

Red-black trees

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Symbol Table Review

Symbol table: key-value pair abstraction.

- Insert a value with specified key.
- Search for value given key.
- Delete value with given key.
- Different implementations
 - Array
 - Linked list
 - BST (binary search tree)

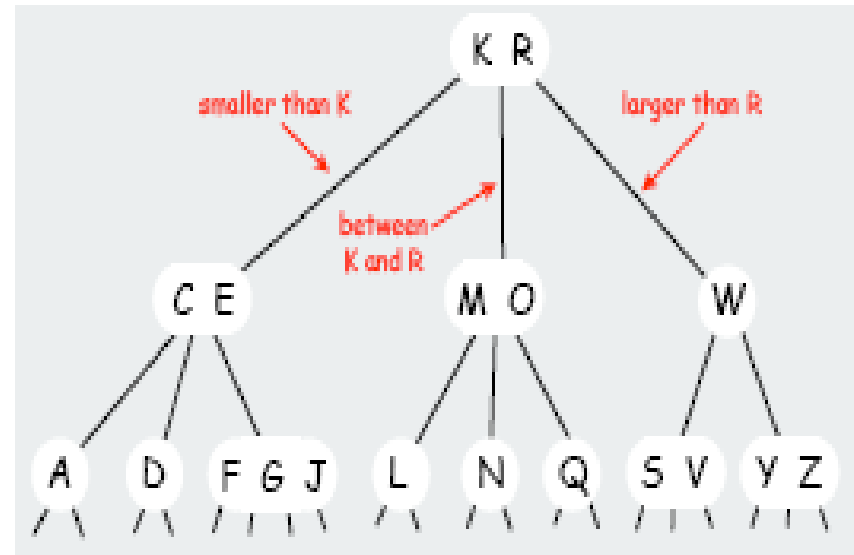
Complexity

implementation	guarantee			average case			ordered iteration?
	search	insert	delete	search	insert	delete	
unordered array	N	N	N	N/2	N/2	N/2	no
ordered array	$\lg N$	N	N	$\lg N$	N/2	N/2	yes
unordered list	N	N	N	N/2	N	N/2	no
ordered list	N	N	N	N/2	N/2	N/2	yes
BST	N	N	N	$1.39 \lg N$	$1.39 \lg N$?	yes
randomized BST	$7 \lg N$	$7 \lg N$	$7 \lg N$	$1.39 \lg N$	$1.39 \lg N$	$1.39 \lg N$	yes

- Randomized BST.
 - Guarantee of $\sim c \lg N$ time per operation (probabilistic).
 - Need subtree count in each node.
 - Need random numbers for each insert/delete op.

