Present value & zero coupon bonds

BOND VALUATION AND ANALYSIS IN PYTHON

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Present Value

- Money compounds from its value today to its value in the future
- This process also works in reverse

Present Value

- We can rearrange our compound interest equation from earlier:

$$FV = PV \times (1+r)^n$$

$$PV = \frac{FV}{(1+r)^n}$$

- Called "discounting to present value" or just "discounting"

Present Value

$$PV \underset{\text{DISCOUNTING}}{\overset{\text{COMPOUNDING}}{\rightleftarrows}} FV$$

- To move from present value to future value, we compound
- To move from future value to present value, we discount

Present Value

- A higher interest rate or longer time period increases the FV
- So a higher interest rate or longer time period must decrease the PV

The pv() function

```python
import numpy_financial as npf
?npf.pv
```

```
Signature: npf.pv(rate, nper, pmt, fv=0)
Docstring: Compute the present value.

Given:
 * a future value, `fv`
 * an interest `rate` compounded once per period, of which there are
 * `nper` total
 * a (fixed) payment, `pmt`

Return: the value now
```

The pv() function

- How much should we invest now at 5% per year to have USD 10,000 in 10 years?

```
import numpy_financial as npf
npf.pv(rate=0.05, nper=10, pmt=0, fv=10000)
```

```
-6139.13
```

```
-npf.pv(rate=0.05, nper=10, pmt=0, fv=10000)
```

```
6139.13
```

The pv() function

- Or, by rearranging our compound interest equation from earlier:

```
pv = 10000 / (1 + 0.05) ** 10
print(pv)
```

```
6139.13
```

Bonds introduction

- Debt instrument, issued by governments and companies
- Investors buy bonds in exchange for interest
- Provide relatively safe and consistent returns
- Are usually less risky and volatile than stocks

Zero coupon bonds

- Pays a single cash-flow called the face value
- Paid at a single point in time called the maturity
- No intermediate cash-flows (called coupons) paid until maturity, hence the name
- Their price is the PV of the single cash-flow

Zero coupon bonds

- Usually issued at a discount to their face value
- This difference is called the yield (measured in percent)

Zero coupon bonds

Let's look at an example of a zero coupon bond that:

- Has a 3 year maturity
- A face value of USD 100
- A yield of 3.5%

What is the price of this bond?

Zero coupon bonds

- Zero coupon bond with a 3 year maturity that yields 3.5% and has a face value of USD 100:

```
import numpy_financial as npf
-npf.pv(rate=0.035, nper=3, pmt=0, fv=100)
```

```
90.19
```

- Or, again by rearranging our compound interest equation from earlier:

```
pv = 100 / (1 + 0.035) ** 3
print(pv)
```

```
90.19
```

Let's practice!

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Coupon paying bonds

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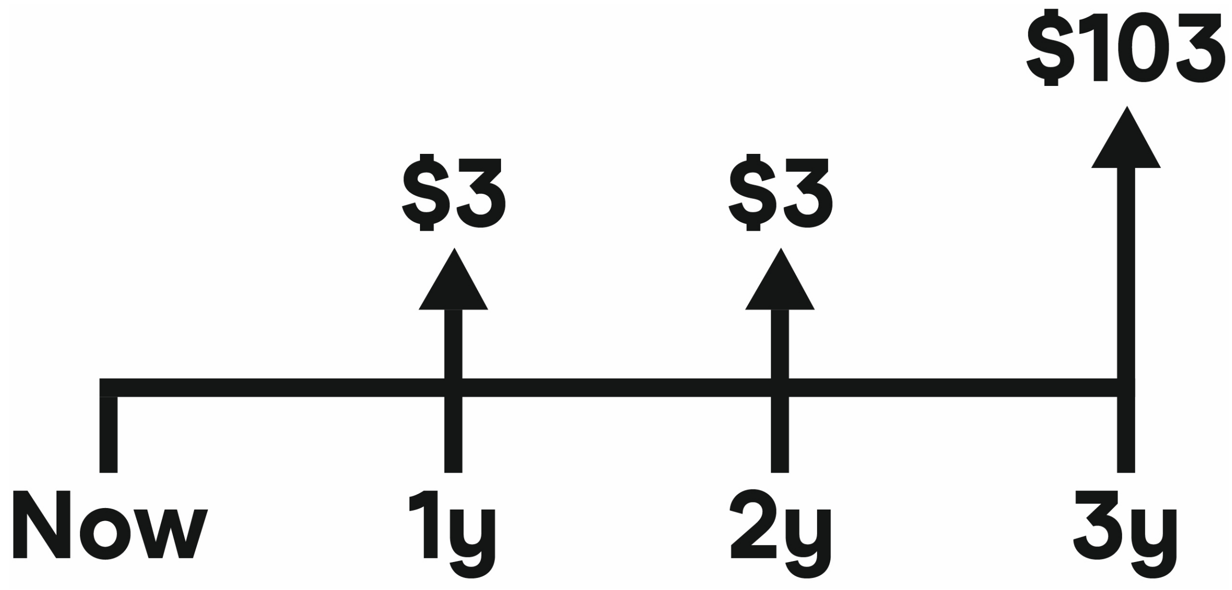
Coupon bond definition

