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MD IFTAKHAR KABIR SAKUR

25th BATCH

COMPUTER AND COMMUNICATION ENGINEERING

International Islamic University Chittagong

COURSE CODE: CCE-2305

COURSE TITLE: Signals and Linear Systems

COURSE TEACHER:

CCE-~~20~~ 2305

Communication

What is Signal?

A signal is an electromagnetic or electrical current that carries data from one system or network to another.

The signal is said to be one-dimensional. A speech signal is an example of a one-dimensional signal whose amplitude varies with time, depending on the spoken word and who speaks it.

When the function depends on two or more variables, the signal is said to be multidimensional.

For example:-

An Image is an example of two dimensional. with the horizontal & vertical coordinates of the image representing the two dimensions.

[Elaborated after 12 pages]

What is a System?

There is always a system associated with the generation of each signal and another system associated with the extraction of information from the signal.

For example:- In speech communication, a sound source or signal excites the vocal tract, which represents a system. The processing of speech signal usually relies on the use of our ears and auditory pathways in the brain. The systems responsible for the production and reception of signals are biological in nature.

They could also be performed using electronic systems that try to emulate or mimic their biological counterparts. For example, the processing of a speech signal may be performed by an automatic speech recognition system in the form of a computer program that recognizes words or phrases.

- There is no unique purpose for a system. Rather the purpose depends on the application of interest.

=) In a communication system, the function of system is to transport the information content of a message signal over a communication channel and deliver it to a destination in a reliable fashion.

For example:- In an automatic speaker recognition system, the function of the system is to extract information from an incoming speech signal for the purpose of recognizing or ~~identifyd~~ identifying the speaker.

- System:- A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals. The interaction between a system and its associated signals is illustrated schematically

→ In an automatic speaker recognition system, the input signal is a speech (voice) signal, the system is a computer, and the output signal is the identity of the speaker.

→ In a communication system, the input signal could be speech signal or computer data. The system itself is made up of the combination of a transmitter, channel, ~~data~~ receiver. And output signal is an estimate of the original message signal.

→ In an aircraft landing system, the input signal is the desired position of the aircraft relative to the runway. The system is the aircraft, and the output signal is a correction to the lateral position of the aircraft.

Overview of specific Systems:-

Input signal → System → Output signal

Message:

Message signal → Transmitter → Transmitted signal → Channel → Received signal → Receiver → Estimate of a Message signal

Transmitter:- The transmitter changes the message signal into a form suitable for transmission over the channel.

Receiver:- The receiver pocesses the channel output to produce an estimate of the message signal.

Communication System

Transmitter is located at one point in space, the receiver is located at some other point in space.

The receiver is located at some other point separate from the transmitter, and the channel is the physical medium that connects them together.

* The transmitter is to convert the message signal produced by a source of information into a form suitable for transmission over the channel.

* The message signal could be a speech signal, television (video) signal, or computer data.

* The channel may be an optical ~~Fir~~ Fiber, Coaxial cable, satellite channel, or mobile radio channel.

=) ট্রান্সমিটের সিগনালে ~~চ্যা~~ channel এর ভিতর দিয়ে আসে। ফলে সেই আসল সিগনালে physical characteristics নষ্ট হয়ে যায়। তাছাড়া, আওয়াজও বহি: সিগনাল এই আসল signal এর সঙ্গে মিশে যায় ও channel ouput কে নষ্ট করে ফেলে। এর ফলে যেই signal টা receive করা হয় সেটা হয় Corrupted version এই transmitted signal এর। Receiver এর ~~ক~~ function হলো এই Contaminate signal কে reconstruct ক

করে একটি পরিচিত Form এ নিয়ে ~~আসা~~, আসে এক যথাযোগ্য স্থানে deliver করে দেয়।

Receiver এর signal-processing role transmitter হতে সম্পূর্ণ বিপরীত ~~ভিন্ন~~। আরও বলতে গেলে, Receiver Channel এর effect এর বিপরীত কাজটা করে।

The communication system can be of an analog or digital type. In signal processing terms, the design of an analog communication system is relatively simple. Specially, the transmitter consists of a modulator. And a receiver consists of a demodulator.

[Modulation is the process of converting the message signal in a form that is compatible with the transmission characteristics of the channel.]

Odd & Even Signal

$\boxed{1}$ $x(t) = 1+t+3t^2+5t^3+9t^4$ ——— (1)

$\rightarrow x(t) = 1+t+3t^2+5t^3+9t^4$ ——— (2)

$\rightarrow x(-t) = 1-t+3(-t)^2+5(-t)^3+9(-t)^4$

$= 1-t+3t^2 - 5t^3+9t^4$ ——— (3)

উপরের (1) নং সমীকরণে t এর মান ধনাত্মক বসালে একটি মান পাওয়া যায়। যা (২) নং সমীকরণে দেওয়া হয়েছে।

আবার, (t) এর মান ঋনাত্মক বসালে আরেকটি মান পাওয়া যায় যা (৩) নং সমীকরণে দেওয়া হয়েছে।

Even মান (জোড় মান) ঃ-

$$x_{even}(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$= \frac{1}{2}\{1+t+3t^2+5t^3+9t^4+1-t+3t^2-5t^3+9t^4\}$$

$$= \frac{1}{2}\{2+6t^2+18t^4\}$$

$= \frac{1}{2} \times 2 \{1 + 3t^2 + 9t^4\}$

$= \{1 + 3t^2 + 9t^4\}$

Odd মান:-

আবার, (2) হতে (3) নং সমীকরণ বিয়োগ করে Odd মান পাওয়া যায়।

$x(t) = \frac{1}{2}\{x(t) - x(-t)\}$

$= \frac{1}{2}\{1 + t + 3t^2 + 5t^3 + 9t^4 - 1 + t - 3t^2 + 5t^3 - 9(t)^4\}$

$= t + 5t^3$

(Ans)

2) $x(t) = 1 + t\cos t + t^2 \sin t + t^3 \sin t \cos t$ —— ①

$\Rightarrow x(t) = 1 + t\cos t + t^2 \sin t + t^3 \sin t \cdot \cos t$ —— ②

$\Rightarrow x(-t) = 1 + (-t)\cdot\cos(-t) + (-t)^2 \sin(-t) + (-t)^3 \cdot \sin(-t) \cdot \cos(-t)$ ---- ③

