

Examen U3 ROBÓTICA

1. Realizar una trayectoria rectilínea cúbica en el tiempo rest-to-rest entre dos puntos P, Q para ser ejecutada por el Origen O6, del efector final.

Solución:

- Determinación de los coeficientes de los polinomios cúbicos para cada componente de la posición.

Si se sabe que X, Y y Z, son una trayectoria cubica en función del tiempo, se puede deducir su posición en el tiempo inicial y el tiempo final del recorrido, así mismo se pueden deducir las velocidades en los mismos puntos, dando analíticamente los siguientes sistemas de ecuaciones.

$$\begin{array}{c}
 \begin{array}{|c|} \hline ax_0^3 + bx_0^2 + cx_0 + dx \\ \hline 3ax_0^2 + 2bx_0 + cx \\ \hline ax_f^3 + bx_f^2 + cx_f + dx \\ \hline 3ax_f^2 + 2bx_f + cx \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline x_0 \\ \hline v_{x0} \\ \hline x_f \\ \hline v_{xf} \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{|c|} \hline ay_0^3 + by_0^2 + cy_0 + dy \\ \hline 3ay_0^2 + 2by_0 + cy \\ \hline ay_f^3 + by_f^2 + cy_f + dy \\ \hline 3ay_f^2 + 2by_f + cy \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline y_0 \\ \hline v_{y0} \\ \hline y_f \\ \hline v_{yf} \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{|c|} \hline az_0^3 + bz_0^2 + cz_0 + dz \\ \hline 3az_0^2 + 2bz_0 + cz \\ \hline az_f^3 + bz_f^2 + cz_f + dz \\ \hline 3az_f^2 + 2bz_f + cz \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline z_0 \\ \hline v_{z0} \\ \hline z_f \\ \hline v_{zf} \\ \hline \end{array}
 \end{array}$$

Donde ai, bi, ci, di con i equivalente a x, y o z respectivamente, son los coeficientes que satisfacen los sistemas.

Para conocer los valores de los coeficientes de la trayectoria se necesita conocer al menos los puntos (x_0, y_0, z_0) y (x_f, y_f, z_f) . Y para que las trayectorias cumplan con la condición *rest-to-rest* se asumen que las velocidades de partida y llegada en cualquier componente x, y & z sean igual a cero. ($V_{x0} = V_{y0} = V_{z0} = V_{xf} = V_{yf} = V_{zf} = 0$).

De los sistemas anteriores se obtiene la siguiente equivalencia . . .

$$\begin{array}{c}
 \begin{array}{|c|c|c|c|c|} \hline t_0^3 & t_0^2 & t_0 & 1 & ax \\ \hline 3t_0^2 & 2t_0 & 1 & 0 & bx \\ \hline t_f^3 & t_f^2 & t_f & 1 & cx \\ \hline 3t_f^2 & 2t_f & 1 & 0 & dx \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline x_0 \\ \hline v_{x0} \\ \hline x_f \\ \hline v_{xf} \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{|c|c|c|c|c|} \hline t_0^3 & t_0^2 & t_0 & 1 & ay \\ \hline 3t_0^2 & 2t_0 & 1 & 0 & by \\ \hline t_f^3 & t_f^2 & t_f & 1 & cy \\ \hline 3t_f^2 & 2t_f & 1 & 0 & dy \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline y_0 \\ \hline v_{y0} \\ \hline y_f \\ \hline v_{yf} \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{|c|c|c|c|c|} \hline t_0^3 & t_0^2 & t_0 & 1 & az \\ \hline 3t_0^2 & 2t_0 & 1 & 0 & bz \\ \hline t_f^3 & t_f^2 & t_f & 1 & cz \\ \hline 3t_f^2 & 2t_f & 1 & 0 & dz \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline z_0 \\ \hline v_{z0} \\ \hline z_f \\ \hline v_{zf} \\ \hline \end{array}
 \end{array}$$

Por lo tanto, los coeficientes son el resultado de los siguientes cálculos:

$$\begin{array}{c}
 \begin{array}{|c|} \hline ax \\ \hline bx \\ \hline cx \\ \hline dx \\ \hline \end{array}
 =
 \begin{pmatrix}
 t_0^3 & t_0^2 & t_0 & 1 \\
 3t_0^2 & 2t_0 & 1 & 0 \\
 t_f^3 & t_f^2 & t_f & 1 \\
 3t_f^2 & 2t_f & 1 & 0
 \end{pmatrix}^{-1}
 \begin{array}{|c|} \hline x_0 \\ \hline v_{x0} \\ \hline x_f \\ \hline v_{xf} \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{|c|} \hline ay \\ \hline by \\ \hline cy \\ \hline dy \\ \hline \end{array}
 =
 \begin{pmatrix}
 t_0^3 & t_0^2 & t_0 & 1 \\
 3t_0^2 & 2t_0 & 1 & 0 \\
 t_f^3 & t_f^2 & t_f & 1 \\
 3t_f^2 & 2t_f & 1 & 0
 \end{pmatrix}^{-1}
 \begin{array}{|c|} \hline y_0 \\ \hline v_{y0} \\ \hline y_f \\ \hline v_{yf} \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{|c|} \hline az \\ \hline bz \\ \hline cz \\ \hline dz \\ \hline \end{array}
 =
 \begin{pmatrix}
 t_0^3 & t_0^2 & t_0 & 1 \\
 3t_0^2 & 2t_0 & 1 & 0 \\
 t_f^3 & t_f^2 & t_f & 1 \\
 3t_f^2 & 2t_f & 1 & 0
 \end{pmatrix}^{-1}
 \begin{array}{|c|} \hline z_0 \\ \hline v_{z0} \\ \hline z_f \\ \hline v_{zf} \\ \hline \end{array}
 \end{array}$$

- Implementación en C++

Función que define que se llevaran a cabo trayectorias cubicas en función del tiempo.

```

458 void Robot::Parametrica2()
478
479     /// Parametrica para rectas con analisis de coeficientes
480     xO6 = ax*t*t*t + bx*t*t + cx*t + dx;
481     vxO6 = 3*ax*t*t + 2*bx*t + cx;
482     axO6 = 6*ax*t + 2*bx;
483
484     /*
485     vxO6 = 0;
486     axO6 = 0;
487     */
488
489     yO6 = ay*t*t*t + by*t*t + cy*t + dy;
490     vyO6 = 3*ay*t*t + 2*by*t + cy;

```

```

491     ayO6 = 6*ay*t + 2*by;
492
493     /*
494     vyO6 = 0;
495     ayO6 = 0;
496     */
497     zO6 = az*t*t*t + bz*t*t + cz*t + dz;
498     vzO6 = 3*az*t*t + 2*bz*t + cz;
499     azO6 = 6*az*t + 2*bz;
500
501     /*
502     vzO6 = 0;
503     azO6 = 0;
504     */
505     rO6.entry(0,0)=xO6;
506     rO6.entry(1,0)=yO6;
507     rO6.entry(2,0)=zO6;
508 }
509

```

Función para guardar y establecer el punto $P(x_0, y_0, z_0)$ y punto $Q(x_f, y_f, z_f)$. Esta función permite guardar más de dos puntos (A,B, C, ...) para una trayectoria compuesta si se quiere.

```

939 void Robot::guardarPosActual()
940 {
941     bool stateExist = false;
942     float tPaP;
943     if(Trayecto.size()>0)
944         if(Trayecto[Trayecto.size()-1].x == xO6 && Trayecto[Trayecto.size()-1].y == yO6 &&
Trayecto[Trayecto.size()-1].z == zO6){
945             stateExist = true;
946         }
947     if(stateExist == false){
948         if(Trayecto.size()==0)
949             tPaP = 0;
950         else
951             tPaP = 3;
952         Trayecto.push_back(vector3d(xO6,yO6,zO6,tPaP));
953         cout << "===== " << endl;
954         cout << "|| Posicion Guardada ||" << endl;
955         cout << "===== " << endl;
956     }
957     else{
958         cout << "===== " << endl;
959         cout << "|| Ya se ha guardado esta posicion ||" << endl;
960         cout << "===== " << endl;
961     }
962     cout << "-----" << endl;
963 }

```

Función para resetear la pila de puntos guardados para la trayectoria.

```

934 void Robot::resetTrayectorias()
935 {
936     Trayecto.clear();
937     cout << "Trayectoria Eliminada" << endl;
938     cout << "-----" << endl;
939 }

```

Función que calcula los coeficientes entre dos puntos guardados (T1, T2)

```

964 void Robot::planTrayectoria(vector3d T1, vector3d T2) // Funciones para planificar y dibujar
trayectorias
965 {
966     Matrix A(4,4);
967     t0=0; tf=T2.t;
968     x0 = T1.x;    xf = T2.x;    v0x = 0;    vfx = 0;
969     y0 = T1.y;    yf = T2.y;    v0y = 0;    vfy = 0;
970     z0 = T1.z;    zf = T2.z;    v0z = 0;    vfz = 0;
971
972     A.entry(0,0) = t0*t0*t0;    A.entry(0,1) = t0*t0;    A.entry(0,2) = t0;    A.entry(0,3) = 1;
973     A.entry(1,0) = 3*t0*t0;    A.entry(1,1) = 2*t0;    A.entry(1,2) = 1;    A.entry(1,3) = 0;
974     A.entry(2,0) = tf*tf*tf;    A.entry(2,1) = tf*tf;    A.entry(2,2) = tf;    A.entry(2,3) = 1;
975     A.entry(3,0) = 3*tf*tf;    A.entry(3,1) = 2*tf;    A.entry(3,2) = 1;    A.entry(3,3) = 0;

```

```

976 Matrix X(4,1), Cx(4,1);
977 Cx.entry(0,0) = x0;
978 Cx.entry(1,0) = v0x;
979 Cx.entry(2,0) = xf;
980 Cx.entry(3,0) = vfx;
981
982 Matrix Y(4,1), Cy(4,1);
983 Cy.entry(0,0) = y0;
984 Cy.entry(1,0) = v0y;
985 Cy.entry(2,0) = yf;
986 Cy.entry(3,0) = vfy;
987
988 Matrix Z(4,1), Cz(4,1);
989 Cz.entry(0,0) = z0;
990 Cz.entry(1,0) = v0z;
991 Cz.entry(2,0) = zf;
992 Cz.entry(3,0) = v fz;
993
994 X = A.inversa()*Cx;
995 ax=X.entry(0,0);
996 bx=X.entry(1,0);
997 cx=X.entry(2,0);
998 dx=X.entry(3,0);
999
1000 cout << "===== " << endl;
1001 cout << "|| Los coeficientes para X son ||" << endl;
1002 cout << "===== " << endl;
1003 X.mostrar();
1004
1005 Y = A.inversa()*Cy;
1006 ay=Y.entry(0,0);
1007 by=Y.entry(1,0);
1008 cy=Y.entry(2,0);
1009 dy=Y.entry(3,0);
1010
1011 cout << "===== " << endl;
1012 cout << "|| Los coeficientes para Y son ||" << endl;
1013 cout << "===== " << endl;
1014 Y.mostrar();
1015
1016 Z = A.inversa()*Cz;
1017 az=Z.entry(0,0);
1018 bz=Z.entry(1,0);
1019 cz=Z.entry(2,0);
1020 dz=Z.entry(3,0);
1021
1022 cout << "===== " << endl;
1023 cout << "|| Los coeficientes para Z son ||" << endl;
1024 cout << "===== " << endl;
1025 Z.mostrar();
1026
1027 }
1028