- Pays regular cash-flows (coupons) during its life
- At maturity it pays both a coupon and the face value
- Coupons are typically paid annually or semi-annually
- The number of coupons paid per year is called the frequency
- Its yield to maturity is the annual return from buying and holding the bond to maturity

Coupon bond example

Take a 3 year bond with a 3% annual coupon, face value of USD 100, and yield of 4%:



WARNING: The coupon is fixed and *doesn't* change!

Coupon bond pricing

We break the bond up into a collection of zero coupon bonds, then price these:

- A 1 year zero coupon bond with a face value of USD 3
- A 2 year zero coupon bond with a face value of USD 3
- A 3 year zero coupon bond with a face value of USD 103

Coupon bond pricing

3 year bond with a 3% annual coupon, face value of USD 100, and yield of 4%

Using our compound interest formula from earlier:

$$\text{1yr ZCB Price: } \frac{3}{(1+0.04)^1} = 2.88$$

$$\text{2yr ZCB Price: } \frac{3}{(1+0.04)^2} = 2.77$$

$$\text{3yr ZCB Price: } \frac{103}{(1+0.04)^3} = 91.57$$

$$\text{Coupon Bond Price: } 2.88 + 2.77 + 91.57 = 97.22$$

Coupon bond formula

More generally, our formula for the price of a coupon bond is:

$$Price = PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + ... + \frac{C}{(1+r)^n} + \frac{P}{(1+r)^n}$$

$$= (\sum_{i=1}^{n} \frac{C}{(1+r)^i}) + \frac{P}{(1+r)^n}$$

- C is the coupon paid in each time period
- r is the yield to maturity of the bond
- P is the face value (or principal) paid at maturity
- n is the number of time periods (typically years)

Using the pv() function

Taking the same 3 year bond with an annual coupon of 3% and yield to maturity of 4%:

```
import numpy_financial as npf
-npf.pv(rate=0.04, nper=3, pmt=3, fv=100)
```

```
97.22
```

We set `pmt` to be positive.

We also put a minus sign before the function.

We set `fv` to 100 not 103.

Let's practice!