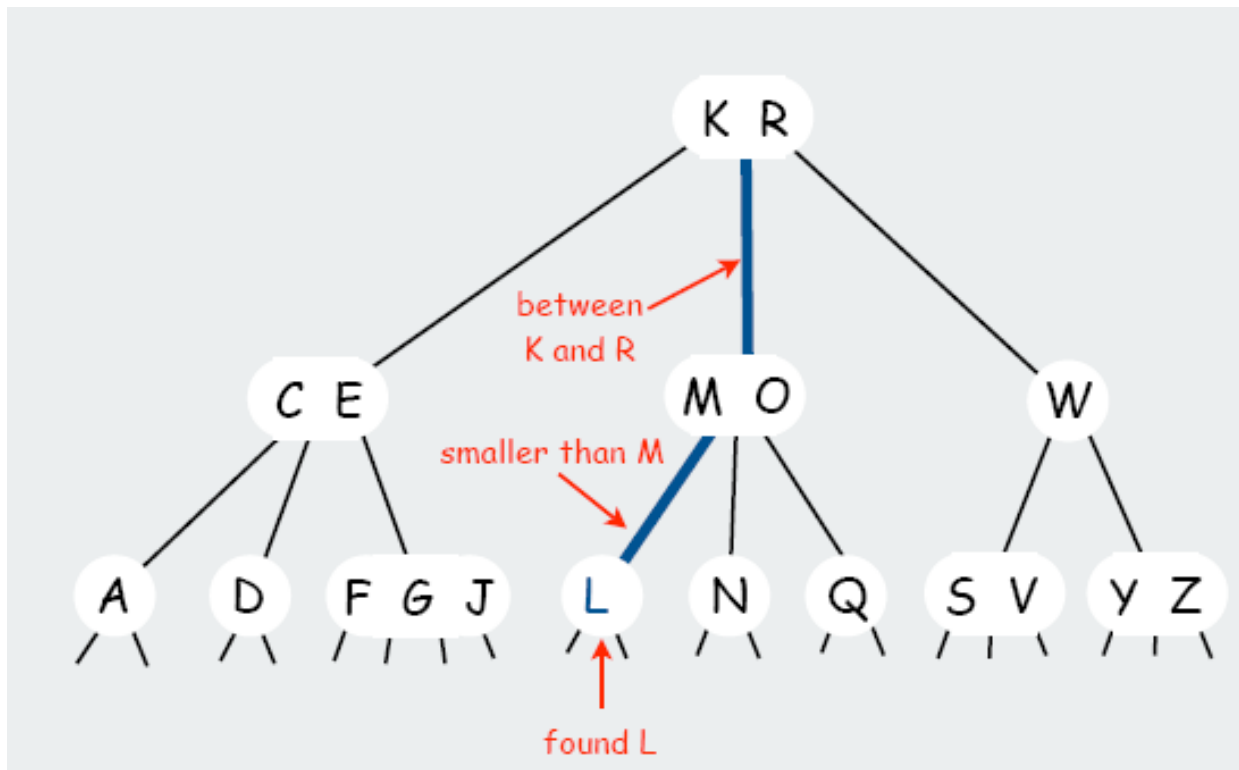
2-3-4 tree

- 2-3-4 tree. Generalize node to allow multiple keys; help to keep tree balanced.
- Perfect balance. Every path from root to leaf has same length.
- Allow 1, 2, or 3 keys per node.
 - 2-node: one key, two children.
 - 3-node: two keys, three children.
 - 4-node: three keys, four children.



