

Warm up: Q. n balls.



n=7

Follow up  
↓

Painter → color them up.

Red Blue

↳ not so good with eyes.

Quant\*  
companies

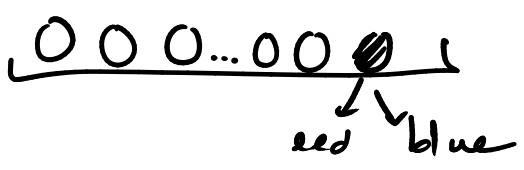
# no of ways → color 'n' balls.

2 consecutive balls with red color.

i/p: n

o/p: —

Approach:



let  $r_n \Rightarrow$  no of n balls sequence where the last ball is red.

$$b_{n+1} = r_n + b_n$$

Recurrence relations

$$r_{n+1} = b_n \Rightarrow r_n = b_{n-1}$$

$b_n \Rightarrow$  no. of n balls seq. where last ball is blue

$$r_1 = 1$$



$$b_1 = 1$$



$$b_{n+1} = b_n + b_{n-1}$$

\* Fibo.

$$ans = r_n + b_n = \underline{b_{n-1}} + \underline{b_n} = \underline{\underline{b_{n+1}}}$$

n=10

return 11<sup>th</sup> fibonacci number |

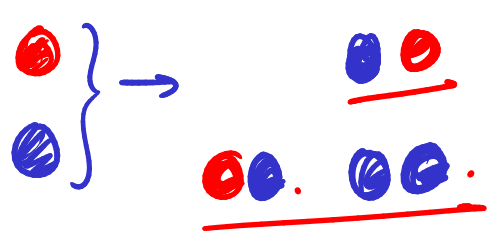
n=1

n=2

n=3

$$b_3 = 1 + 2$$

$$r_3 = 2$$



10

$$b_{10} + r_1$$

1 1 2 3 5 8 13 21 ...

Q. string of length 'n'. character  $\rightarrow$  alphabet. (26 choices)

Variation. At least one consecutive vowels occurrence should be there in a string. # ways.

n = 3

ae x

not a good approach / B.F.

all possible — no vowels are consecutive

Backtracking  
it vowel  $\rightarrow$  26  
21 — — — — — 'n'

did not ask  $\rightarrow$  actual strings

#count

$O(26^n)$

polynomial  $O(n)$

Ans =  $26^n - 21^n$  ??

$C = 21$   
 $V = 5$

$C_1 = 21$   
 $V_1 = 5$

$n \rightarrow \boxed{\text{ans} = C_n + V_n} = C_{n+1}$

$C_{n+1} = C_n + C_{n-1}$

$\begin{cases} C_{n+1} = C_n + V_n \\ V_{n+1} = C_n \end{cases}$   
 $\hookrightarrow V_n = C_{n-1}$

Base cond<sup>n</sup>.  $C_1 = 21$   $V_1 = 5$

Analogy:  $C \rightarrow$  blue ball : 21 shades  
 $V \rightarrow$  red ball : 5 shades

n = 3 all possible strings =  $26^3 = 17576$  aaa

Q. at least 1 consecutive should be present zzz  
= 17576 — all string no consecutive vowels.

$$n=1$$

$$C_1 = 21$$

$$V_1 = 5$$

$$n=2$$

$$C_2 = V_1 + C_1 = 21 + 5 = 26 \times 21$$

$$V_2 = C_1 = 21 \times 5$$

$$\underline{n=3}$$

$$C_3 = V_2 + C_2$$

$$= 26 + 21$$

$$= 47$$

$$V_3 = C_2 = 26$$

$$n=1$$

$$n=2$$

$$C_1 = 21 = 1 \times 21$$

$$C_2 = (C_1 + V_1) \times 21$$

$$V_1 = 5 = 1 \times 5$$

$$V_2 = C_1 \times 5$$

$$\left. \begin{aligned} V_{n+1} &= C_n \times 5 \\ C_{n+1} &= (C_n + V_n) \times 21 \end{aligned} \right\} \begin{aligned} V_1 &= 5 \\ C_1 &= 21 \end{aligned}$$

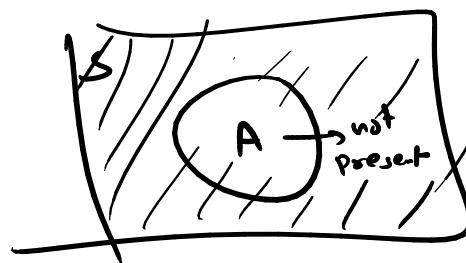
$$\text{ans} = C_n + V_n$$

generic  
ans.  
→

at least 2  
vowels together  
once ↑

ans

$$= \frac{\text{everything} \quad \text{no vowels together}}{\text{26}^n - C_n - V_n}$$