BOND VALUATION AND ANALYSIS IN PYTHON

Bond prices vs. bond yields

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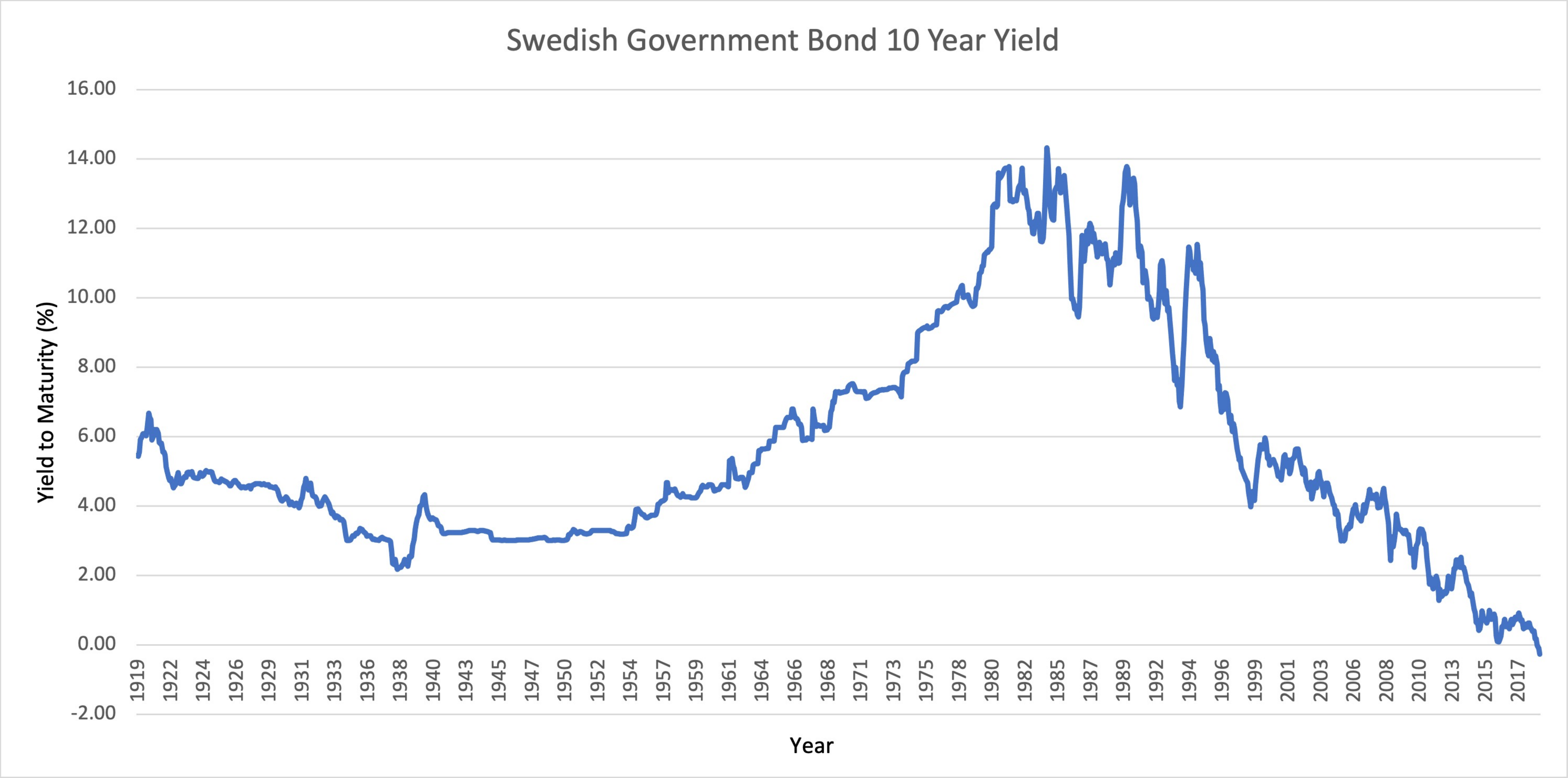
Historical government bond yields



Plotting bond prices vs. yields

```
import numpy as np
import numpy_financial as npf
import pandas as pd
import matplotlib.pyplot as plt
```

```
bond_yields = np.arange(0, 20, 0.1)
print(bond_yields)
```

```
[0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3
...
18.5 18.6 18.7 18.8 18.9 19.0 19.1 19.2 19.3 19.4 19.5 19.6 19.7 19.8 19.9]
```

Plotting bond prices vs. yields

```
bond = pd.DataFrame(bond_yields, columns=['bond_yield'])
print(bond)
```

```
     bond_yield
0           0.0
1           0.1
..          ...
198        19.8
199        19.9
[200 rows x 1 columns]
```