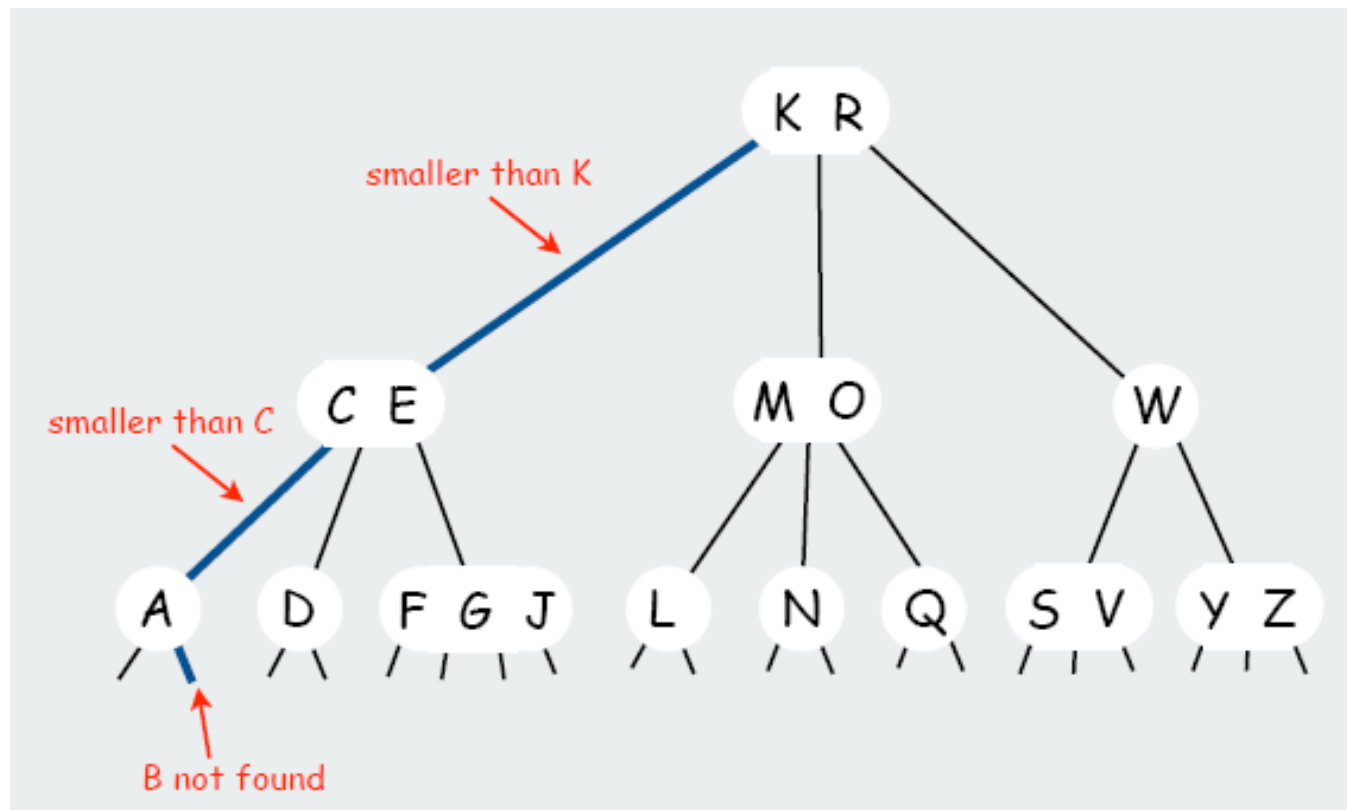
Search

- Compare search key against keys in node.
- Find interval containing search key.
- Ex. Search for L



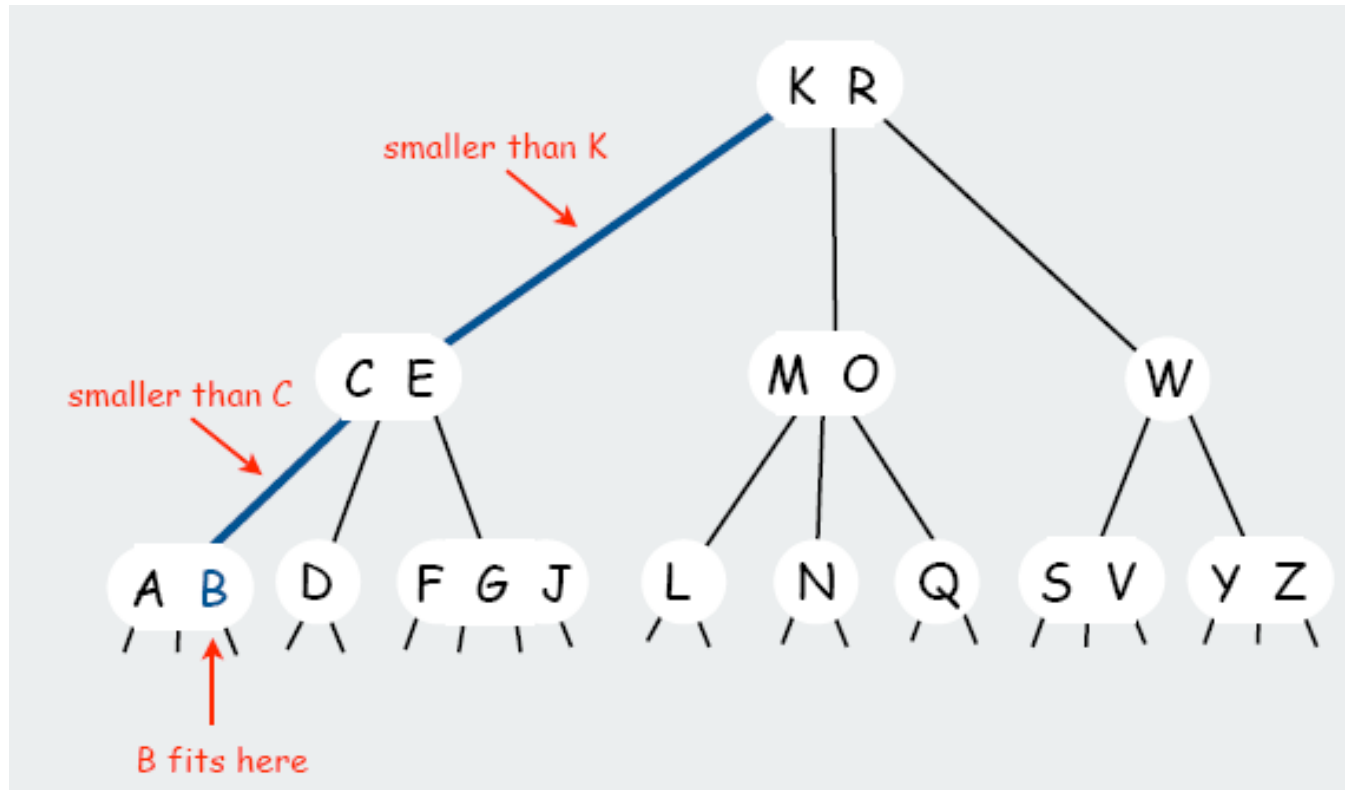
Insert (1)

