



Page NO.

1. 1 to 33 – Mid Term
2. 34 to 60- Final Term

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COURSE TITLE: Digital Signal Processing

COURSE TEACHER:

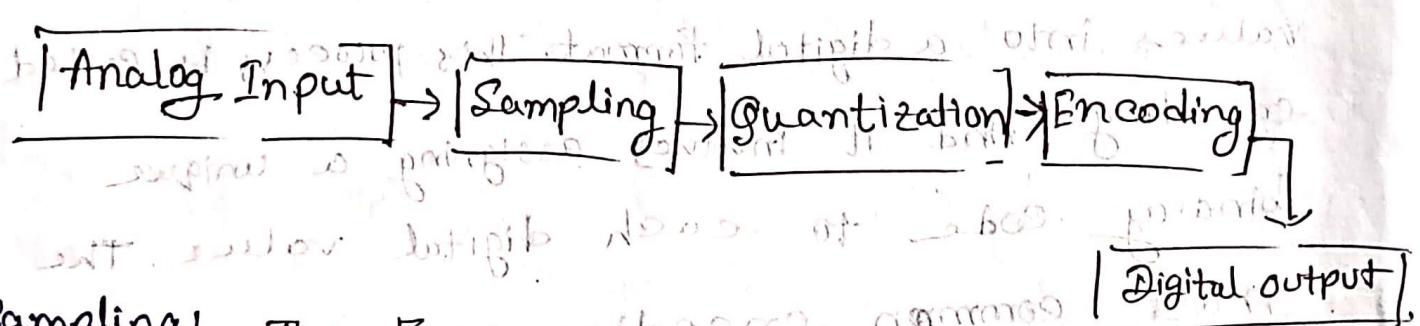
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Q Explain the analog to digital signal conversion process with proper diagram and motive the answer briefly.

⇒ Analog to Digital Conversion (ADC) is the process of converting continuous analog signals into discrete digital signals. The main motive behind this conversion is to facilitate the processing, storage and transmission of signals by digital devices such as computers, smartphones, and digital audio players.

The diagram of ADC process:



Sampling: The first step of ADC is to take samples of the continuous analog signal at fixed time intervals. The process is known as Sampling.

And it also involves measuring the amplitude of the signal at regular intervals.

The sampling rate is expressed in Hertz (Hz) which is the number of samples per second.

Quantization :-

Now, Assigning digital value to each sample. This process is called "Quantization", and it involves rounding the analog value to the nearest digital value. The number of digital values available is determined by the number of bits used for encoding.

For example, if we use 8 bits for encoding we can represent $2^8 = 256$ different values.

Encoding :-

This is the final step to convert the quantized values into a digital format. This process is called encoding, and it involves assigning a unique binary code to each digital value. The most common encoding scheme is binary. Hence each value is represented by a sequence of 0s & 1s.

The accuracy of ADC depends on the sampling rate, the number of bits used for encoding, and the quality of the quantization process. Because of ADC the signals can be manipulated by computers, and other digital devices.

B1(iii) Analog signal $x(t) = 3 \cos 50\pi t$ determine the maximum sampling rate required to avoid aliasing. Suppose that the signal is sampled at the rate $F_s = 25 \text{ Hz}$. What is the discrete time signal obtained after sampling and justify your answer briefly.

~~The maximum sampling rate required to avoid aliasing is the Nyquist-Shannon sampling which is $F_s = 2F_m = 2 \cdot 25 = 50 \text{ Hz}$~~

~~$\therefore F_s = 2 \times 25 = 50 \text{ Hz}$~~

~~Here the signal $x(t) = 3 \cos(50\pi t)$~~

~~\Rightarrow The frequency of the signal $x(t) = 3 \cos(50\pi t)$~~

~~The frequency of the cosine function is 50 Hz .~~

According to the Nyquist Sampling theorem the Sampling rate should be at least twice the maximum frequency component of the signal which is 50 Hz in this case. Therefore, the minimum Sampling rate required to avoid aliasing is 100 Hz .

$$f_s > 2f_{\text{max}} = 250 \text{ Hz}$$

Since the given sampling rate $f_s = 25 \text{ Hz}$. It is not sufficient to avoid aliasing. Aliasing will occur & the reconstructed signal will not accurately represent the original analog signal.

To determine the discrete-time signal obtained after sampling, we can use Pommelot's

$$x_n = x(nT) \quad \text{where} \quad T = 1/F$$

Substituting values $n = mT$ where $m = \text{an integer}$

$$\begin{aligned} x_n &= 3 \cos(50n) \times \delta(nT) \\ &= 3 \cos(2\pi(25)n) \quad [\because 50\pi = 2\pi(25)] \end{aligned}$$

$$\therefore x(n) = 3 \cos(2\pi n) \quad [\because T = 1/F = 1/25 = 0.04 \text{ s}]$$

∴ The resulting discrete-time signal

$$x(n) = 3 \cos(2\pi n)$$

It is a discrete-time cosine signal with a frequency of 0 Hz which is the same as the

DC component of the original analog signal. The high-frequency components of the signal have been lost due to the undersampling, resulting in a lower-frequency discrete-time signal that does not accurately represent the original analog signal. This

demonstrates the effects of aliasing.

- Q(a): Determine the response of the following system to the input signal $x(n)$.

$$x(n) = \begin{cases} 1, & -3 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) $y(n) = x(n-1)$ [unit delay]

(ii) $y(n) = x(n+1)$ [unit advanced system]

(iii) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$ [Moving average filter]

Solve:-

(a) $y(n) = x(n) = ?$ [Identity system]

Solution:- Here, $x(n) = \{-3, -2, -1, 0, 1, 2, 3\}$

In this case output is exactly the same as the input signal. Such system is known as identity system.

(ii)(b) $y(n) = x(n-1)$ [Unit delay system]

\Rightarrow It simply delay the system by one sample unit.

In this case $x(n-1)$ is obtained by shifting $x(n)$ to the right side by 1 unit.

~~$x(n) = \{\dots, 0, -3, -2, -1, 0, 1, 2, 3, 0, \dots\}$~~

(iii)(c) $y(n) = x(n+1)$ [Unit advanced system]

~~$x(n) = \{0, -3, -2, -1, 0, 1, 2, 3, 0, \dots\}$~~

In this case $x(n+1)$ is obtained by shifting $x(n)$ to the left side by 1 unit.

(iv)(d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$ [moving avg. filter]

2(b) (i) $y(n) = x(n) - x(n-1)$

is a causal equation because it describes a cause-and-effect relationship between the current sample $x(n)$ and the previous sample $x(n-1)$.

(ii) $y(n) = \sum_{k=-\infty}^n x(k)$

is a non causal equation because it does not describe a cause and effect relationship between variables.

(iii) $y(n) = x(n^2)$ is a non causal (Cause other change) in the n does not change.

$$\textcircled{V} \quad y(n) = n(2n)$$

\Rightarrow It is a non-causal because it does not describe a cause and effect relationship between variables.

$$\textcircled{V} \quad y(n) = n(n) + 3n(n+4)$$

\Rightarrow It is a causal equation because it describes a cause and effect relationship between the current $n(n)$ and a future sample $n(n+4)$.

$y(n)$ depends on $n(n)$ and $n(n+4)$ which is causal.

{
↳ non causal}

↳ causal example not causal

X-5.1 Inverse Z-transform (easy)

Chapter - 1

Consider the Analog signal:-

$$n_a(t) = 3 \cos 100\pi t$$

(a) Determine the minimum sampling rate required to avoid aliasing.

\Rightarrow

$$n_a(t) = 3 \cos 100\pi t \quad \text{and} \quad \omega = (100\pi) \text{ rad/sec}$$

$$\begin{aligned} \text{we know, } n_a(t) &= A \cos(2\pi f t + \theta) \\ &= 3 \cos(2\pi \cdot 50 \cdot t + \theta) \end{aligned}$$

$$\begin{aligned} \text{we know, } F_s &= 2 \cdot f_{\max} \\ F &= 50 \text{ Hz} \end{aligned}$$

$$\begin{aligned} F_s &= 2 \times 50 \text{ Hz} \\ &= 100 \text{ Hz} \end{aligned}$$

$$e^{j\omega_1 t} + e^{j\omega_2 t} + e^{j\omega_3 t} + \dots$$

b) The signal is sampled at the rate $F_s = 200 \text{ Hz}$. What is the discrete time signal obtained after sampling?

\Rightarrow Here, $F_s = 200 \text{ Hz}$

$$\therefore x(n) = 3 \cos \frac{100\pi}{200} \cdot n$$

$$= 3 \cos \frac{\pi}{2} \cdot n$$

c) Suppose that the signal is sampled at the rate, $F_s = 75 \text{ Hz}$. What is the discrete time signal obtained after sampling?

\Rightarrow Here, $F_s = 75 \text{ Hz}$

$$\therefore x(n) = 3 \cos \frac{100\pi}{75} n$$

$$= 3 \cos \frac{4\pi}{3} n$$

$$= 3 \cos \left(2\pi - \frac{2\pi}{3} \right) n$$

$$= 3 \cos \left(\frac{2\pi}{3} \right) n$$

(d) What is the frequency F or $f_{s/2}$ of sinusoid that yields samples identical to those obtained in part (c)?

\Rightarrow The sampling rate = 0.75 Hz

We know,

$$\begin{aligned} F &= \text{Sampling Frequency} * \text{Sampling Rate} \\ &\quad (\text{Frequency of sinusoid}) \\ &= f \times F_s \\ &= 75f \end{aligned}$$

From (c) we get the Frequency of sinusoid $f = \frac{1}{3}$

$$\begin{aligned} F &= 75 \times \frac{1}{3} \\ &= 25 \text{ Hz} \end{aligned}$$

Clearly, the sinusoidal signal,

$$\begin{aligned} y_{alt}(t) &= 3 \cos 2\pi F t \\ &= 3 \cos 2 \cdot 25 \cdot \pi t \\ &= 3 \cos 50\pi t \end{aligned}$$

$$3 \left(\frac{\pi s}{s} - \pi s \right) \cos \pi t$$

$$3 \cdot \left(\frac{\pi s}{s} \right) \cos \pi t$$

Q2 Consider the Analog signal,

$$x_a(t) = 3 \cos 50\pi t + 10 \sin 300\pi t + \cos 100\pi t$$

What is the Nyquist rate of this signal?

\Rightarrow The frequencies present in signal,

$$x_a(t) = 3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$$

$$f_1 = 25 \text{ Hz}, f_2 = 150 \text{ Hz}, f_3 = 50 \text{ Hz}$$

Here, $f_3 > 2f_m = 300 \text{ Hz}$

The Nyquist rate is, $F_N = 2f_{\max}$

$$\therefore F_N = 2 \times 150 = 300 \text{ Hz}$$

Q3 Consider the Analog signal,

$$x_a(t) = 3 \cos 2000\pi t + 5 \sin 6000\pi t + 10 \cos 12000\pi t$$

(i) What is the Nyquist rate for this signal?