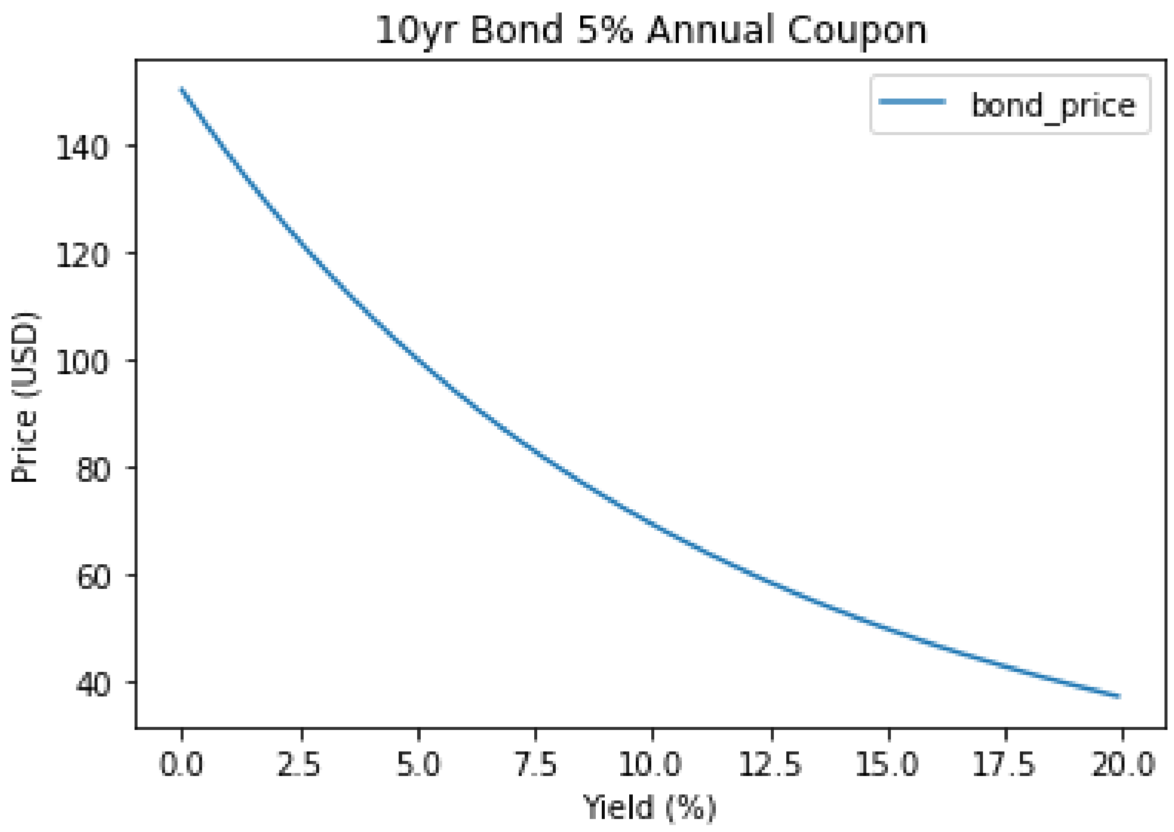
Plotting bond prices vs. yields

```
bond['bond_price'] = -npf.pv(rate=bond['bond_yield'] / 100, nper=10, pmt=5, fv=100)
print(bond)
```

```
     bond_yield  bond_price
0           0.0  150.000000
1           0.1  148.731575
..          ...         ...
198        19.8   37.527719
199        19.9   37.319493
[200 rows x 2 columns]
```

Plotting bond prices vs. yields

```
plt.plot(bond['bond_yield'], bond['bond_price'])
plt.xlabel('Yield (%)')
plt.ylabel('Bond Price (USD)')
plt.title("10 Year Bond 5% Annual Coupon")
plt.show()
```

