2}$$

Date:- 06.08.20

wednesday 7:55 p.m.

## Lecture - 02

### Dynamics of rigid bodies

Dynamics:- (सिफारिश)

Relating to forces producing motion

The part of mechanics in which motions due to the forces associated with it & the proportion of moving object is related is known as dynamics.

Rigid Body (अव्यक्ति वाला वस्तु)

A body is said to be rigid when it is impossible to change its shape by the application of a force however large. For practical purpose all solid bodies are considered as rigid bodies.

Short Form:-

Where two points of a body are fixed on unmovable though the whole body is

movable:

so, particle



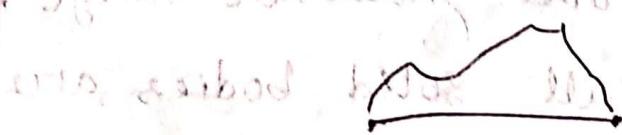
body has to move.

(particle) - sum

The difference between two points of this body can't move though the body is movable.

### Displacement (स्थान):

Displacement is defined as the shortest distance between initial & final position of a body in a particular direction.



F

body has to move

not forward to go

not move outwards

not move outwards

## Velocity (वेग) [Motion along] वेगात्मक

Velocity is defined as the rate of change of displacement with respect to time. If

$\Delta s$  displacement changes with time  $\Delta t$ ,

$$\therefore \text{velocity}, v = \frac{\Delta s}{\Delta t} [\text{ms}^{-1}]$$

$$\therefore \text{acceleration}, a = \frac{\Delta v}{\Delta t} [\text{m s}^{-2}]$$

Instantaneous velocity (विशेष वेग):

It is the limit of the average

velocity as the elapsed (अवधि का) time

approaches zero, or the derivative of  $s(t)$

with respect to  $t$ :

$$\text{inst. } v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\therefore \text{Inst. acceleration}, a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$\Rightarrow$  Like average velocity, instantaneous velocity is a vector with dimension of length per  $t$ .

## ଭ୍ରମେତ୍ର (mass velocity) :-

(momentum) ଅନୁକଳାତିଥିରେ ଭ୍ରମେତ୍ର ହେଉଛି

୧୧. ଏହା ଓ ପରିଚାରକ କ୍ଷୁଣ୍ଟଫଲକେ ଭ୍ରମେତ୍ର ହାଲେ,

## Angular Displacement :- ( $\theta$ )

The angle subtended (ସମୂଦ୍ର) in a particular time interval by an object or particle at a center

of circular path (କ୍ରତ୍ଵକାରୀ ପରିଧି) along

which it is revolving is called

Angular Displacement.



$$\theta = \frac{s}{r} \text{ ଏବଂ } [s = r\theta]$$

ଏହାଟେ  $\theta$  କ୍ଷେତ୍ର ମାତ୍ର ହରାହାତ୍ର ଦେଖିବାରେ

ଦେଉଥା ହେବା ।

বৃত্তিকান: কোনো বৃত্তে ব্যাসার্দি সমান হাত বৃত্তে কেবল  
মে কোন উপন্থ করে তাকে বৃত্তিকান বলে।



মনি Rigid Body এবং মুকুন্দ বৃত্তিকান সাথে একবার

যুক্ত আসু তাইলে কেবল উপন্থ কোণ,

$$\theta = \frac{\text{পারিশি}}{\text{ব্যাসার্দি}} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

And in Degree,

$$2\pi = 360 \text{ degree}$$

$$\therefore 1 \text{ radian} = \frac{360^\circ}{2\pi} = 57.3^\circ \text{ (প্রাপ্ত)}$$

Angular velocity ( $\omega$ ):-

The rate of Change of angular displacement with respect to time is called angular velocity ( $\omega$ )

এই সময়ের সাথে কৈনিক মুন্ত অনুপাতক  
কৈনিক বেজ বলে।

IF  $\Delta\theta$  angular displacement changes  
with time  $\Delta t$ ,  
angular velocity,  $\omega = \frac{\Delta\theta}{\Delta t}$

Instantaneous Angular Velocity:  
The angular velocity of a body rotating at any one moment of time is called as instantaneous velocity.

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular acceleration:

It is the time rate of change of the angular velocity and is usually designated by  $\alpha$  & expressed in radians per second.

Unit system:- SI unit  
Symbol:-  $\text{rads}^{-2}$

$$\alpha = \frac{d\omega}{dt}$$

Ins. angular acceleration:-

It is the rate of an object rotates in a circular path at a particular moment in time.

$$\alpha_i = \lim_{\Delta t \rightarrow 0} \frac{d\omega}{dt} \quad [\text{ms}^{-2}]$$

Angular Momentum :- (कोणीय ऊर्जा)

The angular momentum of a rigid body is defined as the product of the moment of inertia & the angular velocity.

It is analogous (अनुपाती) to linear (लाइनर) momentum & is subject to the fundamental constraints of the conservation principle of angular momentum if there is no external torque on object.

## Angular velocity

क्रीता - विनु वा अक्षात् बोल्ड करते इन्हामान  
वेगाना करात् वृत्तादि घैरु अस्तित्वामा  
होक्ते चूनालके ए विनु वा अक्षात्  
चालाक्य रापार्ट घैरु वेता वल।

$$\text{moment of momentum} = mvr$$

$$\text{So, Angular Momentum, } L = mvr = m(r\omega)r = mr^2\omega$$

$$L = I\omega = I \left( \frac{2\pi N}{t} \right)$$

$$L = \text{Angular Momentum} \rightarrow \text{kg m}^2\text{s}^{-1}$$

$$I = \text{जड़ता भूमध्य} \rightarrow \text{kg m}^2$$

$$\omega = \text{कोटिक वेता} \rightarrow \text{rads}^{-1}$$

$$N = \text{आवर्तन वर्ष्यांग}$$

$$t = \text{समय (s)}$$

$$\text{कार: } ML^2 T^{-1}$$

$$L = \vec{r} \times \vec{p} \rightarrow \text{कर्ता चुरुता}$$

$$\rightarrow \text{राष्ट्रकार वेक्टर}$$

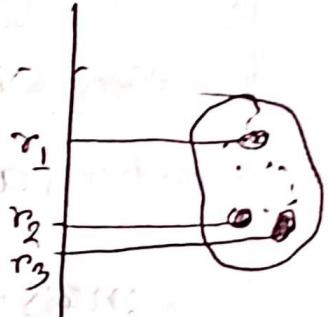
## Moment of Inertia (निर्दिष्ट भूमि) :-

(ग, जड़ता त्रामण)

The sum of product of mass & the square of perpendicular distance of each particle of an object from axis of rotation is called moment of inertia of the object.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 -$$

$$= \{ m_1 r_1^2$$



$$\therefore I = mr^2 \quad [\text{जड़ता त्रामण}]$$

⇒ कोणो निर्दिष्ट अवलोक्ते द्वेष्टे कोणो दृष्टु वर्तुल प्रत्यक्षीकृत कनाड लम्ब दृष्टिकोण एवं प्रत्यक्षित छ एवं उनमल एत मस्तिके एवं अवलोक्ता यास्त्रमध्ये एवं वस्त्र जड़ता त्रामक वाले

Note:-

कोणो निर्दिष्ट अव वर्तावर एकक रामानुजिक द्वेष्टे अवर्तनकृत कोणो दृष्ट वस्त्र जड़ता

પ્રોમાન, સંગ્રહાકારણાને એવી તાત્કાલિકતા  
નીચુંની

અધ્યાત્મ,

જાહૂળત પ્રોમાન,  $I = 2E$

Linear (લાઈનર)

Angular (કોનિક)

$s$

$\theta$

$v$

$\omega$

$a$

$\alpha$

mass  $\rightarrow m$

$I = \text{moment of}$   
 $\text{Inertia}$

Imp (અભિ

L

Force  $\leftarrow F$

$T \rightarrow \text{Torque}$

$$E_k = \frac{1}{2} I \omega^2$$

## Conservation of angular Momentum

(कोणीक अवयवात्र अनुभवत)

For a body rotating about an axis,

Change in angular velocity can be constant about by an external agency called torque. In the case of a system when the external torque is zero, the angular momentum of the system remains constant. This is called the conservation of angular momentum principle.

**short cut:-**

The law of conservation of angular momentum states that, when no external torque acts on an object, no change of angular momentum will occur.

## Centrifugal force

### Torque

### Concept

Considering (स्थिर), a particle of mass (m) moving about an axis in a circular path of radius (r). Let an external force (F) act on the particle along the tangent to the circular path.

$\Rightarrow$  कोना विनु या अक्षके बहाल कर्ते हुनीपूर्णमान कोना कर्ता धारा देक्ते रहे एवं कर्ता उपर प्रभुक धारा देक्ते रहे कर्ता तुलनाके द्वि या अक्षका धारा कर्ता धारा देक्ते रहे।

The moment of the Force =  $F \cdot r$

The moment of Force is called torque & represented by,  $\vec{T}$

$$\therefore \boxed{\vec{T} = F \cdot r = m a r = m \cdot d r = m d r^2}$$

$$\therefore \vec{T} = I \alpha$$

Hence, Torque is equal to product of moment of inertia & angular acceleration. Torque can also be defined by the rate of change of angular momentum.

$$T = \frac{dL}{dt} = \frac{dm}{dt} (I\omega)$$

$$\begin{aligned} m &= I \cdot \frac{d\omega}{dt} + \omega \cdot \frac{dI}{dt} \\ &= I \cdot \alpha + 0 \\ \text{So torque needed to rotate about } &\text{axis} \\ &= I \alpha \end{aligned}$$

∴  $\boxed{T = I\alpha}$

SI unit :-  $\text{kgm}^2\text{s}^{-2}$

Other units : Pound - Force - Feet, lbf. inch,

Common Symbols : M

Dimension :  $ML^2T^{-2}$

S.I. Unit :- newton metre

To

## perpendicular Axes Theorem

(ଲେମ୍ବ ଅନ୍ତ୍ୟ ତପ୍ରାଦ୍ୱାରା)

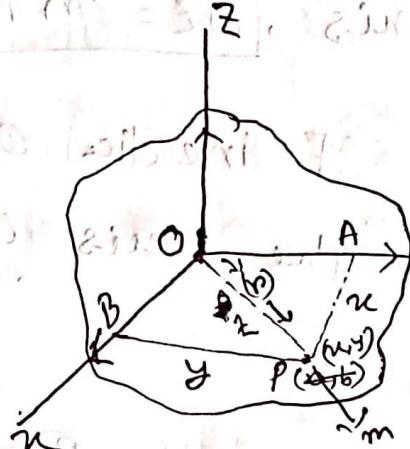
$\Rightarrow$  The moment of inertia of a plane laminar body about an axis perpendicular to the plane is equal to the sum of the moments of inertia about two mutually perpendicular axis in the plane of the lamina such that the three mutually perpendicular axis have a common point of intersection.

$$\Rightarrow I = I_x + I_y$$

$\Rightarrow$  କୋଣୋ ସମତଳ ପାତ୍ର ଏଣେ ଅନ୍ତ୍ୟ ଦ୍ୱାରା ପରିଷ୍ଠାପନ ଲେମ୍ବ ଅନ୍ତ୍ୟ ପାତ୍ରରେ ଏହାର ଶ୍ରମକ ଦୂରେଶ ସମ୍ଭବ ହୁଏ ଏହାର ଅନ୍ତ୍ୟ ଦ୍ୱାରା ଉଚ୍ଚବିନ୍ଦୁ - ଦ୍ୱିତୀୟ ଅନ୍ତ୍ୟ ପାତ୍ର ଅତିଳଙ୍ଗାତ୍ମକ ଶର୍ମନକାରୀ ଅନ୍ତ୍ୟ ମାପରେ ଏହାର ଜ୍ଞାତା ଶ୍ରମକ ଏତା - ସମାନ ।

7.5. State the relation ( $I_z = I_x + I_y$ )

PROOF :-



Considering a plane lamina having the axis  $Ox$  or  $Oy$  in the name of lamina. The axis  $Oz$  passes through  $O$  & is perpendicular to the plane of the lamina.

Let the Lamina be divided into a large number of particles, each of mass( $m$ ). Let a particle of mass  $m$  be at  $p$  with coordinates  $(x, y)$  & situated at a distance  $r$  from the point of intersection of the axis.

$$r^2 = x^2 + y^2 \quad \text{--- (1)}$$

∴ The moment of inertia about point O  
of the particle P.

$$\text{about the axis } \boxed{OZ = m r^2} \quad (2)$$

The moment of inertia of the whole  
lamina about the axis OZ is given

by,

$$I_Z = \sum m r^2 \quad (2)$$

The moment of inertia of the whole  
lamina about the axis OX,  
is given by,

$$I_X = \sum M y^2 \quad (3)$$

Similarly,  $I_OY = \sum m r^2 \quad (4)$

From equation. (3) & (2)

$$I_Z = m (r^2 + y^2)$$

$$= m r^2 + m y^2$$

$$= I_Y + I_X \therefore I_Z = I_Y + I_X$$

*Note:-*

$$I_{\text{min}} = OY \text{ or } OX \text{ or } OZ \text{ or } I_Z = I_Y + I_X$$

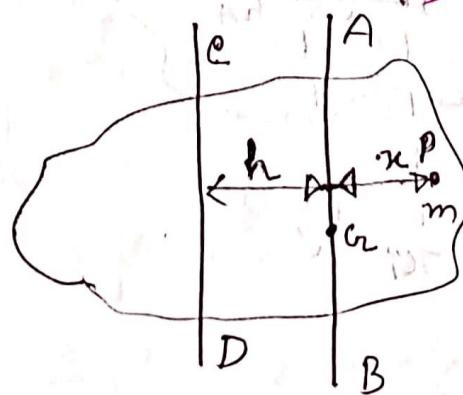
Shortcut:- ~~Integrate from direct~~

The perpendicular axis theorem states that, the moment of inertia of a planar lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it.

### Parallel axes Theorem :-

(समान्तर अक्ष सिद्धान्त)

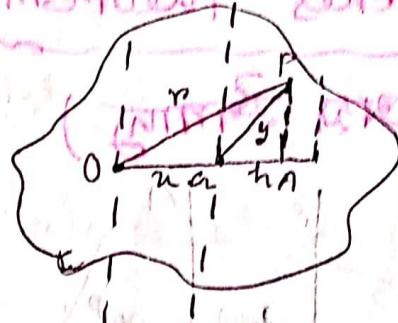
The m



The moment of inertia of a body

about any axis is equal to the sum of the moments of inertia of the body about two parallel axes, passing through the center of mass of the body & the square of the perpendicular distance between the two parallel axes.

$$\therefore I_O = I_G + m r^2$$



Here,

$$OG = r \quad OP = r$$

$$GP = y \quad GA = h$$

mass of P, particle is m.

$$I_0 = \sum m r^2 - \textcircled{1}$$

GPA - q,

$$\begin{aligned} r^2 &= OA^2 + AP^2 \\ &= OG^2 + GA^2 - \textcircled{2} \end{aligned}$$

~~পৰিপ্ৰেক্ষ~~ (পৰিপ্ৰেক্ষ) বিহুন্ত ভৰণমাণ  
GPA কৃত মূল্য,

$$GP^2 = GA^2 + AP^2$$

$$\therefore AP^2 = GP^2 - GA^2 - \textcircled{3}$$

২য় (বা) ১য় পদ্ধতি

$$= (x+h)^2 + y^2 - h^2$$

$$= x^2 + 2xh + h^2 + y^2 - h^2$$

$$= x^2 + 2xh + y^2 - \textcircled{4}$$

১য় পদ্ধতি,

$$I_0 = \sum m r^2 = \sum m(x^2 + 2xh + y^2)$$

$$\begin{aligned} \text{or } I_0 &= \sum m r^2 + \sum 2m x h + \sum my^2 \\ &= m r^2 + I_{\text{c}} + 0 = I_{\text{c}} + mh^2 \end{aligned}$$

Cause,

$$\sum m\ddot{y} = I\alpha$$

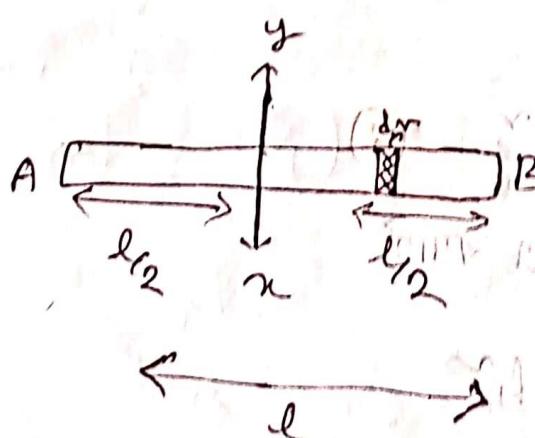
$$\sum m\ddot{h} = 0$$

$$\sum mgh = 0 \Rightarrow mgh$$

$$\sum m\ddot{h} = 0 \quad [g = \text{constant}]$$

### Box Moment of Inertia (क्षेत्रगति):

Proof:-



mass of bar =  $M$

Length of bar =  $l$

Mass per unit =  $M_1$

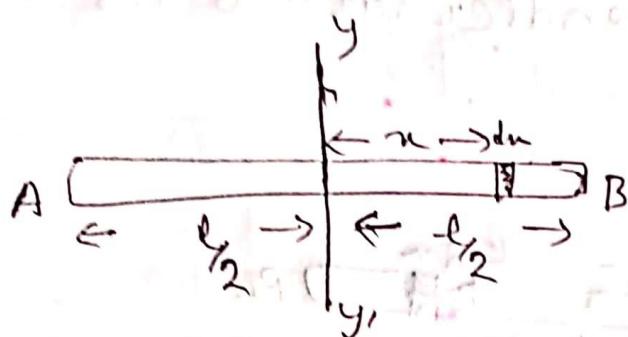
NOW,

Take an element of length  $du$  at

a distance  $x$  from the axis,

Mass of the element =  $(M_L) dx$

$\Rightarrow$



Moment of inertia of the element about the axis  $yy'$

$$= [(M_L) dx] x^2$$

Moment of inertia of the bar AB about the axis  $yy'$

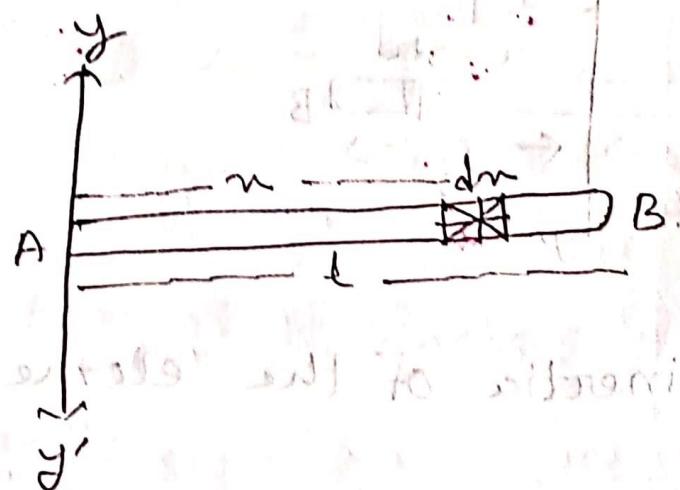
$$= [(M_L) dx] x^2$$

Moment of inertia of the bar AB about the axis  $yy'$

$$I = 2 \int_0^{l/2} (M_L) x^2 dx = \frac{2M}{l} \left[ \frac{x^3}{3} \right]_0^{l/2}$$

$$I = \frac{ML^3}{12} = MK^2 \text{ or, } MK^2 = \frac{ML^3}{12} \therefore K = \frac{l}{2\sqrt{3}}$$

Q) Moment of inertia of a bar about an axis passing through one end and perpendicular to its length.