```

Función que administra los coeficientes a utilizar entre dos puntos de la trayectoria y el cambio de puntos.

```

884 void Robot::mover()
885 {
886     if(Trayecto.size()>=2 && pTrayec < Trayecto.size()-1 && trabajando == true){
887         planTrayectoria(Trayecto[pTrayec],Trayecto[pTrayec+1]);
888         CinematicaInversa2();
889         t =t+0.1;
890         if(t>tf) {pTrayec++; t=0;}
891     }else{
892         pTrayec = 0;
893         trabajando = false;
894     }
895 }
896
897 }

```

2. Calcular las velocidades angulares relativas para cada grado de libertad del manipulador en función del tiempo.

Solución:

- Utilizar el método expuesto en clase, utilizar software de computo simbólico para el cálculo de las derivadas totales de las matrices que resulten del análisis.
Si se sabe que la posición relativa del punto del Origen O4 se puede encontrar con el punto del Origen O6, con la siguiente relación:

$$r_{O4} = r_{O6} - d\hat{k}$$

Con $d = cte$ y $\hat{k} = (0,0,1)$. Entonces se puede realizar el siguiente análisis que proporciona la velocidad lineal de los puntos.

$$\frac{d(r_{O4})}{dt} = v_{O6}$$

Si se sabe que el cálculo de los coeficientes proporciona la Velocidad instantánea al llamar la función de paramétrica en el origen O6 para cada componente en x, y & z. Se tiene.

$$\frac{d(r_{O4})}{dt} = \begin{bmatrix} v_{xO6} \\ v_{yO6} \\ v_{zO6} \end{bmatrix}$$

Al calcular el r_{O4} en base a las transformaciones de $r_{O4} = T_{01} * T_{12} * T_{23} * r_{O4}$, se obtiene.

$$r_{o4} = \begin{bmatrix} x_2 \cos q_1 \cos q_2 - z_4 \cos q_1 \cos q_2 \sin q_3 - z_4 \cos q_1 \cos q_3 \sin q_2 \\ x_2 \cos q_2 \sin q_1 - z_4 \cos q_2 \sin q_1 \sin q_3 - z_4 \cos q_3 \sin q_1 \sin q_2 \\ z_1 - x_2 \sin q_2 - z_4 \cos q_2 \cos q_3 + z_4 \sin q_2 \sin q_3 \\ 1 \end{bmatrix}$$

Y que al derivar respecto al tiempo se obtiene el Jacobiano de r_{O4} por los componentes de las velocidades angulares de los eslabones del 1-3.

$J_{rO4} =$

$z_4 \cos q_2 \sin q_1 \sin q_3 - x_2 \cos q_2 \sin q_1 + z_4 \cos q_3 \sin q_1 \sin q_2$	$z_4 \cos q_1 \sin q_2 \sin q_3 - z_4 \cos q_1 \cos q_2 \cos q_3 - x_2 \cos q_1 \sin q_2$	$z_4 \cos q_1 \sin q_2 \sin q_3 - z_4 \cos q_1 \cos q_2 \cos q_3$
$x_2 \cos q_1 \cos q_2 - z_4 \cos q_1 \cos q_2 \sin q_3 - z_4 \cos q_1 \cos q_3 \sin q_2$	$z_4 \sin q_1 \sin q_2 \sin q_3 - z_4 \cos q_2 \cos q_3 \sin q_1 - x_2 \sin q_1 \sin q_2$	$z_4 \sin q_1 \sin q_2 \sin q_3 - z_4 \cos q_2 \cos q_3 \sin q_1$
0	$z_4 \cos q_2 \sin q_3 - x_2 \cos q_2 + z_4 \cos q_3 \sin q_2$	$z_4 \cos q_2 \sin q_3 + z_4 \cos q_3 \sin q_2$

El cual resulta multiplicado por el vector w_{13} .

$z_4 \cos q_2 \sin q_1 \sin q_3 - x_2 \cos q_2 \sin q_1 + z_4 \cos q_3 \sin q_1 \sin q_2$	$z_4 \cos q_1 \sin q_2 \sin q_3 - z_4 \cos q_1 \cos q_2 \cos q_3 - x_2 \cos q_1 \sin q_2$	$z_4 \cos q_1 \sin q_2 \sin q_3 - z_4 \cos q_1 \cos q_2 \cos q_3$	\dot{q}_1
$x_2 \cos q_1 \cos q_2 - z_4 \cos q_1 \cos q_2 \sin q_3 - z_4 \cos q_1 \cos q_3 \sin q_2$	$z_4 \sin q_1 \sin q_2 \sin q_3 - z_4 \cos q_2 \cos q_3 \sin q_1 - x_2 \sin q_1 \sin q_2$	$z_4 \sin q_1 \sin q_2 \sin q_3 - z_4 \cos q_2 \cos q_3 \sin q_1$	\dot{q}_2
0	$z_4 \cos q_2 \sin q_3 - x_2 \cos q_2 + z_4 \cos q_3 \sin q_2$	$z_4 \cos q_2 \sin q_3 + z_4 \cos q_3 \sin q_2$	\dot{q}_3

Entonces se puede decir que:

$$(J_{rO4}) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} v_{xO6} \\ v_{yO6} \\ v_{zO6} \end{bmatrix}$$

Al despejar el vector de velocidades angulares $\langle \dot{q}_1, \dot{q}_2, \dot{q}_3 \rangle$ se tiene.

$$\dot{W}_{13} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = (J_{T04})^{-1} \begin{pmatrix} v_{x06} \\ v_{y06} \\ v_{z06} \end{pmatrix}$$

Esta equivalencia ya se puede procesar en C++, puesto que se conocen los datos. Para encontrar las velocidades angulares $\langle \dot{q}_4, \dot{q}_5, \dot{q}_6 \rangle$ se toma como base.

$${}^0_3R^{-1} {}^0_6R = {}^3_6R$$

Siendo $R_{06} = R_{01} \cdot R_{12} \cdot R_{23} \cdot R_{34} \cdot R_{45} \cdot R_{56}$, haciendo alusión a R_{ij} como el producto de las rotaciones de z y x en una matriz 3x3. Y suponiendo que R_{06} contiene en el momento del cálculo puras constantes puesto que se conoce la posición de las articulaciones del robot, se tiene.

$${}^0_6R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Entonces al encontrar R_{03}

$$R_{03} = \begin{pmatrix} \cos q_1 & 0 & -\sin q_1 \\ \sin q_1 & 0 & \cos q_1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos q_3 & 0 & -\sin q_3 \\ \sin q_3 & 0 & \cos q_3 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} \cos q_1 \cos q_2 \cos q_3 - \cos q_1 \sin q_2 \sin q_3 & \sin q_1 & -\cos q_1 \cos q_2 \sin q_3 - \cos q_1 \cos q_3 \sin q_2 \\ \cos q_2 \cos q_3 \sin q_1 - \sin q_1 \sin q_2 \sin q_3 & -\cos q_1 & -\cos q_2 \sin q_1 \sin q_3 - \cos q_3 \sin q_1 \sin q_2 \\ -\cos q_2 \sin q_3 - \cos q_3 \sin q_2 & 0 & \sin q_2 \sin q_3 - \cos q_2 \cos q_3 \end{pmatrix}$$

Se obtiene

$${}^3_6R = \begin{pmatrix} \cos q_1 \cos q_2 \cos q_3 - \cos q_1 \sin q_2 \sin q_3 & \sin q_1 & -\cos q_1 \cos q_2 \sin q_3 - \cos q_1 \cos q_3 \sin q_2 \\ \cos q_2 \cos q_3 \sin q_1 - \sin q_1 \sin q_2 \sin q_3 & -\cos q_1 & -\cos q_2 \sin q_1 \sin q_3 - \cos q_3 \sin q_1 \sin q_2 \\ -\cos q_2 \sin q_3 - \cos q_3 \sin q_2 & 0 & \sin q_2 \sin q_3 - \cos q_2 \cos q_3 \end{pmatrix}^{-1} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$${}^3_6R = \begin{pmatrix} \frac{1}{2} \cos(q_1 - q_2 - q_3) + \frac{1}{2} \cos(q_1 + q_2 + q_3) & \frac{1}{2} \sin(q_1 - q_2 - q_3) + \frac{1}{2} \sin(q_1 + q_2 + q_3) & -\sin(q_2 + q_3) \\ \sin q_1 & -\cos q_1 & 0 \\ \frac{1}{2} \sin(q_1 - q_2 - q_3) - \frac{1}{2} \sin(q_1 + q_2 + q_3) & \frac{1}{2} \cos(q_1 + q_2 + q_3) - \frac{1}{2} \cos(q_1 - q_2 - q_3) & -\cos(q_2 + q_3) \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Y si por otra parte R_{36} es el producto de $R_{34} \cdot R_{45} \cdot R_{56}$.

$${}^3_6R = \begin{pmatrix} \cos q_4 & 0 & -\sin q_4 \\ \sin q_4 & 0 & \cos q_4 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \cos q_5 & 0 & -\sin q_5 \\ \sin q_5 & 0 & \cos q_5 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \cos q_6 & -\sin q_6 & 0 \\ \sin q_6 & \cos q_6 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \sin q_4 \sin q_6 + \cos q_4 \cos q_5 \cos q_6 & \cos q_6 \sin q_4 - \cos q_4 \cos q_5 \sin q_6 & -\cos q_4 \sin q_5 \\ \cos q_5 \cos q_6 \sin q_4 - \cos q_4 \sin q_6 & -\cos q_4 \cos q_6 - \cos q_5 \sin q_4 \sin q_6 & -\sin q_4 \sin q_5 \\ -\cos q_6 \sin q_5 & \sin q_5 \sin q_6 & -\cos q_5 \end{pmatrix}$$

Se tiene

$$\begin{pmatrix} \frac{1}{2} \cos(q_1 - q_2 - q_3) + \frac{1}{2} \cos(q_1 + q_2 + q_3) & \frac{1}{2} \sin(q_1 - q_2 - q_3) + \frac{1}{2} \sin(q_1 + q_2 + q_3) & -\sin(q_2 + q_3) \\ \sin q_1 & -\cos q_1 & 0 \\ \frac{1}{2} \sin(q_1 - q_2 - q_3) - \frac{1}{2} \sin(q_1 + q_2 + q_3) & \frac{1}{2} \cos(q_1 + q_2 + q_3) - \frac{1}{2} \cos(q_1 - q_2 - q_3) & -\cos(q_2 + q_3) \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} \sin q_4 \sin q_6 + \cos q_4 \cos q_5 \cos q_6 & \cos q_6 \sin q_4 - \cos q_4 \cos q_5 \sin q_6 & -\cos q_4 \sin q_5 \\ \cos q_5 \cos q_6 \sin q_4 - \cos q_4 \sin q_6 & -\cos q_4 \cos q_6 - \cos q_5 \sin q_4 \sin q_6 & -\sin q_4 \sin q_5 \\ -\cos q_6 \sin q_5 & \sin q_5 \sin q_6 & -\cos q_5 \end{pmatrix}$$

Por algebra lineal se sabe que una matriz $A_{n \times m}$ es igual a una matriz $B_{n \times m}$, si y sólo si las componentes de A son igual a las componentes de B. Dicho lo anterior se eligen 3 posiciones de las matrices de ambos lados y se igualan.

$$\begin{pmatrix} f_1(q_1, q_2, q_3) \\ f_2(q_1, q_2, q_3) \\ f_3(q_1, q_2, q_3) \end{pmatrix} = \begin{pmatrix} g_1(q_4, q_5, q_6) \\ g_2(q_4, q_5, q_6) \\ g_3(q_4, q_5, q_6) \end{pmatrix}$$

Una vez apiladas se les puede sacar el jacobiano el cual resulta en los siguiente.

$$J_F \dot{W}_{13} = J_G \dot{W}_{36}, \quad \dot{W}_{13} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}, \quad \dot{W}_{36} = \begin{pmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{pmatrix}$$

Al despejar se tiene

$$A_2 \dot{W}_{13} = \dot{W}_{36} \quad \text{con} \quad A_2 = \left(J_G^1 \right) J_F$$

Por cuestiones de simplicidad, se calculó el Jacobiano de F y de G en Matlab. Para su comprobación se calculó también A2 y se introdujo en el código.