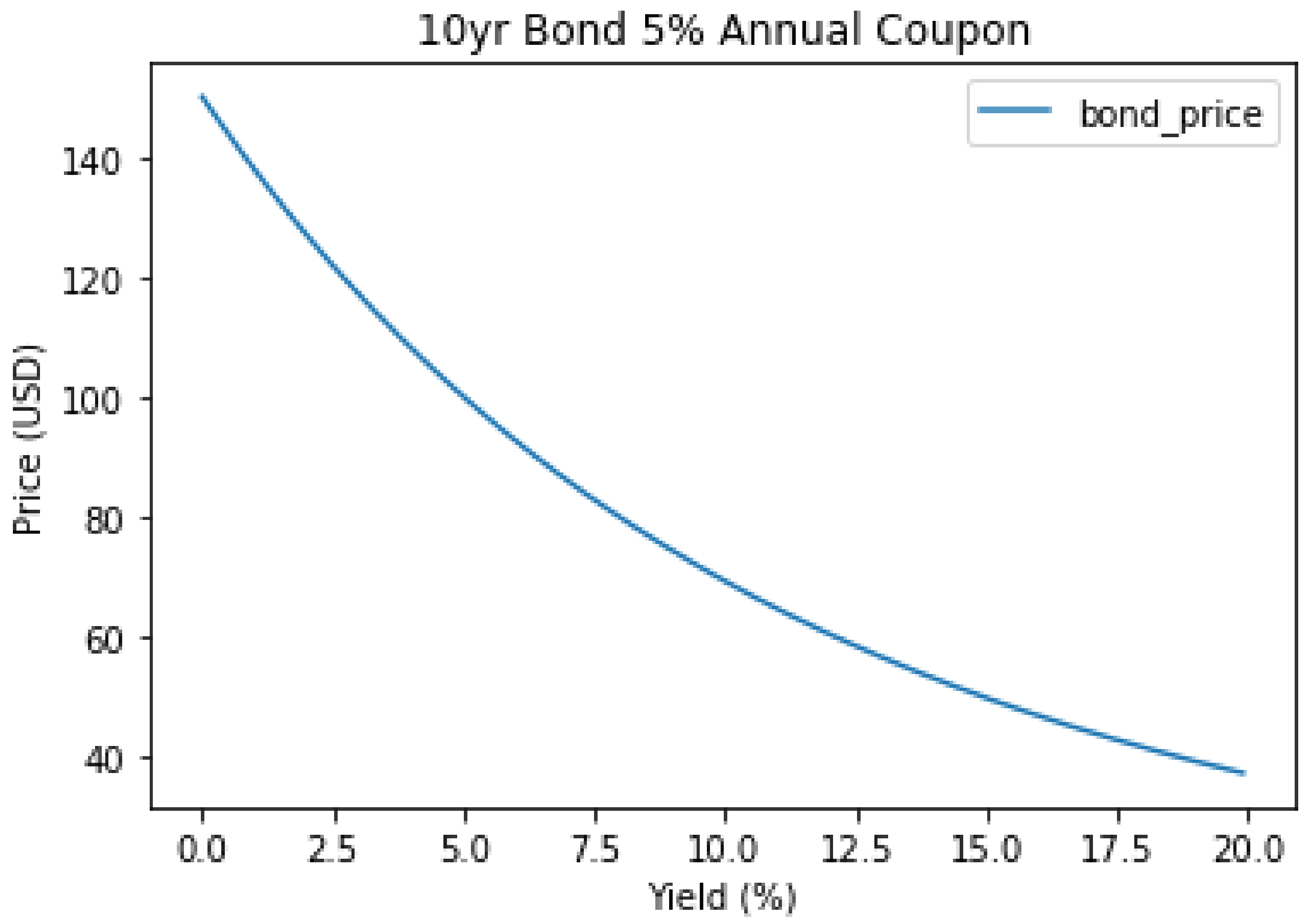


The relationship between price and yield

Prices move inversely to yields

Higher yield = higher discount rate = lower PV

Higher price lowers the return on investment (yield)