Here,

$$I = \int_0^l (M_A) r^2 dm$$

$$\therefore I = \frac{M l^2}{3} \quad \text{--- (3)}$$

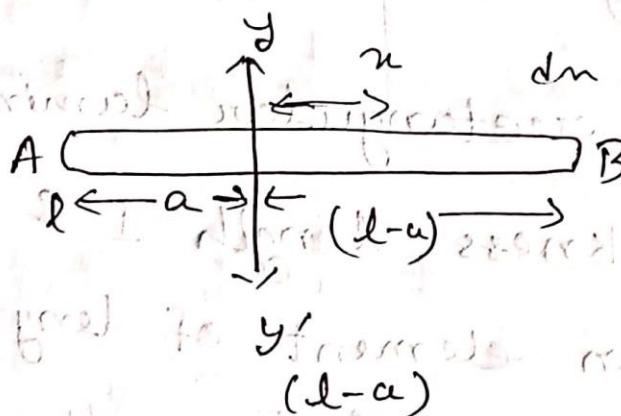
$$\begin{aligned} \text{or, } I &= M_A \left[ \frac{r^3}{3} \right]_0^l \\ &= M_A \cdot \frac{l^3}{3} \end{aligned}$$

$$= \frac{M l^2}{3}$$

But,  $I = M K^2$  so,  $M K^2 = \frac{M l^2}{3}$

$$\therefore K = \frac{l}{\sqrt{3}} \quad \text{Reason: } \textcircled{4} \text{ (opposite to minor axis)}$$

To its length, at a distance from one end:-



$$\text{Here, } I = \int_{-a}^{l-a} (m/x) x^2 dx$$

$$\text{Or, } I = \frac{m}{l} \left[ \frac{x^3}{3} \right]_{-a}^{l-a}$$

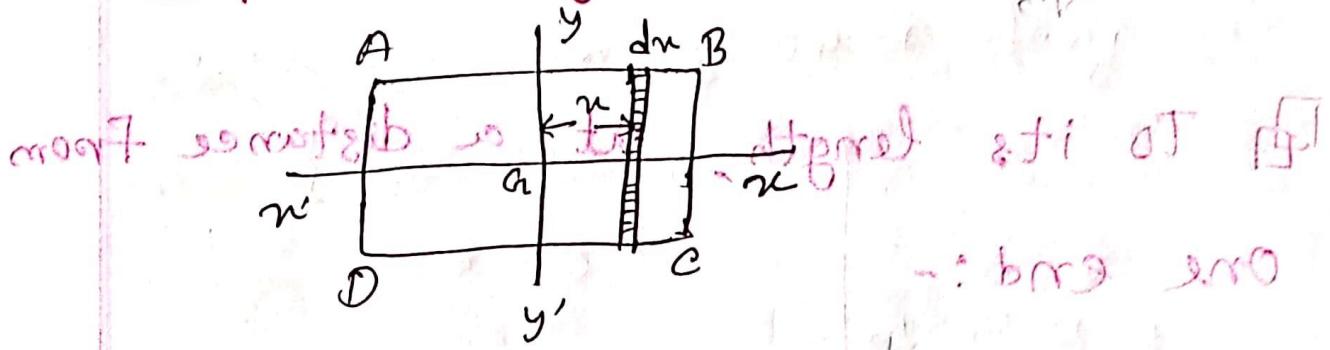
$$\text{Or, } I = \int_{-a}^{l-a} (m/x) x^2 dx$$

$$= m/x \left[ \frac{x^3}{3} \right]_{-a}^{l-a}$$

$$= m/x \cdot \left\{ \frac{1}{3} [l(l-a)^3 - (-a)^3] \right\}$$

$$\therefore I = m \left[ \frac{2}{3} l^2 - la + a^2 \right]$$

From a regular Lamina:-



$\Rightarrow$  Consider a rectangular lamina of uniform thickness, length  $l$  & breadth  $b$ . Consider an element of length  $dr$  at a distance  $r$  from the axis  $yy'$ .

Mass of the lamina =  $M$

Area of the lamina =  $l \times b$

Mass per Unit area =  $\left(\frac{M}{l \times b}\right)$

$$\therefore I = \frac{2M}{R^2} \left[ \frac{R^4}{4} \right]$$

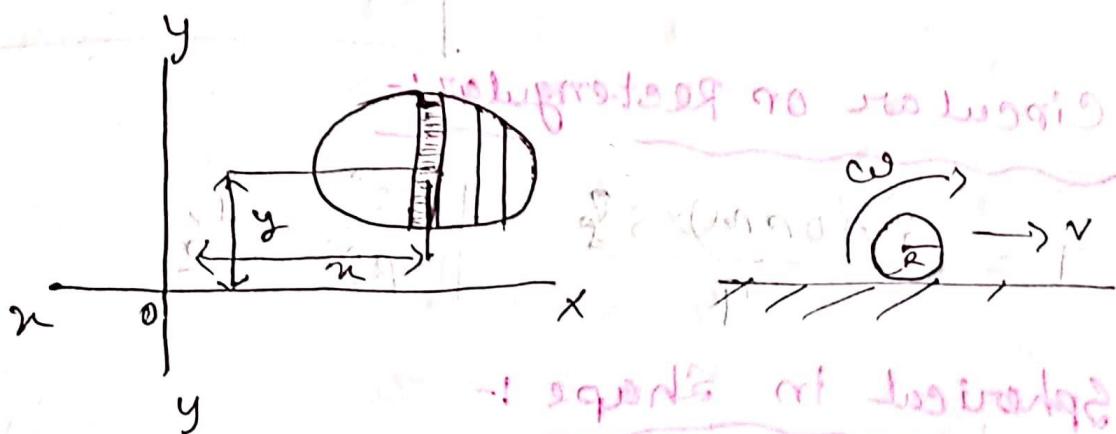
$$= \frac{MR^2}{2}$$

Moment of inertia of a circular disc about its diameter

$$\boxed{\frac{MR^2}{2}}$$

## Routh's Rule:-

In Classical Mechanics, the stretch rule (sometimes referred to as Routh's rule) states that the moment of inertia of a rigid object is unchanged when the object is stretched parallel to an axis of rotation that is a principal axis, provided that the distribution of mass remains unchanged except in the direction parallel to the axis.



Radius of gyration

(गुरुत्वकर्त्रमात्र)

$$v = \omega R ; I = MK^2$$

## For Bodies :-

Square in Shape or Rectangular:-

$$I = \frac{\{(A \text{ or } M) \times s\}}{4}$$

(or)

A = Area of the body

M = Mass of the body

s = Sum of the squares of the couple of semi-boring the one (4R^2)

about which one the m.o.I being calculated.

Circular or Rectangular:-

$$I = \frac{\{(A \text{ or } M) \times s\}}{4}$$

$$\frac{k^2}{R^2} = \frac{1}{2}$$

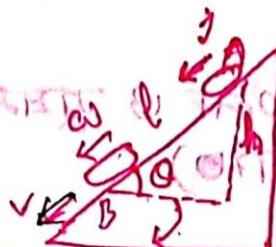
Spherical in Shape :-

$$I = \frac{\{(A \text{ or } M) \times s\}}{5}$$

$$\frac{k^2 R^2}{R^2} = \frac{2}{5}$$

ଏହା କେନେ କୁ ତାଳ ପେଧ ରାଜିଯେ ପାହାଣ ଥାବଲେ

**ପ୍ରତିନି :-**



$$\text{Loss } E_p = mgh$$

$$\sin \theta = \frac{h}{l} \quad \text{or, } h = l \sin \theta$$

$$E_p (\text{Loss of potential Energy}) = mgh \sin\theta$$

Grain in  $E_k$  — ①

$$=\frac{1}{2}mv^2 \left(1 + \frac{k^2}{B^2}\right) - \textcircled{2} = mgl\sin\theta - \textcircled{2}$$

mgl 8m0

$$\text{or, } v^r = \frac{2gl \sin \theta}{1 + \frac{k^r}{R^r}} \quad \text{--- (3)}$$

Derivative w.r.t.  $t$  of equation (3)

$$2v \frac{dv}{dt} = \frac{2gs \sin\theta}{(1 + \frac{R^2}{x^2})} \cdot \frac{dl}{dt}$$

$$\text{Ansatz: } \alpha = \frac{g \sin \theta}{1 + R^2} \quad (4)$$

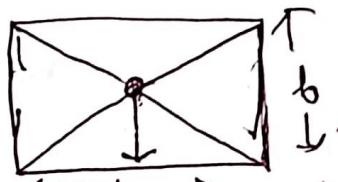
Semi Square

→ one-eighth of 360° circle

$$T_{\text{semi square}} = \frac{2gs \sin \theta}{3} \quad \text{for } \theta = 45^\circ$$

$$= \frac{2gs \sin \theta}{5}$$

□ Routh's rule (সর্বোচ্চ ও সুসম ঘূর্ণনের নিয়ম)



$$I =$$

$$\frac{m}{3} \left\{ \left(\frac{l}{2}\right)^n + \left(\frac{b}{2}\right)^n \right\}$$

$$I = I_x + I_y$$

$$= \frac{mb^2}{12} + \frac{ml^2}{12}$$

$$= \frac{m}{3} \cdot \frac{l^n + b^n}{4} = \frac{m}{12} \cdot (l^n + b^n)$$

$$\frac{1}{n} (n) \quad [\text{sum of other two semi squares}]$$

==

Where,

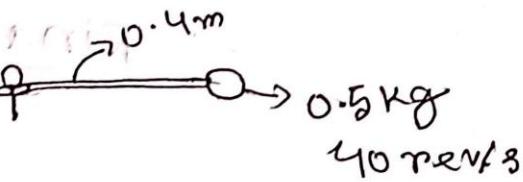
$n=3$  for rectangular body.

$n=4$  for circular/elliptical body.

$n=5$  for spheroidal body.

Rule: If a body is symmetrically shaped about all its three axes, then the moment of inertia about any three axes which passes

through the center of gravity  
of the body can be written  
as;  $I = \{A \text{ or } M \times S\} / 5$  - spherical  
shape



Q A 0.5 kg mass is whirled in a circle  
at the end of a string 0.4 m. long, the  
other end of which is held in the hand.  
If the mass makes 40 revolutions per  
second, what is this its angular momentum?  
If the the number of revolution decreases  
by one in 20 s., Calculate the mean  
value of the torque on the system.

=>

Elasticity (F) ~~Restoration~~

(~~Performance~~) ~~Institution~~

## Elasticity

Elasticity is defined as the property by which a body regains its original position when the forces are withdrawn.

The opposite of elasticity is plasticity. No substance is perfectly elastic or perfectly plastic.

## Stress (F)

When a force is applied on a body, there will be relative displacement of the particles and due to the property of elasticity the particles tend to regain their original position. Stress is defined as the restoring force per unit area.

Normal Stress (नियन्त्रित फोर्स) :-

or Longitudinal Stress :-

Restoring force per unit area  
perpendicular to the surface is called  
Normal / Longitudinal stress.

Tangential Stress :- (सर्फेटी एक्सप्रेस समूह) :-

Restoring force parallel to the surface  
per unit area is called tangential  
stress. (Shearing  $\rightarrow$ )



(Tangential) Stress  
(Tangential  
Stress)

Strain :- (त्रावण्ण) (परिवर्तन)

The ratio (परिवर्तन) of the change in shape  
to the original shape is called

strain. There are three types of  
strains:-

## Longitudinal / Lateral Strain (लंग्यूलर स्ट्रेन) (लैटरल स्ट्रेन)

The ratio of change in length to original length is called longitudinal strain.

$$\left(\frac{\Delta l}{l}\right)$$

$$\left(\frac{\Delta l}{l}\right) = \text{strain}$$

Shearing Strain (विलक्षण स्ट्रेन) Actually, It is defined as the tangent of that angle.

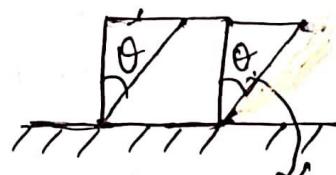
Shear strain is the ratio of the change in deformation (विलक्षण फॉर्म) to its

original length perpendicular to the axes

of the member due to shear stress. Shear stress is stress in parallel to the cross section of the structural member.

section of the structural member.

प्रति दिवस  
जुलाई मास कमाते  
मात्रा है।



It has no  
any unit

Shearing strain.

Angle of shear measured in radians

volume strain: (वॉल्यूम एंस्ट्रेन) (क्षमता विस्तरण)

The ratio of the change in volume to original volume is called volume strain.

Formula :-  $\left( \frac{V}{v} \right)$

Elastic Limit:

The maximum stress which a body exhibits the property of elasticity is called elastic limit.

If the applied force exceeds the maximum stress limit, the body does not regain its original position completely after the external forces are withdrawn.

Original state  
After loading



Initial state  
After unloading  
After loading

Stresses in various units to N/m<sup>2</sup>

## Hooke's Law:-

It states that within the elastic limit, stress is directly proportional to strain.

(front & back)

Stress & Strain

$$\text{Stress} = E \cdot \text{strain} \quad [E = \text{constant}]$$

$$E = \frac{\text{Stress}}{\text{Strain}} \quad [\text{Modulus of elasticity}]$$

$$\text{Or}, \frac{F/A}{l/l} = y \quad [\text{Young's Modulus of elasticity}]$$

$$\text{Or}, \frac{PL}{Al} = y \quad [\text{Tangential strain}]$$

$$n. = \frac{F/A}{\theta} \quad [Shearing stress] \quad [Modulus of Rigidity]$$

$$K = \frac{F/A}{V/V} = \frac{FV}{AV} \quad [\text{Bulk Modulus of elasticity}]$$

[Volume strain]

$K = 0$  at parabolic

Poisson's ratio  
Poisson's ratio

## Poisson's Ratio :-

"Whenever a body is subjected to a force in a particular direction, there is a change in dimensions of the body in the other two perpendicular directions. This is called lateral strain."

=> [যখন কোনো বস্তুর উপর যন্ত্রণা হয় তখন কেবল একটি দিকে অধিক এবং আকার পরিবর্তন ঘটে এবং অক্ষে বাল উন্নত বিকৃতি]

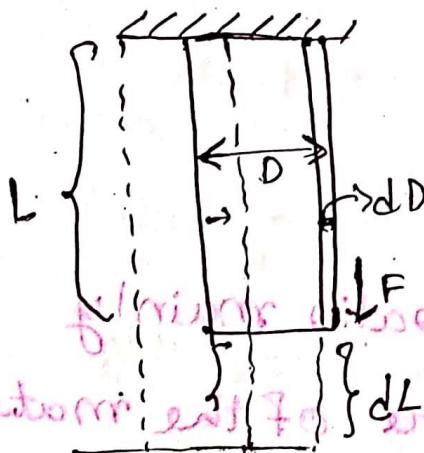
Lateral strain is proportional to the size of the body.

Let the  $(\alpha)$  be the longitudinal strain per unit stress and  $B$  be the lateral strain per unit stress, within the elastic limit,

$$B = \sigma' \alpha$$

$$\text{So, Poisson's ratio, } \sigma' = \frac{B}{\alpha}$$

\* Lateral strain per unit to the longitudinal strain per unit stress is called poison's ratio.



$$\alpha = \frac{dL}{L} \quad [\text{Chay fagto}]$$

$$\beta = \frac{dd}{D} \quad [\text{Ghar fagto}]$$

$$So, \beta = \frac{dd/D}{dL/L}$$

$$\therefore \alpha = \left( \frac{dD}{dL} \right) \left( \frac{dL}{D} \right)$$

The values of  $\alpha$  will varies from 0.2 to 0.4

Initial value :- ( $\alpha_{\text{initial}}$ )

Initial value is the value of  $\alpha$  when the bar is in its original state.