(ii) Assume now that we sample this signal using a sampling rate $F_s = 5000 \text{ samples/s}$ what is

the discrete-time signal obtained after sampling.

$$\Rightarrow x_n = 3 \cos(2\pi f_a t_p) - \frac{5}{2} \cdot 3 \sin(2\pi f_a t_p) + 10 \cos\left(\frac{1}{2}\pi + 2\pi f_a t_p\right)$$

P.T.O

Solution

Q3(i) Here,

$$x_a(t) = 3 \cos 2000\pi t + 5 \sin 6000\pi t + 10 \cos 12000\pi t$$

Here,

$$F_1 = 1000 \text{ Hz} \quad F_2 = 3000 \text{ Hz} \quad F_3 = 6000 \text{ Hz}$$

$$F_{\text{man}} = 6000 \text{ Hz}$$

For Nyquist rate we know,

$$F_3 > 2F_{\text{man}} = 2 \cdot 6000 = 12000 \text{ Hz}$$

$$\therefore \text{Nyquist rate} = 2 \cdot 6000$$

$$= 12000 \text{ Hz}$$

$$f_s = 12 \text{ kHz}$$

Q3(ii)

Here, $F_s = 5000 \text{ samples/s} = 5000 \text{ Hz}$

Here, the analog signal, we know, $x(n) = x_a(nT) = x_a(\frac{n}{f_s})$

$$x_a(t) = 3 \cos 2000\pi t + 5 \sin 6000\pi t + 10 \cos 12000\pi t$$

So, Sampled signal, $x(n) = ?$ after problem.

$$x_a(t) = 3 \cos 2 \cdot \frac{1000}{5000} \pi t + 5 \sin 2 \cdot \frac{3000}{5000} \pi t + 10 \cos 2 \cdot \frac{6000}{5000} \pi t$$

$$= 3 \cos 2 \cdot \frac{1}{5} \pi t + 5 \sin 2 \cdot \frac{3}{5} \pi t + 10 \cos 2 \cdot \frac{6}{5} \pi t$$

$$= 3 \cos 2\pi(2t)$$

Q3(iii) what is the analog signal $y(t)$, that we can reconstruct from the samples if we use ideal interpolation?

\Rightarrow From B we get.

$$3 \cos 2\pi \left(\frac{1}{5}\right)n + 5 \sin 2\pi \left(\frac{3}{5}\right)n + 10 \cos 2\pi \left(\frac{7}{5}\right)n$$
$$= 13 \cos 2\pi \left(\frac{7}{5}\right)n + 5 \sin 2\pi \left(\frac{3}{5}\right)n$$
$$F_2 = f_1 - f_3$$

Digital to Analog Conversion:-

Digital-to Analog Conversion (DAC) is the process of turning a digital signal into an analog signal. A digital signal is made up of binary digits (0's and 1's).

An analog signal on the other hand, is a continuously varying signal like the sound waves produced by the music from a speaker.

To convert a digital signal into an analog signal, we need to recreate the continuously varying signal using a series of discrete values. This is done by taking the digital signal and turning each number into a corresponding analog value.

For example, if the digital signal has a value of 0, it is low voltage and if it has 1 it will be high.

After creating a sequence of analog values that correspond to the original digital signal, we can use those values to drive, such as speaker & motor.

So, we do it by sampling the original analog signal to obtain digital samples, reconstructing the analog signal from the digital samples, and filtering the reconstructed analog signal to remove unwanted high frequency.

Chapter - 02

Basic Signals in Discrete Time Signal
Discrete-time signals

The unit sample sequence is denoted

$$\delta(n) = \begin{cases} 1, & \text{For } n=0 \\ 0, & \text{For } n \neq 0 \end{cases}$$

That means a unit sample sequence is a signal that is zero everywhere except at $n=0$

where its value is unity.

The unit step signal is shown as follows:

$$u(n) = \begin{cases} 1, & \text{For } n \geq 0 \\ 0, & \text{For } n < 0 \end{cases}$$

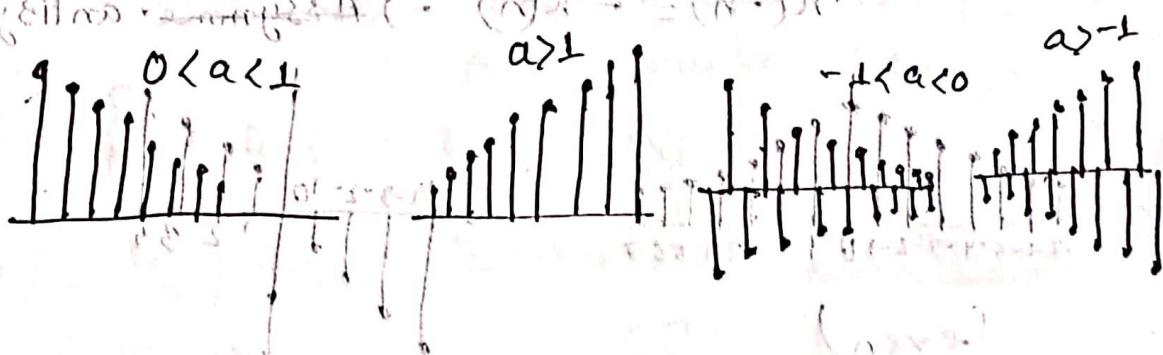
The unit ramp signal, ($x(n) = (n+1)u(n)$)

$$u_T(n) = \begin{cases} 1, & \text{For } n \geq 0 \\ 0, & \text{For } n < 0 \end{cases}$$

The exponential signal,

$$x(n) = a^n \quad \text{For all } n$$

Here, if a is real then $x(n)$ is a real signal.



Classification of Discrete-Time Signals

Energy signals and Power signals: Energy measure of the signal. Amount of work done by someone.

→ Energy Signal:

The signal whose total energy is infinite but the power is finite. This means that the signal has a constant power level over time.

Power Signal:

How quickly a work is done. Rate, speed at which the energy is transferred.

Periodic Signal & Aperiodic

$$x(n+N) = x(n) \quad [\text{For all } n]$$

If there is no value of $n+N$ that satisfies the signal is called nonperiodic or aperiodic.

Symmetric & anti-symmetric (Odd) signals:

A real valued signal,

$$x(-n) = x(n) \rightarrow \text{Symmetric}$$

$$x(-n) = -x(n) \rightarrow \text{Anti-symmetric}$$



Example
2.2.1

Determine the response of the following systems to the input signal

$$x(n) = \begin{cases} n, & -3 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) $y(n) = x(n)$ [identity system]

$$x(n) = \{ \dots, 0, 3, 2, 1, 0, 1, 2, 3, 0, \dots \}$$

(b) $y(n) = x(n-1)$

$$x(n) = \{ \dots, 0, 3, 2, \underset{\uparrow}{1}, 0, 0, 1, 2, 3, 0, \dots \}$$

(c) $y(n) = x(n+1)$ [unit advanced system]

$$x(n) = \{ \dots, 0, 3, 2, 1, 0, 1, 2, 3, 0, \dots \}$$

(d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

[moving average filter]

$\therefore y(n)$ mean value (माध्यमिक मान)

$$y(n) = \frac{1}{3} [x(-1) + x(0) + x(1)]$$

past + present + future

$$x(n) = \{ \dots, 0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0, \dots \}$$

$$\therefore y(0) = \frac{1}{3} [x(-1) + x(0) + x(1)]$$

$$y_1 = \frac{1}{13} [1+0+1] \\ = 2$$

$$y(n) = \{ \dots, 0, 1,$$

$$y(1) = \frac{1}{13} [1+0+2] \\ = 1$$

$$y(2) = \frac{1}{13} [1+2+3] \\ = 2$$

$$y(3) = \frac{1}{13} [2+3] \\ = 5$$

$$y(-1) = \frac{1}{13} [1+2] \\ = 1$$

$$y(-2) = \frac{1}{13} [2]$$

$$y(-3) = \frac{1}{13} [5]$$

$$\therefore n(n) = \{ \dots, 0, 1, \frac{5}{13}, \frac{2}{13}, 1, 0, \frac{1}{13}, 2, \frac{5}{13}, 0, 1, 0, \dots \}$$

$$\{ \dots, 0, 1, \frac{5}{13}, \frac{2}{13}, 1, 0, \frac{1}{13}, 2, \frac{5}{13}, 0, 1, 0, \dots \}$$

$$\text{Ex) } y(n) = \text{median} [x(n+1), x(n), x(n-1)] \quad [\text{median Filter}]$$

$$y(n) = \left\{ -0, 3, 2, 1, 0, 1, \textcircled{2}, 3, 0, \dots \right\}$$

$y(n) = \underbrace{x(n-1)}_{=0}, \underbrace{x(n)}_{=3}, \underbrace{x(n+1)}_{=2}$

$$(8+8+1+0) / 5 = 3.6$$

$$y(0) = \{0, \textcircled{1}, 1\} \quad [(0+1+2+3+4)/5 = 2] \\ = 1$$

$$y(1) = \{0, \textcircled{1}, 2\} \\ = 1$$

$$y(2) = \{1, \textcircled{2}, 3\} \\ = 2$$

$$y(3) = \{\cancel{0}, \cancel{1}, \cancel{2}\} \quad \{0, 2, 3\} \\ = 2$$

$$y(4) = \{\cancel{0}, 0, 1, \cancel{3}\}$$

$$\therefore y(n) = \{0, 4, 2, 1, 1, 1, 2, 3, 0, 0, \dots\} \\ \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \dots\} = (n)$$

(ii) Accumulator ((AP) Sum of all past value)

$$y(n) = \left\{ \begin{array}{l} 0, 1, 2, 3, 0, 1, 2, 3 \\ \vdots \\ y(0), 1, 0, 1, 2, 0, 1 \end{array} \right\} = (15)_B$$

$$y(0) = (0+1+2+3)$$

$$= 6$$

$$y(-1) = (0+1+2) \quad \{ 1(1)0 \} = (0)_B$$

$$= 3$$

$$y(-2) = (0+1)$$

$$= 1$$

$$y(-3) = 0$$

$$y(1) = 0+1+\cancel{2}+3+0$$

$$= 6$$

$$y(2) = 0+1+2+3+0+1$$

$$= 7$$

$$y(3) = 0+1+2+3+0+1+2$$

$$= 9$$

$$\therefore y(n) = \{ 0, 1, 3, 6, 6, 7, 9 \} = (11)_B$$

(Ans)

Block Diagram Representation of Discrete-Time Signals

① Additionschaltung mit einem additiven Verstärker

The diagram shows a circuit with two inputs, $n_1(n)$ and $n_2(n)$, and one output. The output is labeled $g_1 n_1(n) + n_2(n)$. The circuit consists of a summing junction with a gain of g_1 for the first input and a unity-gain inverter for the second input.

② Multiplizierer - 

③ Unit Delay element: - (requires memory, Z^{-1} = unit of delay)

$$n(n) \xrightarrow{\boxed{z^2}} y(n) = n(n-1)$$

④ Unit Advance element! - ($z = \text{unit of advance}$)

$$u(n) \rightarrow [z] \rightarrow y(n) = u(n+1)$$

Classification of Discrete-Time Systems:-

Static vs Dynamic!

Static! - If output depends on input sample:

Dynamic:— Have memory. Can be determined by the input samples in the interval form.

② Time invariant vs Time variant.