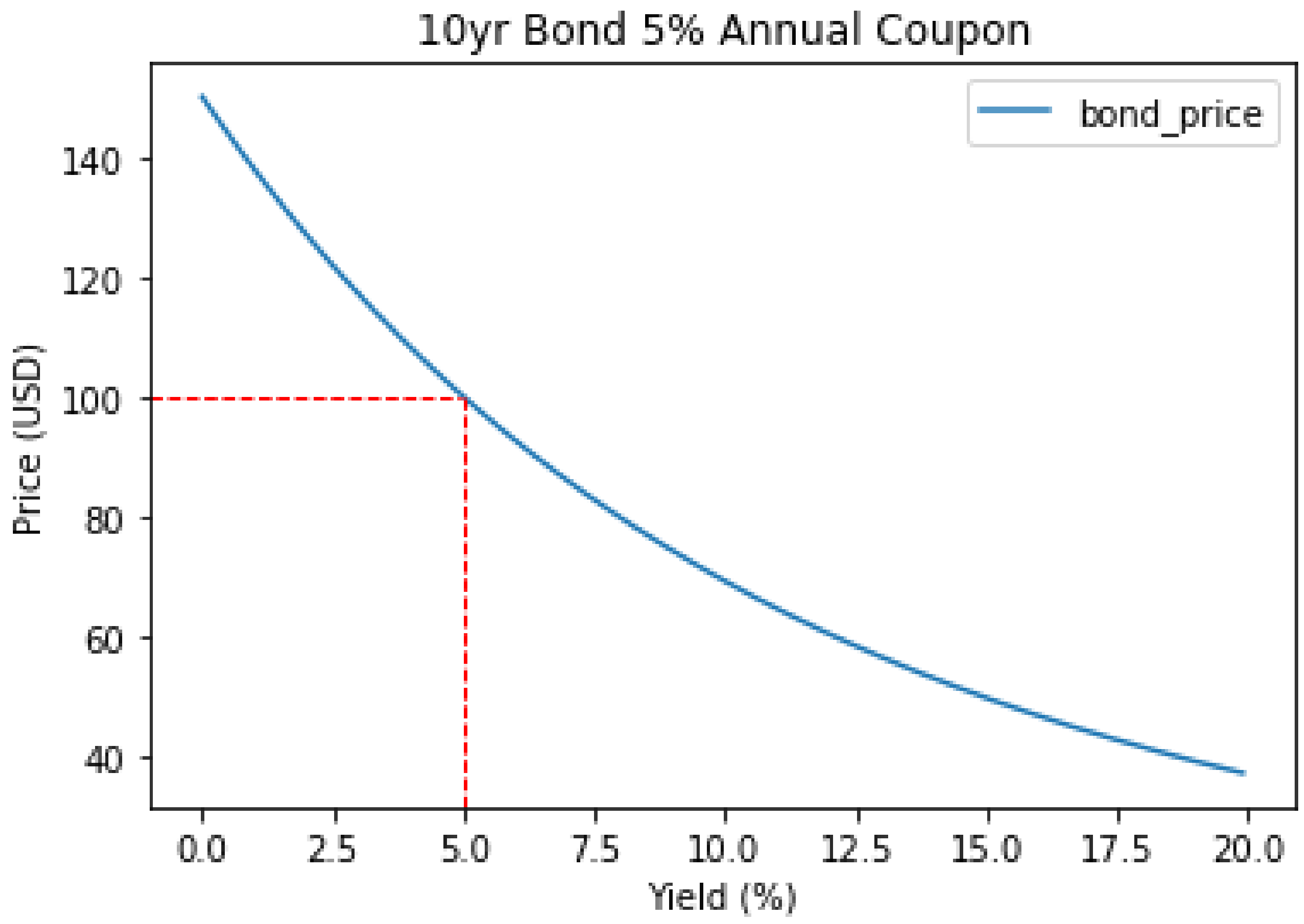
The relationship between price and yield