>> Código Matlab

```
function A=Jacobian
syms t1 t2 t3 t4 t5 t6 x1 x2 x3 x4 x5 x6 z1 z2 z3 z4 z5 z6 q5
syms r00 r01 r02 r10 r11 r12 r20 r21 r22 t1v t2v t3v t4v t5v t6v
T01=[cos(t1) 0 -sin(t1) 0;sin(t1) 0 cos(t1) 0;0 -1 0 z1;0 0 0 1];
T12=[cos(t2) -sin(t2) 0 x2*cos(t2);sin(t2) cos(t2) 0 x2*sin(t2);0 0 1 0; 0 0 0 1];
T23=[cos(t3) 0 -sin(t3) 0;sin(t3) 0 cos(t3) 0; 0 -1 0 0;0 0 0 1];

T34=[cos(t4) 0 -sin(t4) 0;sin(t4) 0 cos(t4) 0;0 -1 0 z4;0 0 0 1];
T45=[cos(t5) 0 -sin(t5) 0;sin(t5) 0 cos(t5) 0;0 -1 0 0;0 0 0 1];
T56=[cos(t6) -sin(t6) 0 0;sin(t6) cos(t6) 0 0; 0 0 1 z6;0 0 0 1];

T02=simplify(T01*T12);
T03=simplify(T02*T23);
T13=simplify(T12*T23);

T04=simplify(T03*T34);
T14=simplify(T13*T34);
T24=simplify(T23*T34);

T05=simplify(T04*T45);
T15=simplify(T14*T45);
T25=simplify(T24*T45);
T35=simplify(T34*T45);

T06=simplify(T05*T56);
T16=simplify(T15*T56);
T26=simplify(T25*T56);
T36=simplify(T35*T56);

R36=T36(1:3,1:3);
R03=T03(1:3,1:3);
R03inv=simplify(inv(R03));
R06=[r00 r01 r02;r10 r11 r12;r20 r21 r22];
C=R03inv*R06;
f1=C(1,1);
g1=R36(1,1);
f2=C(3,1);
g2=R36(3,1);
f3=C(2,3);
g3=R36(2,3);

F=[f1;f2;f3];
G=[g1;g2;g3];

Jg=jacobian(G,[t4,t5,t6])
%ccode(Jg);
Jf=jacobian(F,[t1,t2,t3])
%ccode(Jf);
Jginv=simplify(inv(Jg));
A2=simplify(Jginv*Jf);
%ccode(A2);
```

End

- Implementación en C++

Implementación en Cinematica inversa 1 para las primeras Velocidades angulares contenidas en el \dot{W}_{13}

```
257 bool Robot::CinematicaInversa1() . . .
276
277     /// Analisis de Velocidades Angulares [Eslabones 1-3]
```

```

278
279 Velocidad13.entry(0,0)=vxO6;
280 Velocidad13.entry(1,0)=vyO6;
281 Velocidad13.entry(2,0)=vzO6;
282
283 VAngular13 = Jacobiano13(theta1,theta2,theta3).inversa()*Velocidad13;
284
285 omega1 = VAngular13.entry(0,0);
286 omega2 = VAngular13.entry(1,0);
287 omega3 = VAngular13.entry(2,0);
    . . . }

```

Implementación en Cinematica inversa 2 para las Velocidades angulares contenidas en el \dot{W}_{36} . Se presentan dos formas de calcularlo, la primera con los A2 la cual ya ha sido sintetizada del cálculo de $(Jg^{-1})(Jf)$ y la otra con Jg y Jf de manera explicita sin operación.

```

314 bool Robot::CinematicaInversa2() . . .

```

```

379 for(int fil = 0; fil < 3; fil++){
380     for(int col = 0; col < 3; col++){
381         R06.entry(fil, col) = T06.entry(fil, col);
382     }
383 }
384
385 Matrix _A2(3,3);
386 _A2 = A2(theta1, theta2, theta3, theta4, theta5, theta6, R06);
387
388 //Metodo 1
389 VAngular46 = _A2*VAngular13;
390
391 //Metodo 2
392 //VAngular46 = (Jg(theta4, theta5, theta6).inversa()*Jf(theta1, theta2, theta3, R06))*VAngular13;
393
394 omega4 = VAngular46.entry(0,0);
395 omega5 = VAngular46.entry(1,0);
396 omega6 = VAngular46.entry(2,0);
397
    . . . }

```

Jacobiano13 equivalente a J_{104}

```

743 Matrix Robot::Jacobiano13(float x, float y, float z) // Funciones para Cinematica Inversa, Velocidades
y Aceleraciones Angulares [1-3] . . .
744 Matrix J(3,3);
745 J.entry(0,0)=(z4*cos(y)*sin(x)*sin(z))-1*(x2*cos(y)*sin(x))+(z4*cos(z)*sin(x)*sin(y));
746 J.entry(1,0)=(x2*cos(x)*cos(y))-1*(z4*cos(x)*cos(y)*sin(z))-1*(z4*cos(x)*cos(z)*sin(y));
747 J.entry(2,0)=0;
748
749 J.entry(0,1)=(z4*cos(x)*sin(y)*sin(z))-1*(z4*cos(x)*cos(y)*cos(z))-1*(x2*cos(x)*sin(y));
750 J.entry(1,1)=(z4*sin(x)*sin(y)*sin(z))-1*(z4*cos(y)*cos(z)*sin(x))-1*(x2*sin(x)*sin(y));
751 J.entry(2,1)=(z4*cos(y)*sin(z))-1*(x2*cos(y))+(z4*cos(z)*sin(y));
752
753 J.entry(0,2)=(z4*cos(x)*sin(y)*sin(z))-1*(z4*cos(x)*cos(y)*cos(z));
754 J.entry(1,2)=(z4*sin(x)*sin(y)*sin(z))-1*(z4*cos(y)*cos(z)*sin(x));
755 J.entry(2,2)=(z4*cos(y)*sin(z))+(z4*cos(z)*sin(y));
756 return J;
757 }

```

Jacobiano de G

```

809 Matrix Robot::Jg(float t4, float t5, float t6)

```

```

810 Matrix jg(3,3);
811
812 jg.entry(0,0) = cos(t4)*sin(t6)-cos(t5)*cos(t6)*sin(t4);
813 jg.entry(0,1) = -cos(t4)*cos(t6)*sin(t5);
814 jg.entry(0,2) = cos(t6)*sin(t4)-cos(t4)*cos(t5)*sin(t6);
815 jg.entry(1,0) = 0;
816 jg.entry(1,1) = -cos(t5)*cos(t6);
817 jg.entry(1,2) = sin(t5)*sin(t6);

```

```

818     jg.entry(2,0) = -cos(t4)*sin(t5);
819     jg.entry(2,1) = -cos(t5)*sin(t4);
820     jg.entry(2,2) = 0;
821
822     return jg;
823 }

```

Jacobiano de F

```

824 Matrix Robot::Jf(float t1, float t2, float t3, Matrix R)

```

```

825     Matrix jf(3,3);
826     jf.zero(3,3);
827     if(R.n == 3 && R.m == 3){
828
829         jf.entry(0,0) = R.entry(1,0)*cos(t2+t3)*cos(t1)-R.entry(0,0)*cos(t2+t3)*sin(t1);
830         jf.entry(0,1) = -R.entry(2,0)*cos(t2+t3)-R.entry(0,0)*sin(t2+t3)*cos(t1)-
            R.entry(1,0)*sin(t2+t3)*sin(t1);
831         jf.entry(0,2) = -R.entry(2,0)*cos(t2+t3)-R.entry(0,0)*sin(t2+t3)*cos(t1)-
            R.entry(1,0)*sin(t2+t3)*sin(t1);
832         jf.entry(1,0) = -R.entry(1,0)*sin(t2+t3)*cos(t1)+R.entry(0,0)*sin(t2+t3)*sin(t1);
833         jf.entry(1,1) = R.entry(2,0)*sin(t2+t3)-R.entry(0,0)*cos(t2+t3)*cos(t1)-
            R.entry(1,0)*cos(t2+t3)*sin(t1);
834         jf.entry(1,2) = R.entry(2,0)*sin(t2+t3)-R.entry(0,0)*cos(t2+t3)*cos(t1)-
            R.entry(1,0)*cos(t2+t3)*sin(t1);
835         jf.entry(2,0) = R.entry(0,2)*cos(t1)+R.entry(1,2)*sin(t1);
836         jf.entry(2,1) = 0;
837         jf.entry(2,2) = 0;
838
839     }
840     return jf;
841 }
842

```

Función equivalente a A2, la cual se encuentra sintetizada y sirve para encontrar las aceleraciones angulares.

```

773 Matrix Robot::A2(float t1, float t2, float t3, float t4, float t5, float t6, Matrix R) // Funciones para
Cinematica Inversa, Velocidades y Aceleraciones Angulares [4-6]

```

```

774     Matrix a2;
775     a2.zero(3,3);
776     if(R.n == 3 && R.m == 3){
777
778         a2.entry(0,0) = -
(R.entry(0,2)*cos(t1)*cos(t4)*cos(t6)*sin(t6)+R.entry(1,2)*cos(t4)*cos(t6)*sin(t1)*sin(t6)-
R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)-R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)
*pow(sin(t4),2.0)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*
sin(t5)*sin(t6))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)));
779         a2.entry(0,1) = -
(cos(t5)*sin(t4)*(R.entry(2,0)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6)))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)));
780         a2.entry(0,2) = -
(cos(t5)*sin(t4)*(R.entry(2,0)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6)))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)));
781         a2.entry(1,0) = -
(R.entry(0,2)*cos(t1)*cos(t4)*pow(sin(t6),2.0)+R.entry(1,2)*cos(t4)*sin(t1)*pow(sin(t6),2.0)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t6)*sin(t1)*sin(t4)-R.entry(0,0)*cos(t2+t3)*cos(t4)*sin(t1)*sin(t5)*sin(t6)-
R.entry(0,2)*cos(t1)*cos(t5)*cos(t6)*sin(t4)*sin(t6)-R.entry(1,2)*cos(t5)*cos(t6)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t5)