$= 1 - t\cos t - t^2 \sin t + t^3 \sin t \cdot \cos t$ ---- ③

even:- , ② & ③ যোগ করে পাই,

$\Rightarrow \frac{1}{2}\{1 + t\cos t + t^2 \sin t + t^3 \sin t \cdot \cos t + 1 - t\cos t - t^2 \sin t + t^3 \sin t \cdot \cos t\}$

$\Rightarrow$ ~~...~~ $\frac{1}{2}\left\{2+2t^3 \sin t \cdot \cos t\right\}$

$\Rightarrow 1+t^3 \sin t \cdot \cos t.$

Again,

Odd ~~Numb~~

$$x(t)=\frac{1}{2}\left\{x(t)-x(-t)\right\}$$

$$t^2 \sin t$$

$$=\frac{1}{2}\left\{1+t\cos t + \cancel{t^2\cos t}\ \cancel{t^2\cos t} + t^3\cos t \cdot \sin t - 1 + t\cos t + t^2 \sin t - t^3 \sin t \cdot \cos t\right\}$$

$$=t\cos t + t^2 \sin t$$

(Ans)

Wave দেখে Odd or even নির্ণয়

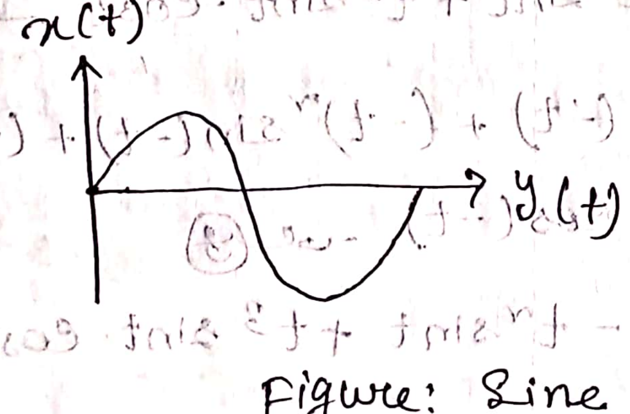


Figure: Sine wave

=> Here, the figure is about sine wave

$x(t) = \sin t$ —— (1)

$x(-t) = \sin(-t) = -\sin t$ —— (2)

=> এখানে, sint তে ধনাত্মক মান বসালে ধনাত্মক হয় এবং ঋণাত্মক মান বসালে ঋণাত্মক হয়। তাহলে sin(t) একটি odd signal।

(Ans)

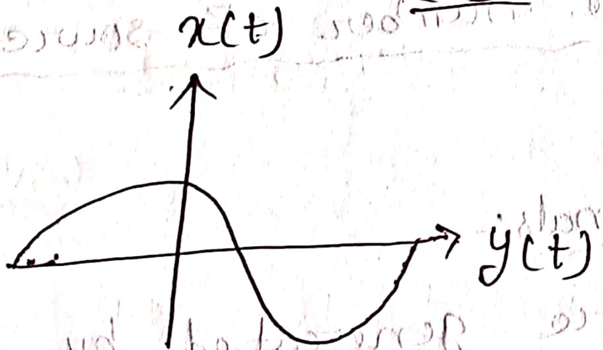


Figure:- Cos wave

=> উপরের চিত্রটি Cos wave এর চিত্র। সুতরাং,

$x(t) = \cos t$ —— (1)

$\Rightarrow x(-t) = \cos(-t)$ —— (2)

$= \cos t$

কোনো সিগনালের Odd বা even হওয়ার কারণ আমরা (t) এর মানের মাধ্যমে জানতে পারি। Cos(t)

তে t এর ধনাত্মক মান বসালে ~~চিহ্নের পরিবর্তন হয়ে~~ ধনাত্মক হয়। এবং ঋনাত্মক মান বসালে চিহ্নের পরিবর্তন হয়ে ~~ঋনাত্মক হয়~~ ধনাত্মক হয়।

তাহলে, cos(t) একটি even signal।

Ans

Signal Classification

Signal can be classified in number of ways.

1) Depending on the number of sources of the signals:-

(i) One channel Signals:-

Signals that are generated by single source are called one channel signals.

Example:- Record of room temperature with respect to time, the audio speaker of a mono speaker etc.

(ii) Multi Channel Signal :-

Signals that are generated by multiple sources are called multi channel signals.

Example:- The audio output of the two stereo speakers, The record of ECG at eight different places in a human body (eight channel signal).

☐ Depending on the number of dependent variables:

(i) one Dimensional Signals:-

A signal which is a function of single independent variable is called one dimensional signal.

Example:- music, speech, heart beat etc.

(ii) Multi-dimensional Signals:-

A signal which is a function of two or more independent variable is called multidimensional signal.

example:-

A photograph, ~~A~~ the motion picture of a black and white TV ~~is~~ ~~on~~

□ Depending on whether the dependent variable is continous or discrete:-

(i) Analog or continuous signals:-

When a signal is defined continuously for any value of independent variable, it is called analog or continuous signal.

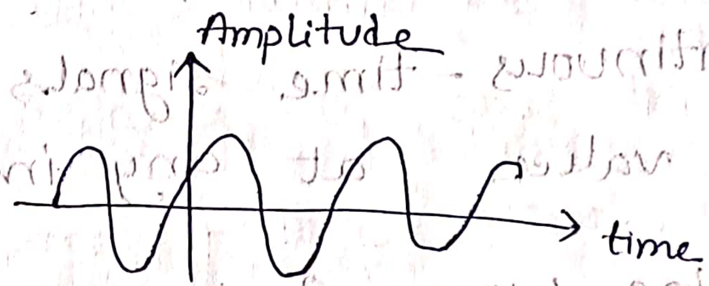
(ii) Discrete signals:-

When a signal is defined for discrete intervals of independent variables, it is called discrete signals.

Most of the discrete signals are either sampled version of analog signals or output of digital systems.

Continuous time & Discrete Time Signals

Continuous Time Signals:-



⇒ A signal is said to be continuous when it is defined for all instants ~~to~~ of time.