- Time invariant don't change with time
- Time variant changes with time

③ Linear vs Non-Linear

→ Linear when a system satisfies

$$T[a_1 u_1(n) + a_2 u_2(n)] = a_1 T[u_1(n)] + a_2 T[u_2(n)]$$

| Math-problem |

(Questions to find S) linear or non linear?

$$\Rightarrow y(n) = n u(n)$$

$$y_1(n) = n u_1(n)$$

$$T[a_1 u_1(n) + a_2 u_2(n)] = F(a_1 u_1(n)) + T$$

$$= n [a_1 u_1(n) + a_2 u_2(n)]$$

$$= n \cdot a_1 u_1(n) + a_2 n \cdot a_2 \cdot a_2 u_2(n)$$

On the other hand,

$$n \{a_1 u_1(n)\} + n \cdot a_2 \cdot u_2(n) = ?$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 n \cdot u_1(n) + a_2 n \cdot u_2(n)$$

$$\text{Q) } y(n) = n(n^2)$$

Here,

$$y_1(n) = n_1(n^2), \quad y_2(n) = n_2(n^2)$$

The output of the system to a linear comb. of $n_1(n)$ & $n_2(n)$

$$\text{So, def } y_3(n) = T[a_1 n_1(n) + a_2 n_2(n)]$$

$$\text{which is } a_1 n_1(n^2) + a_2 n_2(n^2)$$

The linear combination of the two outputs,

$$a_1 y_1(n) + a_2 y_2(n) = a_1 n_1(n^2) + a_2 n_2(n^2)$$

Causal & Non-Causal:-

Causal - n এর value ক্ষমানোর পর Input দিয়ে

Output $y(n)$ এর n এর value

বসম এ রসায়নে ইহা অবহীন Causal. Mathematics on

DSP এ একে Future value হিসেবে বলা হচ্ছে।

Example :-

$$① \rightarrow \text{Causal} + (\text{Non-Causal}) = \text{Causal}$$

$$① y(n) = n(n) - n(n-1)$$

$$\text{এখন, } n=1 \quad \therefore \quad y(1) = n(1) - n(1-1)$$

$$[n(1) - n(0)] = n(1) - n(0) = 1$$

এখন $n(1)$ & $n(0)$ $\rightarrow y(1)$ এর কোন আবশ্যিক মান নাই

Non-causal \rightarrow यदि $y(n) \neq 0$ for $n < 0$ (प्राप्त)

Example:

① $y(n) = x(n) + 3x(n+4)$ \rightarrow $y(n)$ depends on $x(n)$ and $x(n+4)$

$n=1$ \rightarrow $y(1) = x(1) + 3x(5)$

$$\begin{aligned}y(1) &= x(1) + 3x(5) \\&= x(1) + 3x(1+4)\end{aligned}$$

$y(1)$ \rightarrow $x(1)$ and $x(5)$ are future values

($x(1)$ is value of $x(n)$ at $n=1$, $y(n)$ future value)

② $y(n) = x(-n)$ \rightarrow $x(n)$ future value

$$n=1 \rightarrow x(1) + x(-1) = x(1) + x(-1)$$

$$x(n) = x(-1) \rightarrow \text{constant}$$

for $n=-1$, $x(n) = x(1) \rightarrow$ future value

∴ $y(n)$ is non-causal.

Q) Determine if the discrete-time system defined by the difference equation below is causal and non-causal.

$$y(n) = ay(n+1) + bx(n)$$

\Rightarrow Let, $y(n) = y_{\text{ci}}(n) + y_{\text{cs}}(n) \quad \leftarrow \text{Eq. 1}$

First check if $x(n) = c_1 x_1(n) + c_2 x_2(n) \quad \leftarrow \text{Eq. 2}$

using $x(n) = (1-i)^n - (1+i)^n \therefore L = n$

$$y_{\text{cs}} = \sum_{k=0}^{\infty} c_k [c_1 x_1(n-k) + c_2 x_2(n-k)]$$

This is causal since $x(n) = (1-i)^n - (1+i)^n$ is causal.

$$= c_1 y_{2i}^{(1)}(n) + c_2 y_{2i}^{(2)}(n)$$

Now, it is zero-state linear

Let assume $y(-1) = c_1 y_1(-1) + c_2 y_2(-1)$

$$\therefore y_{2i}(n) = a^{(n+1)} [c_1 y_1(-1) + c_2 y_2(-1)]$$

$$= c_1 a^{n+1} y_1(-1) + c_2 y_2 a^{n+1}(-1)$$

$$= c_1 y_{2i}^{(1)}(n) + c_2 y_{2i}^{(2)}(n)$$

\therefore It's zero-input linear

Change out y_0 (as no initial condition)

Chapter-3

$\boxed{z\text{-Transformation and its application to the Analysis of the LTI System}}$

④ The Inverse z-Transform:-

$$u(z) = \sum_{k=-\infty}^{\infty} u(k) \cdot z^{-k}$$

$$\boxed{\square} \quad n(n) \xleftrightarrow{z} X(z)$$

$$n(n-k) \xleftrightarrow{z^{-k}} z^{-k} X(z)$$

$$\boxed{\square} \quad n_1(n) = \begin{cases} 1, 2, 5, 7, 0, 1 \\ 0, 1, 0, 1, 0, 1 \end{cases}$$

$$n_2(n) = \begin{cases} 1, 2, 5, 7, 0, 1 \\ 0, 1, 0, 1, 0, 1 \end{cases}$$

$$n_3(n) = \begin{cases} 0, 0, 1, 2, 5, 0, 7, 0, 1 \\ 0, 1, 0, 1, 0, 1 \end{cases}$$

Ques 3.2.9

Compute the Convolution $n(n)$ of the signals.

$$n_1(n) = \begin{cases} 1, -2, 1 \\ 0, 0, 0 \end{cases}$$

$$n_2(n) = \begin{cases} 1, 0 \leq n \leq 5 \\ 0, \text{ elsewhere} \end{cases}$$

Ans: $n(n) = (S)(n)$

Q) Determine inverse Z-transform of:-

$$n(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

When, (a) ROC :- $|z| < 1$

(b) ROC :- $|z| > 0.5$

(a) $x(z) = \frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$

$$= 1 + \frac{3}{2}z^{-1} + \frac{7}{4}z^{-2} + \frac{15}{8}z^{-3} + \frac{31}{16}z^{-4} + \dots$$

By Comparing this relation with

$$\frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} = \left\{ 1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \dots \right\}$$

(b) $\frac{1}{2}z^{-2} - \frac{3}{2}z^{-1} + 1$

$$= \frac{1}{(2z^2 + 6z^1 + 1)}$$

$$= \frac{32 - 9z^{-1} + 6z^{-2}}{7z^{-2} - 6z^{-3}}$$

— —

Chapter-6

(Frequency Domain Analysis of
LTI Systems)

Chapter-6

[Sampling and reconstruction of Sampling]

Chapter-2

Convolution method

Question: Determine the system's response $y(n)$ graphically and analytically for the following signals using the convolution method.

$$h(n) = \{1, 2, 1, -3\}$$

$$x(n) = \{1, 3, 4, 1, 3\}$$

where $h(n)$ is the impulse response of LTI system
 $x(n)$ is the input signal. Justify the answer.

Solution:-

Determine Z-transform & their ROC of the following discrete time signals.

$$\text{Q2} \quad x(n) = \{ 3, 2, 5, 7 \}$$

So,

$$x(0) = 3$$

$$x(1) = 2$$

$$x(2) = 5$$

$x(3) = 7$; and $x(n) = 0$ for $n < 0$ & for $n > 3$.

By definition of Z-transform,

$$Z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\therefore X(z) = x(0) \cdot z^0 + x(1) \cdot z^{-1} + x(2) \cdot z^{-2} + x(3) \cdot z^{-3}$$

$$= 3 + \frac{2}{z} + \frac{5}{z^2} + \frac{7}{z^3}$$

In $X(z)$, if $z=0$, except the first one all other terms will be infinity.

(2) Determine Z-transform of $x(n) = \{0, 0, 1, 2, 5, 7, 0\}$

$$x(n) = \left\{ \begin{array}{l} 0, 0, 1, 2, 5, 7, 0 \\ \uparrow \end{array} \right\}$$

$$n(0) = 0,$$

$$n(1) = 0$$

$$n(2) = 1$$

$$n(3) = 2$$

$$n(4) = 5$$

$$n(5) = 7$$

$$n(6) = 0$$

$$n(7) = 1$$

We know, $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \{x(n)\}$

$$\begin{aligned} \text{So, } X(z) &= n(0) \cdot z^0 + n(1) \cdot z^1 + n(2) \cdot z^2 + n(3) \cdot z^3 \\ &\quad + n(4) \cdot z^4 + n(5) \cdot z^5 + n(6) \cdot z^6 + n(7) \cdot z^7 \\ &= 0 + 0 + 1 \cdot \frac{1}{z^2} + 2 \cdot \frac{1}{z^3} + 5 \cdot \frac{1}{z^3} + 7 \cdot \frac{1}{z^3} + 0 + \frac{1}{z^7} \end{aligned}$$

Except first two if $z=0$, then all other terms will become infinity.

$$\Leftrightarrow x(n) = \left\{ \begin{array}{l} 2, 4, 5, 7, 3 \\ \uparrow \end{array} \right\}$$

$$n(0) = 5$$

$$n(1) = 7$$

$$n(2) = 3$$

$$n(-1) = 4$$

$$n(-2) = 2$$

we know,

$$z^{-n} = (\sum_{n=0}^{\infty} u(n) \cdot z^{-n}) = \{ (1) + (x) z^{-1} + (x) z^{-2} \} \dots$$

$$\cancel{x(z)} = \cancel{u(0) \cdot z^0 + u(1) z^{-1} + u(2)} \\ x(z) = u(-2) \cdot z^0 + u(-1) \cdot z^1 + u(0) z^2 + u(1) z^{-1} + u(2) z^{-2}$$

$$x(z) = u(-2) \cdot z^0 + u(-1) \cdot z^1 + u(0) z^2 + u(1) z^{-1} + u(2) z^{-2} \\ = 2 \cdot z^0 + 4 \cdot z + 5 \cdot z^2 + \frac{7}{2} z^{-1} + \frac{3}{2} z^{-2} = \{ (1) + (x) z^{-1} \} \dots$$

If $z = \alpha$ & except the third, the rest will be infinity.