$$n=2 \quad \{RB \quad BB \quad BR\} \quad \cancel{RR}$$

$$n=1$$

$$\begin{matrix} B \\ R \end{matrix} \quad \begin{matrix} r_1=1 \\ b_1=1 \end{matrix}$$

$$n=2$$

$$b_2 = r_1 + b_1 = 1 + 1 = 2$$

$$r_2 = b_1 = 1$$

$$\text{ans} = b_2 + r_2 = 2 + 1 = \underline{\underline{3}}$$

callback

DP

# Graphs.

## Weighted graphs.

## Representation.

$u: (v, w)$

adjacency matrix

	1	2	3	4	5
1	0	10			
2	10	0			
3			0		
4				0	
5					0

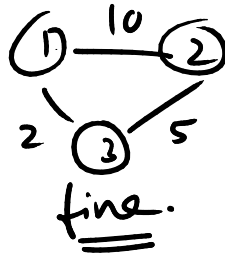
to

from

sense

5 nodes.

airline



adjacent list.

1	(2, 10)
2	(1, 10)
3	
4	
5	

BFS and DFS.

cost:

1	2	3	4	5
$10^8$	$10^8$	$10^8$	$10^8$	$10^8$

Dijkstra

Shortest path.

$$\text{Cost}[v] = \min \left( \text{cost}[u] + g[u][v], \text{cost}[v] \right)$$

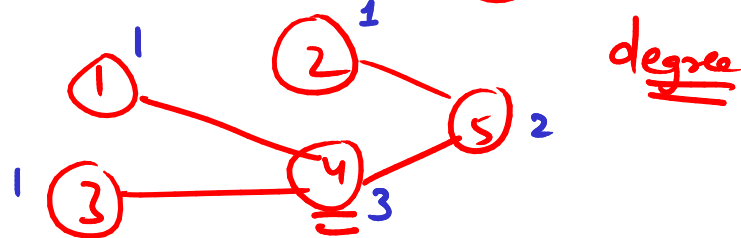
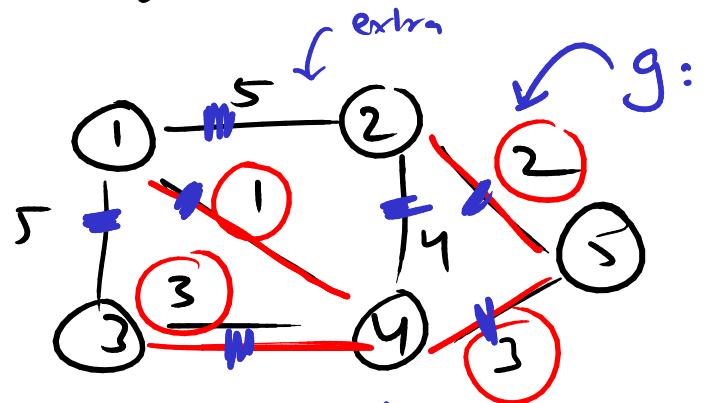
Task: edges | priority to edges with min weight.

pick the cheapest edge first.

1-4  
2-5  
4-5  
3-4

edges

connected.



① weighted undirected graph.

min. Connected graph

$n$  nodes

$n-1$  edges

→ priority on the edge

MINIMUM SPANNING TREE

Σ edge weights

edges sorted as per weight.

Naive represented :

only when ordering  
of edges is imp.

edge { int src  
int dest  
int weight  
}

n vertices

e edges

edge[e] edges ;

7 [ [ ... ] }

BFS and DFS x

'u' → tell me 'v's ??

Fine: Read ?? Tricky.

MST vs. Dijkstra

avg.

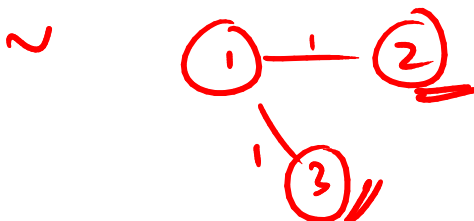
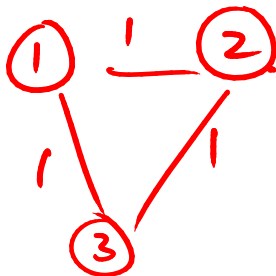
1. MST → greedy | minimize  
graph  
as a  
whole.

Dijkstra → shortest/min cost  
A → B

DP edges  
are

2. n-1 edges in MST.   
Rest are discarded.   
imp should path

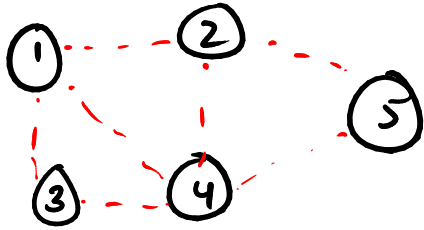
2 to 3 ⇒ cost (2)



edges[]  $\Rightarrow$  all edges are considered discrete.

