Duration

BOND VALUATION AND ANALYSIS IN PYTHON

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The context behind duration

- Prices and yields move inversely
- We don't know how sensitive a bond is to interest rates
- Duration measures interest rate sensitivity

Motivating example

- Take a 5 year & 10 year bond, both with a 5% coupon
- At a 5% yield they both have a price of USD 100:

```
-npf.pv(rate=0.05, nper=5, pmt=5, fv=100)
-npf.pv(rate=0.05, nper=10, pmt=5, fv=100)
```

```
100.00
100.00
```

Motivating example

If interest rates move up to 6%:

```
-npf.pv(rate=0.06, nper=5, pmt=5, fv=100)
-npf.pv(rate=0.06, nper=10, pmt=5, fv=100)
```

```
95.79
92.64
```

- The 5 year bond lost 4.21% of its value, while the 10 year bond lost 7.36%
- The 10 year bond was more sensitive to interest rate changes

What is duration?

- Duration is the % price change for a 1% change in yields (interest rates).
- Higher duration = higher interest rate risk
- Typically used to:
  - Measure interest rate risk
  - Hedge interest rate risk
  - Predict profit & loss as interest rates change

Calculating duration

We will use a simplified formula for duration:

$$Duration = \frac{P_{down} - P_{up}}{2 \times P \times \Delta y}$$

- P_{down} = Bond price at 1% lower yield
- P_{up} = Bond price at 1% higher yield
- P = Bond price at current yield
- Δy = Change in yield (we will use 1%)

Duration example

10 year bond, 5% annual coupon, 4% yield to maturity, what is its duration?

$$Duration = \frac{P_{down} - P_{up}}{2 \times P \times \Delta y}$$

```
price = -npf.pv(rate=0.05, nper=10, pmt=5, fv=100)
price_up = -npf.pv(rate=0.06, nper=10, pmt=5, fv=100)
price_down = -npf.pv(rate=0.04, nper=10, pmt=5, fv=100)
duration = (price_down - price_up) / (2 * price * 0.01)
print(duration)
```

```
7.74
```

A 1% move in interest rates causes a 7.74% change in the bond price.

Summary

- Bonds can behave differently for the same change in yields
- Duration is the % price change of a bond for a 1% change in yields
- Duration measures interest rate sensitivity

Let's practice!

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Factors affecting duration

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Duration as an 'average' time

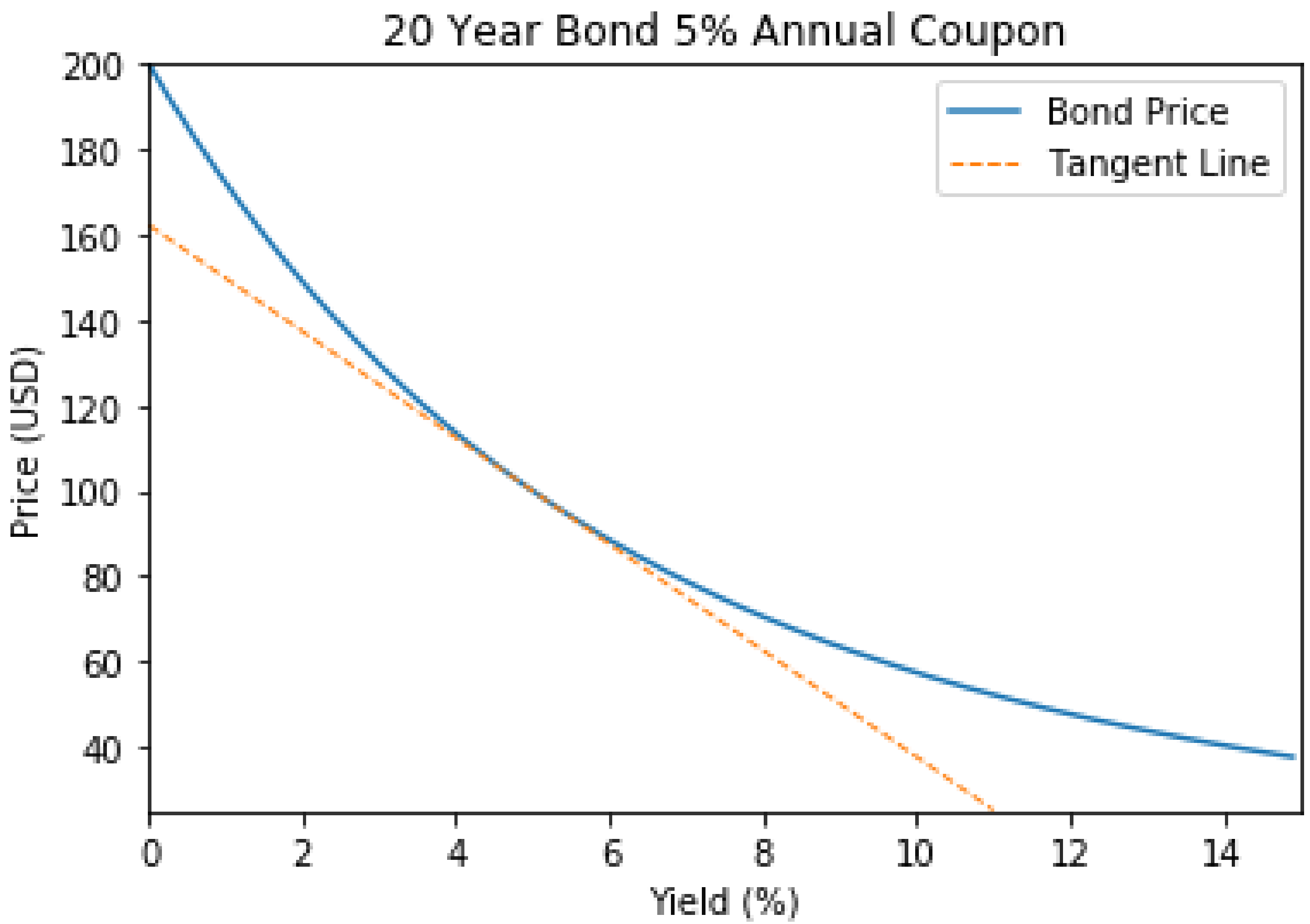
The 'average' time taken to get your money back