```



```

) *sin(t1) *sin(t6) + R.entry(1,0) *sin(t2+t3) *cos(t1) *cos(t4) *cos(t6) *sin(t4) + R.entry(1,0) *cos(t2+t3) *cos(t1) *cos(t4)
) *sin(t5) *sin(t6)) / (pow(cos(t4),2.0) *cos(t6) *sin(t6) - pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6));
782     a2.entry(1,1) =
(cos(t4) * (R.entry(2,0) *sin(t2+t3) *cos(t6) *sin(t4) + R.entry(2,0) *cos(t2+t3) *sin(t5) *sin(t6) -
R.entry(0,0) *cos(t2+t3) *cos(t1) *cos(t6) *sin(t4) - R.entry(1,0) *cos(t2+t3) *cos(t6) *sin(t1) *sin(t4) -
R.entry(2,0) *sin(t2+t3) *cos(t4) *cos(t5) *sin(t6) + R.entry(0,0) *sin(t2+t3) *cos(t1) *sin(t5) *sin(t6) + R.entry(1,0) *sin
(t2+t3) *sin(t1) *sin(t5) *sin(t6) + R.entry(0,0) *cos(t2+t3) *cos(t1) *cos(t4) *cos(t5) *sin(t6) + R.entry(1,0) *cos(t2+t3) *
cos(t4) *cos(t5) *sin(t1) *sin(t6))) / (pow(cos(t4),2.0) *cos(t6) *sin(t6) -
pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6));
783     a2.entry(1,2) =
(cos(t4) * (R.entry(2,0) *sin(t2+t3) *cos(t6) *sin(t4) + R.entry(2,0) *cos(t2+t3) *sin(t5) *sin(t6) -
R.entry(0,0) *cos(t2+t3) *cos(t1) *cos(t6) *sin(t4) - R.entry(1,0) *cos(t2+t3) *cos(t6) *sin(t1) *sin(t4) -
R.entry(2,0) *sin(t2+t3) *cos(t4) *cos(t5) *sin(t6) + R.entry(0,0) *sin(t2+t3) *cos(t1) *sin(t5) *sin(t6) + R.entry(1,0) *sin
(t2+t3) *sin(t1) *sin(t5) *sin(t6) + R.entry(0,0) *cos(t2+t3) *cos(t1) *cos(t4) *cos(t5) *sin(t6) + R.entry(1,0) *cos(t2+t3) *
cos(t4) *cos(t5) *sin(t1) *sin(t6))) / (pow(cos(t4),2.0) *cos(t6) *sin(t6) -
pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6));
784     a2.entry(2,0) = (R.entry(1,2) *pow(cos(t5),2.0) *pow(cos(t6),2.0) *sin(t1) *sin(t4) -
R.entry(1,0) *sin(t2+t3) *cos(t1) *pow(cos(t4),2.0) *cos(t6) + R.entry(1,0) *sin(t2+t3) *cos(t1) *pow(cos(t5),2.0) *cos(t6)
) + R.entry(0,0) *sin(t2+t3) *pow(cos(t4),2.0) *cos(t6) *sin(t1) -
R.entry(0,0) *sin(t2+t3) *pow(cos(t5),2.0) *cos(t6) *sin(t1) + R.entry(0,2) *cos(t1) *pow(cos(t5),2.0) *pow(cos(t6),2.0) *
sin(t4) - R.entry(0,2) *cos(t1) *cos(t4) *cos(t5) *cos(t6) *sin(t6) -
R.entry(1,2) *cos(t4) *cos(t5) *cos(t6) *sin(t1) *sin(t6) -
R.entry(1,0) *cos(t2+t3) *cos(t1) *cos(t4) *cos(t5) *cos(t6) *sin(t5) + R.entry(0,0) *cos(t2+t3) *cos(t4) *cos(t5) *cos(t6) *
sin(t1) *sin(t5) -
R.entry(1,0) *sin(t2+t3) *cos(t1) *cos(t4) *cos(t5) *sin(t4) *sin(t6) + R.entry(0,0) *sin(t2+t3) *cos(t4) *cos(t5) *sin(t1) *
sin(t4) *sin(t6)) / (sin(t5) * (pow(cos(t4),2.0) *cos(t6) *sin(t6) -
pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6)));
785     a2.entry(2,1) = -((pow(cos(t4),2.0) *cos(t6) -
pow(cos(t5),2.0) *cos(t6) + cos(t4) *cos(t5) *sin(t4) *sin(t6)) * (-
R.entry(2,0) *sin(t2+t3) + R.entry(0,0) *cos(t2+t3) *cos(t1) + R.entry(1,0) *cos(t2+t3) *sin(t1))) / (sin(t5) * (pow(cos(t4),
2.0) *cos(t6) *sin(t6) - pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6))) + (cos(t4) *cos(t5)
) *cos(t6) * (R.entry(2,0) *cos(t2+t3) + R.entry(0,0) *sin(t2+t3) *cos(t1) + R.entry(1,0) *sin(t2+t3) *sin(t1))) / (pow(cos(t4)
),2.0) *cos(t6) *sin(t6) - pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6));
786     a2.entry(2,2) = -((pow(cos(t4),2.0) *cos(t6) -
pow(cos(t5),2.0) *cos(t6) + cos(t4) *cos(t5) *sin(t4) *sin(t6)) * (-
R.entry(2,0) *sin(t2+t3) + R.entry(0,0) *cos(t2+t3) *cos(t1) + R.entry(1,0) *cos(t2+t3) *sin(t1))) / (sin(t5) * (pow(cos(t4),
2.0) *cos(t6) *sin(t6) - pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6))) + (cos(t4) *cos(t5)
) *cos(t6) * (R.entry(2,0) *cos(t2+t3) + R.entry(0,0) *sin(t2+t3) *cos(t1) + R.entry(1,0) *sin(t2+t3) *sin(t1))) / (pow(cos(t4)
),2.0) *cos(t6) *sin(t6) - pow(cos(t5),2.0) *cos(t6) *sin(t6) + cos(t4) *cos(t5) *sin(t4) -
cos(t4) *cos(t5) *pow(cos(t6),2.0) *sin(t4) *2.0 + pow(cos(t4),2.0) *pow(cos(t5),2.0) *cos(t6) *sin(t6));
787 }
788     return a2;
789 }

```

3. Calcular las aceleraciones angulares relativas para cada grado de libertad del manipulador en función del tiempo.

Solución:

- Utilizar el método expuesto en clase, utilizar software de cómputo simbólico para el cálculo de las derivadas totales de las matrices que resulten del análisis.
Si se sabe que la posición relativa del punto del Origen O4 se puede encontrar con el punto del Origen O6, con la siguiente relación:

Si del análisis anterior se sabe que:

$$(J_{rO4}) \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \begin{pmatrix} v_{xO6} \\ v_{yO6} \\ v_{zO6} \end{pmatrix} \quad (J_{rO4})(\dot{W}_{13}) = \begin{pmatrix} v_{xO6} \\ v_{yO6} \\ v_{zO6} \end{pmatrix}$$

Entonces al conocer las aceleraciones lineales de la trayectoria se pueden encontrar las aceleraciones angulares para los primeros 3 eslabones.

$$\frac{d(J_{r04})}{dt}(\dot{W}_{13}) + (J_{r04})\frac{d(\dot{W}_{13})}{dt} = \begin{pmatrix} \frac{d(x_{06})}{dt} \\ \frac{d(y_{06})}{dt} \\ \frac{d(z_{06})}{dt} \end{pmatrix}, \quad \text{Si } \frac{d(\dot{W}_{13})}{dt} = \dot{a}_{13}$$

Entonces

$$\frac{d(J_{r04})}{dt}(\dot{W}_{13}) + (J_{r04})(\dot{a}_{13}) = \begin{pmatrix} \frac{d(x_{06})}{dt} \\ \frac{d(y_{06})}{dt} \\ \frac{d(z_{06})}{dt} \end{pmatrix}$$

Si se dice que $J_{r04} = H_{3 \times 3}$, entonces se asume que:

$$\frac{d(J_{r04})}{dt} = \frac{d(H)}{dt} = \begin{array}{|c|c|c|} \hline \frac{d(H_{11})}{dt} & \frac{d(H_{12})}{dt} & \frac{d(H_{13})}{dt} \\ \hline \frac{d(H_{21})}{dt} & \frac{d(H_{22})}{dt} & \frac{d(H_{23})}{dt} \\ \hline \frac{d(H_{31})}{dt} & \frac{d(H_{32})}{dt} & \frac{d(H_{33})}{dt} \\ \hline \end{array}$$

Y si se sabe que

$$\frac{d(H_{ij})}{dt} = J(H_{ij}) \cdot (\dot{W}_{13})$$

Entonces

$$\frac{d(J_{r04})}{dt} = \begin{array}{|c|c|c|} \hline J(H_{11}) \cdot (\dot{W}_{13}) & J(H_{12}) \cdot (\dot{W}_{13}) & J(H_{13}) \cdot (\dot{W}_{13}) \\ \hline J(H_{21}) \cdot (\dot{W}_{13}) & J(H_{22}) \cdot (\dot{W}_{13}) & J(H_{23}) \cdot (\dot{W}_{13}) \\ \hline J(H_{31}) \cdot (\dot{W}_{13}) & J(H_{32}) \cdot (\dot{W}_{13}) & J(H_{33}) \cdot (\dot{W}_{13}) \\ \hline \end{array}$$

Lo cual ya se puede calcular por medio de computo simbólico, en este caso Matlab.

>> Código Matlab para encontrar la derivada del Jacobiano13 o J_{r04} respecto al tiempo

```
function J=Jacobian
syms x y z z1 z2 z3 z4 x2 dx dy dz

T01=[cos(x) 0 -sin(x) 0; sin(x) 0 cos(x) 0; 0 -1 0 z1; 0 0 0 1];

T12=[cos(y) -sin(y) 0 x2*cos(y); sin(y) cos(y) 0 x2*cos(y); 0 0 1 0; 0 0 0 1];

T23=[cos(z) 0 -sin(z) 0; sin(z) 0 cos(z) 0; 0 -1 0 0; 0 0 0 1];
P = [0; 0; z4; 1];

R04= T01*T12*T23*P;

A= jacobian([R04],[x,y,z]);

qv=[dx;dy;dz];

dA00= ccode(jacobian(A(1,1),[x,y,z])*qv)
dA01= ccode(jacobian(A(1,2),[x,y,z])*qv);
dA02= ccode(jacobian(A(1,3),[x,y,z])*qv);

dA10= ccode(jacobian(A(2,1),[x,y,z])*qv);
dA11= ccode(jacobian(A(2,2),[x,y,z])*qv);
dA12= ccode(jacobian(A(2,3),[x,y,z])*qv);
```

```

dA20= ccode(jacobian(A(3,1),[x,y,z])*qv);
dA21= ccode(jacobian(A(3,2),[x,y,z])*qv);
dA22= ccode(jacobian(A(3,3),[x,y,z])*qv);

return
end

```

Así pues al despejar Alpha13 y sustituir el valor de los vectores y matrices en cada punto se logran encontrar las aceleraciones angulares de las primeras 3 articulaciones.

$$\dot{\alpha}_{13} = (J_{r04})^{-1} \left(\begin{pmatrix} \ddot{x}_{06} \\ \ddot{y}_{06} \\ \ddot{z}_{06} \end{pmatrix} - \frac{d(J_{r04})}{dt} (\dot{W}_{13}) \right) = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix}$$

Para encontrar las aceleraciones de los siguientes grados de libertad se toma como base la relación antes mencionada para encontrar las velocidades angulares del 4to al 6to grado de libertad.

$$J_F \dot{W}_{13} = J_G \dot{W}_{36}, \quad \dot{W}_{13} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}, \quad \dot{W}_{36} = \begin{pmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{pmatrix}$$

$$A_2 \dot{W}_{13} = \dot{W}_{36} \quad \text{con} \quad A_2 = (J_G^{-1}) J_F$$

Derivando respecto al tiempo de ambos lados se obtiene

$$A_2 \left(\frac{d(\dot{W}_{13})}{dt} \right) + \frac{d(A_2)}{dt} (\dot{W}_{13}) = \frac{d(\dot{W}_{36})}{dt}$$