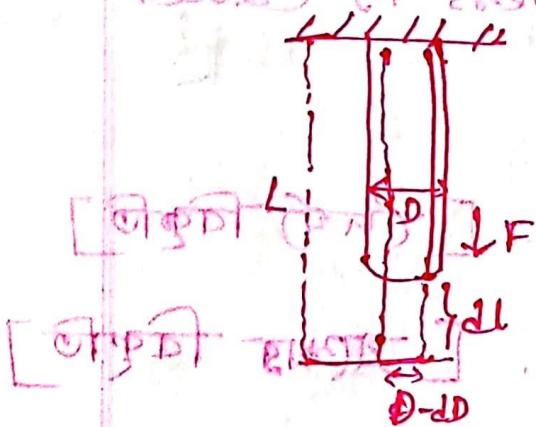
Initial value is the value of  $\alpha$  when the bar is in its original state.

Initial value is the value of  $\alpha$  when the bar is in its original state.

Initial value is the value of  $\alpha$  when the bar is in its original state.

Initial value of the wire:  $\sigma_0$

length  $l$  & cross-sectional area  $A$



The value of poison's ratio mainly depends upon the nature of the material of the body. It has no units as it is a ratio of two numbers.

For most of the substances, the value of  $\nu$  varies between 0.2 to 0.4.

$\Rightarrow$  If the volume of the wire remained unchanged after the force has been applied, then,

Initial volume of the wire,  $V = \pi^2 L r^2$

$$V = \left(\frac{\pi D^2}{4}\right)l \quad \underline{\underline{(ii)}}$$

## Differentiating equation (ii)

$$dv = \frac{\pi}{4} [D^2 dL + 2LD dD]$$

If,  $dv=0$

Then,

$$D^2 dL + 2LD dD = 0$$

$$\frac{dD}{dL} \times \frac{L}{D} = -\frac{1}{2}$$

But,

$$\rho = \left( \frac{dD}{dL} \times \frac{L}{D} \right) = -(-\frac{1}{2})$$

$\rho = \frac{1}{2} \rightarrow$  This is the minimum possible value of poison ratio

$$\rho = -\left( \frac{dD}{dL} \right) \left( \frac{L}{D} \right)$$

at which the bending moment is zero

therefore  $\rho = -\frac{1}{2}$  at equilibrium position

percentage of twist from left end is calculated

$$\frac{\theta}{\theta_0} = 2 \cdot \frac{1}{2} = 100\%$$

## Alternative Method

### The relation between $\gamma$ , $\eta$ & $K$ :-

The relation between  $\gamma$ ,  $\eta$  and  $K$  can also be obtained with the help of the following table.

Consider a unit cube, which is subjected to outward elongational force (प्रसारात्मक बल) on each face. Let ( $\sigma$ ) be the force [ $P$ ] on each face. Let ( $\nu$ ) be the poisson's ratio for the material. In the table, the values of applied stress & the corresponding (प्रतिशत) strains (फ्रैक्चर) produced along the three perpendicular axes are shown. For a stress ( $P$ ) the longitudinal strain produced =  $\frac{P}{\gamma}$  in its own direction & the corresponding strains in the other two perpendicular directions are  $-\frac{\nu P}{\gamma}$  &  $-\frac{\nu P}{\gamma}$ .

## Stress (Along ~~Tension~~) & Strain (along ~~Tension~~)

$\sigma_x$	$\sigma_y$	$\sigma_z$	$\sigma_x$	$\sigma_y$	$\sigma_z$
$+P$	0	0	$+P_y$	$-6P_y/y$	$-6P_y/y$
0	$+P$	0	$-6P_y/y$	$+P_y$	$-6P_y/y$
0	0	$+P$	$-6P_y/y$	$-6P_y/y$	$+P_y$
$P$			$P_y(1-2\zeta)$	$P_y(1-2\zeta)$	$P_y(1-2\zeta)$
$+P$	$+P$	$+P$			
<u>Total volume strain</u>			$\frac{3P}{2y}(1-2\zeta)$		

<u>Shearing force</u>	$+P$	0	0	$+P_y$	$-6P_y/y$	$-6P_y/y$
0	$-P$	(0)	$+6P_y/y$	$-P_y$	$+6P_y/y$	$+6P_y/y$
<u>Sum:-</u>	$+P$	$-P$	0	$P_y(1+6)$	$-P_y(1+6)$	0
Total Shearing Strain:				$-2P_y(1+6)$		

## BULK Modulus: (बाह्यता त्वरिती):

Bulk modulus is used to measure how incompressible (अप्रकाचनीय) a solid is.

Besides, the more the value of  $K$  (Indicate of Bulk modulus) for a material, the higher is its nature to be incompressible. For example, the value

of  $K$  for steel is  $1.6 \times 10^{11} \text{ Nm}^{-2}$ , & the value of  $K$  for glass is  $4 \times 10^{10} \text{ Nm}^{-2}$ .

It is defined as the ratio of the infinitesimal (अनन्तर्गत) pressure increase to the resulting relative decrease of the volume.

Its unit is the = Pascal.

Formula is  $= \frac{\Delta V}{V}$

## Shearing Strain (Deformation):-

tangent of angles and is equal to the length of deformation at its maximum divided by the perpendicular length in the plane of force.

NOW,

$$\text{B.M.}, K = \frac{P}{\text{Strain}}$$

$$\text{So, Strain} = \frac{P}{K}$$

$$\text{Or, } \frac{3P}{y}(1-2G) = P_K$$

$$\text{Or, } K = \frac{Py}{3P(1-2G)}$$

$$\text{Or, } K = \frac{y}{3(1-2G)} \quad (4)$$

NOW,

$$\text{Shearing strain} = \frac{P}{n} \quad [ \text{using (4)} ]$$

From the table,

$$\text{Shearing strain} = \frac{2P}{y} (1+G)$$

$$\text{So, } \frac{P}{n} = \frac{2P(1+G)}{y}$$

$$\text{Or, } n = \frac{y}{2(1+G)} \quad [ \text{divided by } P ] \quad (5)$$

So, From (1) & (2),

$$1 - 2\beta = \frac{y}{3K} \quad \text{--- (iii)}$$

$$1 + 2\beta = \frac{y}{n} \quad \text{--- (iv)}$$

$$2 + 2\beta = \frac{y}{n} \quad \text{--- (iv)}$$

$$3 = \frac{y}{3K} + \frac{y}{n} \quad (SS+1) \text{ eq}$$

$$\text{or, } 3 = y \left( \frac{1}{3K} + \frac{1}{n} \right) \quad (SS+1) \text{ eq}$$

$$\therefore \frac{3}{y} = \frac{1}{3K} + \frac{1}{n} \quad \text{--- (5)} \quad \text{error}$$

Limiting values of  $\beta$  :-  $[-1/8 < 0.5]$

$$\frac{1}{n} = \frac{1}{2(\alpha + \beta)} \quad \text{--- (1)}$$

$$n = \frac{y}{2(\alpha + \beta)} \quad \text{--- (2)}$$

From equation (iii) & (iv)

$$y = 3K(1 - 2\beta) \quad \text{--- (3)}$$

$$y = 2n(1 + \beta) \quad \text{--- (4)}$$

$$3K(1-\lambda^2) \geq 2n(1+\lambda)$$

or  $\lambda^2 = \frac{3K}{2n} - 1$   $\lambda^2 \geq 0$   $\lambda \geq 0$  (in terms of length)

$\lambda$  is positive (positive) so then,  $K & n$  = positive (+)

if  $\lambda$  is negative then,  $K & n$  = negative (-)

Limit: - as  $K & n$  are always positive

$$1 - \lambda^2 > 0$$

from which  $\lambda^2 < 1$  or  $\lambda$  lies between -1 & +1

$$0 < \lambda^2 < 1$$

$$0 < \lambda < 1$$

$\Rightarrow$  If the

(b) If the poisson's ratio is a negative quantity, for  $n & K$  to be positive,

$1 - \lambda^2 > 0$

$$(1 + \lambda)^2 > 0$$

$$\lambda > -1$$

It means the value of  $\lambda$  lies between, -1 & 0.5

Note:-

$\Rightarrow$  In actual practice the value of  $\lambda$

Cannot be negative because the body doesn't expand laterally when it

expands longitudinally (extension). And when  $\sigma = 0.5$ , it means that there is no

(+) change in volume and the body is ~~extensible~~ completely incompressible. ~~at the force~~ it is ~~extending~~ not possible.

In practice, the value of ( $\sigma$ ) for most of the isotropic substances (material) is between 0.2 & 0.4.

The modulus of rigidity ( $E$ )

$\sigma$  = poison's ratio.

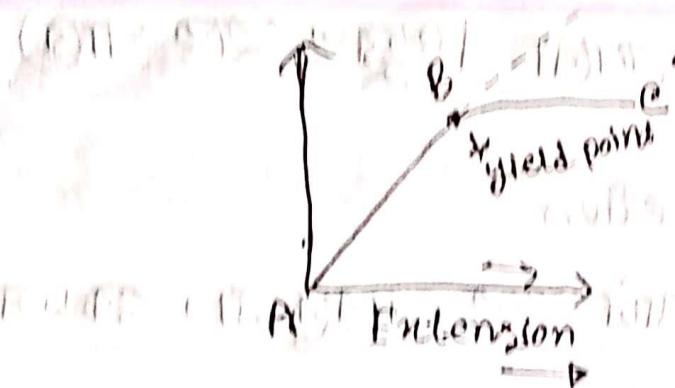
$y$  = Young's modulus.

Yield point ( $\sigma_{yield}$ ):

where a wire is loaded beyond elastic limits, Hooke's law is no longer obeyed.

The extension is more than the corresponding load.

In the above graph B point is yield point.



If the load is removed, the particle don't regain their position.

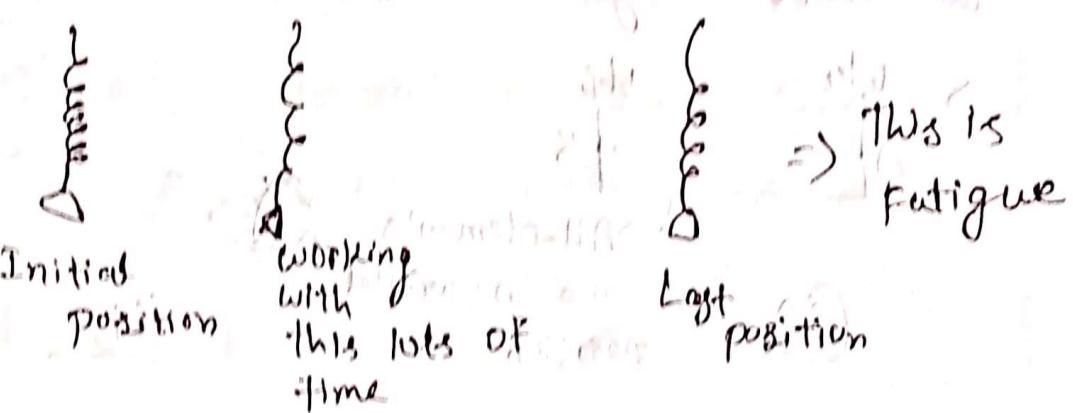
Note:-

जब दबाव नहीं पड़े तो

वस्तु की जगह चल जाती है।

### Elastic Fatigue:-

If a body is continuously subjected to stress & strain it gets fatigued.



### Bending of Beams:-

A beam is defined as a structure of uniform cross-section, whose length is large comparison to its breadth & thickness.

It is subjected to load.

(এটি মুক্তি/সর্ব কলা বিষয়ের প্রতি প্রশ্ন)

### Uniform Cross-section

[কোনও জড়, সূতা ইত্যাদি এর অভিযন্তা সম্মত রেখে রেখে]

Consider a beam of uniform rectangular cross section. Let the beam be subjected to deforming forces so that it bends.



All-elements

(i) are in parallel position.



(ii)

Position No.(i) :-

Let, AB beam be subjected to deforming forces, so that it bends. In the initial position of the beam, the various filaments constituting the beam are in parallel layers of equal length,

so they can remain in straight lines.

position No. (ii) :-

In the final position, above the layer AB, the filaments are elongated (तुरंगमात्रा) while below AB they are compressed (प्रतिमात्रा)

The length of the layer AB remains unaltered. It is called the neutral axis.

The surface combining the neutral axis is called neutral surface.

Mid Term Finish

final Starts from  
Next Page

The filament is elongated

in the first half

and compressed in the second half



**KEEP  
CALM  
ITS TIME FOR THE  
FINAL  
EXAM**

## Lesson:- Surface Tension

Cohesive and Adhesive Force:-

Mutual attraction between the molecules of the same substances is called cohesion (जड़ियांग).

And the force is called as Cohesive Force.

Adhesive Force:-

Mutual attraction between the molecules of different substances is called adhesion (जड़ियांग लकड़ियांग)

And the force is called as Adhesive Force.

Surface Tension:-

These forces are effective only when the distance between the neighbouring molecules is extremely small of the order  $10^{-7}$  cm.

- The greatest distance at which the molecules can attract each other is called molecular

Sphere.

- A sphere having a radius equal to the molecular range, with the molecule at the centre is called the sphere of influence. (Gauss's zone)

### Surface Tension

It is the property of a liquid by which the free surface of a liquid behaves like a stretched membrane.



$\Rightarrow$  Consider an imaginary line AB is dividing the liquid surface the molecules of the liquid in to two parts.

The molecules on one side pull away the molecules on the other

side. Therefore, if the liquid was actually separated into two parts with AB as the boundary line, work would have to be done. The production of a large amount of a free surface requires an increase in the potential energy of the liquid. The undisturbed liquid tries to minimize its potential energy, so, it must take a shape in which its free surface area is as small as possible.

Its surface tends to contract and force per unit length acting on either side of the imaginary line drawn on the liquid surface at rest is called as the surface tension.

$$\text{Surface Tension} = \frac{\text{Force}}{\text{Length}}$$

$$\therefore T = \frac{F}{L}$$

- The unit of surface tension in CGS system is dynes per centimetre
- And in nationalised (स्थानीय)
- MKS system is newtons per metre.
- The dimensions of surface tension are  $[MT^{-2}]$

#### Surface Tension Dependency:-

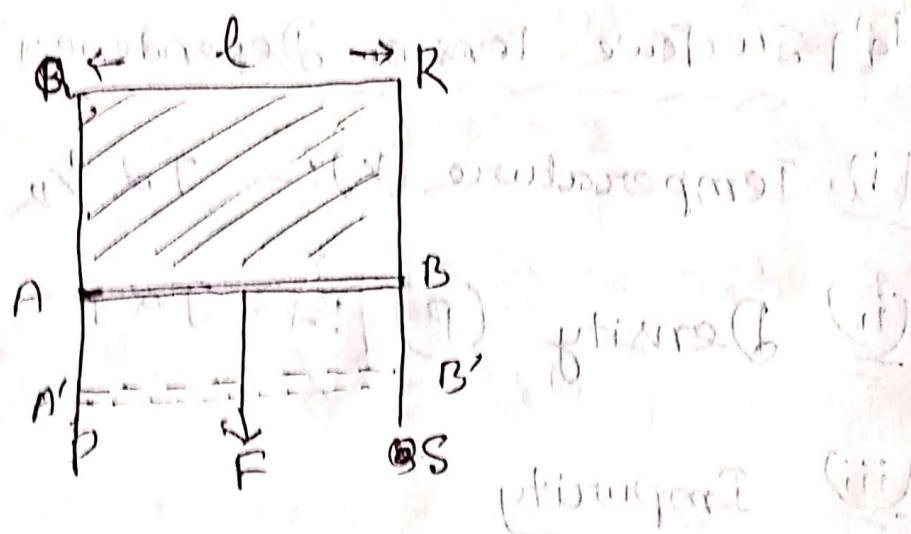
(i) Temperature (K) :-  $T \propto \frac{1}{T}$

(ii) Density ( $\rho$ ) :-  $T \propto \rho$

(iii) Impurity

## Surface Energy & Surface Tension

- As discussed already, a liquid surface contracts spontaneously (जटिलता). Free energy is associated with it and work must be done to extend the surface.
- Let consider a rectangular frame PQRS



with horizontal wire AB. Dip it in a soap solution (जटिलता फैला / रसा). A film is formed across the surface tension ABRQ. The wire AB is pulled up due to surface tension of the film. To keep the wire

equilibrium (स्थिरता) and in its position, a force has to be applied downwards. Let the total force be  $F$ .

So,  $F$  is equal to  $w_1 + w_2$

$w_1$  = weight of the wire

$w_2$  = Extra weight to be used

$F$  will hold the wire at rest in any position, regardless (रिकॉर्डर), of the area of the surface.

When the wire,  $AB$ , is pulled downwards through a distance  $\delta x$ , the area of the film is increased. The molecules in the bulk of the liquid move into the surface layers.

It is assumed that the temperature remains constant

$$\boxed{\text{work done} = F \times \delta x} \quad \text{So, } W = F \cdot \delta x$$

• Total increase in the surface area of

the liquid film =  $2(l \times \Delta n)$

$$= 2(l \times \Delta n)$$

where  $l$  is the length of the wire AB

$$\therefore \text{work done per unit area} = \frac{F \cdot \Delta n}{2l \cdot \Delta n}$$

$$= \frac{F}{2l}$$

(~~length of liquid with size~~)

① The work done per unit area

② gives the increase in potential

energy per unit area of the film.

③ The increase in energy per unit area is called surface energy.

Also the force,  $F = T \times 2l$

Here,  $T$  is the surface tension.

$$\therefore T = \frac{F}{2l}$$

④ It seemed that the temperature remain

const. throughout the process. Actually

the film gets cooled when it is stretched.

Therefore extra energy has to be supplied to restore the original temperature of the

film.

$S = T + H$  (i) (ii) (iii)

$H =$  The quantity of heat absorbed per unit area from atmosphere]

$$\text{Or, } T = S - H \quad [(S - H) = \text{The mechanical part}$$

of the surface eng./  
and transfer ent. and s. The amount of  
work done in increasing  
the surface area of  
the film by one unit]

If the process is

Adiabatic (~~without heat transfer~~),  $S, H = 0$

From eqn (ii)

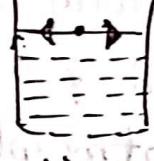
$$T = S$$

The units of free surface energy in CGS system is, ergs per sq cm is

In MKS System:- Joules per sq. metre.