Properties of Z-transform

Linearity:- If $u_1(n)$ & $u_2(n)$ are two input signal and a_1, a_2 constants

then, ~~$\alpha \neq 1$~~ .

$$Z\{a_1 u_1(n) + a_2 u_2(n)\} = a_1 X_1(z) + a_2 X_2(z)$$

Time shifting:- If $u(n)$ is a signal, the shifting by k samples in time domain,

$$Z\{u(n-m)\} = z^m \cdot x(z)$$

$$Z\{u(n+m)\} = z^{-m} x(z)$$

Scaling:- Time reversal in Z-transform of $x(-n)$ is equal to the Complex Conjugate of the Z-transform.
If, $Z\{u(n)\} = x(z)$
 $\therefore Z\{u(-n)\} = x(z^{-1})$

Convolution:-

$$z \{ u_1(n) * u_2(n) \} = u_1(z) * u_2(z)$$

Initial value Theorem:-

Signal $u(n)$ evaluated at $z=1$ is $x(0) = \lim_{z \rightarrow 1^-} (z-1) u(z)$

$$z \{ u(n) \} = X(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$$X(0) = \lim_{z \rightarrow 1^-} (z-1) X(z)$$

Final value Theorem:-

$$u(\infty) = \lim_{z \rightarrow 1^-} (1-z^{-1}) X(z)$$

$$= \lim_{z \rightarrow 1^-} \left(\frac{z-1}{z} \right) X(z)$$

Scaling:-

$$u(n) = 2 \uparrow \{ 1, 2, 3, 4 \}$$

$$u(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

Scale the signal by $\downarrow 2$, $y(z) = \frac{1}{2} u(2z)$

$$y(z) = 2u(2z) = 2 + 2z^{-1} + 6z^{-2} + 8z^{-3}$$

Find the initial & final value of $y(z)$ using scaling

$$\frac{32}{\alpha^3}$$

$$(s) X(s) Y = \{ (m \cdot n) x \} s$$

$$\frac{32}{960}$$

$$(s) X(s) Y = \{ (m+n)x \} s$$

at $(m+n)x$ to maintain the degree of terms

forward \Rightarrow add to output the second part of bsp

$$(s) X = \{ (m)x \} s$$

$$(1-s) X = \{ (n)x \} s$$



**KEEP
CALM
ITS TIME FOR THE
FINAL
EXAM**

lecture 01

Introduction to Analog Filters

Passive Analog Filters (Ideal): (Made up of passive Components. Resistors, Capacitors, Inductors)

Lowpass Filter

Low Frequency signal pass द्वारा फिल्टर द्वारा

High Frequency signal को कमाना द्वारा

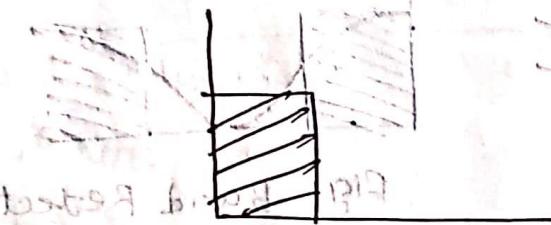


Fig: Lowpass Filter

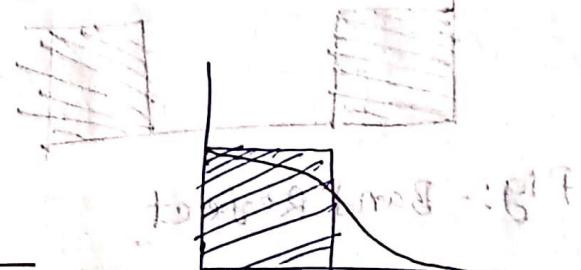


Fig: Lowpass (Realistic)

Highpass Filter

High-pass frequency signal to allow द्वारा, Low

Frequency तो signal को कमाना।

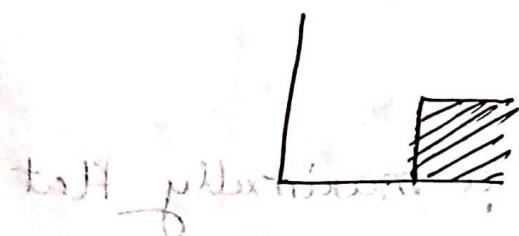


Fig: Highpass Filter

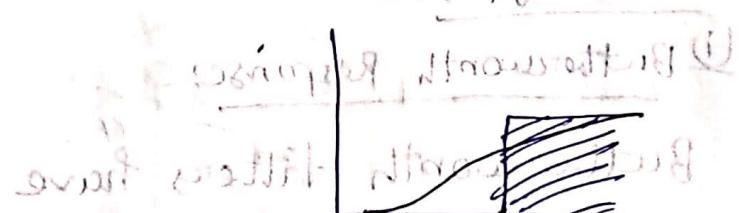
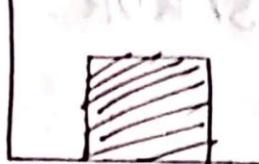


Fig: Highpass (Realistic)

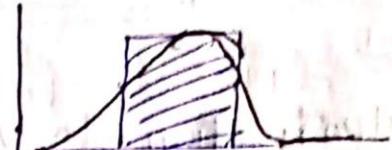
Band-pass Filter

Specific Range Frequency pass द्वारा, Outside Range Frequency बाहर दूरी





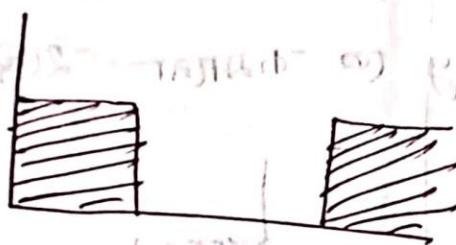
Fig! - Bandpass



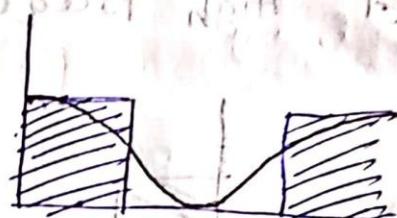
Fig! - Bandpass (realistic)

Band-reject / stop filter:

specific range of frequency with varying Range ω
Frequency pass ω_0 ,



Fig! - Band Reject



Fig! - Band Reject
(realistic)

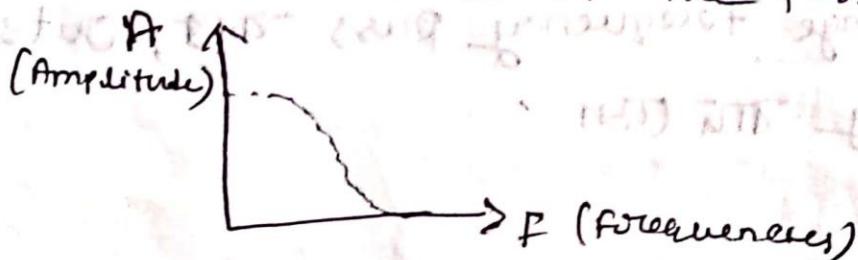
Filter response characteristic:

A graph that shows the amplitude of a filter's output signal as a function of its input function.

Three types:-

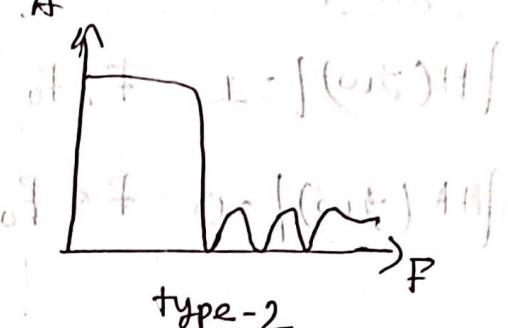
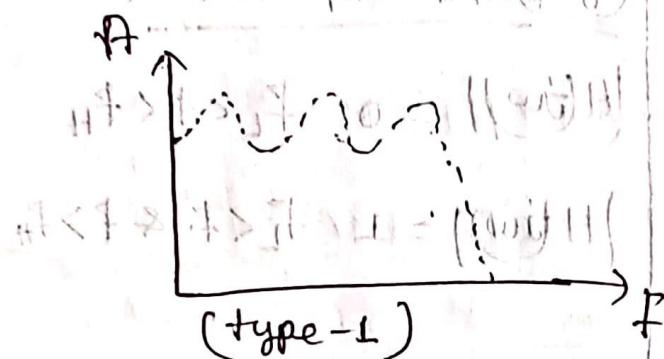
(i) Butterworth Response:-

Butterworth filters have a maximally flat passband. They take long time to attenuate frequencies outside of the pass band.



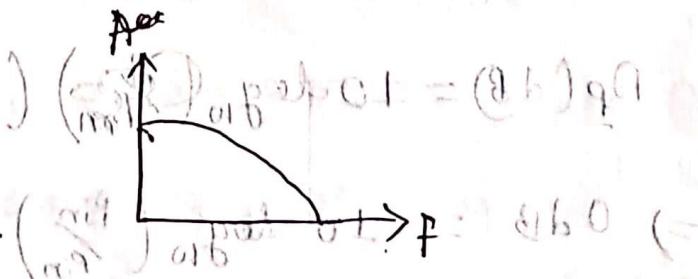
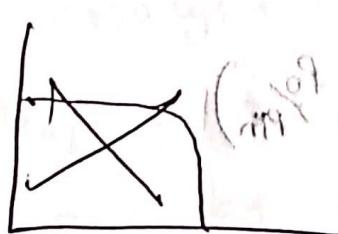
ChebyShev filter: (i) It is steeper roll-off than Butterworth. That means at frequencies outside the passband it attenuates more quickly.

(ii) It has ripples in the passband as a result the amplitude of the output signal is not the same for all frequencies in the passband.

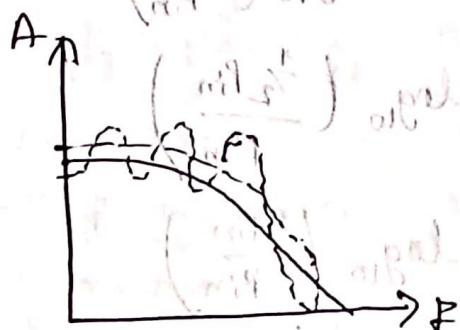


Bessel Filter:

- Constant phase delay $\pi/2$
- Same frequency passband & same amount of delay $\pi/2$
- Chebyshev as well as slow roll-off
- But don't have any ripple



All together!



Frequency transfer function of Filter $H(j\omega)$

① Low-pass Filter:-

$$|H(j\omega)| = 1 \quad f < f_0$$

$$|H(j\omega)| = 0 \quad f > f_0$$

② High-pass Filter:-

$$|H(j\omega)| = 1 \quad f > f_0$$

$$|H(j\omega)| = 0 \quad f < f_0$$

③ Band pass Filter:-

$$|H(j\omega)| = 1 \quad f_L < f < f_H$$

$$|H(j\omega)| = 0 \quad f < f_L \text{ or } f > f_H$$

④ Band-stop Filter:-

$$|H(j\omega)| = 0 \quad f_L < f < f_H$$

$$|H(j\omega)| = 1 \quad f_L < f \text{ or } f > f_H$$

⑤ All pass or phase shift filter:-

$$H(j\omega) = 1$$

For all f has a specific phase response.