- Search to bottom for key.
- Ex. Insert B



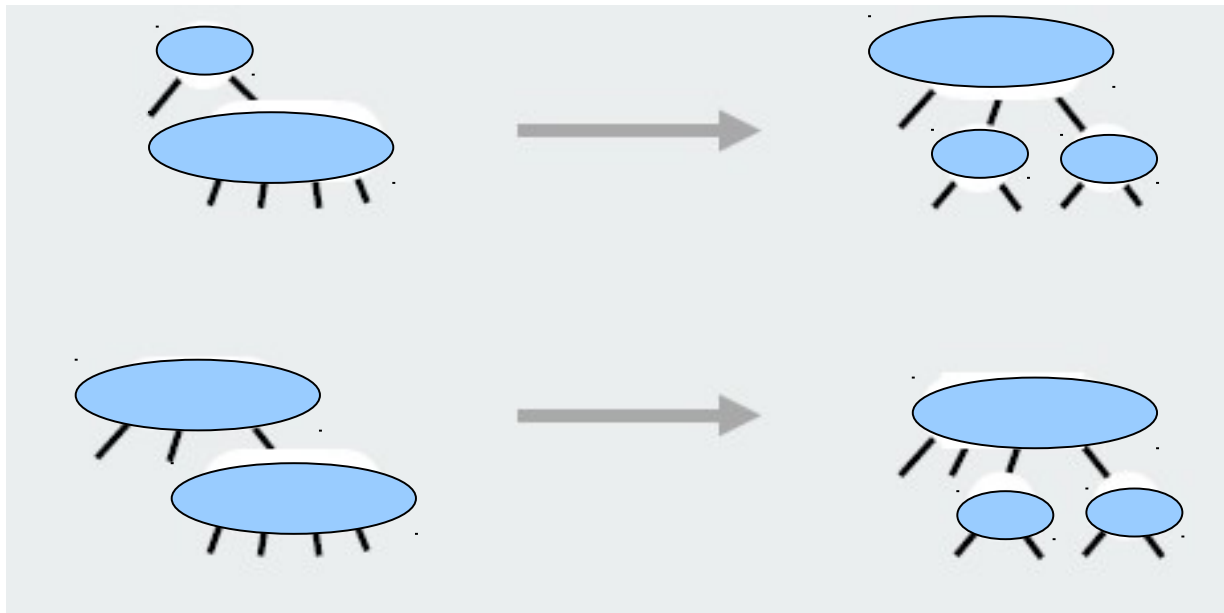
Insert (2)

- 2-node at bottom: convert to 3-node.
- 3-node at bottom: convert to 4-node.
- Ex. Insert B



Transformation

- Local transformations should be applied to keep the tree balanced.
- Ensures that most recently seen node is not a 4-node.
- Transformations to split 4-nodes:



Growth of a tree

Tree grows **up** from the bottom

insert A



insert S

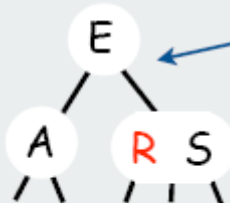


insert E

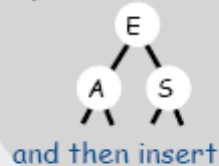


insert R

tree grows
up one level

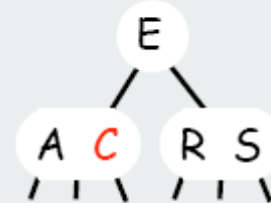


split 4-node to

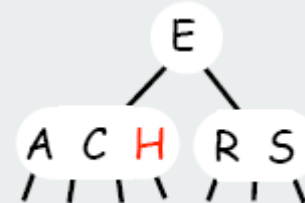


and then insert

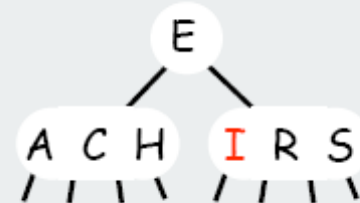
insert C



insert H

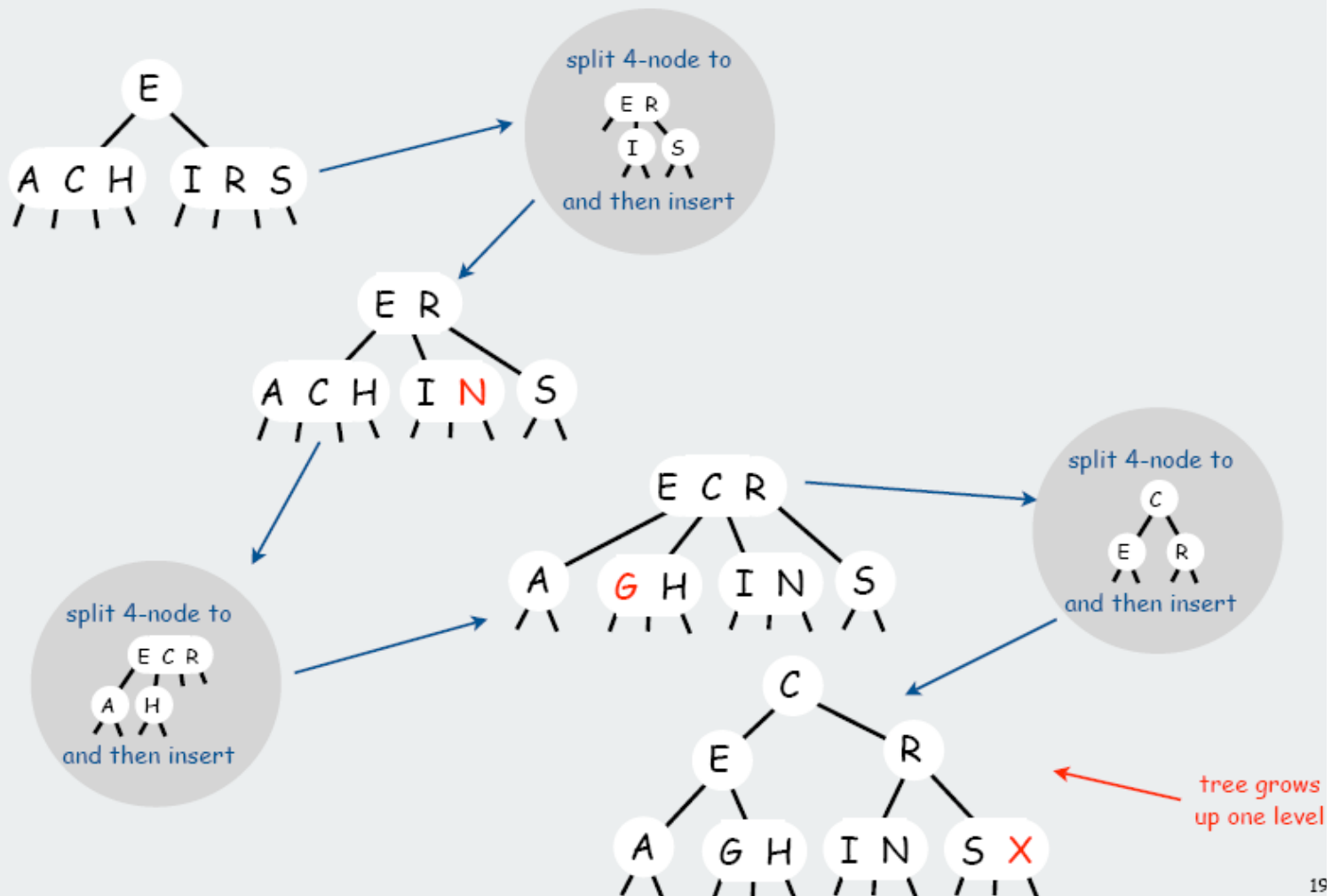


insert I



Growth of a tree (cont.)