⇒ A signal $x(t)$ is said to be a continuous time signal if it is defined for all time ~~to~~ (t).

⇒ A continuous time signal is a function that is continuous, meaning there are no breaks in the signal. For all real value of (t) ~~to~~ you will get a value.

⇒ Can be represented at any instant of the time in its sequence.

⇒ It is the "function of continuous - time variable that has uncountable or infinite set of numbers in its sequence."

=> It is also termed as analog signal.

=> It is a Continuous Function of time defined on the real line (or axis) R.

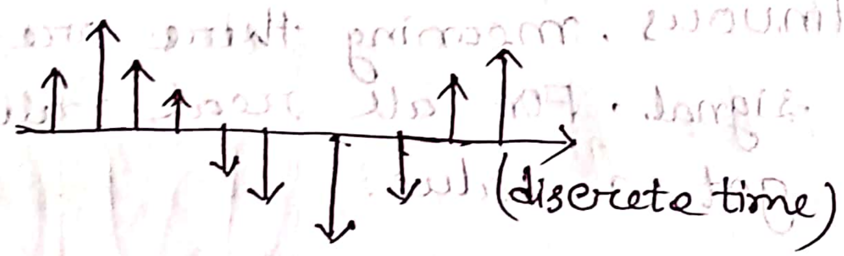
=> ~~has~~ Has continuous amplitude & time.

=> The continuous - time signals will have certain value at any instant of time.

Example:- Sine waves, Cosine waves, temperature & Triangular waves.

Physical parameter:- pressure, temperature, sound and so on.

Discrete time Signal :-



=> A signal is said to be discrete when it is defined at only discrete instant of time.

=> This signal is the "Function of discrete-time variable that has countable or

Finite set OF numbers in its sequence".

It is the digital representation OF Continuous-time signal.

=> It can be represented and defined at certain instants OF time in it sequence.

=> The discrete-time signal is able to define only at the sampling instants.

=> Digital signal can be obtained From the discrete-time signal by quantizing and encoding the sample values.

=> The discrete-time signals are represented with binary bits and stored on the digital medium.

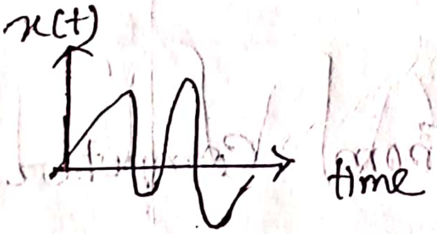
Example!- Output From Computer

Digital Signal :-

Signal which is discrete both in time ~~am~~ and amplitude. In digital signals the value of signal for every discrete time "n" presented by binary codes.

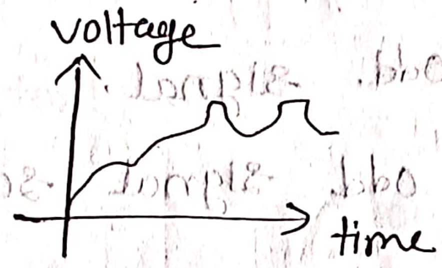
The process of conversion of a discrete time signal to digital signal involves quantization and coding.

Deterministic Signals :-



⇒ A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical Formula are known as deterministic signals.

Non-deterministic Signals:-



A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time.

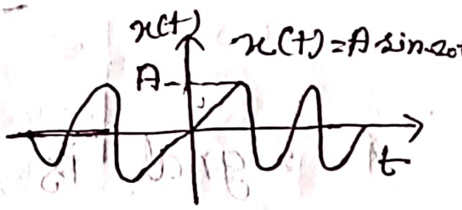
Non-deterministic signals are random in nature. Hence they are called random signals.

Random signals can't be described by a mathematical equation. They are modelled in probabilistic terms.

Odd & Even Signals

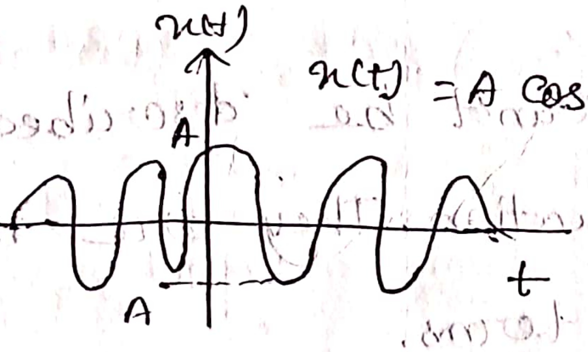
Odd Signals:- When a signal exhibits the antisymmetric with respect to $t=0$, then it is called an odd signal.

Therefore, the odd signal satisfies the condition, $x(-t) = -x(t)$.



even Signals:-

When a signal exhibits the symmetry with respect to $t=0$, then it is called an even signal. Therefore the even signal satisfies the condition, $x(-t) = x(t)$



Amplitude Scaling :-

The amplitude scaling is performed by multiplying the amplitude of the signal by a constant.

$x(t)$ ~~be~~ is a continuous time signal. And $Ax(t)$ is the amplitude scaled version of ~~ext~~ $x(t)$, where A is a constant.

Elementary Signals

Exponential Signals :-

(2) $\to x(t) = A_0 e^{st}$

For simplicity, $A_0 = 1$

So, $x(t) = e^{st}$

s = complex number $= \underbrace{\sigma}_{\text{Real part}} + \underbrace{j\omega}_{\text{Imaginary part}}$

So, $x(t) = e^{(\sigma + j\omega)t}$

From Euler's Formula,

$$e^{ix} = \cos x + i \sin x$$

So, $x(t) = e^{(\sigma + j\omega)t}$

$$x(t) = e^{\sigma t} \cdot e^{j\omega t}$$

Signal, $x(t) = e^{\sigma t} \cdot (\cos\omega t + j \sin\omega t)$

We have two cases depending on omega,

Case 1 :- $\omega = 0$

$$x(t) = e^{\sigma t} \cdot (1 + j \cdot 0)$$

$$x(t) = e^{\sigma t}$$

This sig $x(t)$ is called as real exponential signal.