⑥ Power gain in dB :-



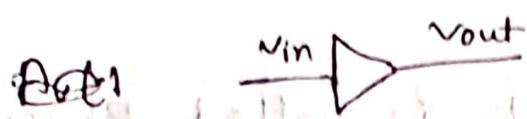
$$A_p(\text{dB}) = 10 \log_{10} \left(\frac{P_o}{P_{in}} \right) \quad (\text{P}_o / \text{P}_{in})$$

$$\Rightarrow 0 \text{ dB} = 10 \log_{10} \left(\frac{P_o}{P_{in}} \right)$$

$$\Rightarrow -3 \text{ dB} = 10 \log_{10} \left(\frac{P_o}{P_{in}} \right)$$

$$\Rightarrow +3 \text{ dB} = 10 \log_{10} \left(\frac{P_o}{P_{in}} \right)$$

$$\text{Voltage gain in dB} = P = \left(\frac{V_o}{V_R}\right)$$



$$Av(\text{dB}) = 20 \log_{10} \left(\frac{V_o}{V_{in}} \right)$$

$$\Rightarrow 0 \text{ dB} = 20 \log_{10} \left(\frac{V_{in}}{V_{in}} \right)$$

$$\Rightarrow -6 \text{ dB} = 20 \log_{10} \left(\frac{1/2 V_{in}}{V_{in}} \right)$$

$$\Rightarrow +6 \text{ dB} = 20 \log_{10} \left(\frac{2 V_{in}}{V_{in}} \right)$$

Lecture-03

Book: A Nagoor Kani

DISCRETE FOURIER Transform (DFT)

Fast Fourier Transform (FFT)

DFT - DFT of a discrete

DFT is developed to convert a continuous function of ω to a discrete function of ω . So that, frequency analysis of discrete time signals can be performed on a digital system. It is obtained by sampling the DTFT of the signal at uniform frequency intervals & the number of samples should be sufficient to avoid aliasing of frequency spectrum. DFT is a sequence of complex numbers.

$$x(k) = x_0, x_1, x_2, x_3, \dots$$

- The plot of magnitude vs k is called Magnitude spectrum.
- The plot of phase vs k is called phase spectrum.
- In general these plots are called Frequency spectrum.

Fast Fourier Transform (FFT):-

DFT [or Compute ~~করা হলো~~ করা হয়ে আছে] a large number of calculation প্রাপ্ত করা হয়। ফল

DFT computation মানে এটা একটা calculation
এবং পদ্ধতি ও বাক্স। যা একটা সময় নিতে,
আস এ-বন্দুর পদ্ধতি এটা FFT এর অধিক,

Definition of DFT:-

$x(n)$ = Discrete time signal of length (L)

$X(k)$ = DFT of $x(n)$ is

N-point DFT of $x(n)$ can be expressed

$$= \text{DFT} \{x(n)\}$$

$$\therefore \text{DFT} \{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-\frac{j2\pi kn}{N}} ; \text{ for } k = 0, 1, 2, \dots, N-1$$

The DFT of $x(n)$ can be expressed,

$$X(k) = \{x(0), x(1), x(2), \dots, x(N-1)\}$$

$$n(2) = \frac{1}{\sqrt{3}} \left[1 + \frac{\cos 2\pi}{4} - j \sin \frac{2\pi}{4} + \cos \frac{2\pi}{2} - j \sin \frac{2\pi}{2} \right]$$

$$= \frac{1}{\sqrt{3}} \left[1 + \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} + \cos \pi - j \sin \pi \right]$$

$$= \frac{1}{\sqrt{3}} \left[1 + 0 - j1 \cdot 0 - 1 - 0 \right]$$

$$= \cancel{\frac{1}{\sqrt{3}} \left(1 - j1 - 1 \right)} = \frac{1}{\sqrt{3}} \{-j\}$$

$$= -j \cdot 0.333$$

$$= 0.333 \angle -\frac{\pi}{2}$$

$$= 0.333 \angle -0.5\pi$$

Amplitude: 0.333

Angle: -0.5π

Lecture - 04

Analog Filter Type Summary:-

① Butterworth Filter:-

Maximally flat passband and a minimally smooth stopband. They are characterized by a constant gain in the passband and monotonically decreasing in the stopband.

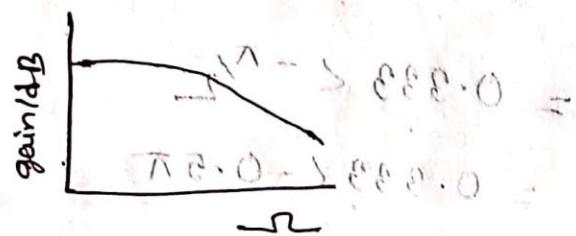


Fig: Butterworth

② Chebyshev Filter:-

Lecture Sheet - 03

Math Design a Butterworth Filter with 1 dB Cutoff at 1 kHz & a minimum attenuation of 40 dB at 5 kHz
Ans- Assume peak passband gain = 1

$$\text{Minimum passband gain} = \frac{1}{\sqrt{1 + \epsilon^2}}$$

$$\text{Ripple } \alpha_{\max} = 20 \log_{10} \sqrt{1 + \epsilon^2} \text{ dB}$$

Minimum Stopband attenuation

$$\alpha_s = -20 \log_{10} \frac{1}{A}$$

$$= 20 \log_{10} A \text{ dB}$$

Design equation:-

$$N > \frac{1}{2} \left[\log_{10} \left(\frac{A^{\infty} - 1}{\epsilon} \right) - \log_{10} \left(\frac{1 - \epsilon}{1 + \epsilon} \right) \right]$$

$$-1 \text{ dB} = 20 \log_{10} \frac{1}{\sqrt{1 + \epsilon^2}}$$

$$\Rightarrow \sqrt{1 + \epsilon^2} = \frac{20 \log_{10}}{-1}$$

$$= 0$$

$$\Rightarrow \epsilon^2 = 1$$

$$\Rightarrow -1 = 20 \log_{10}(1) - 20 \log_{10}(1 + \epsilon^2)$$

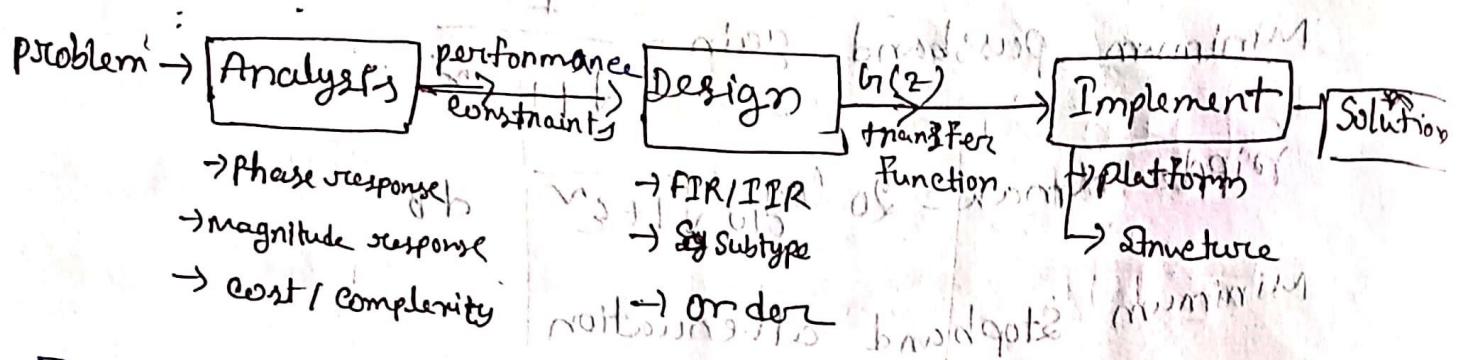
$$\Rightarrow -1 = 0 - 20 \log_{10}(1) + 20 \log_{10}\epsilon^2$$