Waiting longer = more exposed to interest rates

Duration as the slope of the tangent line

Duration is the derivative (rate of change) of price with respect to yield

The slope of the tangent line is the duration



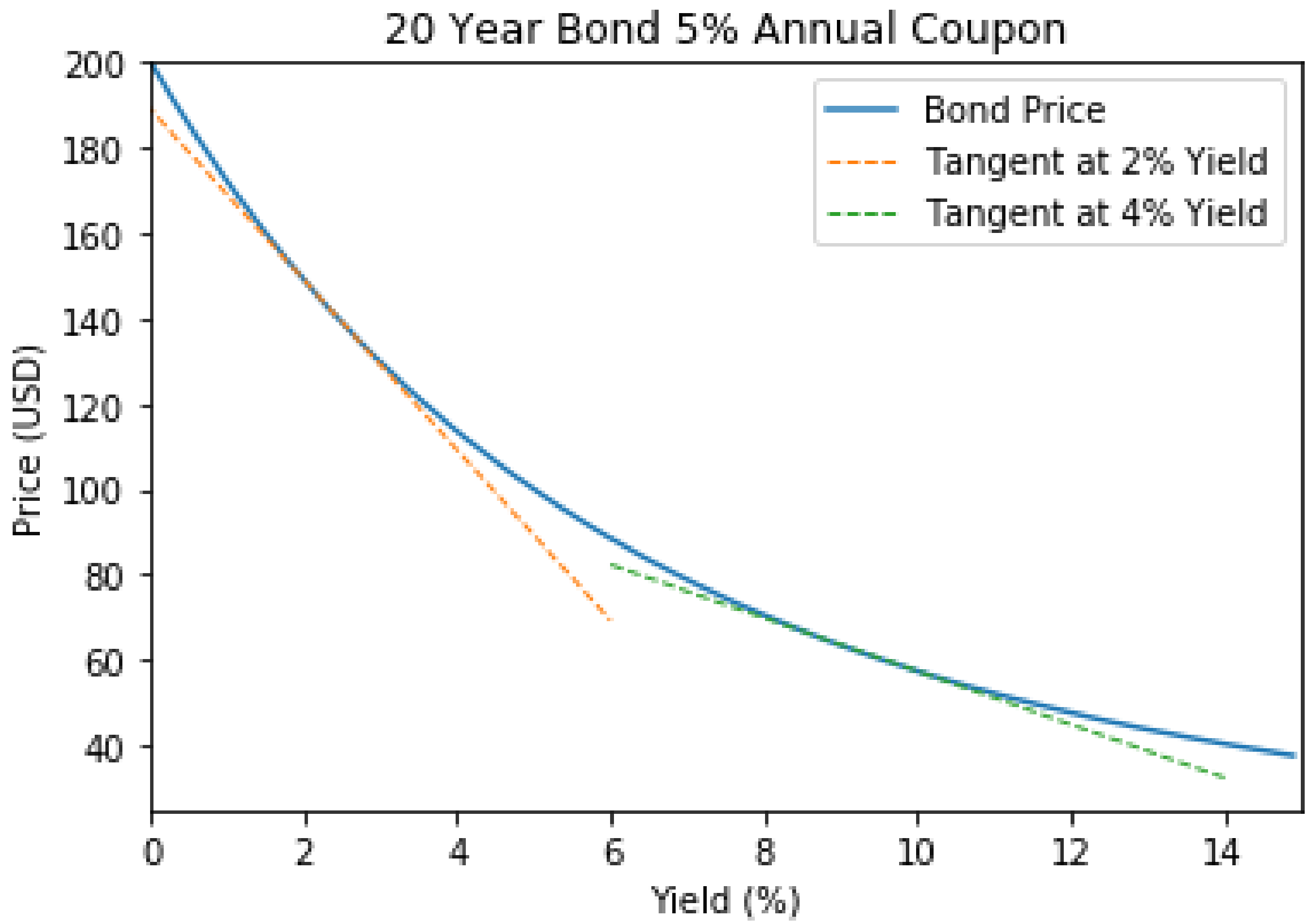
Maturity vs. duration

- Longer maturity = wait longer to get money back
- Wait longer = more exposed to interest rate changes
- Longer maturity = higher duration

Coupon rate vs. duration

- Higher coupon = shorter wait to get money back 'on average'
- So less exposed to interest rate changes
- Therefore higher coupon = lower duration
- Zero coupon bonds have higher duration than coupon bonds

Bond yield vs. duration



- Bond price curve is steeper for lower yields
- Lower yields = higher sensitivity to interest rates = higher duration

Ways of investigating duration