Ans

## Pressure Difference Across & Spherical Surface :-



(i) When the liquid surface is plane, the resultant force of surface tension on a molecule is zero.



(ii) When the liquid surface is Convex (outward), the resultant force is outward.



(iii) When the liquid surface is Concave, the resultant force is inward.

Thus for curved liquid surface the pressure on Concave side is greater than on the Convex side.

The difference of pressure depends upon the surface tension of the liquid and the radius of curvature of its surface.

Example :- A liquid drop of radius  $R$  breaks up into 64 small drops, calculate the

Main Formulae =

Change in Energy required =  $(\text{Surface Tension}) \times (\text{Increase in Area})$

$$\Delta E = T \times A = \text{constant}$$

$$\Delta E = \frac{4 \pi R^2 \sigma}{64} = \frac{4 \pi R^2 \sigma}{64} \times 64 = 4 \pi R^2 \sigma$$

### Angle of Contact

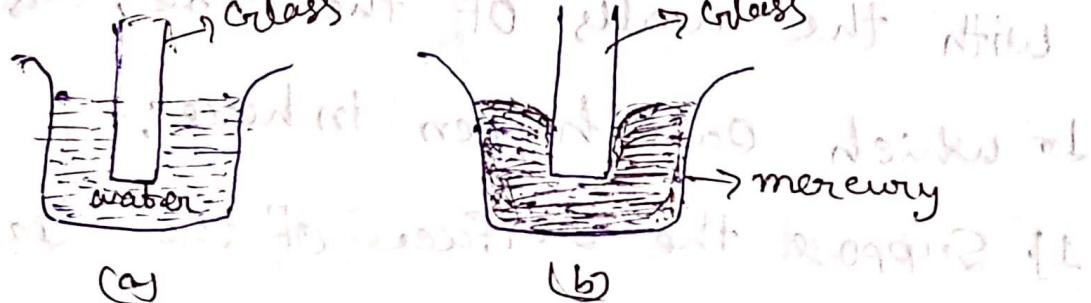
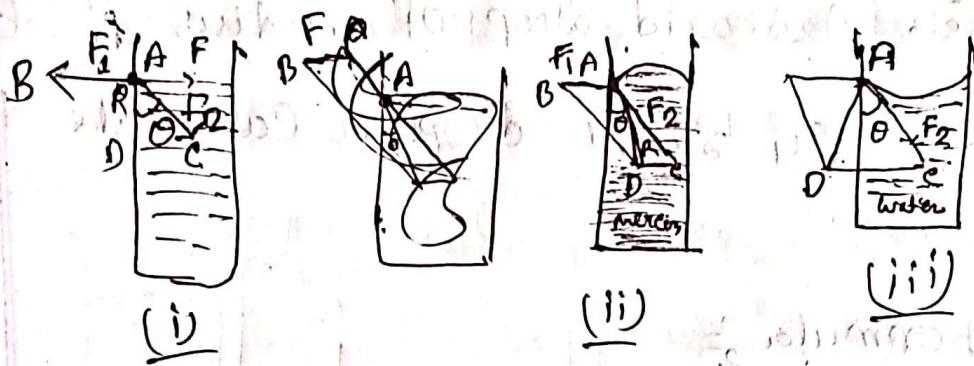


Diagram (a) is made of attraction

Diagram (b) is made of repulsion



$\Rightarrow$  Considering a liquid in a glass tube.

The molecule, at A experiences,

- (i) Force of adhesion (लगावण)  $F_1$  between it(A) & the glass molecules.
- (ii) Force of cohesion  $F_1$  between it and the molecules of the liquid.

#  $F_1$  acts along AC making an angle of  $45^\circ$  with the walls of the tube, causes

which can happen in here:-

1. Suppose the surface of the liquid is horizontal as shown in (i)  $F_2$  is the force of cohesion acting along AC.

The component of  $F_2$  along AE is  $F_2 \sin\theta$ .

$F_1$  is the force of adhesion along AB.

$$F_1 = F_2 \sin\theta$$

Thus the molecule A does not experience any force in the horizontal direction.

The only force is in the vertical direction and is equal to  $F_2 \cos\theta$

Where,  $\theta = 45^\circ$

And the angle of contact is  $90^\circ$

$$P_r V_r = P_a V_a + P_b V_b \text{ or, } P_r \times \frac{4}{3} \pi r^2 = P_a \times \frac{4}{3} \pi a^2 + P_b \times \frac{4}{3} \pi b^2$$

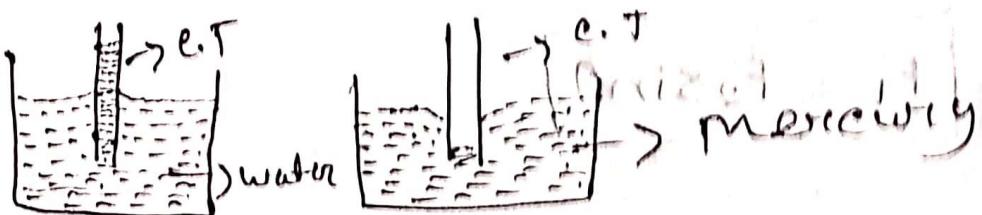
$$\text{or, } P_r P_r = P_a a^2 + P_b b^2$$

Capillarity (कपिलरी)

When a capillary tube (कपिलरी तालि) of fine bore (उमडा भुजा) is dipped (उवाटा द्या) in water, water rises in the tube.

A tube with a fine bore is called a Capillary tube.

on the But if the same tube is dipped in mercury, there is depression of mercury level in the tube.



The property of rise or depression of a liquid inside a capillary tube is called Capillarity. It is one of the most important effects of surface tension.

In general, the liquids that wet the glass, rise inside the capillary tube with while those which do not wet the glass show a depression inside the capillary tube.

Expression for surface tension is

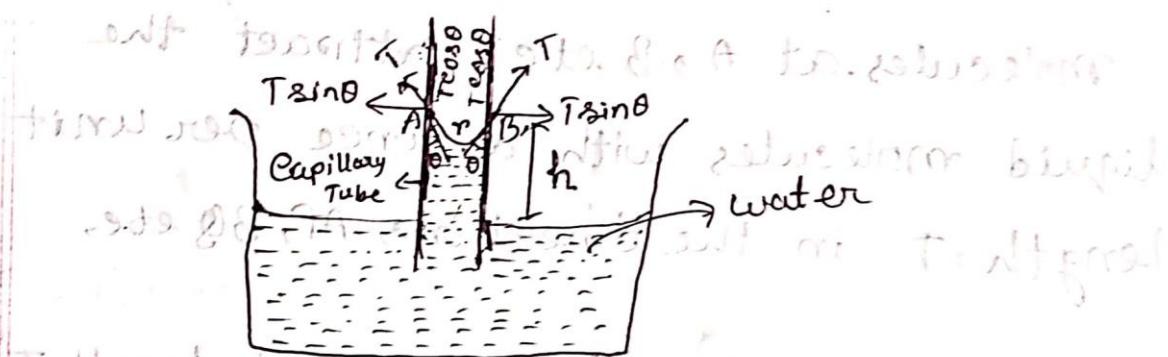
$\sigma = \frac{F}{l} = \frac{W}{l} = \frac{mg}{l}$

[To be continued]

## Surface Tension

### Expression for Surface Tension

(जल की सतह का तनाव)



Tangential force per unit length due to surface tension.

$\Rightarrow$  Consider a capillary tube of radius  $r$  dipped in water. Due to surface tension, the water rises inside the capillary tube.

$h$  = height of water in tube

The meniscus is concave. (point A, B)

T acts tangential to the surface with the wall of the capillary tube.

The force per unit length, T refers to the

force experienced by the molecule at A.  
due to,

1] The force of adhesion due to glass.

molecules.

2] The force of cohesion due to liquid

molecules at A, B etc. attract the  
liquid molecules with a force per unit  
length  $T$  in the directions AP, BQ etc.

$\Rightarrow$  To resolve the force per unit length  $T$   
along AB into two rectangular Components.

(i)  $T \cos \theta$  acting vertically upwards.

(ii)  $T \sin \theta$  acting in the horizontal  
direction.

After removing out of liquid  $T$   
adheres with  $70^{\circ}$ . Thus

at  $70^{\circ}$   $T$  is  $1.732 T$  times less than  $T$ .



Force per Unit length =  $T \cos \theta$

Circumference =  $2\pi r$

Total upward force =  $T \cos \theta \times 2\pi r$

This upward force balances the weight of the liquid column in the capillary tube.

Height of the liquid column =  $h$

Volume of the liquid in the meniscus (मूल्यान)

$$= (\pi r^2 \times r) -$$

$$= \frac{\pi r^3}{3}$$

Volume of the liquid in the column

$$= \pi r^2 h + \frac{\pi r^3}{3} = \pi r^2 \left( h + \frac{r}{3} \right)$$

Density of the liquid =  $\rho$

Weight of the liquid =  $\pi r^2 (h + \frac{r}{3}) \rho \cdot g$

Equating (i) & (ii) & solving for  $h$  -

$$2\pi r T \cos \theta = \pi r^2 (h + \frac{r}{3}) \rho \cdot g$$

$$= 4\sigma r, T = \frac{\sigma(r(h + \frac{r}{3})) P.g}{2 \cos \theta}$$

when the angle of contact is zero,

$$\cos \theta = 1$$

$$4\sigma r, T = \frac{\sigma(r(h + \frac{r}{3})) P.g}{2}$$

### Determination of surface tension:

(अनुपातिकीय)

Surface tension of water can be determined experimentally using capillary method.

The height of the liquid column & the internal diameter of the capillary tube is found with the help of a travelling microscope.

For a particular tube, the radius is  $r$ , the height of water is  $h$  and the density of water is  $P$ . Then the surface tension of water at room temperature

$$T = \frac{\rho(h + \frac{r}{3})\rho g}{2\cos\theta}$$

Taking,  $\theta = 0$ ,  $\cos\theta = 1$

$$T = \frac{\rho(h + \frac{r}{3})\rho g}{2\cos\theta} \text{ at dist from A}$$

Surface tension of water can be determined.

In the actual experiment, three capillary tubes of different diameters are taken and the mean value of surface tension is calculated.

This method suffers from a number of drawbacks.

1] The angle of contact has been taken to be zero, because it can't be measured inside the tube.

2] The tube may not be uniform in area of cross-section.

(iii) The tube may not be cleaned.

Example of Capillary action

Q A pen nib is split and the tip to provide the narrow capillary and the ink is drawn up on the pen nib continuously, so why is it not wet?

Q In oil lamps & blower bulb the ink is drawn up through the capillaries of the wick.

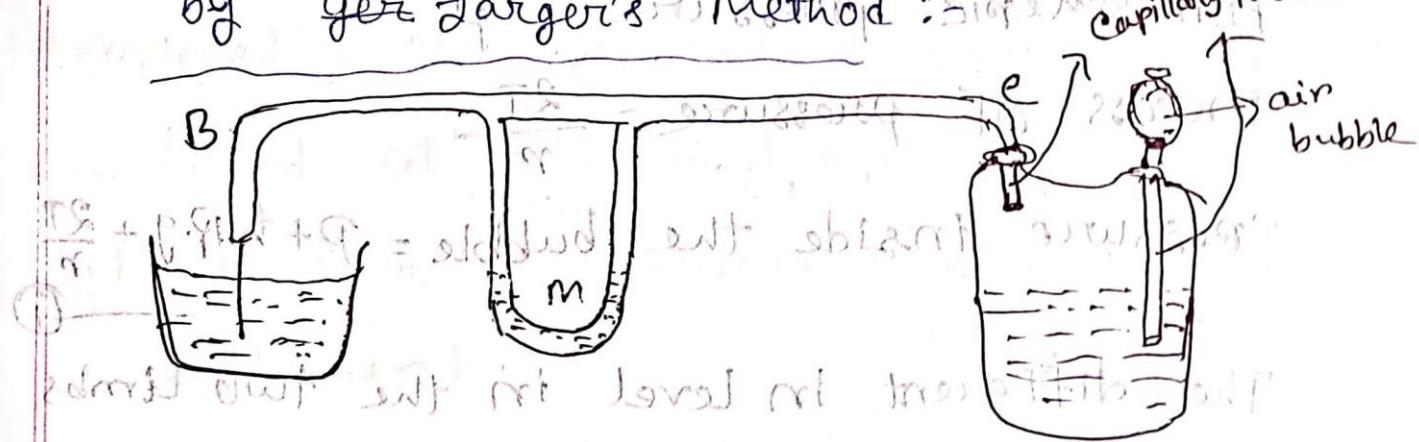
Q pores in the blotting paper act as capillaries, when the blotting paper is kept in contact with ink, the ink absorbed.

Q Walls get dampened in rainy season due to their absorption of water by the thick hy. capillary action.

5) Leaves, trunk & branches of a tree possess fine capillaries. Sap & water rises even up to the top most leaves by capillary action.

6) Sandy soils are dry whereas clay soils are damp. The interspaces between the particles of the clay form finer capillaries and water rises to the surface quickly.

□ Determine surface tension of a liquid by Ger Langer's method : ~~using capillary tube~~



The apparatus (2)

D is Woulf's bottle. On one side it is fitted

with a ~~70~~ thistle funnel and other side it is connected to a tube BC.

The tube BC is fitted with a manometer M. The end of the tube BC is joined to a capillary tube. Two capillary tube liquid

One is from C & the other is inside the water which surface tension is to be determined.

For stop cock is opened the pressure of bottle increases here,

Excess of pressure inside the bubble,

$$= \frac{2T}{r}$$

The pressure of the liquid column

$$= h \rho g$$

Atmospheric pressure =  $p_{atm}$

Excess of pressure =  $\frac{2T}{r}$

pressure inside the bubble =  $p + h \rho g + \frac{2T}{r}$  ①

The difference in level in the two limbs of the manometer when the bubble

just bursts =  $h \rho g$

## Density of the liquid in the manometer

and  $P_1 = P_2$

Pressure of air in the manometer,

$$P_1 = P + hP_2 g \quad \text{--- (ii)}$$

Equating (i) and (ii)

$$P + hP_1 g + \frac{2T}{r} = P + hP_2 g$$

$$\frac{2T}{r} = (hP_2 - hP_1)g.$$

$$ST = \frac{rgP}{2} (hP_2 - hP_1)$$

The radius ( $r$ ) of the capillary tube is measured with a travelling microscope

This method can be usefully

employed (i) to find the surface tension of a liquid at various temperatures.

(ii) to find the surface tension of molten metal and

(iii) to compare the surface tension of two liquids at the same temperature.

math problem is solved

Q1 A liquid drop of radius ( $R$ ) breaks up into 64 small drops. Calculate the change in Energy.

⇒ Let,

the surface tension of the liquid be  $T$ .

Radius of the bigger drop =  $R$

Surface area of the bigger drop =  $4\pi R^2$

Volume of the bigger drop =  $\frac{4}{3}\pi R^3$

Let, the radius of each small drop be,  $r$

$$\therefore \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

∴  $R^3 = 64r^3$

$$\therefore R = 8r$$

∴  $\frac{R}{r} = 8$

$$\therefore \text{surface area ratio} = \frac{R}{r} = 8$$

Surface area of each small drop,

$$S = 4\pi r^2$$

looking to go back to original  
surface area

$$= 4\pi \left(\frac{R}{64}\right)^2 = 4\pi \frac{R^2}{64} = \frac{\pi R^2}{16}$$

Surface area of 64 small drops,

$$= \frac{64 \times \pi R^2}{16} = 4\pi R^2$$

Increase in surface area =

$$16\pi R^2 - 4\pi R^2$$

to increase surface area

$$= 12\pi R^2$$

Change in energy = (Surface tension) x

(Surface Area)

$$= T \times 12\pi R^2$$

$$= 12\pi R^2 T$$

$$\frac{12\pi R^2 T}{\pi R^2} = 12T$$

(Ans)

$$12T = 12 \times 0.072 = 0.864$$

(2) Calculate the amount of energy needed to break a drop of petrol of volume  $10^{-6} \text{ m}^3$  into a thousand millions drops of equal size. Surface Tension of petrol is  $26 \times 10^{-3} \text{ N m}^{-1}$ .

$\Rightarrow$  Let, the radius of petrol's drop be = R.

The volume of a drop of petrol =  $10^{-6} \text{ m}^3$ .

The surface tension of petrol,  $T = 26 \times 10^{-3} \text{ N m}^{-1}$

We will convert it into thousand

million drops =  $10^9$

What Amount of energy needed = ?

Here,

$$\frac{4}{3} \pi R^3 = 10^{-6}$$

$$R^3 = \frac{3 \times 10^{-6}}{4\pi}$$

$$\text{Or, } R = \left( \frac{3}{4\pi} \right)^{1/3} \times 10^{-2} \text{ m.}$$

$$= 1.465 \times 10^{-3}$$

~~If let,  $\sigma$  constant in glass~~

Also,

The radius of small drops  $= r_1 = R$

So, volume of  $10^9$ 's drop of petrol

$$\text{be} = \frac{4}{3} \pi r^3 \times 10^9$$

Here,

$$\frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi r^3 \times 10^9$$

$$\text{Or, } 1.32 \times 10^{-8} = \frac{4}{3} \times \pi r^3 \times 10^9$$

$$\text{Or, Or, } R^3 = 10^9 \times r^3$$

$$\text{Or, } r^3 = \frac{R^3}{10^9} = \frac{(1.465 \times 10^{-3})^3}{10^9}$$

$$\text{Or, } r^3 = 3.144 \times 10^{-18}$$

$$\text{Or, } r = 5.32 \times 10^{-9}$$

Increase in surface area

$$A = 10^9 \times 4\pi r^2 - 4\pi R^2$$

$$= 10^9 \times 4\pi (10^9 r^2 - R^2)$$

$$= 4\pi \left\{ 10^9 (5.32 \times 10^{-9})^2 - (1.465 \times 10^{-3})^2 \right\}$$

$$= 4\pi$$

Calculate the work done in spraying a spherical drop of mercury of radius  $10^{-3}$  m into a million drops of equal size. Surface tension of mercury =  $550 \times 10^{-3}$  Nm $^{-1}$ .