$$\Rightarrow -1 = 20 \log_{10}\epsilon^2$$

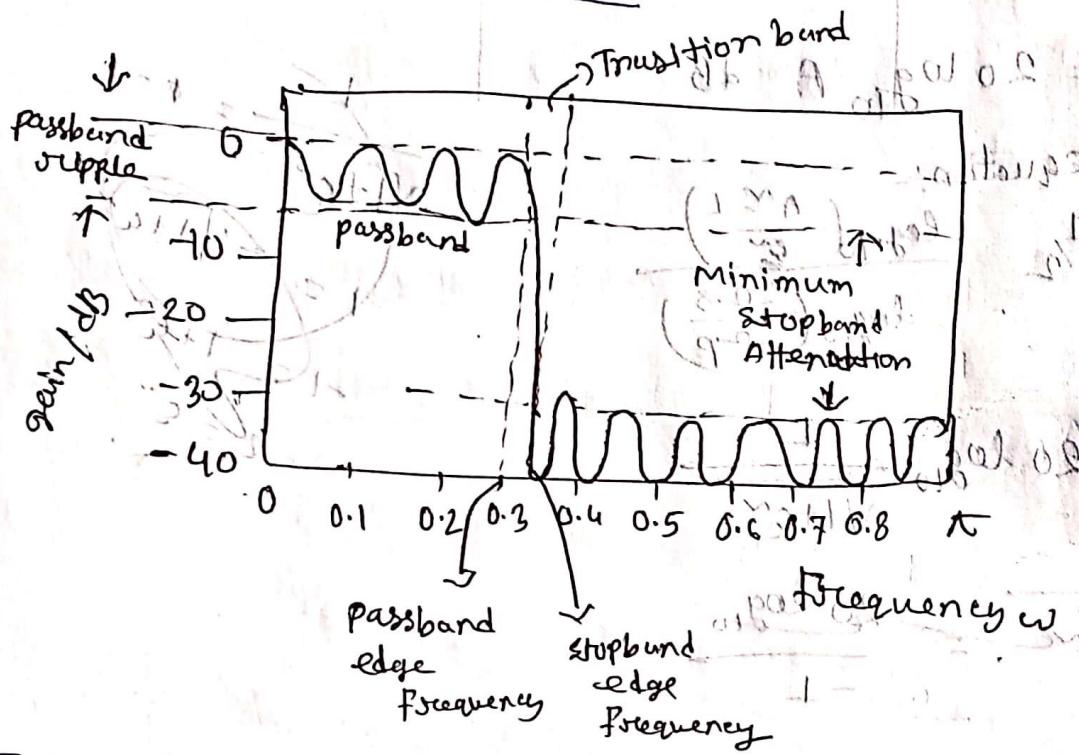
Filter Design Specification:-

→ Motions of filter design methodology → required filter

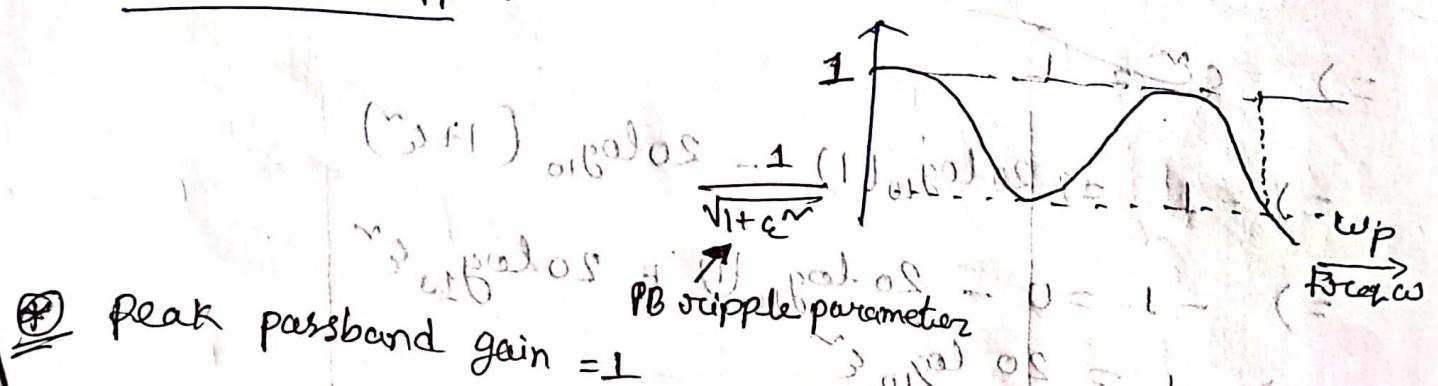
Design process



Performance Constraints:-



Passband ripples:-

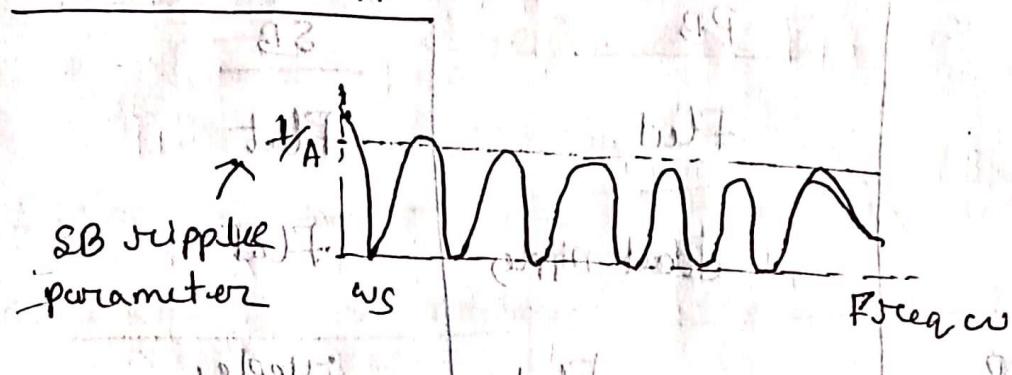


Peak passband gain = 1

$$\therefore \text{Minimum passband gain} = \frac{1}{\sqrt{1+\epsilon^2}}$$

$$\Rightarrow \text{ripple } \alpha_{\text{max}} = 20 \log_{10} \sqrt{1+\epsilon^2} \text{ dB}$$

Stopband Ripple



→ peak passband gain is $A \times$ larger than peak stopband gain.

→ Minimum stopband attenuation, $\alpha_s = -20 \log_{10} \frac{1}{A}$

$$\text{know got } \alpha_s \text{ as } \alpha_s = 20 \log_{10} A \text{ in dB}$$

FIR vs IIR

FIR

① NO Feedback (just zeros)

② Always stable

③ Can be linear phase

④ Unrelated to time filtering

⑤ Not low complexity

⑥ phase ~~not~~ important use FIR

IIR

① Feedback (poles & zeros)

② May be unstable

③ Difficult to control phase

④ Derive from analog prototype

⑤ Fast Low Comp.

⑥ Not important → use FIR

④ Analog Filter Design

Basic choices:-

More ripples \rightarrow Narrow transition band

Family	PB	SB
Butterworth	Flat	Flat
Chebyshev - I	Flat (with ripples)	Flat
Chebyshev - II	Flat	Ripples
Elliptical	Ripples	Ripples

⑤ Butterworth filters:-

Maximally flat in pass & stop band

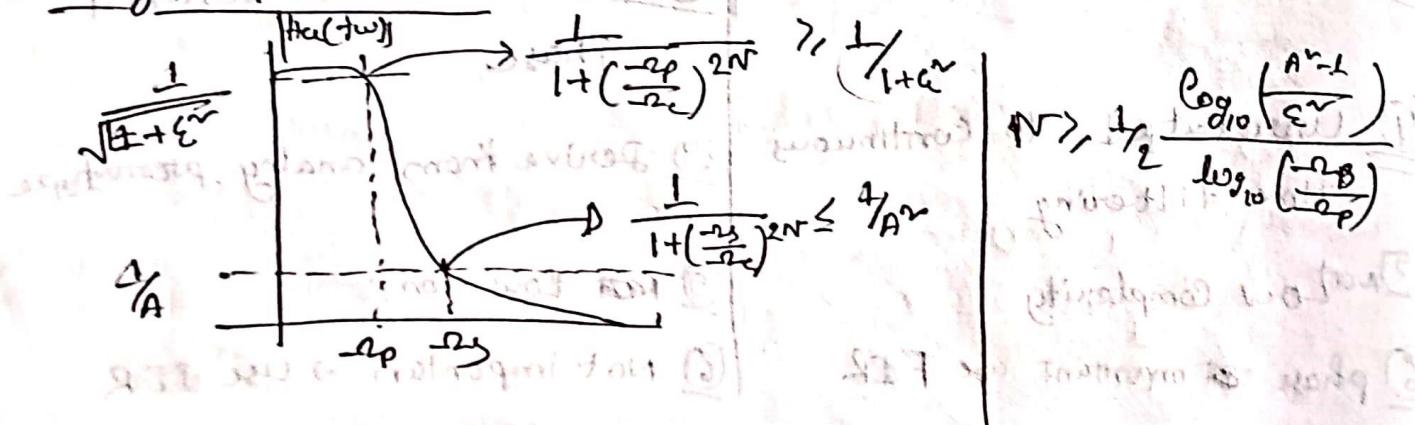
$$\text{Magnitude response, } |H_a(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}} \quad [N = \text{filter order}]$$

(i) $\omega_c \ll \omega_s$ then, $|H_a(j\omega)|^2 \rightarrow 1$

(ii) $\omega_c = \omega_s$ then, $|H_a(j\omega)|^2 = \frac{1}{2}$

(iii) $\omega_c \gg \omega_s$ then, $|H_a(j\omega)|^2 \rightarrow \left(\frac{\omega_c}{\omega_s}\right)^{2N}$

⑥ Design Specification



K_1

selectivity ≥ 1

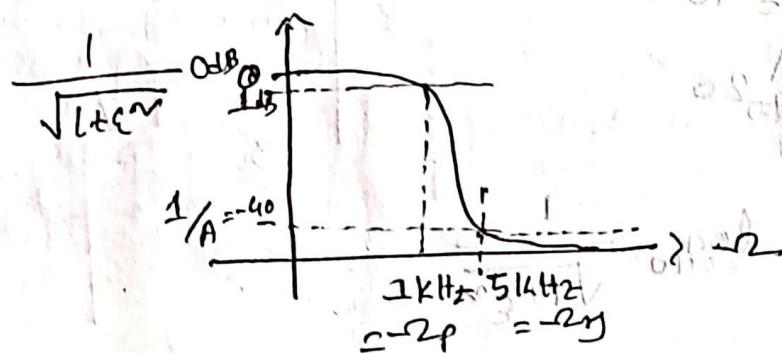
$$K = \frac{-2p}{-2s} = \frac{1}{1+\zeta^2} \times A^2$$

$$k_1 = \frac{\epsilon}{\sqrt{A^2 - 1}}$$

discrimination ≥ 1

Q) Design a Butterworth filter with + dB cutoff at 1 kHz & a minimum attenuation of 40 dB at 5 kHz

\Rightarrow



~~$$-\text{dB} = \frac{1}{\sqrt{1+\zeta^2}}$$~~

=

$$-1dB = -0.282$$

~~$$-2dB = 20 \log_{10} \frac{1}{\sqrt{1+\zeta^2}}$$~~

~~$$-40dB = 20 \log_{10} \frac{1}{A}$$~~

\Rightarrow

$$-\frac{40}{20} \text{ dB} = \log_{10} \frac{1}{A}$$

$$\Rightarrow -2 \text{ dB} = \log_{10} \frac{1}{A}$$

$$\Rightarrow (10)^{-2} \text{ dB} = \frac{1}{A}$$

$$\Rightarrow -\frac{100}{100} = \frac{1}{A} \Rightarrow A = 100$$

N = 4, 3.28

(Ans)

⇒ Chebyshov-1 Filter

Equiripple in passband (Flat in stopband)

→ minimize maximum error

Formulae:- $|H_a(j-2)|^2 = \frac{1}{1 + \epsilon^2 T_N^2\left(\frac{\omega}{\omega_p}\right)}$

Chebyshov
Polynomial
of order N,
 $T_N(\omega) = \begin{cases} \cos(N \cos^{-1} \omega) & |\omega| \leq 1 \\ \cosh(N \cosh^{-1} \omega) & |\omega| > 1 \end{cases}$

ϵ = Desired passband ripple

ω_p, ω_s = Minimum stopband attenuation

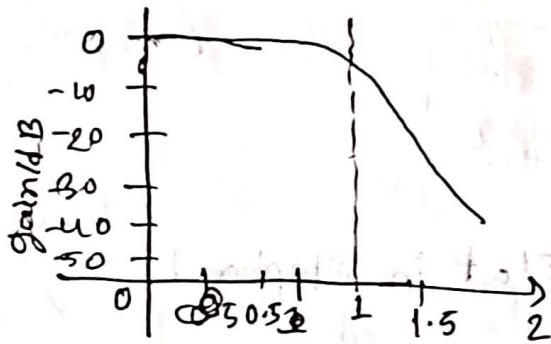
$$\frac{1}{A^2} = \frac{1}{1 + \epsilon^2 T_N^2\left(\frac{\omega_s}{\omega_p}\right)}$$

$$= \frac{1}{1 + \epsilon^2 [\cosh(N \cosh^{-1} \omega)]^2} \quad [|\omega| > 1]$$

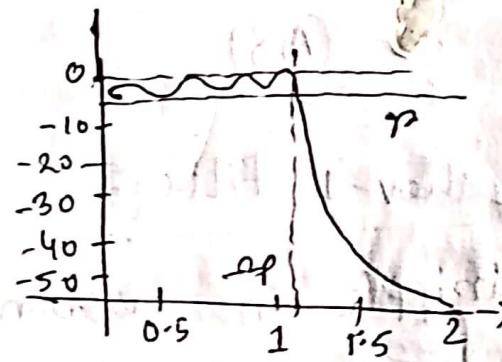
$$\Rightarrow N \geq \frac{\cosh^{-1}\left(\frac{\sqrt{A^2 - 1}}{\epsilon}\right)}{\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)} \rightarrow K_1 \text{ determination}$$

$\rightarrow K_2$, selectivity

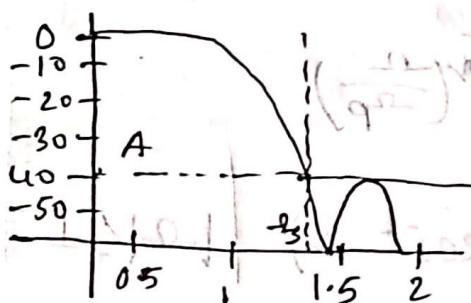
Analog Filter type summary:-



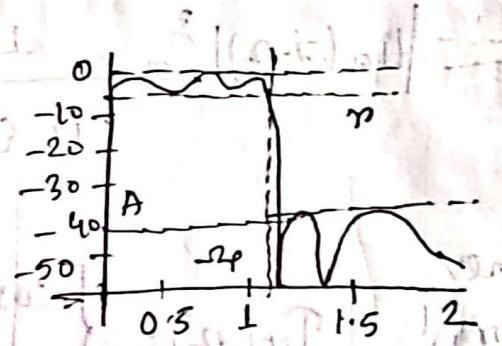
Butterworth



Chebychev-I



Chebychev-II



Elliptical

Bilinear Transformation

$$\text{solution : } S = \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\text{inverse, } z = \frac{1 + s}{1 - s}$$