Depending on sigma (in Real exp.) we have two cases:-

1| $\sigma > 0$ (σ is (+) positive)

2| $\sigma < 0$ (σ is (−) negative)

Case 1: $\omega \neq 0$

On this we have three cases,

1| $\sigma = 0$

Signal, $x(t) = \cos\omega t + j\sin\omega t$

2| $\sigma > 0$

$x(t) = e^{\sigma t}(\cos\omega t + j\sin\omega t)$

$\underset{\text{Real}}{\underline{Re}}(x(1)) = e^{\sigma t} \cdot \cos\omega t$

$Im(x\,(w)) = e^{\sigma t} \cdot \sin\omega t$

3| $\sigma < 0$

Sinusoidal Signal:-

$$x(t) = A\cos(\omega_0 t \pm \phi)$$

$$\text{or, } A\sin(\omega_0 t \pm \phi)$$

$\omega_0 \rightarrow$ Fundamental Frequency

$\phi \rightarrow$ Phase shift

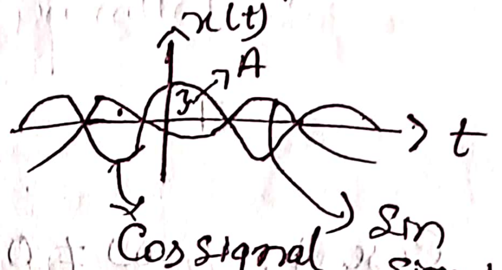
$t \rightarrow$ Time

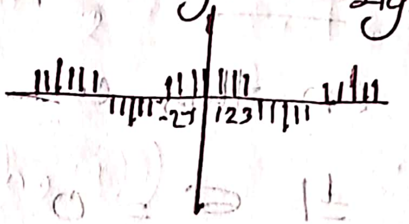
$A \rightarrow$ Amplitude

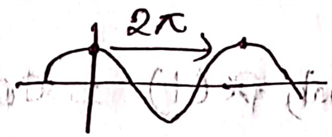
$$\omega_0 = 2\pi F$$

$$F = \frac{\cancel{2\pi}\,\omega_0}{2\pi}$$

$$T = 1/F = \frac{2\pi}{\omega_0}$$

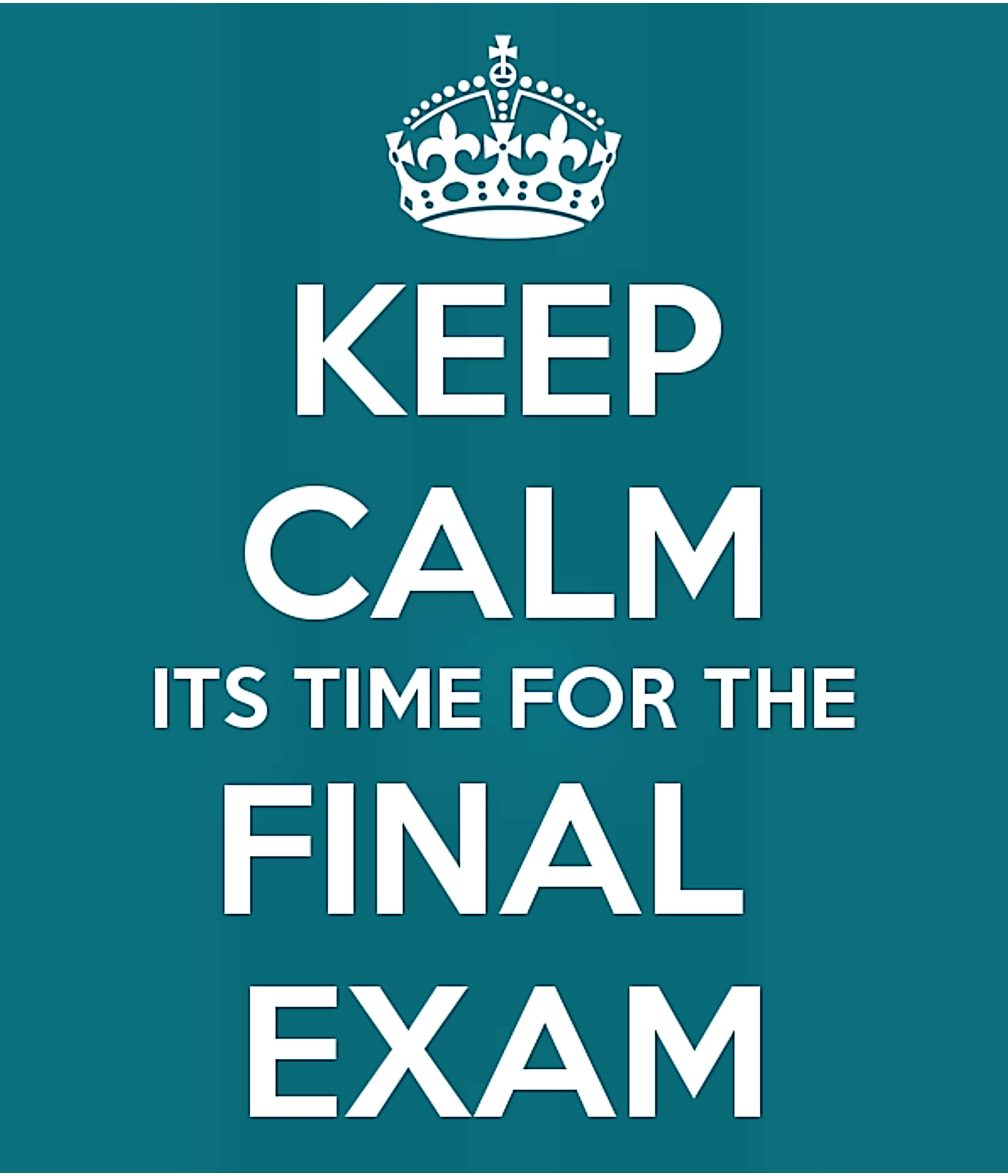






$$T = 2\pi$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$


KEEP
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ITS TIME FOR THE
FINAL
EXAM

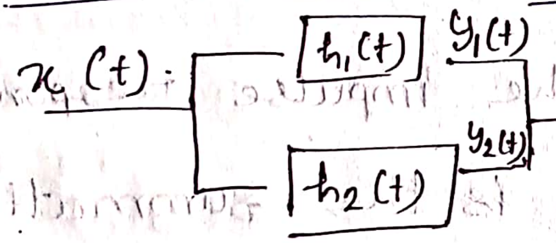
Interconnection of LTI Systems

→ To make something more advance we connect several machines or component in signal processing it's the same processing.