$\Rightarrow$  Here,  $A \times F = \text{work against}$

The radius of mercury,  $R = 10^{-3}$  m.

Total drops =  $10^6$

So,  $\frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$

$$\frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$$

$$\text{Or, } R^3 = 10^6 \times r^3$$

$$\text{Or, } R = 10^2 \times r$$

$$\text{Or, } r = \frac{R}{100} = \frac{10^{-3}}{10^2} = 10^{-5} \text{ m.}$$

Increase in surface area

$$\Delta A = 10^6 \times 4\pi r^2 - 4\pi R^2 = 4\pi (10^6 \times r^2 - R^2)$$

$$A = 4\pi \left\{ 10^6 \times (10^{-5})^2 - (10^{-3})^2 \right\}$$

$$= 4\pi \times 99 \times 10^{-6} \text{ m}^2$$

$$= 1.244 \times 10^{-3}$$

$$\text{Work done} = T \times A$$

$$= 550 \times 10^{-3} \times 1.244 \times 10^{-3}$$

$$= 6.892 \times 10^{-6}$$

Q Calculate the amount of energy needed to break a drop of water of diameter  $2 \times 10^{-3} \text{ m}$ . into  $10^9$  droplets of equal size. Surface tension of water  $72 \times 10^{-3} \text{ Nm}^{-1}$ .

$$\Rightarrow \text{Here, } \frac{D_{\text{old}}}{R_{\text{new}}} = \frac{2}{10} = \frac{1}{5}$$

The diameter is  $= 2 \times 10^{-3} \text{ m}$

$$\text{So, Radius, } R = 10^{-3} \text{ m.}$$

Droplets of equal size  $= 10^9$

Surface Tension,  $T = 72 \times 10^{-3} \text{ Nm}^{-1}$

$\frac{4}{3}\pi R^3$

$$= 10^9 \times \frac{4}{3}\pi r^3$$

$$\text{Or, } R^3 =$$

$$\text{Or, } r^3 = \frac{R^3}{10^9} \text{ (Ans)}$$

$$\text{Or, } r^3 = \frac{R^3}{10^9} \text{ (Ans)}$$

$$\text{Or, } r = \frac{R}{10^3} = \frac{10^{-3}}{10^3} = 10^{-3-3} = 10^{-6} \text{ m.}$$

$A(80,0)$

Increase in surface area

$$A = 10^9 \times 4\pi r^2 - 4\pi R^2 \text{ To calculate}$$

$$= 4\pi \left( 10^9 \times r^2 - R^2 \right)$$

$$= 4\pi \left\{ 10^9 \times (10^{-6})^2 - (10^{-3})^2 \right\}$$

$$= 4\pi \left( \frac{10^{9-12} \times 10^{-12}}{(10^{-3}-10^{-6})} \right)$$

$$= 4\pi \left( \frac{10^{-3}}{10^{-6}} \right)$$

$$= 1.255 \times 10^{-2}$$

Excess pressure,  
 1 side (Inside/Outside) =  $\frac{2T}{r}$   
 2 sides (Both inside + outside) =  $\frac{4T}{r}$  □

e. Amount of energy =  $A \times T$

$$E_{\text{max}} = (1.258 \times 10^{-2} \times 72 \times 10^{-3}) J$$

$$= 9.039 \times 10^{-4} J.$$

(Ans) □

Q An air bubble of radius 0.1 mm is situated just below the surface of water.

Calculate the gauge pressure (excess of pressure) inside the air bubble. Surface

Tension of water =  $7.2 \times 10^{-2} \text{ N/m}$

$\Rightarrow$  Excess of pressure =  $\frac{2T}{r}$

$$= \frac{(7.2 \times 10^{-2} \times 2)(0.1 \div 10^3)}{\pi r^2}$$

$$\begin{aligned} &= \frac{7.2 \times 10^{-2} \times 2}{(0.1 \div 10^3) \pi} = 1.44 \times 10^3 \\ &= 1440 \text{ Nm}^{-2} \end{aligned}$$

(Ans)

Q] In an experiment for determining the surface tension of water by capillary rise, a capillary tube of diameter 1 mm is used. The height of water in the capillary tube was found to be 3 cm. Calculate the surface tension of water. Take density of water  $10^3 \text{ kg m}^{-3}$ .

Note:-

$$\text{Excess pressure} = h \rho g$$

$$\frac{2T}{r} = h \rho g$$

The pressure of liquid column

$$\text{Diameter} = 1 \text{ mm}$$

$$\text{Radius, } r = 0.5 \text{ mm} = \frac{0.5}{1000} \text{ m}$$

$$= 5 \times 10^{-4} \text{ m}$$

$$\text{The height of water, } h = 3 \text{ cm}$$

$$= 3 \times 10^{-2} \text{ m}$$

$$\text{Density of water, } \rho = 10^3 \text{ kg m}^{-3}$$

So, the difference in water level,

$$h = (h_1 - h_2)$$

$$\text{Ans} \quad h = (20.39 - 14.694) \text{ m.}$$

$$= 14.696 \text{ m.}$$

$$= (2.939 - 1.47) \times 10^{-3}$$

$$= 1.469 \times 10^{-3} \text{ m.}$$

(Ans)

A glass tube of internal radius  $5 \times 10^{-3} \text{ m}$  is dipped vertically into a vessel containing mercury such that the lower end of the tube is  $10^{-2} \text{ m}$ . below the surface of mercury. Calculate the gauge pressure of air inside the tube to blow a hemispherical bubble at the lower end of the tube. Surface tension

Surface tension of mercury =  $3.5 \times 10^{-1} \text{ Nm}^{-1}$ . Density of mercury

Mercury =  $1.36 \times 10^1 \text{ kg m}^{-3}$ .

$\Rightarrow$  Hence,

Internal radius of the glass tube,  $r = 5 \times 10^{-9} \text{ m}$ .

Lower end of that tube,  $h = 10^{-2} \text{ m}$ .

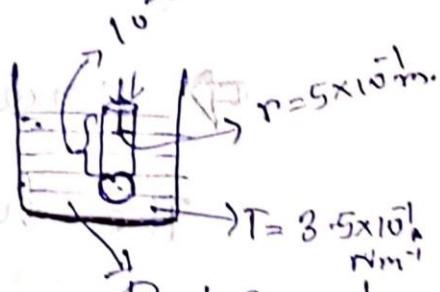
Surface tension of mercury,  $T = 3.5 \times 10^{-1} \text{ Nm}^{-1}$

Density of mercury,  $\rho = 1.36 \times 10^1 \text{ kg m}^{-3}$

Gravity,  $g = 9.8 \text{ ms}^{-2}$

Gauge pressure = ?

$$\text{Gauge pressure} = \frac{2T}{r} + h \rho g$$



$$= \frac{2 \times 3.5 \times 10^{-1}}{5 \times 10^{-9}} + 10^{-2} \times 1.36 \times 10^1 \times 9.8$$

$$= \frac{2 \times 3.5 \times 10^{-1} + (5 \times 10^{-9})(10^{-2} \times 1.36 \times 10^1 \times 9.8)}{5 \times 10^{-9}}$$

$$= 27328.1472.8 \text{ Nm}^{-2}$$

(Ans)

In a capillary tube, water rises to a height of 0.1 m. In the same capillary tube mercury is depressed by  $3.42 \times 10^{-2}$  m.

Angle of Contact for water =  $0^\circ$

Angle of Contact for mercury =  $135^\circ$

Calculate the surface tension of mercury given the surface tension of water as  $72 \times 10^{-3}$  N/m. Density of mercury =  $13.6 \times 10^3$  kg/m<sup>3</sup>.

Here, Density of  $P_1 = 1000$  kg/m<sup>3</sup>.

The height of water,  $h = 0.1$  m.

The depression of mercury =  $3.42 \times 10^{-2}$  m.

At Angle of Contact,

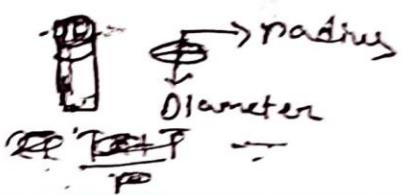
(Assumption for water,  $\theta_1 = 0^\circ$ )

(Assumption for mercury,  $\theta_2 = 135^\circ$ )

Surface Tension of water  $T_1 = 72 \times 10^{-3}$  N/m

Density of mercury,  $P_2 = 13.6 \times 10^3$  kg/m<sup>3</sup>

(Depression अर्थात् निचैर्या  
negative)



The Surface tension of mercury = ? ,  $T_2 = ?$

Note:- दोनों स्तरों पर विपरीत विकार के कारण,

$$\text{लेखन के लिए}, \frac{2T h_{10}}{r} = h \rho g$$

किन्तु यहां, Angle of Contact  $\theta$  का उपयोग करते हैं

$$\text{लेखन के लिए}, \frac{2T \times \cos \theta_1}{r} = h \rho g$$

इसीलिए  $\theta_1 = 90^\circ$  का उपयोग करते हैं

$$\text{लेखन के लिए}, \frac{h_1 \rho_1 g \cdot r}{2 \cos \theta_1} \quad \text{.....(i)}$$

$$\text{लेखन के लिए}, \frac{h_2 \rho_2 g \cdot r}{2 \cos \theta_2} \quad \text{.....(ii)}$$

Here,  $\theta_1 = 90^\circ$  का उपयोग करते हैं

$$\frac{T_2}{T_1} = \left( \frac{h_2}{h_1} \right) \times \left( \frac{\rho_2}{\rho_1} \right) \times \left( \frac{\cos \theta_1}{\cos \theta_2} \right)$$

$$\text{Or, } T_2 = T_1 \left( \frac{h_2}{h_1} \right) \times \left( \frac{\rho_2}{\rho_1} \right) \times \left( \frac{\cos \theta_1}{\cos \theta_2} \right)$$

$$\text{Or, } T_2 = 72 \times 10^{-3} \times \frac{-3.42 \times 10^{-2} \times 13.6 \times 10^3 \cos 0^\circ}{0.1 \times 1000 \times \cos 135^\circ}$$

$$\text{Or, } T_2 = 4.73 \times 10^{-1} \quad \text{(Ans)}$$

In Jaegeir's experiment, a capillary tube of internal diameter  $5 \times 10^{-4}$  m. dips  $3 \times 10^{-1}$  m. inside water contacted in a beaker. The difference in level of water manometer when the bubble is released in  $0.09$  m. calculate the surface tension of water.

$$\Rightarrow \text{Here, Density of } P_1 \& P_2 = 10^3 \text{ kg/m}^3 \text{ g/cm}^3$$

The diameter of a capillary tube =  $5 \times 10^{-4}$  m.  
 Radius,  $r = 2.5 \times 10^{-4}$  m

$$\text{Height of water, } h = 3 \times 10^{-2} \text{ m.}$$

The difference in level of water manometer,  $H = 0.09$  m.

$$\text{Surface Tension, } T = ?$$

We know,

$$\frac{2T}{r} + hP_1g + p = \rho + HP_2g$$

$$\frac{2T}{r} + 0.09 \times 10^3 \times 9.81 = 0.09 \times 10^3$$

EFFECT OF CENTRIFUGAL FORCE  
(Unit 3) (20 - marks)

$$\text{Or, } \frac{2T}{r} = H\rho_2 g - h\rho_1 g$$

$$\text{Or, } T = \frac{r(H\rho_2 g - h\rho_1 g)}{2}$$

$$\text{Or, } T = \frac{(2.5 \times 10^4)(0.09 \times 10^3 \times 9.8 - 3 \times 10^{-2} \times 10^3 \times 9.8)}{2}$$

$$\text{Or, } T = \frac{(2.5 \times 10^4 \times 9.8)}{2} \times \frac{(6.09 \times 10^3 - 3 \times 10^{-2} \times 10^3)}{2}$$

$$= 0.1102$$

~~$$\text{Shear force } f_s = 1.102 \times 10^{-1} \text{ N/m.}$$~~

~~$$\text{Bending } M = 0.0735 \text{ Nm}^{-1}$$~~

~~$$\text{Or, } M = 73.5 \times 10^{-3} \text{ Nm}^{-1}$$~~

Answer (Ans)

bending moment

To avoid excessive bending moment in bridge a

spring deflection becomes difficult with regards

and it is hence necessary to have a

constant bending moment along the entire

length of the bridge so that the deflection

viscous  $\rightarrow$  जलत्

Incompressible  $\rightarrow$  अप्रसंत्वम्

## Lesson - 02 (Final)

### Fluid Dynamics and viscosity

1. Eqn of C

2. B. Theorem

3. viscosity

4. Stokes law

#### Introduction:-

In the case of a liquid at rest, the hydrostatic pressure at any point inside the liquid is given by,

$$P = h \rho g$$

[ $h$  = height of water]

↳ This is only for Liquid

A fluid in motion possesses various forms of energy like kinetic energy, potential energy and gravitational energy. In the case of ideal liquids, it is assumed that the viscous forces are completely absent.

and the liquid is highly incompressible.

## Stream Line Motion and Rate of Flow:

~~length of pipe~~ ~~to unit~~ ~~to cross section~~

~~area of pipe~~ ~~to calculate~~

~~length of pipe~~ ~~to calculate~~

~~length of pipe~~ ~~to calculate~~

$v = \text{velocity of water}$

$a = \text{Area of pipe} / \text{Area of cross section}$

~~length of pipe~~

~~length of pipe~~ ~~to calculate~~

$s = v t$  ~~length of pipe~~ ~~to calculate~~

$s = v t a$  ~~length of pipe~~ ~~to calculate~~

$\Rightarrow s = \text{volume of liquid per unit time}$

$\Rightarrow s = v t a = (v a) t$  ~~length of pipe~~ ~~to calculate~~

~~length of pipe~~ ~~to calculate~~

• যদি কোন স্থানে এক জমাত রসান্বেদ লিড

flow হয় তবে সেটা Stream line or steady.

Stream line motion, upper surface এর বেগ

এবং নিচের বেগ Lower surface এর বেগ

এবং কোন বেগ

এবং কোন বেগ

In case of stream line motion, at any instant, the tangent to curve gives the direction of the flow of the liquid.

The stream line may be straight or curved. The condition for stream line motion depends on the velocity of flow of the liquid.

If the velocity does not exceed a certain limiting value called the critical velocity.

If the velocity is not higher than the

Critical velocity, the flow is said to be stream line.

And if it is higher than critical velocity the motion is said to be turbulent.

For laminar flow Reynold's number  $Re$  should

be less than 2000. According to Reynold,

the critical velocity of a liquid is given

by,

$$Re = \frac{\rho v D}{\eta}$$

$$\text{OR, } Re = \frac{\rho \cdot v \cdot D}{\eta}$$

Reynold ଏଣ୍ଟି ନାମାଙ୍କଳ ମତ ଧାର୍ଜନେ  
 Velocity ଓ ଏଣ୍ଟି ଧାର୍ଜନେ ।

~~critical~~  
 $V_e = \text{velocity of flow}$

$Re$  = Reynold's number

$\rho$  = Density of the fluid

$D$  = Diameter of the tube

$\eta$  = Coefficient of viscosity of the liquid

To (~~in air~~) ~~nitrobenzene~~ which smells like benzene.  
In turbulent flow, the path of the particles during their motion is zig-zag.

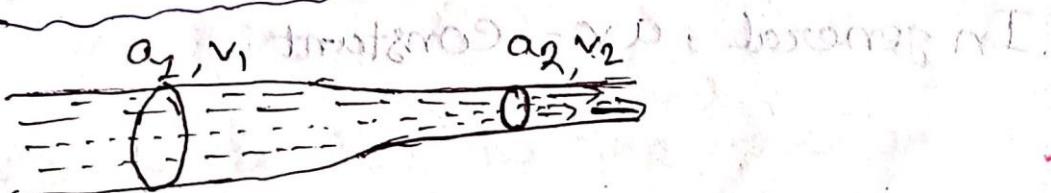
तारा अक्षर layer-५ Flow  $25 \text{ cm}^3$ , प्रतीक्षा particles  
तारा छिप्प layer-६ Flow  $25 \text{ cm}^3$

Mr. = A. brass: for present busid. To Inverness.

**Example:-**

The flow of river's water is a stream line motion in ~~natural~~ general time.

## Equation of continuity :-



wall velocity (afar)

$$\text{speed mass } \frac{\text{volume}}{\text{time}} = \rho v$$

বেগ বেগ,

বাহির থেকে  
বেগ

বাহির থেকে  $a_1 > a_2$

বাহির থেকে  $a_2 < a_1 \Rightarrow v_2 > v_1$   $\leftarrow$  velocity  
বাহির থেকে প্রস্তুত হলে, equation of continuity  
বাহির থেকে প্রস্তুত

As there is no accumulation (মান কর) of  
the liquid at any point, the amount

of liquid flowing per second is the  
same at all cross-sections of the tube.

Amount of liquid flowing per second,  $A = a_1 v_1$

Amount of " " "  $B = a_2 v_2$

So,  $a_1 v_1 = a_2 v_2 \leftrightarrow$  The equation of  
continuity.