Frequency axis, $s = j\omega$

$$\therefore z = \frac{1 + j\omega}{1 - j\omega}$$

$$\text{Poles, } s = 6 + j\omega \rightarrow z = \frac{(1+6) + j\omega}{(1+6) - j\omega}$$

$$\therefore |z|^2 = \frac{1 + 36 + \omega^2 + 12j\omega}{1 - 36 + \omega^2 - 12j\omega}$$

CT \leftrightarrow DT Frequency relation:-

$$\Rightarrow Z = e^{j\omega}$$

$$\Rightarrow S = \frac{1 - e^{-j\omega}}{1 + e^{-j\omega}} = \frac{2 j \sin \frac{\omega}{2}}{2 \cos \frac{\omega}{2}} = j \tan \frac{\omega}{2}$$

$$-2 = \tan(\frac{\omega}{2})$$

$$\omega = 2 \tan^{-1} \Omega$$

④ Infinite range of CT Frequency, $-\infty < \omega < \infty$

maps to finite DT Freq range $-\pi < \omega < \pi$

⑤ Non linear, $\frac{d}{d\omega} \Omega \rightarrow \infty$ as $\omega \rightarrow \pi$

Lecture sheet - 4

Filter specifications:

S_p = peak passband deviation

S_p = Stopband deviation

ω_{p1} = Passband edge Frequency
($2\pi f_p$)

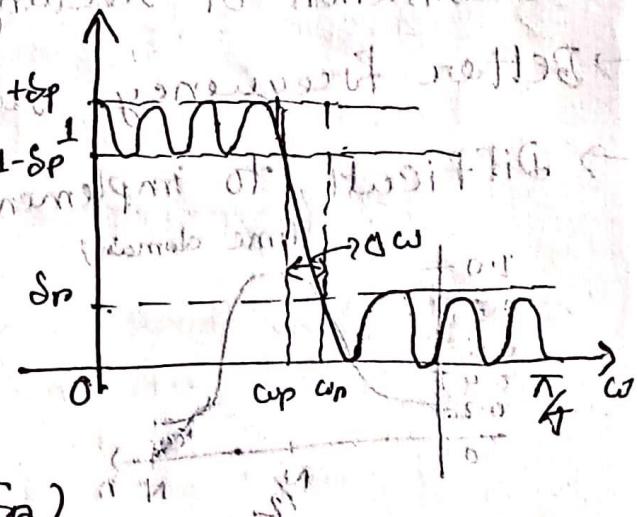
ω_{s1} = Stopband edge Frequency ($2\pi f_s$)

ω_S = Sampling Frequency ($\omega_S = 2\pi f_S$)

FIR Coefficient calculation:-

$$y(n) = \sum_{k=0}^m h(k) \cdot x(n-k)$$

$$H(z) = \sum_{k=0}^m h(k) \cdot z^{-k}$$



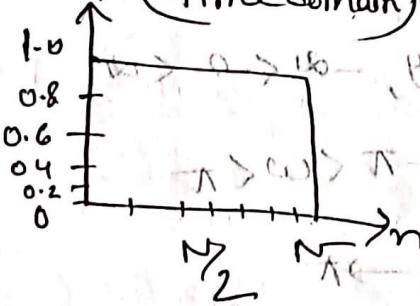
1) Hamming window

NRM

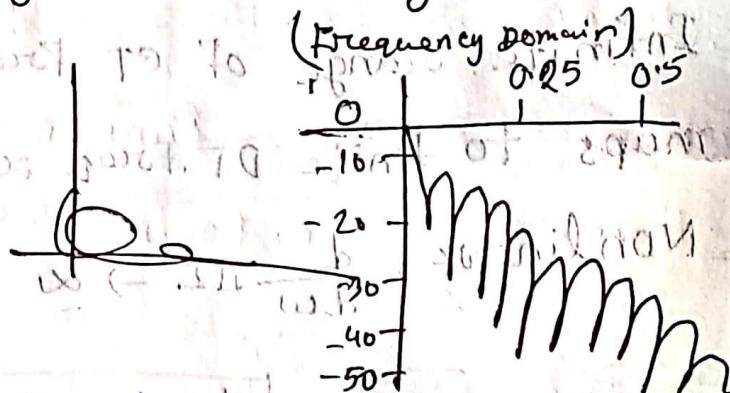
Window Method

Rectangular:- Simplest type of window, value 1 for specified range & zero = 0 for outside range. Window has very easy to implement. Have the disadvantage of having poor frequency resolution.

$w(n)$ (Time domain)

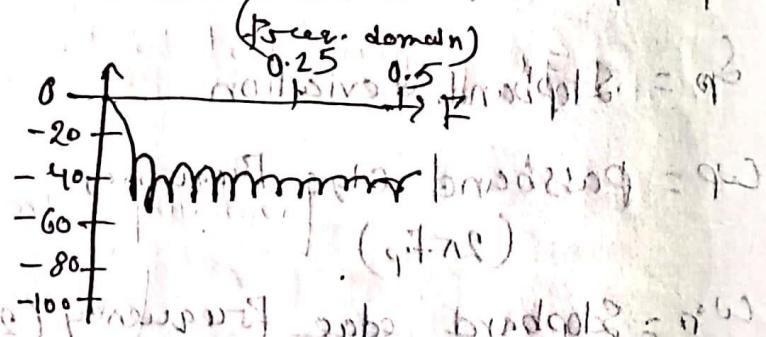
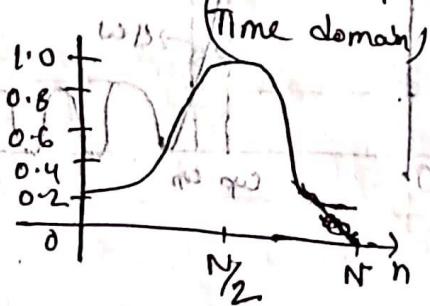


(Frequency domain)



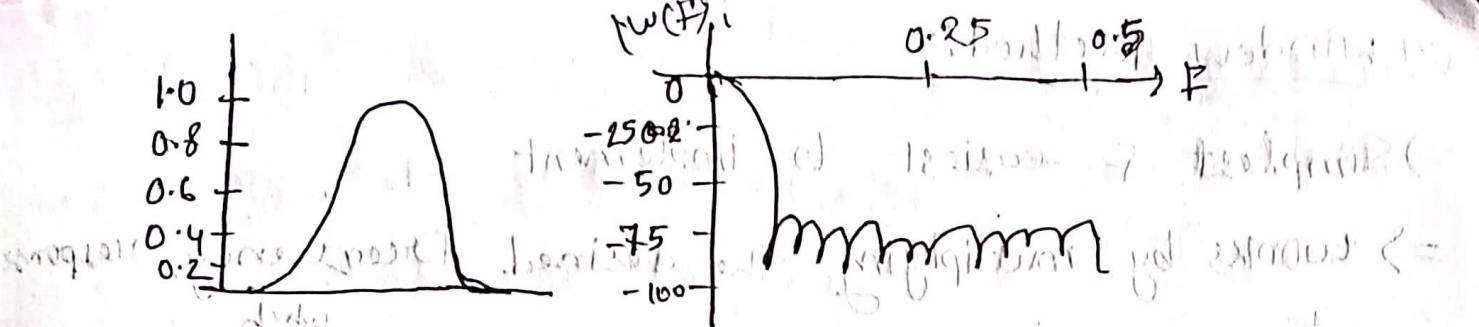
2) Hamming window

- Combination of rectangular function & cosine function
- Better frequency resolution than Rectangular.
- Difficult to implement.



(3) Blackman

- Used for spectral analysis.
- Combination of rectangular & window & cosine function.
- If frequency resolution is the most important factor, then hamming / blackman good choice.



(Time domain) \leftrightarrow (Frequency domain)

Hamming window (eqn & math)

$$\text{For-1st} \rightarrow w(n) = \begin{cases} 0.54 + 0.46 \cos(2\pi n/N) & ; 0 \leq n \leq N \\ 0 & \text{for, from elsewhere} \end{cases}$$

Formula-2:-

$$\rightarrow \Delta F = 3.32/N \quad \rightarrow \text{Wobin's result for minimum}$$

$$\rightarrow N = \text{filter order}$$

$$\rightarrow \Delta F = \text{Normalized transition width} = \frac{\Delta f}{f_s} \quad \rightarrow \text{Transition width}$$

Comparison of the window, optimum & frequency sampling methods:

Optimal Method:-

FIR Filter Coefficients, Compute

N value & making Filter with good amplitude response.

Most accurate than other three

Can achieve the desired Frequency Response with no ripples

Difficult to implement

Linear equines Computation is expensive

(2) Window Method -

- ⇒ Simplest & easiest to implement.
- ⇒ works by multiplying the desired frequency response ^{which} + tapers the response at the edges to reduce ripples in frequency response.
- ⇒ Can introduce significant ripples in the frequency if window function is not chosen carefully.

(3) Frequency Sampling method -

- Compromise between the window & optimum methods.
- ⇒ It is less difficult than optimum but more difficult than window.
- ⇒ works by sampling desired frequency at a finite number of points.
- Coefficients are calculated using DFT.

uses:

→ ease of implementation → window

→ Accuracy → optimum

→ Compromises between ease of implementation

& accuracy → frequency Sampling method

~~1~~
 voltage across resistor $\frac{1}{2}$ $\text{A} \cdot \text{V}$ $\text{sin}(t + \tan^{-1}(\frac{\omega_0}{\sqrt{1+\omega_0^2}}))$
 at. ω_0 start freq. and the damping ratio $\zeta = \frac{-\pi + \tan^{-1}(\frac{\omega_0}{\sqrt{1+\omega_0^2}})}{4\pi}$
 above magnitude $\sqrt{1+\omega_0^2} = \sqrt{1 + (\frac{1}{2})^2} = \frac{\sqrt{5}}{2}$
 so required val of above start freq. $\omega_0 = \tan^{-1}(\frac{1}{2})$
 ab. at resonance of previous step $\rightarrow 8.7 = 8.13$

$$\text{DFT} \rightarrow 9.19$$

most simple to determine to resonant frequency

(a) FIR Filter (Method Solution PFT) (3ca), 3

to calculate the required no. of iteration of
8. Butterworth (circuit)
of filter designed with orders of 40 steps

methodology of plotting the magnitude and phase

magnitude in steps, profit fit $\frac{1}{2} \log \frac{1}{\sqrt{1+e^{2s}}}$
phase error of steps and result obtained

phase error of steps and result obtained

and turn, so, M to make the profit fit

and convert \Rightarrow 81 to steps $\frac{1}{2} \log \frac{1}{\sqrt{1+e^{2s}}}$ = 0.78

resulted at $e^{2s} = 1.414$

Multirate

Multirate

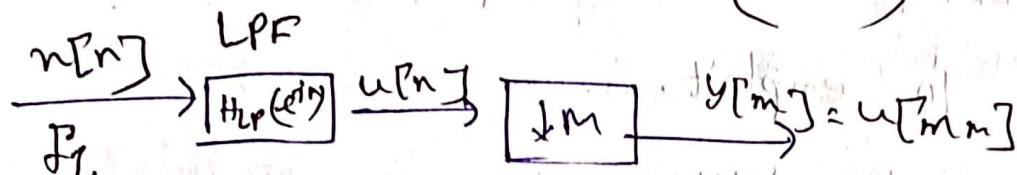
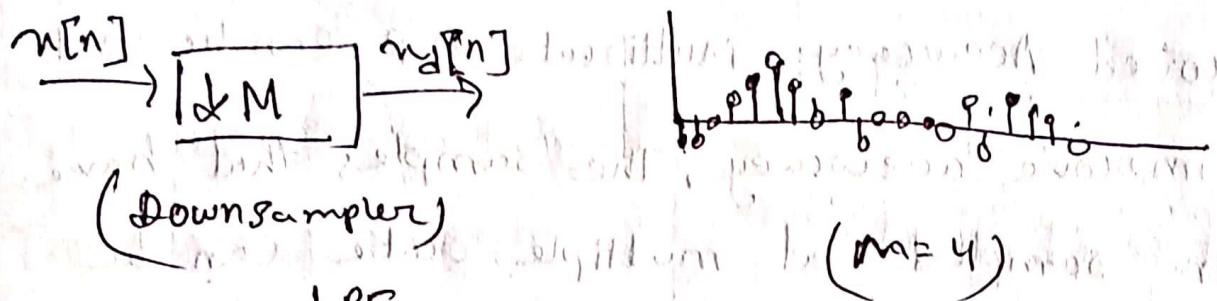
Multirate means 'multiple sampling rate'. DSP system uses ~~it~~ in case signal at one point rate has to use systems that expects a different rate.