Si

$$\begin{aligned} \frac{d(\dot{W}_{13})}{dt} &= \dot{\alpha}_{13} \\ \frac{d(\dot{W}_{36})}{dt} &= \dot{\alpha}_{36} \\ \frac{d(A_2)}{dt} &= \dot{A}_2 \end{aligned}$$

La ecuación se simplifica en

$$A_2 \dot{\alpha}_{13} + \dot{A}_2 \dot{W}_{13} = \dot{\alpha}_{36}$$

De la cual sólo hace falta conocer \dot{A}_2

Tomando a

$$A_2 = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Entonces se puede simplificar las columnas de la matriz en

$$\mathbf{a}_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$$

$$\mathbf{a}_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$$

$$\mathbf{a}_3 = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

Entonces expandiendo se tiene que \dot{A}_2 es igual a

$$\frac{d}{dt}A_2 = \left(\frac{d}{dt}\mathbf{a}_1 \quad \frac{d}{dt}\mathbf{a}_2 \quad \frac{d}{dt}\mathbf{a}_3 \right)$$

$$\frac{d}{dt}\mathbf{a}_1 = \begin{pmatrix} \frac{d(a_{11})}{dt} \\ \frac{d(a_{21})}{dt} \\ \frac{d(a_{31})}{dt} \end{pmatrix}$$

$$\frac{d}{dt}\mathbf{a}_2 = \begin{pmatrix} \frac{d(a_{12})}{dt} \\ \frac{d(a_{22})}{dt} \\ \frac{d(a_{32})}{dt} \end{pmatrix}$$

$$\frac{d}{dt}\mathbf{a}_3 = \begin{pmatrix} \frac{d(a_{13})}{dt} \\ \frac{d(a_{23})}{dt} \\ \frac{d(a_{33})}{dt} \end{pmatrix}$$

Si anteriormente se ha mencionado el uso del jacobiano para componentes de una matriz se puede deducir que

$$\frac{da_{ij}}{dt} = \frac{\partial a_{ij}}{\partial q_1} \dot{q}_1 + \frac{\partial a_{ij}}{\partial q_2} \dot{q}_2 + \frac{\partial a_{ij}}{\partial q_3} \dot{q}_3 + \frac{\partial a_{ij}}{\partial q_4} \dot{q}_4 + \frac{\partial a_{ij}}{\partial q_5} \dot{q}_5 + \frac{\partial a_{ij}}{\partial q_6} \dot{q}_6$$

Que es equivalente a esta notación

$$\frac{da_{ij}}{dt} = J(a_{ij}) \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{pmatrix}$$

Con ello se puede decir que \dot{A}_2 utilizando Jacobianos es igual a:

$$\frac{d}{dt}A_2 = \left(J_{a_1}\dot{\mathbf{q}} \quad J_{a_2}\dot{\mathbf{q}} \quad J_{a_3}\dot{\mathbf{q}} \right)$$

El cual se puede encontrar con la ayuda de Matlab, retomando el ejercicio para encontrar las velocidades angulares de los grados de libertad 3-6, del ejercicio 2.

>> Complementación código Matlab

```

A2=simplify(Jginv*Jf);
%ccode(A2);

a1=[A2(1,1); A2(2,1); A2(3,1)];
a2=[A2(1,2); A2(2,2); A2(3,2)];
a3=[A2(1,3); A2(2,3); A2(3,3)];

Ja1=jacobian(a1,[t1,t2,t3,t4,t5,t6]);
Ja2=jacobian(a2,[t1,t2,t3,t4,t5,t6]);
Ja3=jacobian(a3,[t1,t2,t3,t4,t5,t6]);

qv=[t1v;t2v;t3v;t4v;t5v;t6v];
dA2=[Ja1*qv,Ja2*qv,Ja3*qv]
ccode(dA2)

```

Con lo cual ya sólo se necesitaría sustituir dA2 en para encontrar

$$A_2 \ddot{\alpha}_{13} + \dot{A}_2 \dot{W}_{13} = \ddot{\alpha}_{36}$$

- Implementación en C++

Implementación en Cinematica inversa 1 para las primeras Aceleraciones angulares contenidas en el vector

 $\dot{\alpha}_{13}$

```

257 bool Robot::CinematicaInversa1() . . .
288
289     /// Analisis de Aceleraciones Angulares [Eslabones 1-3]
290
291     Aceleracion13.entry(0,0)=axO6;
292     Aceleracion13.entry(1,0)=ayO6;
293     Aceleracion13.entry(2,0)=azO6;
294
295     Matrix dA13(3,3);
296     dA13 = dJacobian13(theta1, theta2, theta3, omega1, omega2, omega3);
297     AAngular13 = (Jacobiano13(theta1, theta2, theta3).inversa())*(Aceleracion13 - dA13*VAngular13);
298
299     alpha1 = AAngular13.entry(0,0);
300     alpha2 = AAngular13.entry(1,0);
301     alpha3 = AAngular13.entry(2,0);
302
303     return true;
304 }
. . . }

```

Implementación en Cinematica inversa 2 para las siguientes Aceleraciones angulares contenidas en el vector

 $\ddot{\alpha}_{36}$

```

314 bool Robot::CinematicaInversa2() . . .
398     /// Analisis de Aceleraciones Angulares [Eslabones 4-6]
399
400     Matrix DA2(3,3);
401     DA2 = dA2(theta1, theta2, theta3, theta4, theta5, theta6, omega1, omega2, omega3, omega4, omega5,
402         omega6, R06);
403
404     AAngular46 = A2*AAngular13 + DA2*VAngular13;
405     alpha4 = AAngular46.entry(0,0);
406     alpha5 = AAngular46.entry(1,0);
407     alpha6 = AAngular46.entry(2,0);
. . . }

```

Función que contiene la matriz resultante del código en Matlab para

 $\frac{d(J_{R04})}{dt}$

```

758 Matrix Robot::dJacobian13(float x, float y, float z, float dx, float dy, float dz) {
759     Matrix dJ(3,3);

```

```

760    dJ.entry(0,0) = -dy*(z4*(sin(x)*sin(y)*sin(z)-cos(y)*cos(z)*sin(x))-
x2*sin(x)*sin(y))+dx*(z4*(cos(x)*cos(y)*sin(z)+cos(x)*cos(z)*sin(y))-x2*cos(x)*cos(y))-
dz*z4*(sin(x)*sin(y)*sin(z)-cos(y)*cos(z)*sin(x));
761    dJ.entry(0,1) = -dx*(z4*(sin(x)*sin(y)*sin(z)-cos(y)*cos(z)*sin(x))-
x2*sin(x)*sin(y))+dy*(z4*(cos(x)*cos(y)*sin(z)+cos(x)*cos(z)*sin(y))-
x2*cos(x)*cos(y))+dz*z4*(cos(x)*cos(y)*sin(z)+cos(x)*cos(z)*sin(y));
762    dJ.entry(0,2) =
dy*z4*(cos(x)*cos(y)*sin(z)+cos(x)*cos(z)*sin(y))+dz*z4*(cos(x)*cos(y)*sin(z)+cos(x)*cos(z)*sin(y))-
dx*z4*(sin(x)*sin(y)*sin(z)-cos(y)*cos(z)*sin(x));
763
764    dJ.entry(1,0) = dx*(z4*(cos(y)*sin(x)*sin(z)+cos(z)*sin(x)*sin(y))-
x2*cos(y)*sin(x))+dy*(z4*(cos(x)*sin(y)*sin(z)-cos(x)*cos(y)*cos(z))-
x2*cos(x)*sin(y))+dz*z4*(cos(x)*sin(y)*sin(z)-cos(x)*cos(y)*cos(z));
765    dJ.entry(1,1) = dy*(z4*(cos(y)*sin(x)*sin(z)+cos(z)*sin(x)*sin(y))-
x2*cos(y)*sin(x))+dx*(z4*(cos(x)*sin(y)*sin(z)-cos(x)*cos(y)*cos(z))-
x2*cos(x)*sin(y))+dz*z4*(cos(y)*sin(x)*sin(z)+cos(z)*sin(x)*sin(y));
766    dJ.entry(1,2) = dx*z4*(cos(x)*sin(y)*sin(z)-
cos(x)*cos(y)*cos(z))+dy*z4*(cos(y)*sin(x)*sin(z)+cos(z)*sin(x)*sin(y))+dz*z4*(cos(y)*sin(x)*sin(z)+cos(z)*sin(x)
)*sin(y));
767
768    dJ.entry(2,0) = 0;
769    dJ.entry(2,1) = dy*(z4*(cos(y)*cos(z)-sin(y)*sin(z))+x2*cos(y))+dz*z4*(cos(y)*cos(z)-sin(y)*sin(z));
770    dJ.entry(2,2) = dy*z4*(cos(y)*cos(z)-sin(y)*sin(z))+dz*z4*(cos(y)*cos(z)-sin(y)*sin(z));
771    return dJ;
772 }

```

Función que contiene la matriz resultante del código de Matlab para A_2 .

```

790    Matrix Robot::dA2(float t1, float t2, float t3, float t4, float t5, float t6, float t1v, float t2v, float
t3v, float t4v, float t5v, float t6v, Matrix R)
791    Matrix da2;
792    da2.identity(3);
793    if(R.n == 3 && R.m == 3){
794
795        da2.entry(0,0) =
t4v*((R.entry(0,2)*cos(t1)*cos(t6)*sin(t4)*sin(t6)+R.entry(1,2)*cos(t6)*sin(t1)*sin(t4)*sin(t6)+R.entry(0,2)*cos
(t1)*cos(t4)*cos(t5)*pow(cos(t6),2.0)+R.entry(1,2)*cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t1)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*pow(cos(t5),2.0)*sin(t6)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.
0)*pow(cos(t5),2.0)*sin(t1)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*pow(sin(t4),2.0)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*pow(cos(t5),2.0)*sin(t1)*pow(sin(t4),2.0)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4
)*cos(t5)*cos(t6)*sin(t4)*2.0+R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t5)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*cos(t6)*sin(t1)*sin(t4)*2.0-
R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*sin(t1)*sin(t5)*sin(t6))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))-
(1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0))*(-
pow(cos(t4),2.0)*cos(t5)+cos(t5)*pow(sin(t4),2.0)+pow(cos(t4),2.0)*cos(t5)*pow(cos(t6),2.0)*2.0-
cos(t5)*pow(cos(t6),2.0)*pow(sin(t4),2.0)*2.0+cos(t4)*cos(t6)*sin(t4)*sin(t6)*2.0+cos(t4)*pow(cos(t5),2.0)*cos(t
6)*sin(t4)*sin(t6)*2.0)*(R.entry(0,2)*cos(t1)*cos(t4)*cos(t6)*sin(t6)+R.entry(1,2)*cos(t4)*cos(t6)*sin(t1)*sin(t
6)-R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)-R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)
)*pow(sin(t4),2.0)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*pow(cos(t5),2.0)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*
sin(t5)*sin(t6))/(sin(t5))-t6v*((R.entry(0,2)*cos(t1)*cos(t4)*pow(cos(t6),2.0)-
R.entry(0,2)*cos(t1)*cos(t4)*pow(sin(t6),2.0)+R.entry(1,2)*cos(t4)*pow(cos(t6),2.0)*sin(t1)-
R.entry(1,2)*cos(t4)*sin(t1)*pow(sin(t6),2.0)+R.entry(0,2)*cos(t1)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*2.0+R.entry(1
,2)*cos(t5)*cos(t6)*sin(t1)*sin(t4)*sin(t6)*2.0+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*pow(sin(t4),2.0)*sin(t6)
-
R.entry(0,0)*sin(t2+t3)*cos(t5)*sin(t1)*pow(sin(t4),2.0)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos
(t5),2.0)*cos(t6)*sin(t4)-R.entry(0,0)*sin(t2+t3)*cos(t4)*pow(cos(t5),2.0)*cos(t6)*sin(t1)*sin(t4)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t5)*cos(t6)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t5)*cos(t6)*sin(t1)*
sin(t4)*sin(t5))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))-
(1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0))*(pow(cos(t4)
,2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*pow(cos(t6),2.0)-
pow(cos(t4),2.0)*pow(sin(t6),2.0)+pow(cos(t5),2.0)*pow(sin(t6),2.0)+pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(cos(t6)
),2.0)-
pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0)*(R.entry(0,2)*co
s(t1)*cos(t4)*cos(t6)*sin(t6)+R.entry(1,2)*cos(t4)*cos(t6)*sin(t1)*sin(t6)-
R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)-R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-