We can investigate the different factors affecting duration by:

- Varying one factor and directly calculating the duration
- Plotting a price/yield graph and seeing where it is most steep
- Plotting a duration/factor graph

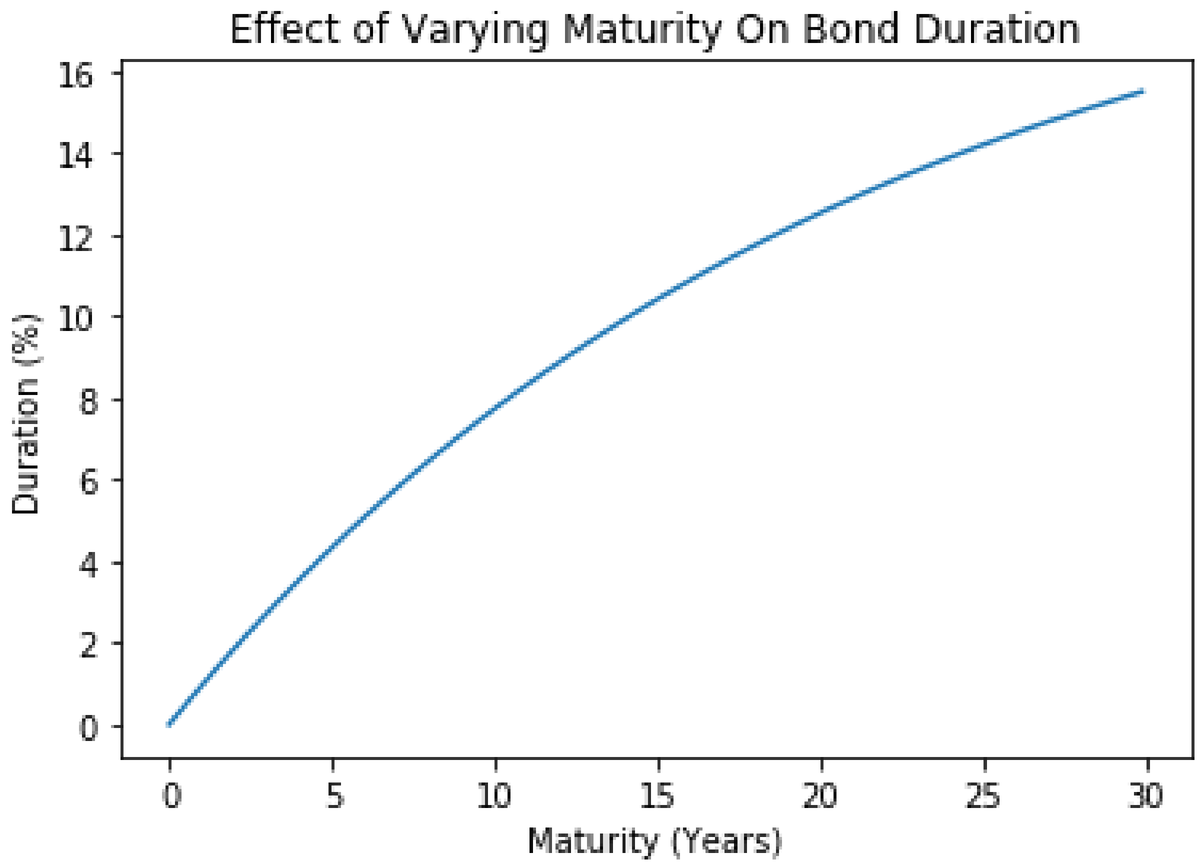
Plotting bond maturity against duration

```
import numpy as np
import numpy_financial as npf
import pandas as pd
import matplotlib.pyplot as plt
```

```
bond_maturity = np.arange(0, 30, 0.1)
bond = pd.DataFrame(bond_maturity, columns=['bond_maturity'])
bond['price'] = -npf.pv(rate=0.05, nper=bond['bond_maturity'], pmt=5, fv=100)
bond['price_up'] = -npf.pv(rate=0.05 + 0.01, nper=bond['bond_maturity'], pmt=5, fv=100)
bond['price_down'] = -npf.pv(rate=0.05 - 0.01, nper=bond['bond_maturity'], pmt=5, fv=100)
bond['duration'] = (bond['price_down'] - bond['price_up']) / (2 * bond['price'] * 0.01)
```