In general,  $a v = \text{constant}$

বাহির থেকে প্রস্তুত হলে,  $a_1 v_1 = a_2 v_2$

বাহির থেকে প্রস্তুত হলে,  $a_1 v_1 = a_2 v_2$

## Energy of a liquid in Motion

1) Liquid में विद्युत ऊर्जा का रूप विद्युत ऊर्जा है।

2) इसका अर्थ वह energy है।

विद्युत ऊर्जा का अर्थ वह energy है।

विद्युत ऊर्जा का अर्थ वह energy है।

विद्युत ऊर्जा का अर्थ वह energy है।

But the kinetic energy per unit weight

is called velocity head and is equal to

$$\Rightarrow \frac{\frac{1}{2}mv^2}{mg} \Leftarrow (\bar{w} = \text{weight} = mg)$$

(\*) This energy is due  
to the position  
motion of the liquid.

$$\text{Or}, \frac{\frac{1}{2}v^2}{g} \quad \text{Or}, \frac{v^2}{2g}$$

Liquid - एवं विद्युत ऊर्जा का अर्थ वह energy है।

3) Calculation

$$\text{So, velocity head} = \frac{v^2}{2g}$$

2) Potential Energy:-

This energy is due to the position of the liquid with respect to the ground level.

Exerted  $\rightarrow$  दबाव

So, we know,  $P.E. = mgh$

So, potential head  $= \frac{mgh}{mg} = h$

The potential energy per

unit weight

3) pressure Energy:

The energy is due to the pressure exerted

(दबाव) on the liquid while it is flowing.

This energy is  $= PV$   $\downarrow$  volume  
 $\downarrow$  pressure

so, The pressure energy per unit weight

$$\Rightarrow \frac{PV}{mg}$$

$$\text{But we, } P = \frac{m}{V} \therefore \frac{1}{P} \cdot \frac{1}{\rho} = \frac{V}{m}$$

so, pressure head  $\Rightarrow \frac{P}{\rho g}$

at top level head at sub at bottom level head at bottom

head at top level head at bottom

So, now if we add all ~~three~~ ~~ener~~ values of three energies; we will get Total energy per unit mass at any point is

Given by :-

$$\left[ \frac{v^2}{2} + gh + \frac{P}{\rho g} \right]$$

[when the flow is along the stream line]

If we calculate with total energy per unit weight :-

$$\left[ \frac{v^2}{2g} + h + \frac{P}{\rho g} \right]$$

Constant

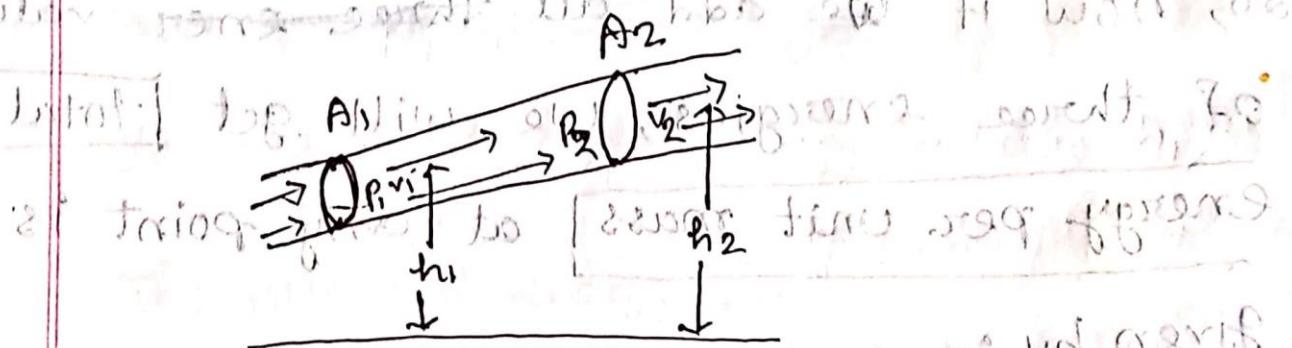
In Bernoulli's Theorem:-

Liquid at flow and steady at non-turbulent रूप तरे अन्यु शुल्कार्. (मानव)

अमान रुपा।

[and man बिल्डिंग ग्रेड] वृत्ति ग्रेड मान बुड़ी

ग्रेड ( $\eta - \eta_1$ ) = 90% मान ( $\eta - \eta_1$ ) = 90%.



The motion of the liquid is streamline.

Let the pressure of the liquid at the

Cross-section at  $A_1 = a_1$ , and at  $A_2 = a_2$

The velocity of flow of the liquid at  $A_1$

is  $v_1$  and  $A_2$  is  $v_2$ .

work done per second on the liquid

entering  $A_1$ ,  $w_1 = P_1 a_1 v_1$

leaving  $A_2$ ,  $w_2 = P_2 a_2 v_2$

we know

$w = F S$

$w$ . done per time,  $\frac{w}{t} = \frac{F S}{t}$

Also,  $P = F/A$  &  $F = P A$

so,  $\frac{w}{t} = \frac{P A S}{t} \Rightarrow [S/t = v]$

$\therefore w = P A V$

Net work done on the liquid,

$$w = w_1 - w_2 = P_1 a_1 v_1 - P_2 a_2 v_2$$

But,  $a_1 v_1 = a_2 v_2$  [Cause it is in streamline]  
[Equation of continuity]

$$\therefore w = (P_1 - P_2) a_1 v_1 = (P_1 - P_2) a_2 v_2$$

This work done on the liquid contributes  
for the changes in kinetic energy & gravitational energy.

Note:-

Gravitational energy =  $\rho g h A t + \frac{1}{2} \rho V_0 (g - g) A t$

From potential energy,  $P = mgh$

As we also know,  $m = \rho V \rightarrow V = \frac{m}{\rho}$  [ $\rho = \frac{m}{V}$ ]

We know, volume,  $V = \frac{1}{6} \pi r^2 h$  (area · length)

Now,  $\frac{mgh}{t} = \frac{\rho Vgh}{t}$  [ $\therefore m = \rho V$ ]

Now,  $= \frac{\rho \cdot A \cdot l \cdot g \cdot h}{t} \Rightarrow \rho agh \cdot \frac{l}{t} = \rho agh \cdot v$   
(i) unit per time (kg/m² s)  $\downarrow$  velocity

Now, change in

gravitational energy,  $E_1 = (\rho_1 V_1) \rho g \cdot (h_2 - h_1)$

Kinetic energy,  $E_2 = \frac{1}{2} m v^2$  (this is what we know)

$$= \frac{1}{2} \rho \cdot \rho_1 V_1 \cdot v^2$$

total velocity

$$= \frac{1}{2} (\rho_1 V_1) \rho (v_2^2 - v_1^2)$$

$$\text{So, } W = B_1 + E_2$$

$$\text{or, } (P_1 - P_2) \alpha_1 v_1 = (\alpha_1 v_1) \rho g (h_2 - h_1) + \frac{1}{2} (\alpha_1 v_1) \rho (v_2^2 - v_1^2)$$

$$\text{or, } P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\text{or, } \frac{v_1^2}{2} + gh_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + \frac{P_2}{\rho} + gh_2$$

$$\text{Or, } \frac{v_1^2}{2} + gh + \frac{P}{\rho} = \text{constant} \quad \underline{\text{work}} \quad (ii)$$

This equation represents Bernoulli's equation

Dividing equation (ii) by  $g$ .

$$\left( \frac{v^2}{2g} \right) + h + \left( \frac{P}{g\rho} \right) = \text{constant}$$

Here,  $\left( \frac{v^2}{2g} \right)$  = The velocity head

$h$  = gravitational head

$\left( \frac{P}{g\rho} \right)$  = pressure head

## Special Case: Venturiometer

When it is a horizontal PIPE,  
and the cross-section is constant,

$$\frac{v^2}{2} + \frac{P}{\rho} = \text{constant}$$

$$\text{or, } P + \frac{1}{2} \rho v^2 = \text{constant}$$

$P$  = Static pressure

$\frac{1}{2} \rho v^2$  = Dynamic pressure

Velocity (पर्याप्ति)

A venturiometer is a horizontal tube. It has different cross sections at A and B. It is used to find the rate of flow of a liquid when the motion of the liquid crossing through any cross-section of the pipe is constant.

It means that, the amount of liquid crossing per second at A is equal to the amount of liquid crossing per second at B. But

$$\frac{P_1 - P_2}{\rho} = \frac{P_2}{\rho} - \frac{P_1}{\rho} = g\alpha - gh = g\alpha$$

$$= -gh$$

the area of cross-section at A is large.

### 7. Law of hydrostatic pressure:-

The difference in pressures between two points A and B can be calculated by applying Bernoulli's principle.

The liquid in the vessel is at rest. Therefore,  $v=0$ . The total energy per unit mass at A,

$$= g(h+\alpha) + \frac{P_1}{\rho} \quad \text{(i)}$$

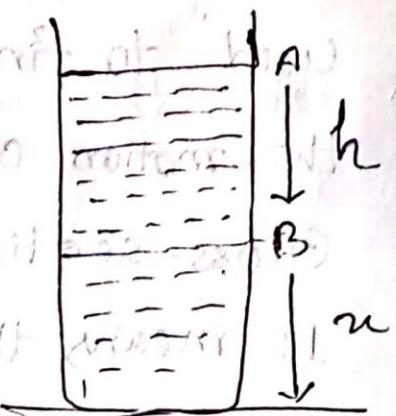
Total energy per unit mass at B,

$$= g\alpha + \frac{P_2}{\rho} \quad \text{(ii)}$$

Equating (i) & (ii)

$$g(h+\alpha) + \frac{P_1}{\rho} = g\alpha + \frac{P_2}{\rho}$$

$$P_2 - P_1 = g\rho\alpha h$$

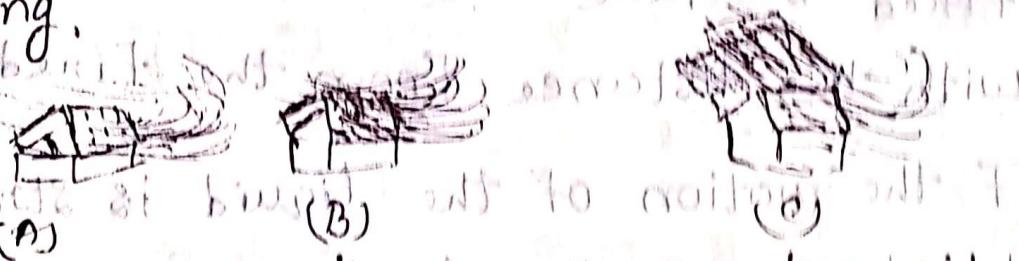


~~Structures built with brick and stone~~ ~~and~~ ~~concrete~~ ~~are~~ ~~most~~ ~~dur~~

### 8) Blowing of Roofs

Due to wind, storm or cyclone, the roofs are blown off. When a high velocity wind blows over the roof, there is considerable lowering of pressure on the roof.

As the pressure on the lower side of the roof is higher, roofs are easily blown off without damaging the walls of the building.



Due to wind, storm or cyclone, the roofs are blown off without damaging the building.

The sharp edges of the building should not protrude to outside as it increases the wind resistance.

Streamline  $\rightarrow$  Fluid flow but free from turbulent.

### Viscosity (जलता)

Whenever a liquid flows on a horizontal surface, the velocities of the different layers of the liquid parallel to the fixed surface are different and increase with the distance from the fixed surface.

If the motion of the liquid is streamline, the layer of the liquid in contact with the fixed surface is stationary. The velocity of any layer increases with the distance from the fixed layer. If any two layers are considered, the upper layer tends to accelerate the motion of the lower layer and the lower layer tends to retard the motion of the upper layer. The

two layers together tends to destroy their relative motion. As if there is a backward tangential force. (মদি অগ্রলঞ্চ ইতি প্রাণীক ইহ  
অপরিবর্তনকীল সুস্থিত হয়ে থাকা অঙ্গ ইহ স্থিতি  
মহে অপরিবর্তনকীল সুস্থি ইয়ে দৃশ্য বাঢ়ে এবং সুস্থিত  
অগ্রলঞ্চ কো বৃক্ষ পান্তি। মদি দুর্ঘটি সুস্থিত সুস্থিতে  
বিহু ইহ, উপরে সুস্থি নিচে সুস্থিত কোকে বৃক্ষ  
কুতু ছাইতে প্রথম নিচে সুস্থি উপরে সুস্থি কোকে  
কমাতে চাইবে। এই কুতু, একে অপরে রাতিরে নিচে  
নিচের ইতি অমান কুতু চাইবে, ~~সমস্ত আদৃত~~ অ  
স্থানক্রিক এবং একে অপরে আপক্রিক ইতিরে  
বৃক্ষ কুতু চাইবে)।

An external force is required to overcome this backward drag and to maintain the relative velocity between the different layers of the liquid. This property by virtue (ক্ষত) of which

a liquid opposes the relative motion between the different layers is called viscosity or internal friction.

Generally, thin liquids like alcohol, water, spirit etc. are less viscous.

Whereas, thick liquids like Coal tar, Castor oil, glycerine etc. are more viscous.

According to Newton, the backward tangential force  $F$  on any layer is dependent on,

(i)  $F \propto A$ , the area of the layer.

(ii)  $F \propto \frac{du}{dn}$ , the velocity gradient at the layer,

$$F \propto A \cdot \frac{du}{dn} \rightarrow \left[ \begin{array}{l} \text{Distance अथवा तारे} \\ \text{velocity अथवा वेग} \end{array} \right]$$

or,  $F = -\eta A \frac{du}{dn}$

Here,  $\eta$  is the coefficient of viscosity of the fluid and  $\frac{du}{dn}$

iii)   $\frac{du}{dx}$  is called the viscosity gradient or change of velocity with distance.

iv)  The negative sign shows that the force is acting opposite to the direction of velocity.

v)   $\eta$  is called the coefficient (stiffn.) of viscosity of the fluid and  $\frac{da}{dx}$  is called change in the velocity with distance. The negative sign ~~shows~~ shows that the force is acting opposite to the direction of velocity.

$$\text{If, } A=1 \text{ & } \frac{du}{dx}=1 \text{ then, } F = -\eta$$

Therefore, the coefficient of viscosity is defined as the tangential force required to maintain a unit velocity gradient between two layer of area one unit each.

Units, IF  $A = 1 \text{ sq. cm}$ ,

$$\frac{du}{dx} = \frac{1 \text{ cm/s}}{1 \text{ cm}}$$

and  $F = 1 \text{ dyne}$ , then [যালো এসে গুণ করুন]

$$\eta = \frac{F}{A \cdot \frac{du}{dx}} \rightarrow \frac{\text{dyne}}{\text{cm}^2 \cdot \text{s}^{-1}}$$

Numerically,  $\eta = 1 \text{ poise}$

The coefficient of viscosity (স্থিরতা পুনর্গুণ)

of a liquid is one poise, if a force  
of  $1 \text{ dyne}$  is required to maintain a  
velocity gradient of one unit between  
two layers of area  $1 \text{ sq cm}$  each.

Dimensions of coefficient of viscosity:-

$$[\eta] = \frac{[F]}{[A] \left[ \frac{du}{dx} \right]} = \frac{[MLT^{-2}]}{[L^2][T^{-1}]} = [M^{-1}L^{-1}T^{-1}]$$

Unit of  $\eta$  in SI is  $\text{kg/m.s}$ , or  $\text{Ns/m}^2$

## Stoke's law:

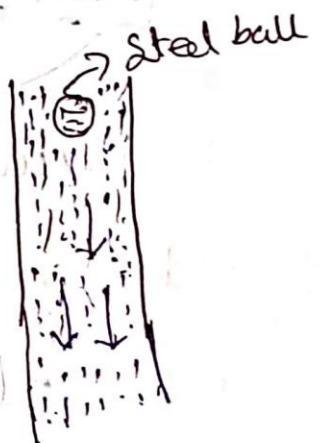
When a small ball is left gently on the surface of a long vertical column of a viscous liquid, the ball moves vertically downward. Initially, the ball is accelerated due to gravity. The motion of the ball is opposed by the viscosity of the liquid. Consequently, the resultant force acting on the ball becomes zero and it moves with a constant velocity called the terminal velocity  $v$ .

The resistive force  $F$ , acting on the ball depends on,

(i) The radius of the ball,  $r$

(ii) The terminal velocity,  $v$

(iii) Coefficient of viscosity,  $\eta$ .



Expressing dimensionally, Form

$$[F] = K [n^a] [v^b] [n^c] \quad (i)$$

$$[MLT^{-2}] = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c$$

$$[MLT^{-2}] = [M]^c [L]^{a+b-c} [T]^{b-c} \\ \rightarrow [M]^1 [L]^4 [T]^{-2}$$

Equating the powers

$$c = 1$$

$$-b - c = -2$$

$$\frac{b}{2} = 1 \\ b = 2$$

$$a + b - c = 1$$

$$a = 1$$

Substituting these values in equation (i)

$$F = Knvn$$

But,  $K = 6\pi$   $\rightarrow$   $K = 6\pi \times 0.2 \times 1 \rightarrow K = 12\pi$

Force,  $F = 6\pi n n v$

Coefficient of viscosity of a liquid

Can be determined by Stoke's Formulae,

Suppose,

Radius of the ball =  $r$ .

Terminal velocity =  $v$

Coefficient of viscosity =  $\eta$

Density of the ball =  $\rho$

Density of the liquid =  $\sigma$

Net weight of the ball =  $\frac{4}{3} \pi r^3 (\rho - \sigma) g$  — (i)

Upward force due to viscosity,  $6\pi\eta rv$

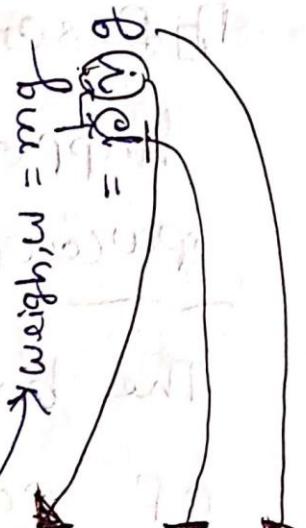
$$6\pi\eta rv = \frac{4}{3} \pi r^3 (\rho - \sigma) g — (ii)$$

$$\text{Terminal velocity, } v = \frac{2r^2 (\rho - \sigma) g}{9\eta}$$

$$\text{Co. Viscosity, } \eta = \frac{2rv(\rho - \sigma)g}{9v}$$

Hence  $\eta$  can be determined.