Bond premium vs. bond discount

Premium: Price > 100, Yield < Coupon

Discount: Price < 100, Yield > Coupon

Par: Price = 100, Yield = Coupon

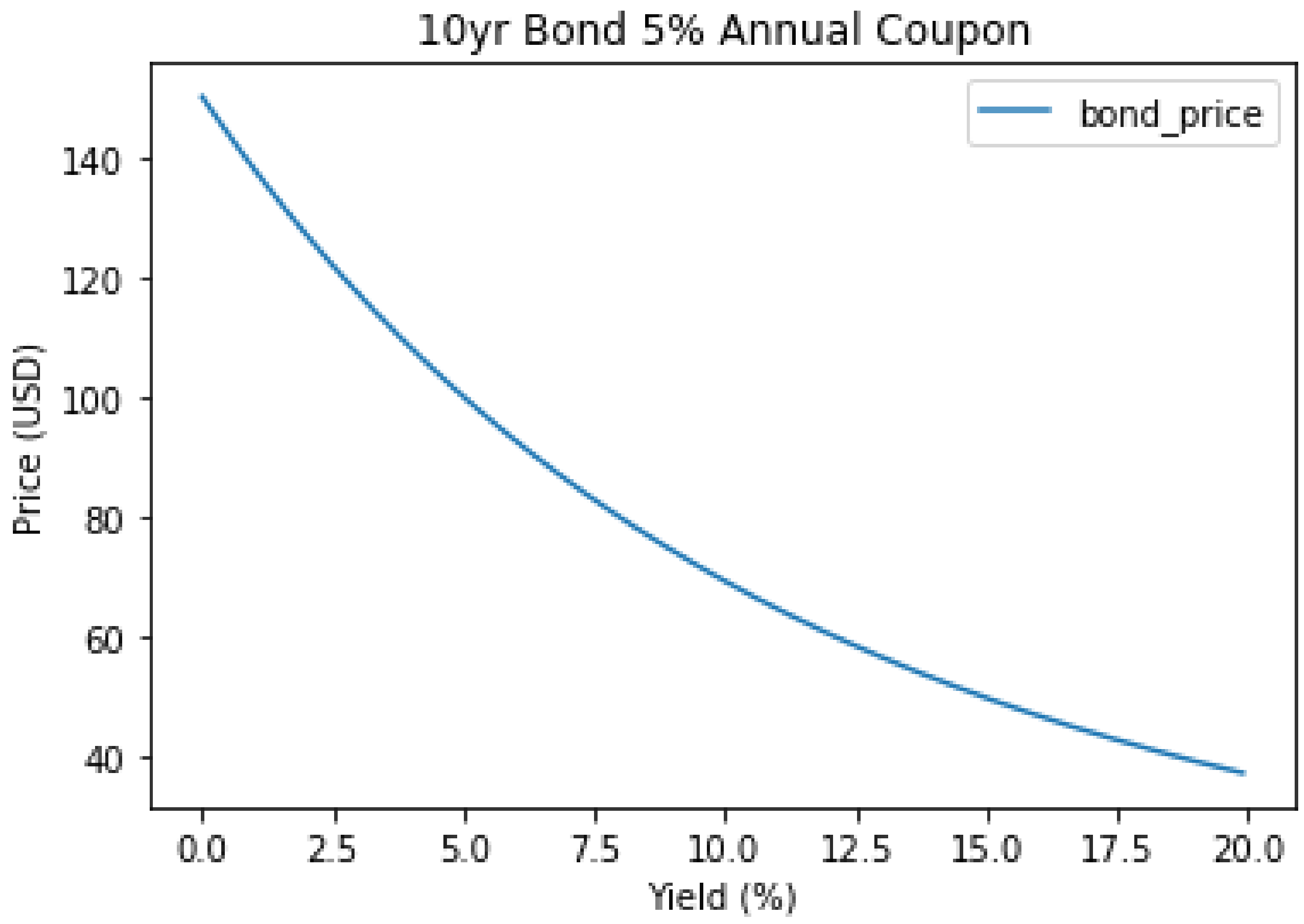


The relationship between price and yield

Price/yield relationship is non-linear

The line we have plotted is not a straight line

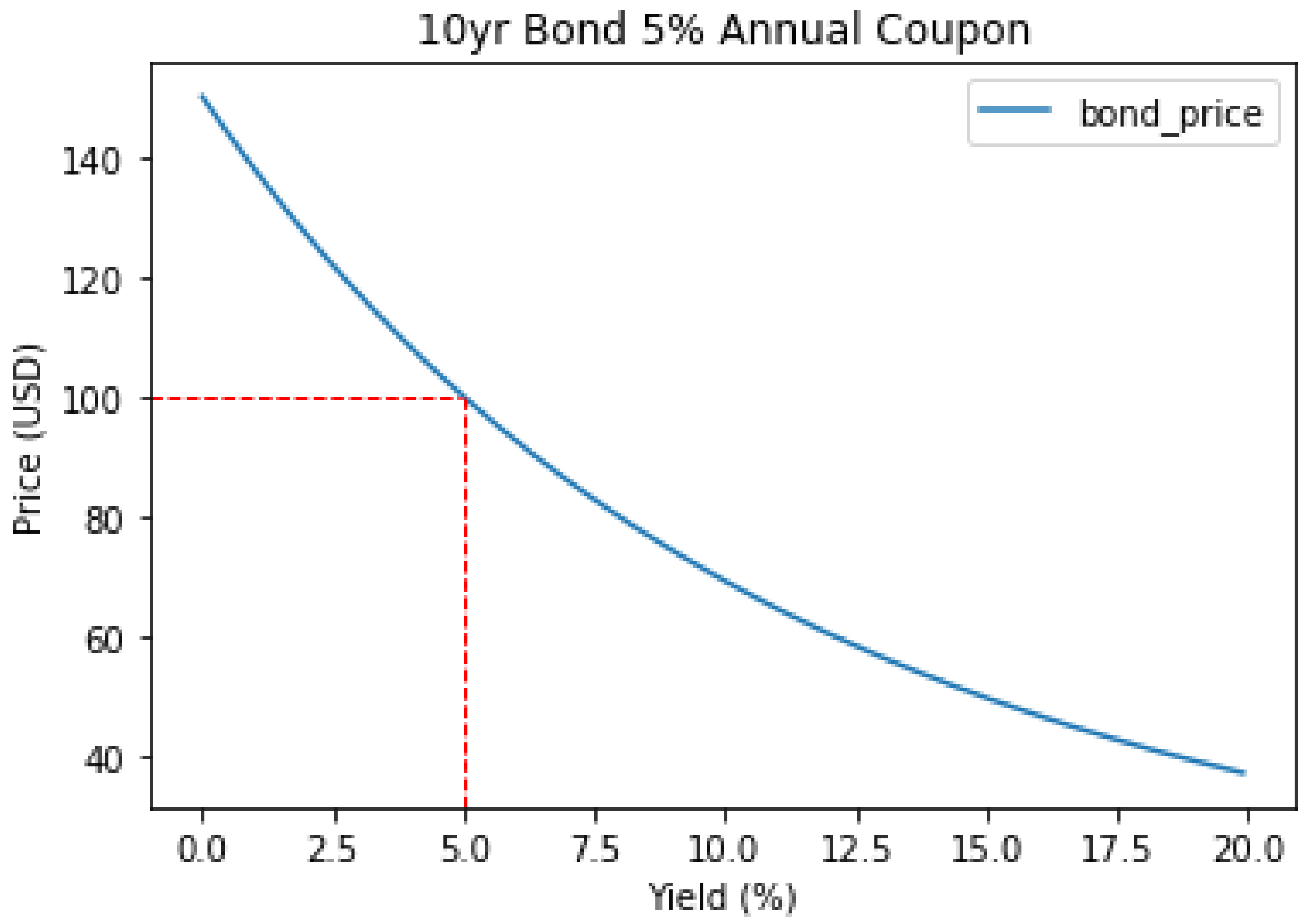
This is due to something called convexity