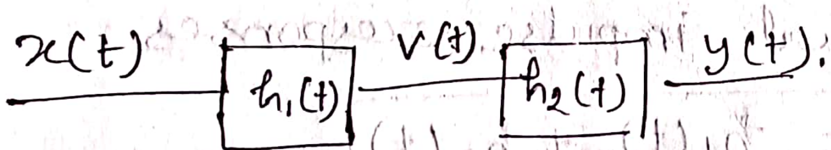
There are two common interconnection:-

1| parallel : Here we have two or more systems connected to each other.

2| Cascade or Series :- Here we have two or more system chained back to back.



→ parallel



→ Series

~~Diagr.~~ Figure:-

✱ To find Impulse response from several blocks always start from the output & try to find their relation from input & output.

* LTI Systems → output is basically input convolved with the impulse response the input for the top and bottom branches is the same.

* Go back to the Figure OF "parallel" :-

$$y(t) = y_1(t) + y_2(t)$$

$$\Rightarrow y(t) = x(t) * h_1(t) + x(t) * h_2(t) \quad \left[\text{See LTI System}\right]$$

$$\Rightarrow y(t) = x(t) * \left(h_1(t) + h_2(t)\right)$$

* For parallel structure the impulse response for the whole system, is the summation of all individual impulse responses

$$\Rightarrow \boxed{h(t) = h_1(t) + h_2(t)}$$

Scanned with CamScanner CamScanner

~~Cas~~ Cascade

Or, Series :- (Find the impulse response)

[See Figure of Series]

- Start from output find a relation from output to input.

- Here, $y(t)$ basically the input to the second system, calling it as $v(t)$.

So, $y(t) = v(t) * h_2(t)$

$v(t)$:- It is the output from LTI system so, replace $v(t)$ by $x(t)$ convolved by $h_1(t)$

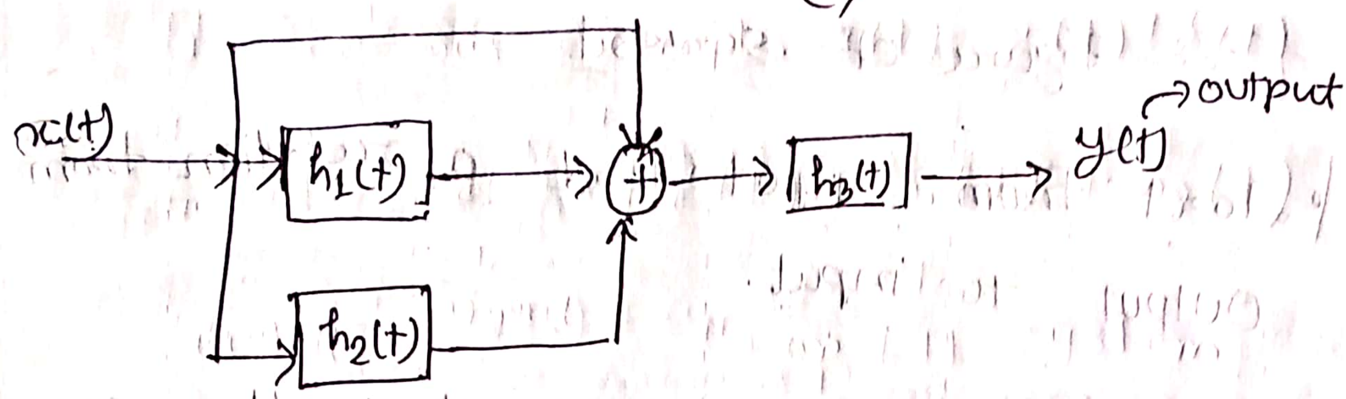
So, $y(t) = x(t) * \underbrace{h_1(t) * h_2(t)}$

Impulse response of the whole system.

⊛ · So, for a series structure the impulse response for the whole system is the convolution of all individual impulse responses.

$$\Rightarrow \boxed{h(t) = h_1(t) * h_2(t)}$$

Example :- (what is the impulsion of it)



Let's call the signal as v(t)

and $y(t) = v(t) * h_3(t)$

$v(t) = x(t) + (x(t) * h_1(t) + x(t) * h_2(t))$ — (1)

=) Here a convolution missing from the first term. so we can't factor out x(t) convolved to

So a trick must be used :-

Remark :-

$x(t) * \delta(t - t_0) = x(t - t_0)$

=) By convolving x(t) with a shifted delta function we get the shifted version of x(t) from

$x(t) * \delta(t) = x(t)$ — (2)

So, writing (2) in (1)

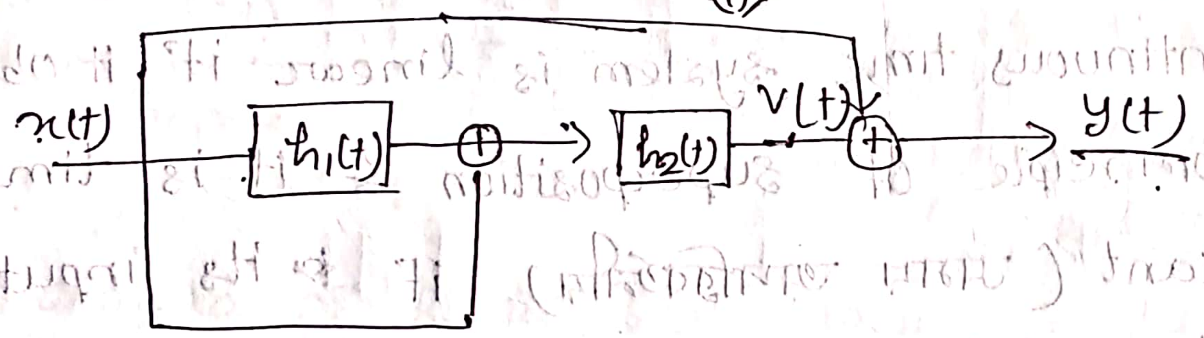
$v(t) = x(t) * \delta(t) + x(t) * h_1(t) + x(t) * h_2(t)$