```

```

R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)
*pow(sin(t4),2.0)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*pow(cos(t5),2.0)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*
sin(t5)*sin(t6))/sin(t5))-
t5v*((R.entry(0,2)*cos(t1)*pow(cos(t6),2.0)*sin(t4)*sin(t5)+R.entry(1,2)*pow(cos(t6),2.0)*sin(t1)*sin(t4)*sin(t5)
)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*pow(cos(t5),2.0)*sin(t4)*sin(t6)+R.entry(0,0)*cos(t2+t3)*pow(cos(t5),2.0)*sin(t1)
)*sin(t4)*sin(t6)+R.entry(1,0)*cos(t2+t3)*cos(t1)*sin(t4)*pow(sin(t5),2.0)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t
1)*cos(t6)*pow(sin(t4),2.0)*sin(t5)-R.entry(0,0)*cos(t2+t3)*sin(t1)*sin(t4)*pow(sin(t5),2.0)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t6)*sin(t1)*pow(sin(t4),2.0)*sin(t5)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t4)*sin(t5)*sin(t6)*2.0+R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(
t5)*sin(t1)*sin(t4)*sin(t5)*sin(t6)*2.0)/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6))-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(cos(t4)*sin(t
4)*sin(t5)-cos(t4)*pow(cos(t6),2.0)*sin(t4)*sin(t5)*2.0-
cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0+pow(cos(t4),2.0)*cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0)*1.0/pow(pow(cos(t4),2.
0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*sin(t6),2.0)*(R.entry(0,2)
)*cos(t1)*cos(t4)*cos(t6)*sin(t6)+R.entry(1,2)*cos(t5)*sin(t1)*sin(t6)-
R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)-R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)
*pow(sin(t4),2.0)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*pow(cos(t5),2.0)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*
sin(t5)*sin(t6))/sin(t5)-
(cos(t5)*1.0/pow(sin(t5),2.0)*(R.entry(0,2)*cos(t1)*cos(t4)*cos(t6)*sin(t6)+R.entry(1,2)*cos(t4)*cos(t6)*sin(t1)
)*sin(t6)-R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)-
R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)
*pow(sin(t4),2.0)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*pow(cos(t5),2.0)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*
sin(t5)*sin(t6))/sin(t5)-
(t1v*(R.entry(1,2)*cos(t1)*cos(t4)*cos(t6)*sin(t6)-R.entry(0,2)*cos(t4)*cos(t6)*sin(t1)*sin(t6)-
R.entry(1,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)+R.entry(0,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)+R.en
try(0,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)+R.entry(1,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)*pow
(sin(t4),2.0)-R.entry(0,0)*sin(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)-
R.entry(1,0)*sin(t2+t3)*cos(t4)*pow(cos(t5),2.0)*sin(t1)*sin(t4)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t5)
*sin(t4)*sin(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*sin(t5)*sin(t6))/sin(t5)*(pow(cos(t4)
,2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(t2v*(R.entry(1
,0)*cos(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)-
R.entry(0,0)*cos(t2+t3)*cos(t5)*cos(t6)*sin(t1)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*sin(t1)*sin(t4)
)*sin(t5)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t4)*pow(cos
(t5),2.0)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6))/sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(t3v*(R.entry(1
,0)*cos(t2+t3)*cos(t1)*cos(t5)*cos(t6)*pow(sin(t4),2.0)-
R.entry(0,0)*cos(t2+t3)*cos(t5)*cos(t6)*sin(t1)*pow(sin(t4),2.0)+R.entry(0,0)*sin(t2+t3)*cos(t5)*sin(t1)*sin(t4)
)*sin(t5)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*sin(t4)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t4)*pow(cos
(t5),2.0)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t5)*sin(t6))/sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))));
796      da2.entry(0,1) = -
t6v*((R.entry(0,2)*cos(t1)*cos(t4)*cos(t6)*sin(t6)*2.0+R.entry(1,2)*cos(t4)*cos(t6)*sin(t1)*sin(t6)*2.0-
R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)+R.entry(0,2)*cos(t1)*cos(t5)*sin(t4)*pow(sin(t6),2.0)-
R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)+R.entry(1,2)*cos(t5)*sin(t1)*sin(t4)*pow(sin(t6),2.0)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*sin(t4)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t4)*sin(t1)*sin(t4)*sin(t6)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*cos(t5)*cos(t6)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t5)
)*cos(t6)*sin(t1)+R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t6)*sin(t5)-
R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t6)*sin(t1)*sin(t5))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))-
1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(pow(cos(t4)
,2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*pow(cos(t6),2.0)-
pow(cos(t4),2.0)*pow(sin(t6),2.0)+pow(cos(t5),2.0)*pow(sin(t6),2.0)+pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(cos(t6)
),2.0)-
pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0)*(R.entry(0,2)*co
s(t1)*cos(t4)*pow(sin(t6),2.0)+R.entry(1,2)*cos(t4)*sin(t1)*pow(sin(t6),2.0)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t6)*sin(t1)*sin(t4)-R.entry(0,0)*cos(t2+t3)*cos(t4)*sin(t1)*sin(t5)*sin(t6)-

```


[illegible]


```

6)*sin(t4)*sin(t6)*2.0)*(R.entry(1,2)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*cos(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*cos(t6)
)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t6)*sin(t1)-
R.entry(0,0)*sin(t2+t3)*pow(cos(t5),2.0)*cos(t6)*sin(t1)+R.entry(0,2)*cos(t1)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*
sin(t4)-R.entry(0,2)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t6)-
R.entry(1,2)*cos(t4)*cos(t5)*cos(t6)*sin(t1)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*cos(t6)*
sin(t1)*sin(t5)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t4)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*
sin(t4)*sin(t6))/sin(t5))-t6v*(-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*sin(t6)
)+R.entry(0,2)*cos(t1)*cos(t4)*cos(t5)*pow(cos(t6),2.0)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*sin(t1)*sin(t6)
)-R.entry(0,0)*sin(t2+t3)*pow(cos(t5),2.0)*sin(t1)*sin(t6)-
R.entry(0,2)*cos(t1)*cos(t4)*cos(t5)*pow(sin(t6),2.0)+R.entry(1,2)*cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t1)-
R.entry(1,2)*cos(t4)*cos(t5)*sin(t1)*pow(sin(t6),2.0)+R.entry(0,2)*cos(t1)*pow(cos(t5),2.0)*cos(t6)*sin(t4)*sin(
t6)*2.0+R.entry(1,2)*pow(cos(t5),2.0)*cos(t6)*sin(t1)*sin(t4)*sin(t6)*2.0+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)
)*cos(t5)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t5)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*cos(t6)*sin(t1)*sin(t4)+R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*sin(t1)*
sin(t5)*sin(t6))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(1.0/pow(pow(co
s(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))* (pow(cos(t4)
,2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*pow(cos(t6),2.0)-
pow(cos(t4),2.0)*pow(sin(t6),2.0)+pow(cos(t5),2.0)*pow(sin(t6),2.0)+pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(cos(t6)
,2.0)-
pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0)*(R.entry(1,2)*po
w(cos(t5),2.0)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*cos(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*cos(t6)
)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t6)*sin(t1)-
R.entry(0,0)*sin(t2+t3)*pow(cos(t5),2.0)*cos(t6)*sin(t1)+R.entry(0,2)*cos(t1)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*
sin(t4)-R.entry(0,2)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t6)-
R.entry(1,2)*cos(t4)*cos(t5)*cos(t6)*sin(t1)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*cos(t6)*
sin(t1)*sin(t5)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t4)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*
sin(t4)*sin(t6))/sin(t5))+t5v*((R.entry(0,0)*sin(t2+t3)*cos(t5)*cos(t6)*sin(t1)*sin(t5)*2.0+R.entry(0,2)*cos(t1)
)*cos(t4)*cos(t6)*sin(t5)*sin(t6)+R.entry(1,2)*cos(t4)*cos(t6)*sin(t1)*sin(t5)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*pow(cos(t5),2.0)*cos(t6)+R.entry(0,0)*cos(t2+t3)*cos(t4)*pow(cos(t5),2.0)
)*cos(t6)*sin(t1)+R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t6)*pow(sin(t5),2.0)-
R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t6)*sin(t1)*pow(sin(t5),2.0)-
R.entry(0,2)*cos(t1)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*sin(t5)*2.0-
R.entry(1,2)*cos(t5)*pow(cos(t6),2.0)*sin(t1)*sin(t4)*sin(t5)*2.0-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t5)*cos(t6)*sin(t5)*2.0-
R.entry(0,0)*sin(t2+t3)*cos(t4)*sin(t1)*sin(t4)*sin(t5)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*sin(t4)*
sin(t5)*sin(t6))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(cos(t4)*sin(t
4)*sin(t5)-cos(t4)*pow(cos(t6),2.0)*sin(t4)*sin(t5)*2.0-
cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0+pow(cos(t4),2.0)*cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0)*1.0/pow(pow(cos(t4),2.
0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(R.entry(1,2)
)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*cos(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*cos(t6)
)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t6)*sin(t1)-
R.entry(0,0)*sin(t2+t3)*pow(cos(t5),2.0)*cos(t6)*sin(t1)+R.entry(0,2)*cos(t1)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*
sin(t4)-R.entry(0,2)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t6)-
R.entry(1,2)*cos(t4)*cos(t5)*cos(t6)*sin(t1)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*cos(t6)*
sin(t1)*sin(t5)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t4)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*
sin(t4)*sin(t6))/sin(t5)-
(cos(t5)*1.0/pow(sin(t5),2.0)*(R.entry(1,2)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*cos(t6)+R.entry(1,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*cos(t6)
)+R.entry(0,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t6)*sin(t1)-
R.entry(0,0)*sin(t2+t3)*pow(cos(t5),2.0)*cos(t6)*sin(t1)+R.entry(0,2)*cos(t1)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*
sin(t4)-R.entry(0,2)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t6)-
R.entry(1,2)*cos(t4)*cos(t5)*cos(t6)*sin(t1)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*cos(t6)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*cos(t6)*
sin(t1)*sin(t5)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t4)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*
sin(t4)*sin(t6))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+t1v*(-
R.entry(0,2)*pow(cos(t5),2.0)*pow(cos(t6),2.0)*sin(t1)*sin(t4)+R.entry(0,0)*sin(t2+t3)*cos(t1)*pow(cos(t4),2.0)*
cos(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t1)*pow(cos(t5),2.0)*cos(t6)+R.entry(1,0)*sin(t2+t3)*pow(cos(t4),2.0)*cos(t6)*sin(t1)
) -