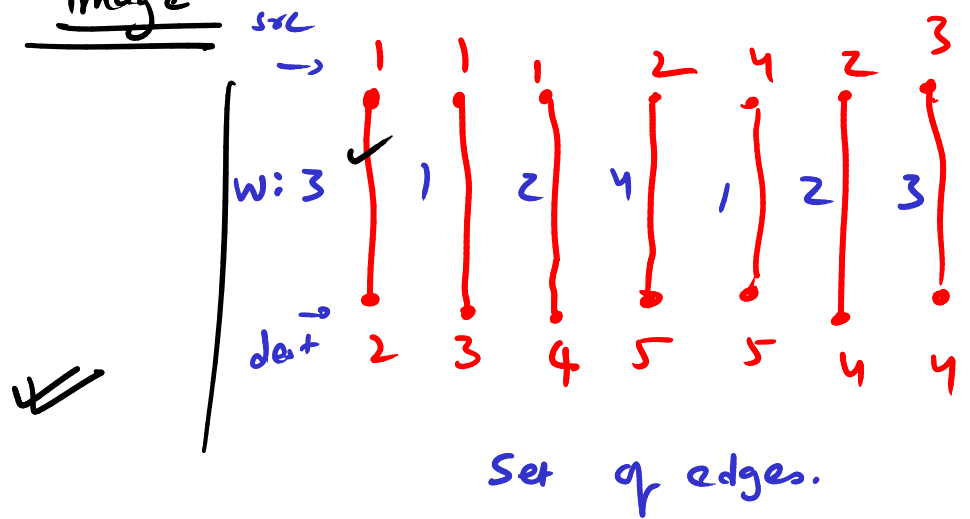
n nodes  $\Rightarrow$  nodes are discrete.

no neighborhood concept.



set of nodes

Image



Connectivity.

sets of nodes. :  $\swarrow$  {1} {2} {3} {4} {5}

parent/head of the set. {1,2} {3} {4} {5}

$\downarrow$   
root.

index : 1 2 3 4 5  $\swarrow$  nodes  
 root : 2 2 3 3 5  $\swarrow$  head of the set

Initial cond<sup>n</sup>:  
 every node belongs to its set.

add\_edge(root, u, v)

root-of-u = root(u)

root-of-v = root(v)

root[root-of-v] = root-of-u  
 (3,4) (1,2) (5)

(2,1)  
 $\downarrow \downarrow$   
 2 1

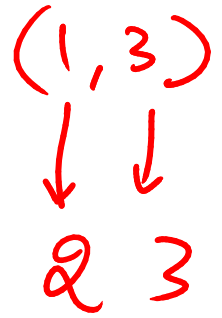
root[1] = 2

1-3

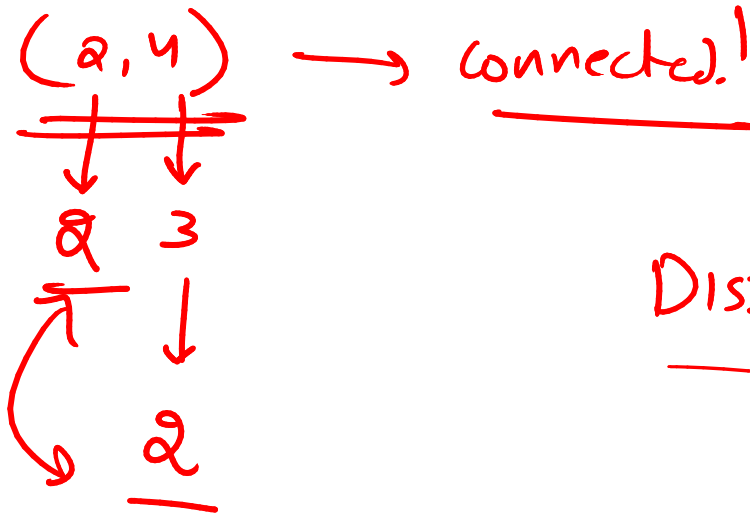
Somehow ensure: 4 & 2 are automatically connected.

how  $\Rightarrow$  2 and 4 are connected!

$\text{root}[u] \neq u \rightarrow$  its not a parent.



$\text{root}[3] = 2$



connected!

DISJOINT SET UNION

DSU

How to utilize DSU?

Kruskal's algo.

1. edges  $\rightarrow$  sort them based on edge weight.

2. root array  $\rightarrow$  1 to  $n \Rightarrow$  all set their own parent.

3. Traverse each edge in sequence:

if  $\text{disconnected}(u, v)$ :

$\text{root of } u = \text{root}(u)$

$\text{root of } v = \text{root}(v)$

$\text{root}[\text{root\_of\_}u] = \text{root\_of\_}v$

else: continue.

$n-1$   
times.  
==