Now, we have a common factor,

$$\Rightarrow v(t) = x(t) * [\delta(t) + h_1(t) + h_2(t)]$$

So, $\underbrace{y(t)}_{\text{Output}} = \underbrace{x(t)}_{\text{input}} * \underbrace{[\delta(t) + h_1(t) + h_2(t)] * h_3(t)}_{\text{(Impulse response of the whole system)}}$

<u>Example (2):</u>– (what is the impulsion of it)



Here

$$y(t) = x(t) + v(t) \quad \text{—(1)} \quad \cancel{v(t)}$$

Here,

$$v(t) = z(t) * h_2(t) \quad \text{—(2)}$$

| $z(t)$ = Summation of two branches
| So, $z(t) = x(t) * h_1(t) + x(t)$ —(3)

using (3) in (2)

$$\text{So, } v(t) = \left(x(t) * h_1(t) + x(t)\right) + h_2(t)$$

$$\Rightarrow v(t) = x(t) * h_1(t) * h_2(t) + x(t) * h_2(t) \quad \text{—(4)}$$

using eqn (4) in (1)

$y(t) = x(t) + x(t) * h_1(t) * h_2(t) \cdot x(t)$

So, like previous example $x(t) = x(t) * \delta(t)$

So, $\underbrace{y(t)}_{\text{output}} = \underbrace{x(t)}_{\text{input}} * [\underbrace{\delta(t) + h_1(t) + h_2(t)}_{\text{For LTI system, this is the impulsion}}]$

LTI System

A continuous time system is linear if it obeys the principle of superposition & it is time invariant (সময় অপরিবর্তনীয়) if its input-output relationship does not change with time.

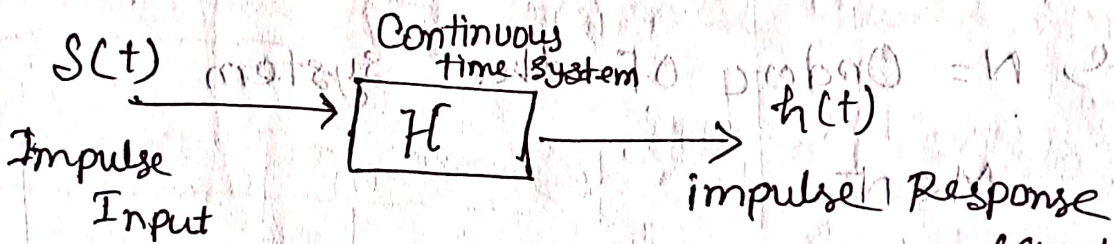
When a continuous time system satisfies the properties of linearity (রৈখিকতা) and time invariance then it is called as LTI (Linear Time Invariant system).

Impulse ~~Rets~~ Response:-

(যেমন Continuous Time System এর Input $\delta(t)$ হলে...)

$\Rightarrow$ When the input to a continuous time system is a unit impulse signal $\delta(t)$ then the output is called an impulse response of the system & it is denoted by $h(t)$.

$\therefore$ Impulse Response, $h(t) = H\{\delta(t)\}$



Pdf (Signal And Sys.. → (A Nagoon Kani) page-58)

Formula of RL, RC, RLC circuit & the mathematical equation governing them:-

① RL circuit:- $\frac{d\,y(t)}{d(t)} + \cancel{L \cdot \frac{d}{dt}}\ \frac{R}{L} \cdot y(t) = \frac{1}{L} \cdot x(t)$

② RC circuit:- $\frac{d \cdot y(t)}{dt} + \frac{1}{RC} \cdot y(t) = \frac{1}{R} \cdot \frac{d\,x(t)}{dt}$

③ RLC circuit:- $\frac{d^2 y(t)}{dt^2} + \frac{R}{L} \frac{d\,y(t)}{d(t)} + \frac{1}{LC}\, y(t) = \frac{1}{L} \frac{d\,x(t)}{dt}$

The relation of input-output of an LTI System:-

$$a_0 \frac{d^N}{dt^N} \cdot y(t) + a_1 \frac{d^{N-1}}{dt^{N-1}} \cdot y(t) + a_2 \frac{d^{N-2}}{dt^{N-2}} \cdot y(t) + \dots$$

$$+ a_{N-1} \frac{d}{dt} y(t) + a_N y(t)$$

$$= b_0 \frac{d^M}{dt^M} \cdot x(t) + b_1 \frac{d^{(M-1)}}{dt^{(M-1)}} \cdot x(t) + b_2 \frac{d^{M-2}}{dt^{M-2}} \cdot x(t)$$

$$+ \dots + b_{M-1} \cdot \frac{d}{dt} x(t) + b_M x(t)$$

Here, N = Order of the system

$M \leq N$

$a_0 = 1$

Interconnection OF LTI System

① ~~CasCade~~ interconnection :-

⊛ LTI Systems and their properties :-

in term of impulse Response :-

1) Memory less :-

(i) Discrete Time Signal (DTS) : impulse Response, $h(n)=0 ; n \neq 0$

(ii) Continuous Time Signal (CTS) " " $\boxed{h(t)=0 ; t \neq 0}$

[2] Casual :-

(i) DTS :- $h(n) = 0 ; \quad n<0$ [For n value less than zero]

(ii) CTS :- $h(t) = 0; \quad t<0$ [F t value less than zero]

3) Stability :-

(i) DTS :- $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$ [Only if the impulse response is absolutely summable]

(ii) CTS :- $\int_{-\infty}^{\infty} |h(z)| \, dz < \infty$ [Only if its ~~simple~~ impulse response is absolutely integreatable]

Problem Solving of Previous Lesson

① Problems:–

$h(n) = \left(\frac{1}{2}\right)^n u(n)$? Stable ? Casual ?