```

[illegible]

[illegible]

[illegible]

[illegible]

```

R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6))/sin(t5))-
t4v*((cos(t4)*cos(t5)*sin(t1)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6))/sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))-
(cos(t5)*sin(t4))*(-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t6)*sin(t1)-
R.entry(2,0)*sin(t2+t3)*cos(t5)*sin(t4)*sin(t6)+R.entry(1,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*sin(t6)+R.entry(
0,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t6))/sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(cos(t5)*sin(t4
)*1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0))*(-
pow(cos(t4),2.0)*cos(t5)+cos(t5)*pow(sin(t4),2.0)+pow(cos(t4),2.0)*cos(t5)*pow(cos(t6),2.0)*
cos(t5)*pow(cos(t6),2.0)*pow(sin(t4),2.0)*2.0+cos(t4)*cos(t6)*sin(t4)*sin(t6)*2.0+cos(t4)*pow(cos(t5),2.0)*cos(t
6)*sin(t4)*sin(t6)*2.0)*(R.entry(2,0)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6))/sin(t5))-t6v*((cos(t5)*sin(t4)*(R.entry(2,0)*cos(t2+t3)*cos(t6)*sin(t5)-
R.entry(2,0)*sin(t2+t3)*sin(t4)*sin(t6)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*cos(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*sin(t4)*sin(t6)+R.entry(0,0)*sin
(t2+t3)*cos(t1)*cos(t6)*sin(t5)+R.entry(1,0)*cos(t2+t3)*sin(t1)*sin(t4)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t6)*
sin(t1)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*cos(t6)+R.entry(1,0)*cos(t2+t3)*cos(t4)*cos(t5)*
cos(t6)*sin(t1)))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0))*(-
cos(t5)*sin(t4)*1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(pow(cos(t4
),2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*pow(cos(t6),2.0)-
pow(cos(t4),2.0)*pow(sin(t6),2.0)+pow(cos(t5),2.0)*pow(sin(t6),2.0)+pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(cos(t6
),2.0))-
pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0)*(R.entry(2,0)*si
n(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6))/sin(t5))-t2v*cos(t5)*sin(t4)*(R.entry(2,0)*cos(t2+t3)*cos(t6)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*sin(t5)*sin(t6)-
R.entry(2,0)*cos(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*cos(t6)*sin(t4)+R.entry(0,0)*cos
(t2+t3)*cos(t1)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t6)*sin(t1)*sin(t4)+R.entry(1,0)*cos(t2+t3)*sin(t1)*
sin(t5)*sin(t6)-R.entry(1,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6))/sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(t1v*cos(t5)*si
n(t4)*(R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(0,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(1,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*sin(t1)*sin(t5)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t4)*cos(t5)*sin(t1)*sin(t6))
)/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))));
802      da2.entry(2,1) =
t5v*((cos(t4)*(R.entry(2,0)*cos(t2+t3)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*cos(t5)*sin(t6)+R.entry(1
,0)*sin(t2+t3)*cos(t5)*sin(t1)*sin(t6)+R.entry(2,0)*sin(t2+t3)*cos(t4)*sin(t5)*sin(t6)-
R.entry(1,0)*cos(t2+t3)*cos(t4)*sin(t1)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*sin(t5)*sin(t6)))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))+cos(t4)*(cos(t4)
)*sin(t4)*sin(t5)-cos(t4)*pow(cos(t6),2.0)*sin(t4)*sin(t5)*2.0-
cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0+pow(cos(t4),2.0)*cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0)*1.0/pow(pow(cos(t4),2.
0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(R.entry(2,0
)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-

```

```
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*cos(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*sin(t4)*sin(t6)+R.entry(0,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6)))+t6v*((cos(t4)*(R.entry(2,0)*cos(t2+t3)*cos(t6)*sin(t5)-
R.entry(2,0)*sin(t2+t3)*sin(t4)*sin(t6))-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*cos(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*sin(t4)*sin(t6)+R.entry(0,0)*sin
(t2+t3)*cos(t1)*cos(t6)*sin(t5)+R.entry(1,0)*cos(t2+t3)*sin(t1)*sin(t4)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t6)*
sin(t1)*sin(t5)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*cos(t6)+R.entry(1,0)*cos(t2+t3)*cos(t4)*cos(t5)*
cos(t6)*sin(t1))/((pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))-
cos(t4)*1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(pow(cos(t4)
,2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*pow(cos(t6),2.0)-
pow(cos(t4),2.0)*pow(sin(t6),2.0)+pow(cos(t5),2.0)*pow(sin(t6),2.0)+pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(cos(t6)
),2.0)-
pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0)*(R.entry(2,0)*si
n(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6)))-
t4v*((sin(t4)*(R.entry(2,0)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6)))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))+cos(t4)*(-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t6)*sin(t1)-
R.entry(2,0)*sin(t2+t3)*cos(t5)*sin(t4)*sin(t6)+R.entry(1,0)*cos(t2+t3)*cos(t5)*sin(t1)*sin(t4)*sin(t6)+R.entry(
0,0)*cos(t2+t3)*cos(t1)*cos(t5)*sin(t4)*sin(t6)))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))-
cos(t4)*1.0/pow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0))*(-
pow(cos(t4),2.0)*cos(t5)+cos(t5)*pow(sin(t4),2.0)+pow(cos(t4),2.0)*cos(t5)*pow(cos(t6),2.0)*2.0-
cos(t5)*pow(cos(t6),2.0)*pow(sin(t4),2.0)*2.0+cos(t4)*cos(t6)*sin(t4)*sin(t6)*2.0+cos(t4)*pow(cos(t5),2.0)*cos(t
6)*sin(t4)*sin(t6)*2.0*(R.entry(2,0)*sin(t2+t3)*cos(t6)*sin(t4)+R.entry(2,0)*cos(t2+t3)*sin(t5)*sin(t6)-
R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t6)*sin(t4)-R.entry(1,0)*cos(t2+t3)*cos(t6)*sin(t1)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin
(t2+t3)*sin(t1)*sin(t5)*sin(t6)+R.entry(0,0)*cos(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)+R.entry(1,0)*cos(t2+t3)*
cos(t4)*cos(t5)*sin(t1)*sin(t6)))+(t2v*cos(t4)*(R.entry(2,0)*cos(t2+t3)*cos(t6)*sin(t4)-
R.entry(2,0)*sin(t2+t3)*sin(t5)*sin(t6)-
R.entry(2,0)*cos(t2+t3)*cos(t4)*cos(t5)*sin(t6)+R.entry(0,0)*sin(t2+t3)*cos(t1)*cos(t6)*sin(t4)+R.entry(0,0)*cos
(t2+t3)*cos(t1)*sin(t5)*sin(t6)+R.entry(1,0)*sin(t2+t3)*cos(t6)*sin(t1)*sin(t4)+R.entry(1,0)*cos(t2+t3)*sin(t1)*
sin(t5)*sin(t6)-R.entry(1,0)*sin(t2+t3)*cos(t4)*cos(t5)*sin(t1)*sin(t6)-
R.entry(0,0)*sin(t2+t3)*cos(t1)*cos(t4)*cos(t5)*sin(t6)))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6));
803      da2.entry(2,2) = -t5v*((cos(t5)*cos(t6)*sin(t5)*2.0-cos(t4)*sin(t4)*sin(t5)*sin(t6))*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1)))/(sin(t5)*(pow(cos(t4),
2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(pow(cos(t4),2
.0)*cos(t6)-pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6))*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1))*(cos(t4)*sin(t4)*sin(t5)
)-cos(t4)*pow(cos(t6),2.0)*sin(t4)*sin(t5)*2.0-
cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0+pow(cos(t4),2.0)*cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0*1.0/pow(pow(cos(t4),2.
0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0))/sin(t5)-
(cos(t5)*1.0/pow(sin(t5),2.0)*(pow(cos(t4),2.0)*cos(t6)-
pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6))*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1)))/(pow(cos(t4),2.0)*cos(
t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))+cos(t4)*cos(t6)
*sin(t5)*(R.entry(2,0)*cos(t2+t3)+R.entry(0,0)*sin(t2+t3)*cos(t1)+R.entry(1,0)*sin(t2+t3)*sin(t1)))/(pow(cos(t4)
```

```

,2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))-
cos(t4)*cos(t5)*cos(t6)*(R.entry(2,0)*cos(t2+t3)+R.entry(0,0)*sin(t2+t3)*cos(t1)+R.entry(1,0)*sin(t2+t3)*sin(t1)
)*(cos(t4)*sin(t4)*sin(t5)-cos(t4)*pow(cos(t6),2.0)*sin(t4)*sin(t5)*2.0-
cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0+pow(cos(t4),2.0)*cos(t5)*cos(t6)*sin(t5)*sin(t6)*2.0)*1.0/pow(pow(cos(t4),2.
0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))-
tlv*((R.entry(1,0)*cos(t2+t3)*cos(t1)-R.entry(0,0)*cos(t2+t3)*sin(t1))*(pow(cos(t4),2.0)*cos(t6)-
pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6)))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))-
(cos(t4)*cos(t5)*cos(t6)*(R.entry(1,0)*sin(t2+t3)*cos(t1)-
R.entry(0,0)*sin(t2+t3)*sin(t1)))/(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+t2v*((pow(cos(
t4),2.0)*cos(t6)-
pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6))*(R.entry(2,0)*cos(t2+t3)+R.entry(0,0)*sin(t2+t3)*cos(t
1)+R.entry(1,0)*sin(t2+t3)*sin(t1)))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(cos(t4)*cos(t5)
)*cos(t6)*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1)))/(pow(cos(t4),2.0)*cos(
t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+t3v*((pow(cos(
t4),2.0)*cos(t6)-
pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6))*(R.entry(2,0)*cos(t2+t3)+R.entry(0,0)*sin(t2+t3)*cos(t
1)+R.entry(1,0)*sin(t2+t3)*sin(t1)))/(sin(t5)*(pow(cos(t4),2.0)*cos(t6)*sin(t6)-
pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(cos(t4)*cos(t5)
)*cos(t6)*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1)))/(pow(cos(t4),2.0)*cos(
t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))-t6v*(((-
pow(cos(t4),2.0)*sin(t6)+pow(cos(t5),2.0)*sin(t6)+cos(t4)*cos(t5)*cos(t6)*sin(t4))*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1)))/(sin(t5)*(pow(cos(t4),
2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))+(cos(t4)*cos(t5)
)*sin(t6)*(R.entry(2,0)*cos(t2+t3)+R.entry(0,0)*sin(t2+t3)*cos(t1)+R.entry(1,0)*sin(t2+t3)*sin(t1)))/(pow(cos(t4)
),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))-
((pow(cos(t4),2.0)*cos(t6)-pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6))*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1))*1.0/pow(pow(cos(t4),2.0
)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))*1.0/pow(cos(t4)
),2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6))*1.0/pow(cos(t4)
),2.0)-
pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0)/sin(t5)+cos(t4)
)*cos(t5)*cos(t6)*(R.entry(2,0)*cos(t2+t3)+R.entry(0,0)*sin(t2+t3)*cos(t1)+R.entry(1,0)*sin(t2+t3)*sin(t1))*1.0/p
ow(pow(cos(t4),2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(pow(cos(t4)
),2.0)*pow(cos(t6),2.0)-pow(cos(t5),2.0)*pow(cos(t6),2.0)-
pow(cos(t4),2.0)*pow(sin(t6),2.0)+pow(cos(t5),2.0)*pow(sin(t6),2.0)+pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(cos(t6)
),2.0)-pow(cos(t4),2.0)*pow(cos(t5),2.0)*pow(sin(t6),2.0)+cos(t4)*cos(t5)*cos(t6)*sin(t4)*sin(t6)*4.0))+t4v*(((-
pow(cos(t4),2.0)*cos(t5)*sin(t6)+cos(t5)*pow(sin(t4),2.0)*sin(t6)+cos(t4)*cos(t6)*sin(t4)*2.0)*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1)))/(sin(t5)*(pow(cos(t4),
2.0)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6)))-
((pow(cos(t4),2.0)*cos(t6)-pow(cos(t5),2.0)*cos(t6)+cos(t4)*cos(t5)*sin(t4)*sin(t6))*(-
R.entry(2,0)*sin(t2+t3)+R.entry(0,0)*cos(t2+t3)*cos(t1)+R.entry(1,0)*cos(t2+t3)*sin(t1))*1.0/pow(pow(cos(t4),2.0
)*cos(t6)*sin(t6)-pow(cos(t5),2.0)*cos(t6)*sin(t6)+cos(t4)*cos(t5)*sin(t4)-
cos(t4)*cos(t5)*pow(cos(t6),2.0)*sin(t4)*2.0+pow(cos(t4),2.0)*pow(cos(t5),2.0)*cos(t6)*sin(t6),2.0)*(-
pow(cos(t4),2.0)*cos(t5)+cos(t5)*pow(sin(t4),2.0)+pow(cos(t4),2.0)*cos(t5)*pow(cos(t6),2.0)*2.0-
cos(t5)*pow(cos(t6),2.0)*pow(sin(t4),2.0)*2.0+cos(t4)*cos(t6)*sin(t4)*sin(t6)*2.0+cos(t4)*pow(cos(t5),2.0)*cos(t
6)*sin(t4)*sin(t6)*2.0));
805 }
807     return da2;
808 }