Plotting bond maturity against duration

```
plt.plot(bond['bond_maturity'], bond['duration'])
plt.xlabel('Maturity (Years)')
plt.ylabel('Duration (%)')
plt.title("Effect of Varying Maturity On Bond Duration")
plt.show()
```



Summary

The duration of a bond will increase for a:

- Higher maturity
- Lower coupon rate
- Lower level of yields

Let's practice!

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Dollar duration & bond price prediction

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Dollar duration

- Duration = % change in bond price for 1% change in yields
- Dollar duration = $ change in bond price for 1% change in yields:
- Tells us how much money we make or lose for a change in interest rates

$$\text{Dollar Duration} = \text{Duration} \times \text{Bond Price} \times 0.01$$

DV01

- DV01 = $ change in bond price for 0.01% change in yields.
- 0.01% = 1% of 1% = 1 basis point
- Short for "dollar value of one basis point"

$$\mathrm{DV01} = \mathrm{Duration} \times \mathrm{Bond\ Price} \times 0.0001$$

Dollar duration example

Bond with a price of USD 92.28 and duration of 7.98%:

```
dollar_duration = 92.28 * 7.98 * 0.01
print("Dollar Duration: ", dollar_duration)
```

```
Dollar Duration: 7.36
```

```
DV01 = 92.28 * 7.98 * 0.0001
print("DV01: ", DV01)
```

```
DV01: 0.0736
```

Creating a duration neutral portfolio

- Protect a portfolio from interest rate changes
- Often called "hedging"
- First calculate DV01 of portfolio and hedging instrument
- Find quantity of bond to have equal DV01 to hedge

Hedging DV01 example

- Your existing portfolio has a DV01 of USD 10,000
- Bond has a price of USD 92.28 and DV01 of USD 0.0736

```
portfolio_dv01 = 10000
bond_dv01 = 0.0736
```

```
hedge_quantity = portfolio_dv01 / bond_dv01
print("Number of bonds to sell: ", hedge_quantity)
```

```
Number of bonds to sell: 135,869
```

Hedging DV01 example

- Your existing portfolio has a DV01 of USD 10,000
- Bond has a price of USD 92.28 and DV01 of USD 0.0736

```
bond_price = 92.28
hedge_amount = hedge_quantity * bond_price
print("Dollar amount to sell: USD", hedge_amount)
```

```
Dollar amount to sell: USD 12,538,043
```

Bond price prediction

- Dollar duration can be used to predict bond price changes:

$$\text{Price Change} = -100 \times \text{Dollar Duration} \times \Delta y$$

- Useful to quickly predict how a bond or portfolio will behave

Price prediction example

- 10 year bond with 4% coupon and 5% yield, price USD 92.28, dollar duration USD 7.36

Estimated bond price change if interest rates drop 3%:

```
-100 * 7.36 * -0.03
```

```
22.08
```

Actual change from repricing the bond:

```
-npf.pv(rate=0.02, nper=10, pmt=4, fv=100) - 92.28
```

```
25.69
```

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