Sol Stable:–

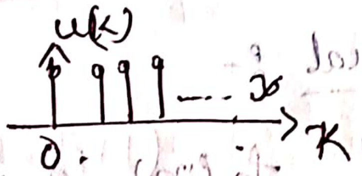
$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty \quad \text{—①}$$

$$\Rightarrow h(k) = \left(\frac{1}{2}\right)^k u(k) \quad \text{—②}$$

② কে ① এ বসাই,

$$\sum_{k=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^k u(k)\right|$$

$1 \rightarrow k = 0$ to ∞



$$\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \cdot 1 = \frac{1}{1-\frac{1}{2}} \quad \Bigg| \quad \sum_{n=0}^{N} a^n = \frac{1}{1-a}$$

$$= 2 < \infty \rightarrow \text{Stable}$$

Casual:

$h(n) = \left(\frac{1}{2}\right)^n u(n) \rightarrow 1;\ n = 0$ to ∞

$0;\ n < 0$

For $n < 0$; $h(n) = \left(\frac{1}{2}\right)^n \times 0$

$h(n) = 0$

For $n<0$; $h(n)=0$ → The given system is Casual system

Convolution

Topics :-

(1) Convolution
(2) Discrete or linear convolution
(3) Procedure for evacuating linear Convolution
(4) Methods of performing linear Convolution

— o —

Convolution:- A convolution is an integral that expresses the amount of overlap of one function when it is shifted over another function.

It is also a mathematical tool, used to calculate the output of an LTI system when the impulse response & input is available.

There are five Important Options in this course:-

(1) Shifting (Both amplitude & time shifting)
(2) Scaling → (" " " Scaling)
→ (" " " reversal)

3] Differentiation

4] Integration

5] Convolution

Example of Convolution:-

Output, $y(t)$ = [Convolution] $h(t)$ [OF LTI System] $*$ $x(t)$ [Input of LTI System]

[Convolution operator]

$$\boxed{= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau} \quad \text{---(1)}$$

$(\tau \rightarrow \text{Tau})$

Explanation:-

→ we have two signals ⇒ $x(t)$ & $h(t)$.

→ we will have to replace ⇒ $t=\tau$ $t=\tau$
τ in T's place   ⇒ (1)   ⇒ (2)
$x(\tau)$ $h(\tau)$

→ we need to fix one signal to remain on its own position. ⇒ $x(\tau)$ ⇒ will be on its own position.

→ Another signal will move by time reversal operation ⇒ $h(\tau)$ ⇒ (2) (3)

Time reversal operation (scaling)

$h(-\tau)$

→ Now the movable signal will perform the time shifting operation

↓ Time shifting

$\Rightarrow h[-(\tau - t)]$

Here the ~~variable~~

Time Rev. & Time shifting operation works only against $t = \tau$ not against minus tau.

So, $h[-(\tau - t)] = h(t-\tau)$

So, Now multiplying with another signal,

$\Rightarrow$ 4

$x(\tau) \cdot h(t-\tau)$

$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \, d\tau$ $\Rightarrow$ 5

These are the 5 steps related with Convolution operation

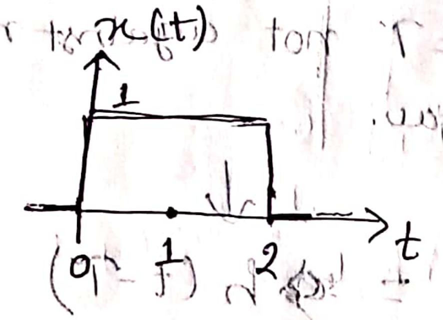
Example:-

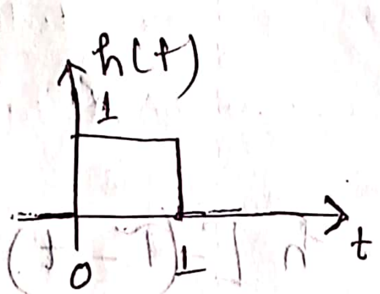
$x(t) \leftarrow$ input of LTI system

$h(t) \leftarrow$ impulse respons of LTI S.