```


6. Realizar una visualización tridimensional en tiempo real en C++ y la API OpenGL. Posición de Home

```

"C:\Users\User Admi\Desktop\Robot Posiciones - c
=====
|| Posicion r06 ||
=====
192
-1.38533e-005
321
=====
|| Angulos con C.I. ||
=====
theta1 : 0
theta2 : 0
theta3 : 0
theta4 : -3.14159
theta5 : 4.71239
theta6 : 3.14159
=====
|| Velocidades Angulares ||
=====
ohmega1 : 0
ohmega2 : 0
ohmega3 : 0
ohmega4 : 0
ohmega5 : 0
ohmega6 : 0
=====
|| Aceleraciones Angulares ||
=====
alpha1 : 0
alpha2 : 0
alpha3 : 0
alpha4 : 0
alpha5 : 0
alpha6 : 0

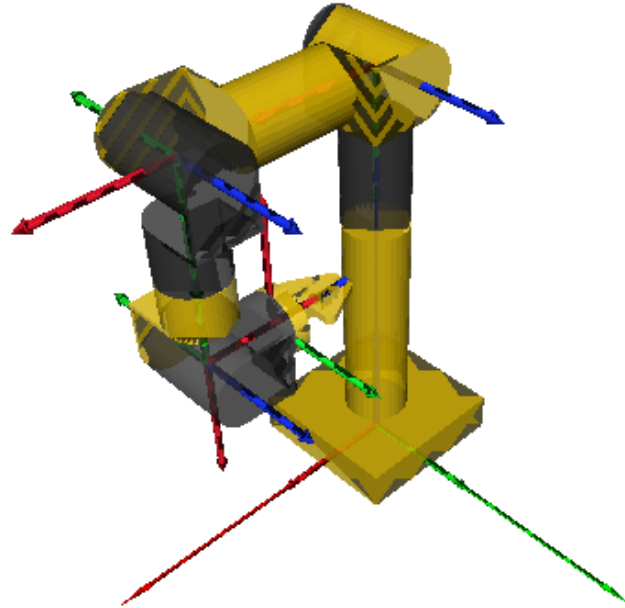
```

TECLA G

```

=====
|| Posicion Guardada ||
=====

```

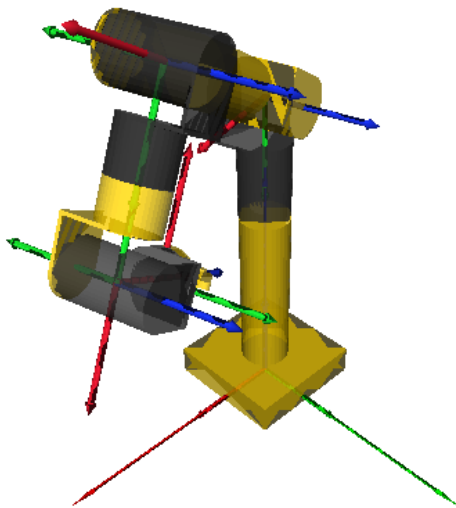


Movimiento de sus articulaciones

Articulación	Tecla
1	" 1 ", " 2 "
2	" 3 ", " 4 "
3	" 5 ", " 6 "
4	" 7 ", " 8 "
5	" 9 ", " 0 "
6	" O ", " P "

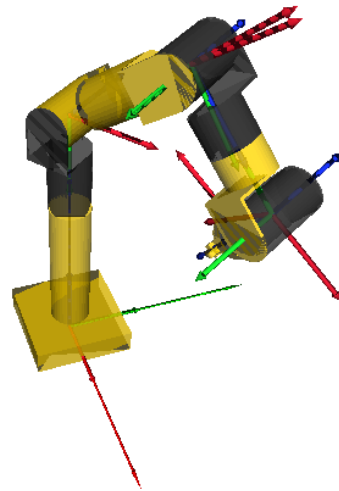
TECLA G

```
=====
|| Posicion Guardada ||
=====
```



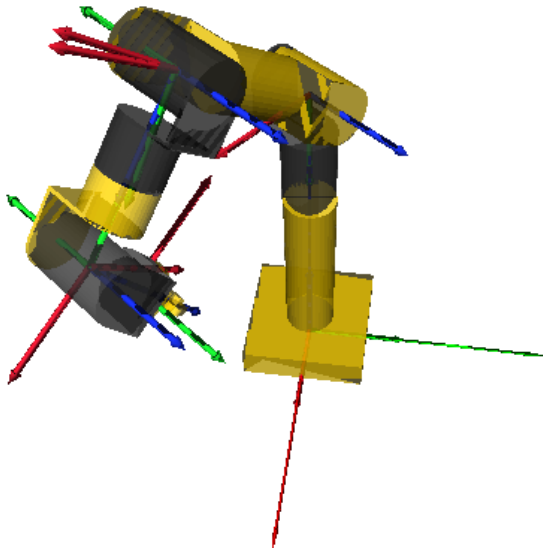
TECLA G

```
=====
|| Posicion Guardada ||
=====
```



TECLA G

```
=====
|| Posicion Guardada ||
=====
```



TECLA T

```
=====
|| Trayectoria a seguir ||
=====
[seg: 0] a (192,-1.38533e-005,321)->
[seg: 3] a (328.572,289.095,370.74)->
[seg: 3] a (375.727,-224.42,370.74)->
[seg: 3] a (202.327,-224.442,304.717)

Tiempo estimado de trayectoria [seg]: 9
```

TECLA M

Coeficientes para la trayectoria cubica de (304.346,84.2302,464.271)->(286.426,250.26,481.946) y el analisis de sus velocidades angulares y aceleraciones angulares en un punto aleatorio del recorrido.

(304.346,84.2302,464.271)->(286.426,250.26,481.946)

=====

|| Los coeficientes para X son ||

=====

1.32738

-5.97319

0

304.346

=====

|| Los coeficientes para Y son ||

=====

-12.2985

55.3432

0

84.2302

=====

|| Los coeficientes para Z son ||

=====

-1.30927

5.89172

0

464.271

=====

|| Posicion r06 ||

=====

304.346

84.2302

464.271

=====

|| Angulos con C.I. ||

=====

theta1 : 0.138796

theta2 : -0.739441

theta3 : 0.266181

theta4 : -2.65506

theta5 : 4.09977

theta6 : 3.5348

=====

|| Velocidades Angulares ||

=====

ohmega1 : 9.13558e-008

ohmega2 : -2.14575e-007

ohmega3 : -1.7919e-007

ohmega4 : -7.39528e-008

ohmega5 : 3.28603e-007

ohmega6 : 2.70023e-007

=====

|| Aceleraciones Angulares ||

=====

alpha1 : -0.127725

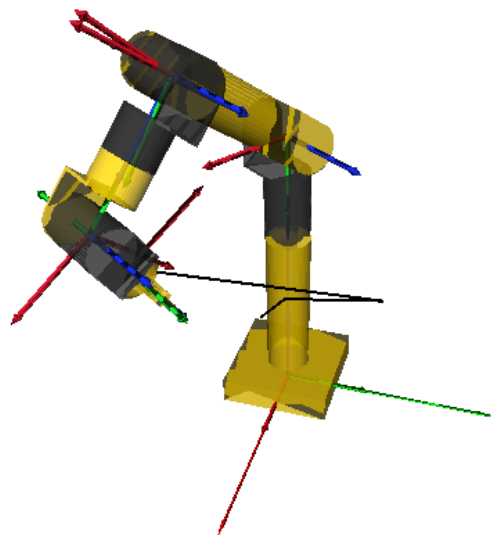
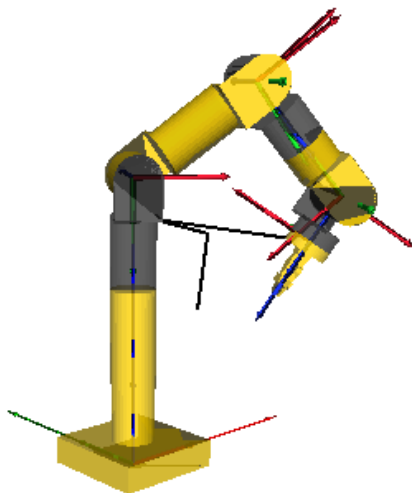
alpha2 : 0.164686

alpha3 : 0.280226

alpha4 : 0.0686877

alpha5 : -0.366065

alpha6 : -0.317164



TECLA M

Coeficientes para la trayectoria cubica de (286.426,250.26,481.946)->(315.029,-205.156,479.092) y un punto instantáneo en el corrimiento, con sus velocidades y aceleraciones angulares

(286.426,250.26,481.946)->(315.029,-205.156,479.092)

|| Los coeficientes para X son ||

-2.11875

9.53436

0

286.426

|| Los coeficientes para Y son ||

33.7345

-151.805

0

250.26

|| Los coeficientes para Z son ||

0.211434

-0.951457

0

481.946

|| Posicion r06 ||

286.426

250.26

481.946

|| Angulos con C.I. ||

theta1 : 0.573457

theta2 : -0.716172

theta3 : 0.0919845

theta4 : -2.47246

theta5 : 4.10024

theta6 : 3.86484

|| Velocidades Angulares ||

ohmega1 : 1.518e-007

ohmega2 : 1.36205e-007

ohmega3 : -1.56715e-007

ohmega4 : 6.53857e-008

ohmega5 : -3.8945e-008

ohmega6 : 1.00571e-007

|| Aceleraciones Angulares ||

alpha1 : -0.250931

alpha2 : -0.0658449

alpha3 : 0.222744

alpha4 : -0.0545176

alpha5 : -0.0320944

alpha6 : -0.25947