So, the rate needs to be increased or decreased & some processing is required to do so.

Resampling:— process of converting a signal from one sampling rate to another. This is often done to increase or decrease the resolution of the signal or to change the sampling rate to match the requirement of particular application.

Downsampling:

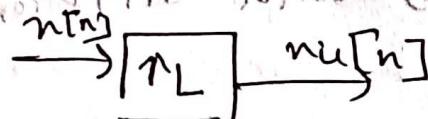
- process of removing samples,
- without 'Low pass' filtering, a signal is downsampled
- Downsampled when the signal is oversampled.
- Decimation:— Combined operation of both Filtering & Downsampling.
- To downsample by a factor of M, we must keep every $\frac{1}{M}$ th sample as it is & remove the $(M-1)$ samples in between.



Unsampling: Increase sampling rate of a signal.

This can be done inserting new samples between the existing samples.

$$n_L[n] = \begin{cases} n[n/L], & n=0, \pm 1, \pm 2, \dots \\ 0 & \text{otherwise} \end{cases}$$



Symbol Upsampler

Advantage of Multirate DSP:-

(i) Reduce Computational Complexity:

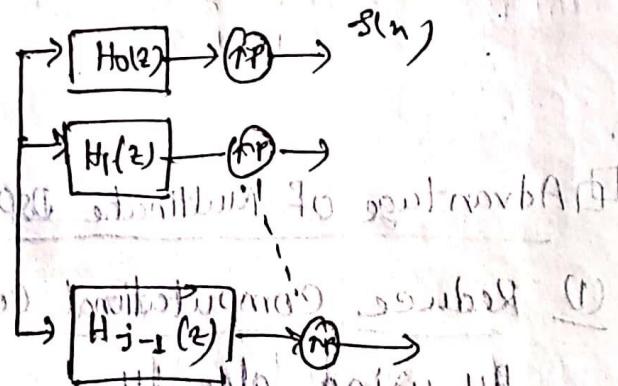
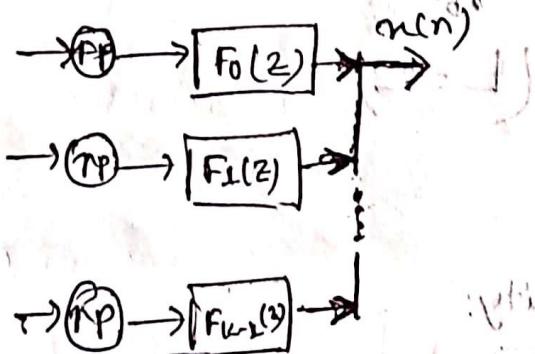
By using algorithm. Algorithms can be implemented using fewer operations when they are applied to signals that have been sampled at multiple rates.

(2) Improved Accuracy: Multirate DSP can be used to improve accuracy. The samples that have been sampled at multiple rates can be designed to exploit.

(3) Increased Flexibility: Algorithms can be implemented in a variety of ways, depending on the specific requirements of the application.

Application of Multirate DSP:

- Used to design for phase shifter.
- Interfacing of digital systems with different sampling rates.
- Implementation of Digital filter bank.



(a) Synthesis Filter Bank

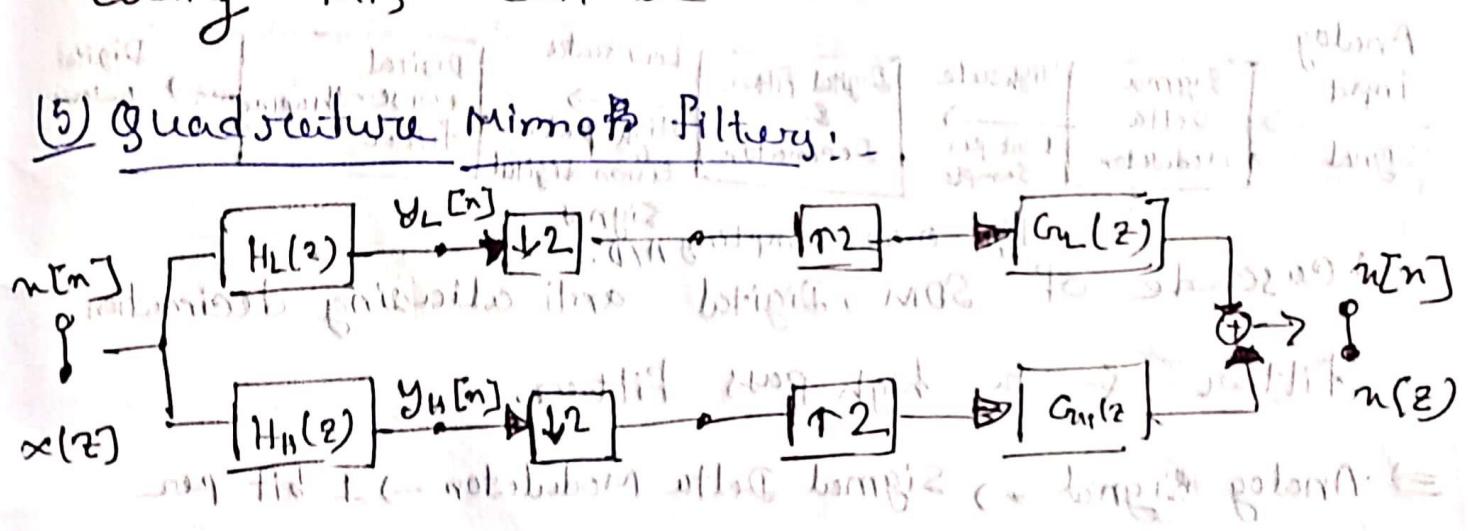
(b) Analysis Filter Bank

defined for designing most practical filter banks of multirate systems.

(4) Subband Coding of Speech Signals

A technique used to compress speech signals. The speech signal is divided into a number of frequency bands, and each band is encoded separately. By using quantization, differential coding this can be done.

(5) Quadrature Mirrored Filtering:



=> Split into two output with bandwidth half of the original bandwidth. The output QMF bank after being processed (encoding, decoding, individual amplification etc) is recombined to a single signal using the synthesis filter bank also composed of QMFs.

(6) Transmultiplexor

Devices used for Converting Time Division

Multiplexed (TDM) signal & Frequency Division

Multiplexed (FDM) band division of 40

(7) Oversampling A/D & D/A conversion

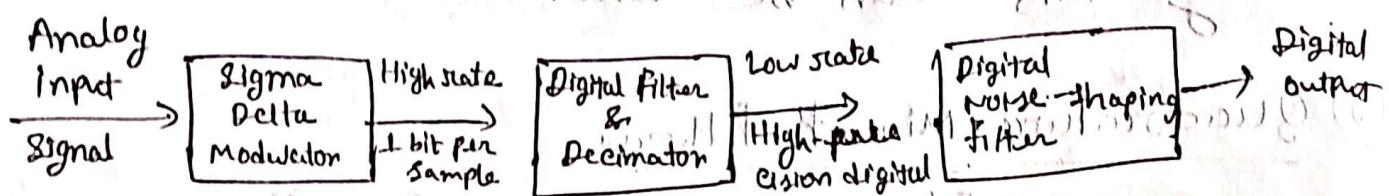


Fig: oversampling A/D.

⇒ Cascade of SDM, Digital anti aliasing decimation

Filter & a high pass filter.

⇒ Analog Signal → Sigma Delta Modulator → 1 bit per sample output at a very high sampling rate

To filter noise after the output of digital noise shaping filter provides digital output to attenuate sampling rate high precision low pass filter output the quantization

Noise at lower frequencies) having high pass filter but with high frequencies, b

D/A Conversion

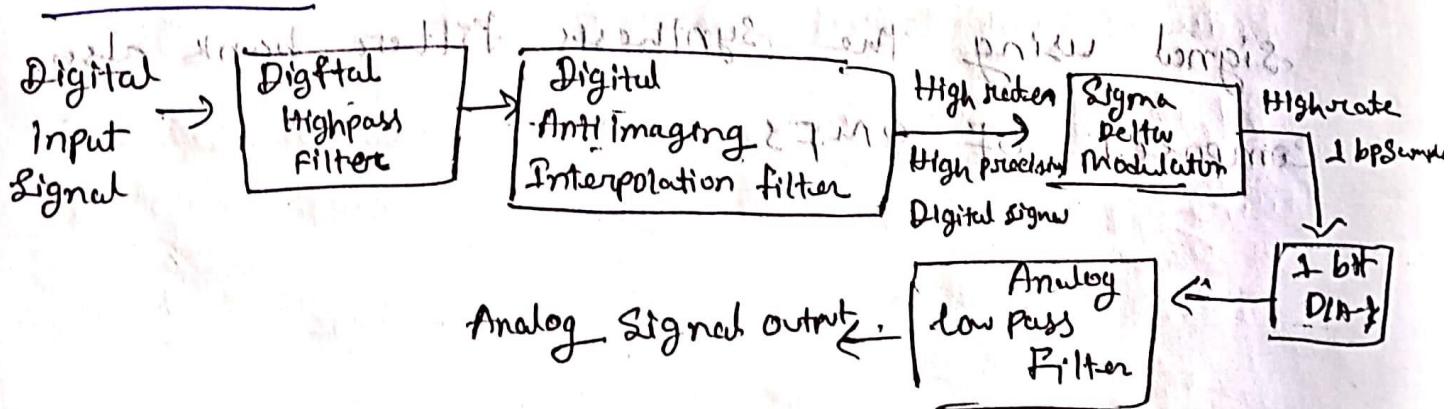


Fig: D/A Conversion.

- Digital signal passed \rightarrow Highpass Filter
- output Fed to Digital interpolators
- High sampling rate signal is the input of SDM.
- SDM provides high sampling rate.
- One bit per sample output.
- Analog Low pass Filter makes low Frequency.
- Analog Filters.