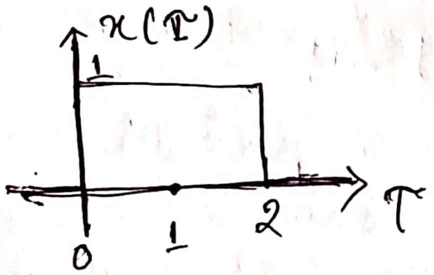
$y(t) = ?$

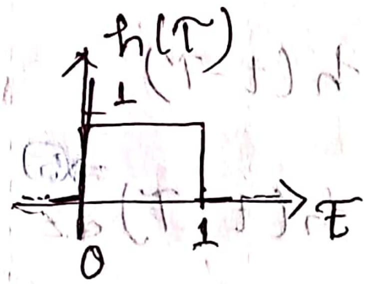
Solve:- wave form :-



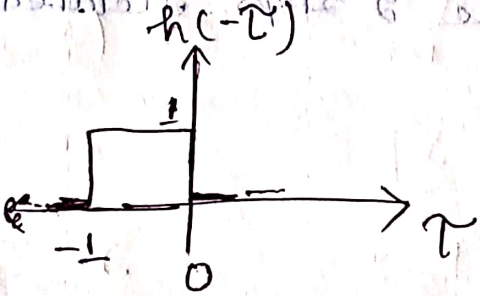


$\Rightarrow$ Replacing t with τ



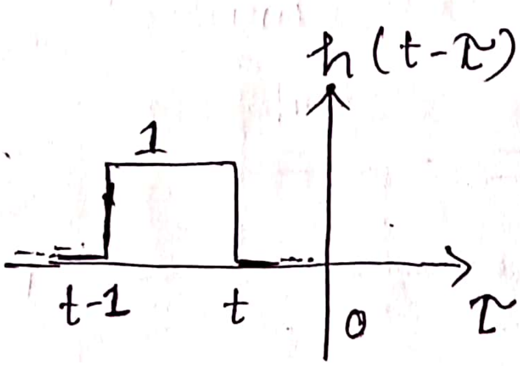


~~Time shifting or~~ Time reverse wave form



$\downarrow$

↓ Time shifting



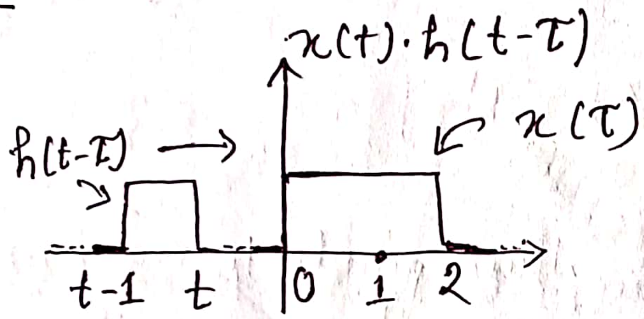
$x(\tau)\cdot h(t-\tau)$

$y(t) = \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\,dt$

$y(t)=0$ when $t<0$

↓ Step ④

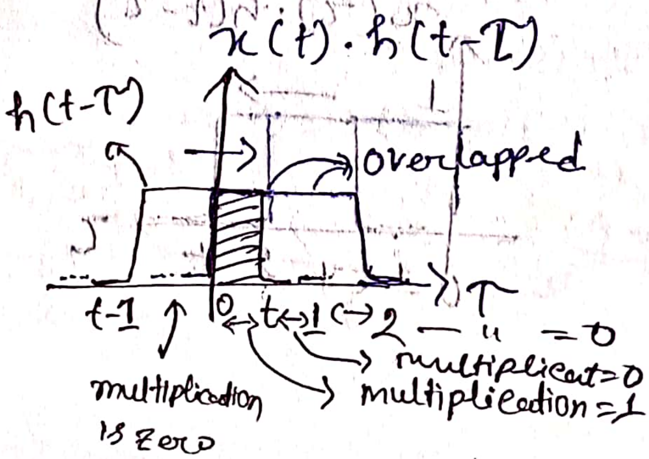
<u>Case 1:-</u>



(*) In case-1 time $\boxed{t<0}$. So there is no overlap between the two signal $h(t-\tau)$ & $x(\tau)$. As there is no overlap the multiplication is going to be 0.

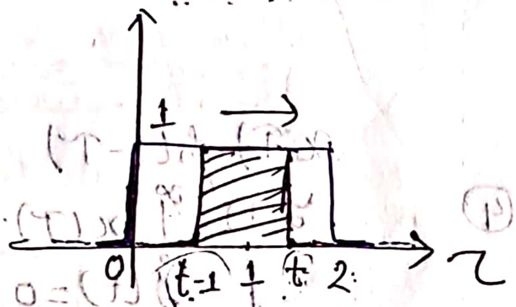
<u>Case - 2:-</u> $\boxed{0<t<1}$

So, her 0 to t is only non zero.



$$y(t) = \int_0^t 1\, d\tau = [\tau]_0^t = t$$

Case 3:- ~~t>1~~ $1 < t < 2$



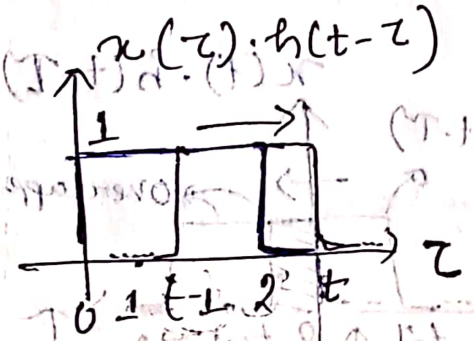
$$y(t) = \int_{t-1}^{t} 1\, d\tau$$

$$= [\tau]_{t-1}^{t}$$

$$= t - t + 1$$

$$= 1$$

Case 4:- $2 < t < 3$



$$y(t) = \int_{t-1}^{2} 1\, d\tau$$

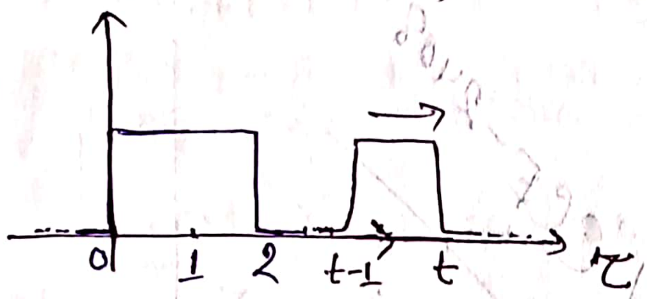
$$= [\tau]_{t-1}^{2}$$

$$= 2 - t + 1$$

$$= 3 - t$$

Case 5:- $t > 3$

$x(\tau) \cdot h(t-\tau)$



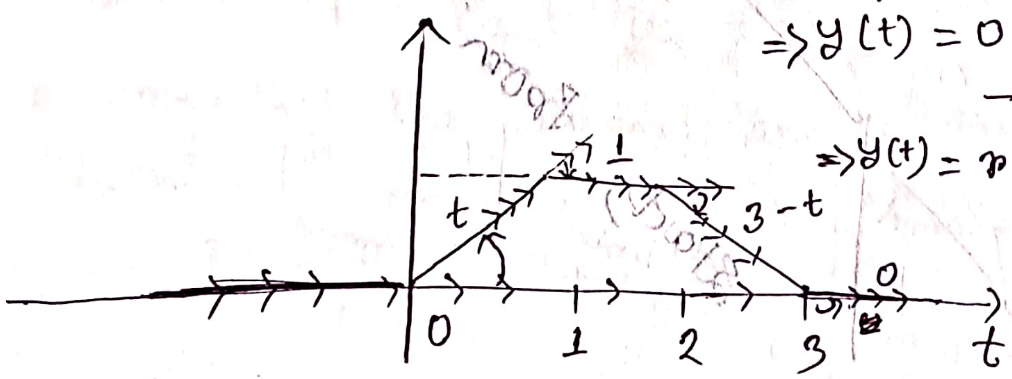
No overlaping

so, $y(t) = 0$

so, output, $y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & t > 3 \end{cases}$

(Ans)

Expression of wave form:-



RAM signal

$\Rightarrow y(t) = 0 + 1 \cdot r(t-0) - 1 \cdot r(t-1) - 1 \cdot r(t-2) + 1 \cdot r(t-3)$

$\Rightarrow y(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$

(Ans)