Stoke's law is applicable to determine coefficient of viscosity of highly viscous liquids.



Describe with liquid's law, what happens when someone jumps with parachute:-

The body of jumper & the direction of parachute is the reason why someone lands successfully.

বাতাস এবং প্রাণমুক্ত আটকে থাকে ঠিক  
direction কৈছে দিবে।

এখন ক্ষেত্র velocity reaction মডেল same  
হচ্ছে তখন ক্ষেত্র একটি fixed velocity  
প্রচলিত হয়ে আছে ক্ষেত্রে আমরা বলি

Terminal velocity, এজন অস্থির পদার্থ মনে  
হো যে এই নিম্নীক বেগে খিলাফে land  
করবে।

To progress to top of page at

## Math Solve

Example:- 7.1 :- ( Theory : Stream line motion )  
and Rate of Flow  
(*for गतिशील*)

Water flows through a horizontal pipe

line of varying cross section at the rate

of  $0.2 \text{ m}^3\text{s}^{-1}$ . Calculate the velocity of water

at a point where the area of cross section

of the pipe is  $0.02 \text{ m}^2$ .

$\Rightarrow$

$$\frac{A = 0.02 \text{ m}^2}{Q = 0.2 \text{ m}^3\text{s}^{-1}} \Rightarrow$$

Here,

Area of cross section =  $0.02 \text{ m}^2$

Rate of Flow =  $0.2 \text{ m}^3\text{s}^{-1}$

velocity =  $v$  ?  
we know,

Ratio of flow =  $a v$

$$\text{or, } v = \frac{\text{Ratio of flow}}{a}$$

$$= \frac{0.2}{0.02} = 10 \text{ ms}^{-1}$$

(Ans)

**Ex-2** A pitot tube is fixed on the wing

of an aeroplane to measure the speed  
of the aeroplane. The tube contains a  
liquid of density  $800 \text{ kg m}^{-3}$ . The difference  
in level between the two limbs is  
 $0.5 \text{ m}$ . Density of air =  $1.293 \text{ kg m}^{-3}$ . Calculate  
the speed of the aeroplane.



Here,

The density of liquid,  $\rho_1 = 800 \text{ kg m}^{-3}$

The difference in level between two  
limbs,  $H = 0.5 \text{ m}$ .

Density of air,  $\rho_2 = 1.293 \text{ kg m}^{-3}$

velocity of plane,  $v = ?$

We know,

$$\frac{1}{2} \rho_2 v^2 = \rho_1 H g$$

$$\text{or, } v^2 = \frac{2 \rho_1 H g}{\rho_2}$$

$$\text{or, } v = \sqrt{\frac{2P_2 \times 11.7g}{P_1}} = \sqrt{\frac{0.1800 \times 0.5 \times 9.8}{1.293}}$$

$$= 77.87 \text{ m/s}$$

(Ans)

Example 7.4:-

Calculate the speed at which the velocity head of a stream of water is equal to 0.5 m of Hg. (Assume atmospheric pressure to be 760 mm of height measured when the manometer density is 13.6 cm Hg of Hg. So, the pressure of any gas is the pressure exerted by it on the walls of the container that can raise the manometer to 13.6 cm in the barometer.)

Now (Notes) The standard atmospheric pressure of 760 mm is height measured when the manometer density is 13.6 cm Hg of Hg. So, the pressure of any gas is the pressure exerted by it on the walls of the container that can raise the manometer to 13.6 cm in the barometer.)

∴ We know,

$$\text{Velocity head} = \frac{v^2}{2g}$$

$$0.5 \text{ m of Hg} = 0.5 \text{ m} \times 13.6 \text{ cm of water}$$

$$\text{Or, } v = \sqrt{v \cdot H \times 2 \times g \times 13.6}$$

$$\text{Or, } v = \sqrt{v \cdot H \times 2 \times g \times 13.6}$$

$$\text{Or, } v = \sqrt{0.5 \times 2 \times 9.8 \times 13.6}$$

$$\text{Or, } v = 11.54 \text{ m/s}$$

(Ans) 70 m/s  $\Rightarrow$  70 km/h

A railway engine is fitted with a tube whose one end is inside a reservoir of water in between the rails. The other end of the tube is 4m above the surface of water in the reservoir. Calculate the speed with which the water pushes out of the proper upper end, if the engine is moving with a speed of  $108 \text{ km hr}^{-1}$ .

$\Rightarrow$  As the train is moving

So there are both kinetic & potential

energy available.

So,

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$

or, Here,

$$\text{or, } gh_1 + \frac{1}{2}v_1^2 = gh_2 + \frac{1}{2}v_2^2$$

$$\text{or, } g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

$$\text{or, } \frac{1}{2}v_1^2 = g(h_2 - h_1) + \frac{1}{2}v_2^2$$

$$\text{or, } v_1^2 = 2g(h_2 - h_1) + v_2^2$$

$$\text{or, } v_1 = \sqrt{2g(h_2 - h_1) + v_2^2} \quad \text{(i)}$$

Here,

$$v_1 = ?$$

$$(h_2 - h_1) = 4 \text{ m. } (h_1 - h_2) = 4 \text{ or, } (h_2 - h_1) = -4$$

$$g = 9.8 \text{ m s}^{-2}$$

$$v_2 = 108 \text{ km/hr} = \frac{108 \times 1000}{3600} = 30 \text{ m s}^{-1}$$

$$\text{So, } v_1 = \sqrt{2 \times 9.8 \times 4 + (108)^2} (30)^2 = 28.66 \text{ m s}^{-1}$$

(Ans)

Example - 7.7:-  
 A water flows through a horizontal pipe line of varying cross-section. At a point where the pressure of water is  $0.05 \text{ m}$  of Hg. the velocity of flow is  $0.25 \text{ ms}^{-1}$ . Calculate the pressure at another point where the velocity of flow is  $0.4 \text{ ms}^{-1}$ .

Density of water is  $10^3 \text{ kg m}^{-3}$ .

$$\Rightarrow \frac{P_1}{\rho} + \frac{1}{2} V_1^2 = \frac{P_2}{\rho} + \frac{1}{2} V_2^2$$

$$P_1 = 0.05 \text{ Hg}$$

$$V_1 = 0.25 \text{ ms}^{-1}$$

$$V_2 = 0.4 \text{ ms}^{-1}$$

$$\rho = 10^3 \text{ kg m}^{-3}$$

Here,  $V_1 = 0.25 \text{ ms}^{-1}$ ;  $V_2 = 0.4 \text{ ms}^{-1}$   $\rho = 10^3 \text{ kg m}^{-3}$   
 pressure of water,  $P_1 = 0.05 \text{ Hg}$

$$= 0.05 \times 13.6 \times 10^3 \times 9.8 \text{ N m}^{-2}$$

$$= 6664 \text{ N m}^{-2}$$

∴ we know,

$$\frac{P_1}{\rho} + \frac{1}{2} V_1^2 = \frac{P_2}{\rho} + \frac{1}{2} V_2^2$$

Question:-  $0.05 \text{ m} \text{ of Hg}$   
 $\text{Nm}^{-2}$

Or,  $P_2 = P_1 + \frac{1}{2} (v_1^2 - v_2^2) \cdot P$

Or,  $P_2 = 6664 + \frac{1}{2} [(0.25)^2 - (0.4)^2] \times 10^3$

Or,  $P_2 = ?$  (using result from 2)

Or,  $\frac{P_1}{P} = \frac{P_2}{P} + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$

Or,  $\frac{P_1}{P} = \frac{P_2}{P} + \frac{1}{2} (v_2^2 - v_1^2)$

Or,  $\frac{P_2}{P} = \frac{P_1}{P} + \frac{1}{2} (v_1^2 - v_2^2)$

Or,  $\frac{P_2}{P} = \frac{P_1}{P} + \frac{1}{2} (v_1^2 - v_2^2)$

Or,  $P_2 = P_1 + \frac{1}{2} \cdot P (v_1^2 - v_2^2)$

Or,  $P_2 = 6664 + \frac{1}{2} \times 10^3 \{ (0.25)^2 - (0.4)^2 \}$

Or,  $P_2 = 6664 + 48.75$

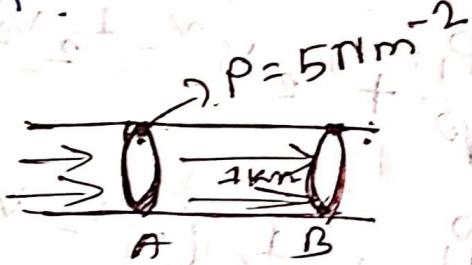
Or,  $P_2 = 6615.25 \text{ Nm}^{-2} (\text{Ans})$

Or,  $P_2 = \frac{6615.25}{13.6 \times 10^3}$

Example :- 7.8 :-

In a horizontal pipe line of uniform area of cross section, the pressure falls by  $5 \text{ Nm}^{-2}$  between two points separated by a distance of 1 km. What is the change in kinetic energy per kg of the oil flowing at these points? Density of oil  $\rho = 800 \text{ kgm}^{-3}$

$\Rightarrow$



According to Bernoulli's principle,

$$\frac{P}{\rho} + gh + \frac{1}{2} v^2 = \text{Constant}$$

As the pipe line is horizontal,

$$\frac{P}{\rho} + \frac{1}{2} v^2 = \text{Constant}$$

As there are two points,

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2$$

$$\text{or, } \frac{(P_1 - P_2)}{\rho} = \frac{1}{2}(v_2^2 - v_1^2) \quad \text{--- (i)}$$

Here,

$$P_1 - P_2 = 5 \text{ Nm}^{-2}$$

$$\rho = 800 \text{ kgm}^{-3}$$

$\frac{1}{2}(v_2^2 - v_1^2)$  = changes  
in kinetic  
energy.

∴ change in kinetic energy per kg,

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{(P_1 - P_2)}{\rho}$$

$$= \frac{5}{800} \text{ J/kg}$$

$$= 6.25 \times 10^{-3} \text{ J/kg}$$

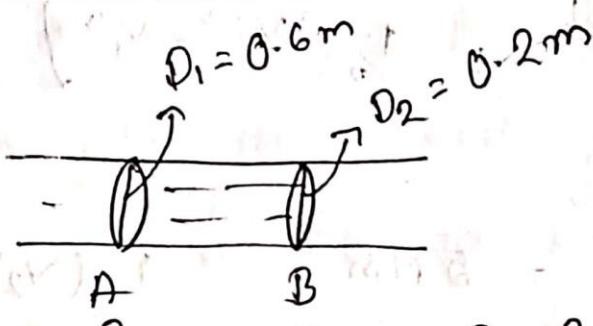
(Ans)

Example:- 7.10 (सम्पूर्ण विवर)

water is flowing through a horizontal pipe line. At two points A and B. The diameters are 0.6 m and 0.2 m. The pressure difference between the points A and B is 1 metre column of water. Calculate the volume of water flowing per

Second.

Ans:-



$$P_1 - P_2 \Rightarrow P_1 - P_2 = 1 \text{ metre column of water}$$

Here,

Diameter,  $D_1 = 0.6 \text{ m}$ .

$$D_2 = 0.2 \text{ m}.$$

The pressure difference,  $(P_1 - P_2) =$

$$= 1 \text{ metre column of water.}$$

$$= 1 \times 10^3 \times 9.8$$

$$\approx 9.8 \times 10^3 \text{ Nm}^{-2}$$

The volume of water,  $V = ?$

$\Rightarrow$  For A,

$$\text{Rate of flow, } Q = a_1 a_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^2 - a_2^2)}}$$

$$\text{Here, } a_1 = \frac{\pi \times (0.6)^2}{4} = 0.09\pi \text{ m}^2$$

$$a_2 = \frac{\pi (0.2)^2}{4} = 0.01\pi \text{ m}^2$$

$$P_1 - P_2 = (1 \times 10^3 \times 9.8) \text{ Nm}^{-2}$$

$$\alpha_1^r - \alpha_2^r = 48 \times 10^{-4} \text{ m}^2$$

The horizontal range,

$$l = v_x t = v_x \sqrt{\frac{2h_2}{g}} = (\sqrt{2gh_1}) \sqrt{\frac{2h_2}{g}}$$

$$= 2\sqrt{h_1 h_2}$$

$$h_2 = H - h_1$$

$$l = 2\sqrt{h_1(H-h_1)}$$

$$l^2 = 4h_1H - 4h_1^2$$

On Differentiating,

$$2l \left( \frac{dl}{dh_1} \right) = 4H - 8h_1$$

For  $l$  to be maximum,  $\frac{dl}{dh_1} = 0$

$$\therefore 4H - 8h_1 = 0$$

$$h_1 = \frac{H}{2}$$

$$h_2 = \frac{H}{2}$$

and, therefore, for the range of the liquid to be maximum, the orifice must be at half the height of the liquid column.

Example - 7.11:- Find the limiting velocity of a rain drop.

Ques Find the limiting velocity of a rain drop. Assume, diameter =  $10^{-3}$  m.

Density of air relative to water =  $1.3 \times 10^{-3}$  kgm $^{-3}$

Coefficient of viscosity of air =  $1.81 \times 10^{-8}$  S.I. units

Density of water, =  $10^3$  kgm $^{-3}$ .

$$\text{Diameter} = 10^{-3}$$

$\Rightarrow$  According to Stoke's law,

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$\text{Or, } v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2r^2\rho \left[ 1 - \left( \frac{\sigma}{\rho} \right) \right] g}{9\eta}$$

Here,  $r = 5 \times 10^{-4}$  m.

$$\rho = 10^3 \text{ kgm}^{-3}$$

$$\sigma/\rho = 1.3 \times 10^{-3}$$

$$\eta = 1.81 \times 10^{-8} \text{ S.I. units.}$$

$$v = \frac{2 \times (5 \times 10^{-4})^2 \times 10^3 [1 - 1.3 \times 10^{-3}] \times 9.8}{9 \times 1.81 \times 10^{-8}}$$
$$= 30.04 \text{ ms}^{-1} \quad (\text{Ans})$$

## Lecture-03 (Final)

### Thermodynamics

#### (তHERMODYNAMICS)

##### Thermodynamic System

A thermodynamic system is a body of matter and/or radiation confined (সীমাবদ্ধ) in space by walls with defined permeabilities, which separate it from its surroundings.

The surroundings may include

Other thermodynamic systems, or physical systems that are not thermodynamic systems. (অন্য, পরিবেশ, অভিস্থৰ নিয়ে আলোচনা করা হবে বা আলোচনা করা পায় (পর্যবেক্ষণ))

Systems are classified as follows:-

##### Isolated System (ফিল্ট্র সিস্টেম)

If there is no interchange of energy or matter between the system and surroundings

then the system is called isolated system.

( इत्तु उक्ति केवल एक सिस्टम रहत ना पात्र हो तो

Isolated System का लिखित ग्रन्थ यह है)



→ (इसके)

Closed System (वद्ध मिस्टेम) :-

If no energy or matter crosses the boundary, then the system is called closed system.

(कोई ऊर्जा और देहांशु यानुकूलीय या आपसे विद्युत

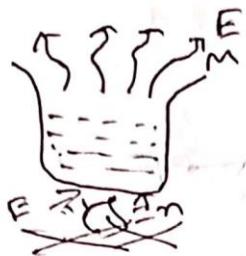
रहत ना तो एक वद्ध मिस्टेम है)



Open System (उद्धुक्त मिस्टेम)

If there is an interchange of energy or matter between the system and surroundings, then the system is called

open system. (জ্ব ও কার্ডি প্রেসুর পদ্ধতিতে মাঝ  
অবাস্থা প্রেসুর মাঝ)



### Thermodynamic process

A thermodynamic process is the ~~or~~ energetic development of a thermodynamic system proceeding (পরিয়ে মাঝ) from an initial state to a final state.

Different thermodynamic processes are:-

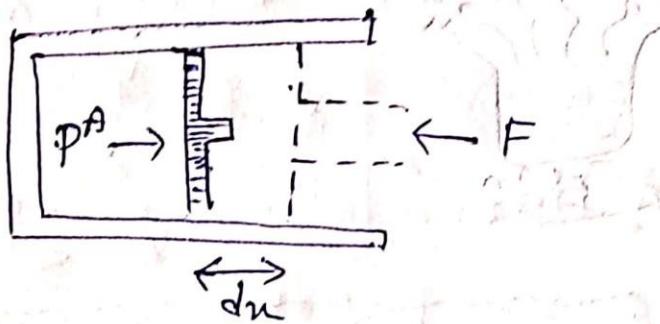
#### 1. Isochoric process (সম্ভূত প্রক্রিয়া):-

If during a process, the volume is constant then the process is called isochoric process.

#### 2. Isobaric process (সমচাপ প্রক্রিয়া):-

যে অপরাধীয় প্রক্রিয়ায় সিস্টেমটি চলে (কোনো

દર્શિકાનું રૂપનું તાત્કાલ પ્રમાણે પ્રતિષ્ઠાન વાળી।



### 3. Isothermal process (મસ્તક પ્રક્રિયા):-

IF during a process the temperature is constant then the process is called isothermal process.

An isothermal process is a change of a system, in which the temperature remains constant ( $\Delta T = 0$ ). This typically occurs when a system is in contact with an outside thermal reservoir, and the change occurs slowly enough to allow the system to continually adjust to the temperature of the reservoir through heat exchange.

system whereas if the temperature of the system is slightly greater than that of the surroundings there will be a flow of heat in the opposite direction. Such process is therefore reversible process whence no dissipative force is present.

### Irreversible processes-

If there is a finite temperature difference between system and surroundings the direction of the heat flow can't be reversed by an infinitesimal change in temperature of the system and the process is irreversible.

### Concept of 1st law of Thermodynamics:-

Scientist named Joule discovered this.

### 1st law of thermodynamics:-

Work done on a system is equal to the

When heat is supplied to any system then a part of that helps to increase the internal energy of the system and the remaining part of the energy is used by the system to do external work on the environment.