The relationship between price and yield

Summary of key points:

- Prices and yields move inversely
- Premium: Price > 100, Yield < Coupon
- Discount: Price < 100, Yield > Coupon
- Par: Price = 100, Yield = Coupon
- The price/yield relationship is non-linear



Let's practice!

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Calculating bond yields

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Yield calculation motivation

- Yield to maturity tells us our expected return on investment
- Can be useful for comparing bonds with different characteristics

Zero coupon bond yield formula

Recall the formula for the price of a zero coupon bond:

$$PV = \frac{FV}{(1+r)^n}$$

We can rearrange this equation to solve for yield (r):

$$FV = PV \times (1+r)^n$$

$$\frac{FV}{PV} = (1+r)^n$$

$$\sqrt[n]{\frac{FV}{PV}} = (1+r)$$

$$\sqrt[n]{\frac{FV}{PV}} - 1 = r$$

Zero coupon bond yield example

Let's look at the same zero coupon bond from earlier working backwards:

- Has a 3 year maturity
- A face value of USD 100
- A price of USD 90.19

What is the yield of this bond?

Zero coupon bond yield calculation

3 year zero coupon bond, price USD 90.19, face value USD 100:

$$r = \sqrt[n]{\frac{FV}{PV}} - 1$$

```
ytm = (100 / 90.19) ** (1/3) - 1
print(ytm)
```

```
0.035
```

We will use `ytm` for 'yield to maturity'.

Coupon bond yield formula?

Coupon bond formula:

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + ... + \frac{C}{(1+r)^n} + \frac{P}{(1+r)^n}$$

$$= \sum_{i=1}^{n} \frac{C}{(1+r)^i} + \frac{P}{(1+r)^n}$$

This equation cannot be rearranged in terms of r

We use trial and error to find r

This is how the `npf.rate()` function works

Coupon bond yield example

Let's consider our coupon paying bond earlier which:

- has a maturity of 3 years
- pays a 3% annual coupon
- has a price of USD 97.22

What is its yield to maturity?

Coupon bond yield calculation

3 year coupon bond, 3% annual coupon with a price of USD 97.22:

```
import numpy_financial as npf
npf.rate(nper=3, pmt=3, pv=-97.22, fv=100)
```

```
0.04
```

Remember we need to set the PV to a negative number.

This is because the price of the bond is money we pay (negative cash-flow).

Let's practice!