(मध्ये कोणता सिस्टेम असावित प्रकाराचा असू शकत नाही आणि असावित किंवा अंतर सिस्टेम असू शकील असावित हृद्दिते प्रकाराचा अंतर सुट्टा सिस्टेम एवढी वायरिंग (environment) असत आणि वायरिंग काज घेणार वाढते)

Explanation:-

Explanation of above:- here,  $\Delta U = U_2 - U_1$   $U_1$  = Internal energy of the system.

The change in internal energy. If Q amount of heat is added to the system then  $U_2$  will become the internal energy.

Thus the first law of the thermodynamics gives,

$Q = \Delta U + w \Rightarrow$  It is the mathematical form OF the First law of thermodynamics.

For infinitesimal reversible process,

the first law takes the form,

$$dQ = dU + dW$$

$$\text{or, } dQ = dU + PdV$$

চৰণ আয়তন

here,

$dQ$  = সিলেক্ট কৰা কোষিশ

$dU$  = অভিকৃতি কৰা হুক্ম

$dW$  = System কৰা হুক্ম

কৰা

### Application of first law of thermodynamics

Molar Specific Heat :- (মোলার প্রদার্ঘ প্রস্তাৱ)

The amount of heat needed to increase  
the temperature one Kelvin of one mole

gas is called molar specific heat.

(মোলার প্রদার্ঘ এক মোলের এক কেলভিন হৃদ্ধি কৰতে

প্রয়োকনীয় পদক্ষেপ এবং পদার্ঘ মোলার আপেক্ষিক

আপ বা (মোলার প্রদার্ঘ প্রস্তাৱ বলে)।

নথি দৰিদ্ৰতাৰ জন্য এটা খুব বৃহৎ পৰিমাণ হ'ল।

নথি দৰিদ্ৰতাৰ জন্য এটা খুব বৃহৎ পৰিমাণ হ'ল।

নথি দৰিদ্ৰতাৰ জন্য এটা খুব বৃহৎ পৰিমাণ হ'ল।

Molar specific heat at constant pressure :-

(ମିଥ୍ୟାକୁ ଠାରେ ଡ୍ରାମେଡ ମୋଲାଟ୍ ଆପେକ୍ଷିତ ତାପ) [CP]

At constant pressure, the amount of heat

needed to increase the temperature one Kelvin of one mole gas is called molar

specific heat, at  $1 \text{ mol}$  व्याप्र temp.  $1 \text{ K}$  (केलिन) पाढ़ाते गोले खिल गए मरुद्धर तास मिठे हुए ठंडे

ରାଜ୍ଯ ଚାପ



## Formulae -

$$C_p = \frac{\Delta Q}{m \Delta T}$$

Molar Specific heat at Constant volume :- ( $c_v$ )

(ମିଶ୍ର ଆମ୍ବାତନ ମାଲାଟୁ ଆମିଶିଳ ପାଗ)

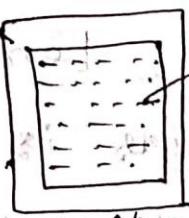
A The amount of heat needed to increase the temperature one Kelvin of one mole gas is called molar specific heat.

(1 mol വായുമുണ്ട് Temperature നിൽക്കുന്ന ആവശ്യത്തിൽ

[1] (വാളുതോ ചോലേ മേ പരമിന്മാൻ അപ ദേഡ്മാ

നാജോ അക്കേ  $C_V$  എന്ന)

1 mol വായുമുണ്ട്  $\rightarrow$  1 mol വായുമുണ്ട്  
uniform bedding



(കൂടിയാണ് അത് ഏറ്റവും കുറവാണ്)  $\rightarrow$  തോ ദിശയിൽ പിന്തും കുറവാണ്  
കുറവാണ് തോ ദിശയിൽ പിന്തും കുറവാണ്

$$C_V = \frac{dQ}{mdT} \quad \text{or, } C_V = \frac{dQ}{mKT}$$

### Relation between $C_P$ and $C_V$

Let us consider a cylinder containing one mole gas. A frictionless piston is attached to the cylinder.

P = Pressure; V = Volume; T = Temperature

U = Internal Energy of gas,

Formulas :-

$$dQ = dU + PdV \quad \text{--- (1)}$$

$$dU = C_V dT \quad \text{--- (2)}$$

We know that, the amount of heat needed to increase the temperature one Kelvin of one mole gas is called molar specific heat at constant volume,  $C_V$  and increase in temperature  $dT$  at pressure  $P$ .

$$\text{So, } C_P = \frac{dQ}{dT}$$

$$\text{Or, } dQ = C_P dT \quad \dots (3)$$

If  $R$  is the molar gas constant, then for one mole gas we get,

$$PV = RT$$

$$\text{Or, } PdV = RdT \quad \dots (4)$$

From (2), (3) and (4) we put those values into equation (1),

$$C_P dT = C_V dT + R dT$$

$$\text{Or, } C_P = C_V + R$$

$$\text{Or, } C_P - C_V = R$$

$$\textcircled{1} \quad dQ = dU + PdV$$

$$\textcircled{2} \quad C_V dT + PdV = 0$$

$$\textcircled{3} \quad dT = \frac{PdV + Vdp}{R}$$

$$\textcircled{4} \quad C_V \left( \frac{PdV + Vdp}{R} \right) + PdV = 0$$

$$\textcircled{5} \quad Vdp + \gamma PdV = 0 \quad [\text{where, } \gamma = (C_p/C_v)]$$

$$\textcircled{6} \quad \frac{dp}{p} + \gamma \frac{dv}{v} = 0$$

$$\textcircled{7} \quad \ln p + \gamma \ln v = \text{constant}$$

$$\textcircled{8} \quad \ln p + \ln v^\gamma = " \quad \text{or} \quad \ln(pv^\gamma) = "$$

$$\textcircled{9} \quad \ln(pv^\gamma) = "$$

$$\textcircled{10} \quad pv^\gamma = "$$

Here,

$$C_p > C_v \quad [\text{Always}]$$

So,

$$C_p - C_v = R \quad \text{--- (i)}$$

$$\frac{C_p}{C_v} = \gamma \quad \text{--- (ii)}$$

$R$  is a constant = 8.314

$$1 \text{ प्रामाण्यक } = 1.67$$

$$2 \text{ " } = 1.41$$

$$3 \text{ " } = 1.33$$

value of  $\gamma$ .

### Second Law of Thermodynamics

To convert thermal power into others we need an engine here. This is what we call as Thermo-engine.

Different scientist have stated the law in different forms:

#### 1) Carnot's Statement :-

No engine can be built which can extract a fixed amount of heat and will convert totally into work.

#### 2) Clasius's Statement :-

It is impossible for a self acting machine

unaided by external agency, convey heat from one body at a lower temperature to another body at a higher temperature

### 3. Planck's Statement :-

It is impossible to construct an engine which can extract heat continuously from a source of heat and completely transform it into work.

### 4. Kelvin's Statement :-

continuous flow of energy can't be obtained from an object cooling it than the coolest part of its surroundings.

### Heat Engine and Efficiency of Heat Engine

(उष्टीने इंजिन एवं उष्टीने संकेतन दोष)

### Heat Engine :-

Any mechanism for the conversion of

heat into mechanical power is called a heat engine or any suitable device which can convert heat into mechanical work is called heat engine.

### Efficiency of heat Engine:-

$$\eta = \frac{W}{Q}$$

The efficiency of heat engine is defined as the ratio of work done during a cycle to the heat absorbed during the cycle.

Thus if  $W$  is the amount of work obtained from heat engine in one cycle at the expense of amount of heat, then its efficiency  $\eta$  is defined as,

$$\eta = \frac{W}{Q} = (Q_1 - Q_2) / Q_1$$

$$= 1 - (Q_2 / Q_1)$$

## Carnot's Heat Engine and its Efficiency:-

French scientist Sadi Carnot discovered this in 1824.

### In Carnot's heat engine:-

A system carried through a Carnot cycle is the prototype of all cyclic heat engines. Feedways that is <sup>commonly</sup> They receive an input of heat at one or more higher temperatures, do

mechanical work on their surroundings and reject at some lower temperature.

When any working substance is carried through a cyclic process, there is no change in its internal energy in any complete cycle and from the first law the net flow of heat  $Q$  into the substance in any completed cycle is equal to the workdone  $W$  by the engine per cycle,

$$W = Q_1 - Q_2$$

Here,

$Q_1$  = The heat flowing in the system per cycle

$Q_2$  = The heat flowing out of the system per cycle.

Thermal efficiency :-

$$\eta = \frac{\text{work output}}{\text{heat input}} = \frac{W}{Q_1}$$

$\rightarrow$  [Heat output / Heat input]  $\rightarrow$  [Output / Input]  
heat converted into work per cycle  
heat drawn from the heat source into the system per cycle

$$\text{or } \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{--- (i)}$$

[For Carnot's engine the heat flowing in or out of the system is proportional to the temperature to the heat reservoir]

$$Q \propto T$$

$$\text{or, } \frac{Q}{T} = \text{constant}$$

If  $T_1$  and  $T_2$  are the temperatures of the source and sink respectively, then

$$\left(\frac{Q_1}{T_1}\right) = \left(\frac{Q_2}{T_2}\right)$$

$$\text{So, } \frac{Q_1}{Q_2} = \frac{T_2}{T_1}$$

From equation ① we get,

$$\eta = 1 - \frac{T_1}{T_2}$$

$$\text{or, } \eta = \frac{T_2 - T_1}{T_2 + T_1}$$

As efficiency (कार्यकारी) is expressed in term of percentage,

$$\eta = \frac{(T_2 - T_1)}{T_2} \times 100\%$$

In the same way,

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$$

Note:- As,  $T_2 > T_1$ , so we can't get 100% from any engine.

If the difference between the source of heat and the receiver of heat will be much then the efficiency will be much too. But wouldn't it be much more than 20% or 25%, sometimes 50%.

### Entropy

The disability of converting energy of a system or unavailability of energy to convert into work is entropy.

(কোন পিষ্টেমেষ কাজিষ ক্ষমতার অক্ষমতা বা অসম্ভাব্যতার ক্ষমতার কাজিষ অসম্ভাব্যতা অনুরূপ এবং ক্ষমতার ক্ষমতা অনুরূপ)

- ⊗ It has not been possible to measure the absolute magnitude of entropy. If anybody absorbs or rejects heat then its entropy is changed.
- ⊗ The change of entropy is measured by the rate of absorption or rejection of heat by the system with respect to temperature.

(ক্রেটার জিএলপি অসমানোর পার্শ্ব দ্রব্য দ্রব্য পরিমাণ

অফিল হয়ে পরিষ্ঠিত হৃষি কার অন্তর্ভুক্ত

সর্বিক্ষণ পরিমাণ হৃষি (ডে) ১৭৫ সেল কার্বন

If any system absorbs or rejects a given amount of heat at Temperature T, then the Change of entropy

$$ds = \frac{dq}{T} \quad | ds = \text{জূহীত হা/ অক্ষিত হা}$$

$$\therefore \Delta S = \frac{q_2 - q_1}{T} = \frac{0}{T} = 0$$

অর্থাৎ, অন্তর্ভুক্ত প্রক্রিয়া গ্রহণ করে এবং প্রক্রিয়া করে আসে, তখন এই প্রক্রিয়াটি শূন্য অভিযন্তা।

### Change of Entropy

1) Change of entropy in reversible adiabatic process :- (অন্তর্ভুক্ত প্রক্রিয়া শূন্য)

By definition, the heat absorbed in a reversible adiabatic is zero,

$$dq_1 = 0 \quad \text{কারণ প্রক্রিয়া শূন্য}$$

Hence For such process entropy change

is given by, কারণ প্রক্রিয়া শূন্য

$$dS = \frac{dQ_r}{T} = 0$$

where,  $S = \text{constant}$

As the  $S$  is a constant and such process is called isentropic.

### ~~Change of Entropy in~~

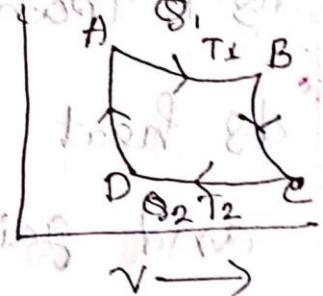
Reversible thermal process :-

(प्रतियानी प्रक्रिया का अनुपरिवर्तन)

In reversible isothermal process the temperature

remains constant and for such a process

$$S = \int \frac{dS}{T}$$



In AB,

The entropy increases  $= \frac{Q_1}{T_1}$

In CD,

$$\frac{Q_2}{T_2}$$

$$\therefore \text{Entropy Change, } \Delta S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

But in reversible process,

$$\Delta S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

But in reversible,  $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$$\therefore \Delta S = \frac{Q_1}{T_1} - \frac{Q_1}{T_1} = 0$$

$\therefore$  Entropy didn't change.

### changes of Entropy in irreversible process:-

Let ~~too~~ the temperatures of two objects are respectively  $T_1$  and  $T_2$ . If  $T_1 > T_2$ , then heat will flow from the warm object to the cold object.

$d\varphi$  is transferred from warm to cold one.

$d\varphi$  heat is lost by warm object and gain by cold object,

Therefore,  $= \text{decrease in entropy of warm object} + \text{increase in entropy of cold object}$

$-\frac{d\varphi}{T_1} = \text{Decrease of entropy of warm object}$

$\frac{d\varphi}{T_2} = \text{Increase of entropy of cold object.}$

Changes in Entropy,  $ds = (-\frac{dQ}{T_1}) + (\frac{dQ}{T_2})$

Since,  $T_1 > T_2$

$ds > 0$

So, irreversible entropy is always positive.

### Heat Death of the Universe

Everything in nature tries to acquire the state of equilibrium (সমত).

As the systems, or when objects stop sharing

temperature between them is called as thermal equilibrium.

As the systems proceed towards the equilibrium their entropy also increases. The entropy

of a system becomes maximum when we

do not get any work from it.

Since everything in nature wants to attain equilibrium so the entropy of nature is

gradually increases increasing  
of the universe  
so, the entropy when it will attain the  
highest point then everything will attain  
the same temperature.

As a result heat energy will not be  
possible to convert into mechanical energy.

It is said to be as the heat death of  
the Universe.

### The Third law of thermodynamics

"The entropy of a system approaches a  
constant value as its temperature approaches  
absolute zero."

This is because a system at zero temperature  
exists in its ground state, so that  
its entropy is determined only by the  
degeneracy of the ground state.

↓  
(उत्तम)

### Math Solve

① A quantity of air at 27°C and atmospheric pressure is suddenly compressed to half its original volume. Find the final pressure and temperature.

⇒ Here,

$$\text{Initial pressure, } P_1 = 1 \text{ atm}$$

$$\text{Initial Temperature, } T_1 = 27^\circ\text{C}$$

$$= (27 + 273) \text{ K} \\ = 300 \text{ K}$$

$$\gamma = 1.4$$

Let,

$$\text{Initial volume, } V_1 = V$$

$$\text{Final volume, } V_2 = \frac{V}{2}$$

$$\text{Final Pressure, } P_2 = ?$$

$$\text{Final Temperature, } T_2 = ?$$

⇒ we know that,

During sudden compression the process is adiabatic.

Hence,

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

or,  $P_2 = \frac{P_1 V_1^\gamma}{V_2^\gamma}$

$$\text{or, } P_2 = \left( \frac{1 \times V}{\frac{V_2^\gamma}{2}} \right)^{1.4}$$

$$= (2)^{1.4} = 2.639 \text{ atm}$$

Again,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{or, } T_2 = T_1 \left( \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} \right)$$

$$\text{or, } T_2 = 300 \times \left( \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} \right)^{1.4}$$

$$\text{or, } T_2 = 300 \times \left( \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} \right)^{1.4-1}$$

$$\text{or, } T_2 = 300 \times (2)^{1.4-1}$$

$$= 395.85 \text{ K}$$

(Ans)

### Problem-2:

Find the efficiency of the Carnot's

engine working between the steam point  
and the ice point

$\Rightarrow$  Here, initial state

Initial Temperature,  $T_1$  (Steam point) =  $100^\circ\text{C}$

$$= (100 + 273)\text{K} = 373\text{K}$$

Final Temperature,  $T_2$  (Ice point) =  $0^\circ\text{C} = (0 + 273)\text{K}$

Efficiency,  $\eta = ?$

We know that,

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{273}{373}$$

$$= \frac{100}{373} = 0.268$$

$$\therefore \eta = \frac{100}{373} \times 100\% = 26.8\%$$

(Ans)

Problem - 03:-

: Solution

A Carnot's engine whose temperature of the source is 400K takes 200 calories of heat at this temperature and rejects 150 calories of heat to the sink. What is the temperature of sink? Also calculate the efficiency of the engine?

→ Here,  $Q_1 = 200 \text{ Cal}$

Initial heat,  $Q_1 = 200 \text{ Cal}$

Final heat,  $Q_2 = 150 \text{ Cal}$

Initial Temperature,  $T_1 = 400 \text{ K}$

Final Temperature,  $T_2 = ?$

⇒ We know that,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{or, } T_2 = \frac{T_1 \cdot Q_2}{Q_1}$$

$$\text{or, } T_2 = \frac{400 \times 150}{200} = 300 \text{ K}$$

Again,

$$\begin{aligned}\text{Efficiency, } \eta &= 1 - \left( \frac{T_2}{T_1} \right) \\ &= 1 - \left( \frac{300\text{K}}{400\text{K}} \right) \\ &= \frac{1}{4} = 0.25 \times 100\% \\ &= 25\%\end{aligned}$$

(Ans)

(4) If a system absorbs 1100J heat and work is done 300J on the system, Find the internal energy of the system.

$\Rightarrow$  Here, Heat,  $Q = 1100\text{J}$

work done,  $w = 300\text{J}$

Internal energy,  $U = ?$

We know that,

$$\begin{aligned}U &= Q - w \\ &= (1100 - 300)\text{J} \\ &= 800\text{J}\end{aligned}$$

(Ans)