Tree grows **up** from the bottom



Complexity

implementation	guarantee			average case			ordered iteration?
	search	insert	delete	search	insert	delete	
unordered array	N	N	N	N/2	N/2	N/2	no
ordered array	lg N	N	N	lg N	N/2	N/2	yes
unordered list	N	N	N	N/2	N	N/2	no
ordered list	N	N	N	N/2	N/2	N/2	yes
BST	N	N	N	1.38 lg N	1.38 lg N	?	yes
randomized BST	7 lg N	7 lg N	7 lg N	1.38 lg N	1.38 lg N	1.38 lg N	yes
2-3-4 tree	$c \lg N$	$c \lg N$		$c \lg N$	$c \lg N$		yes

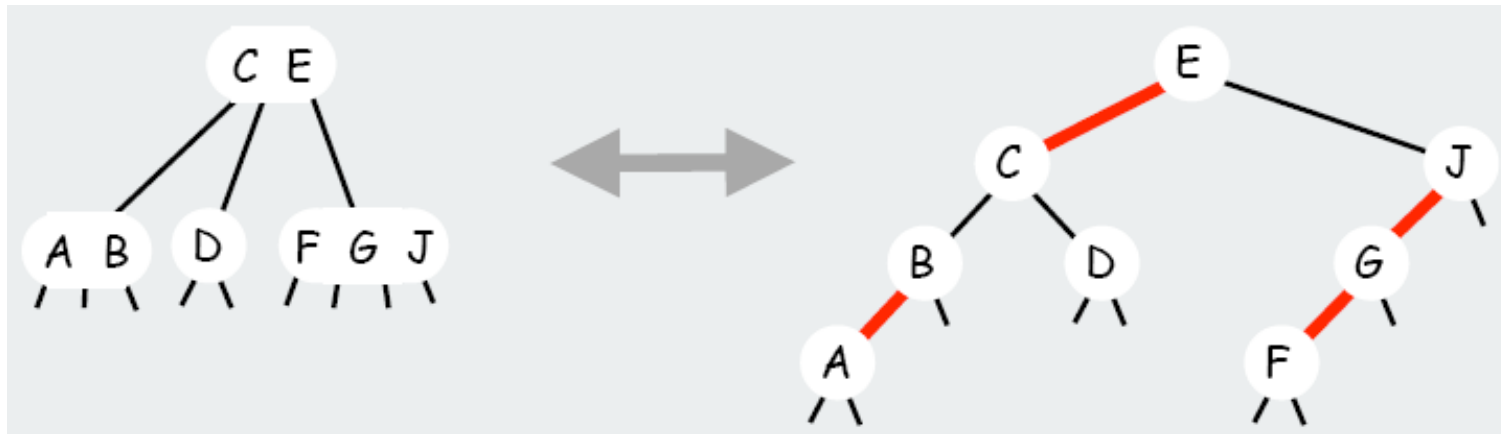
- Worst case: lg N [all 2-nodes]
- Best case: $\log_4 N = 1/2 \lg N$ [all 4-nodes]
- Between 10 and 20 for a million nodes.
- Between 15 and 30 for a billion nodes.

Red-black tree

- Represent 2-3-4 tree as a BST.
- Use "internal" left-leaning edges for 3- and 4- nodes.



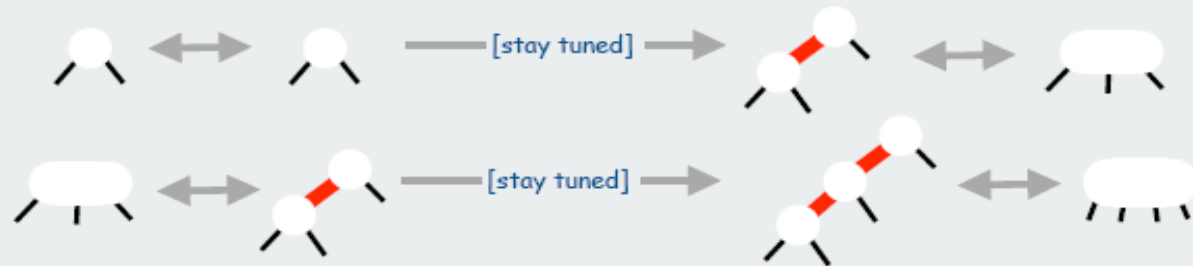
- 1-1 correspondence between 2-3-4 and left-leaning red-black trees.



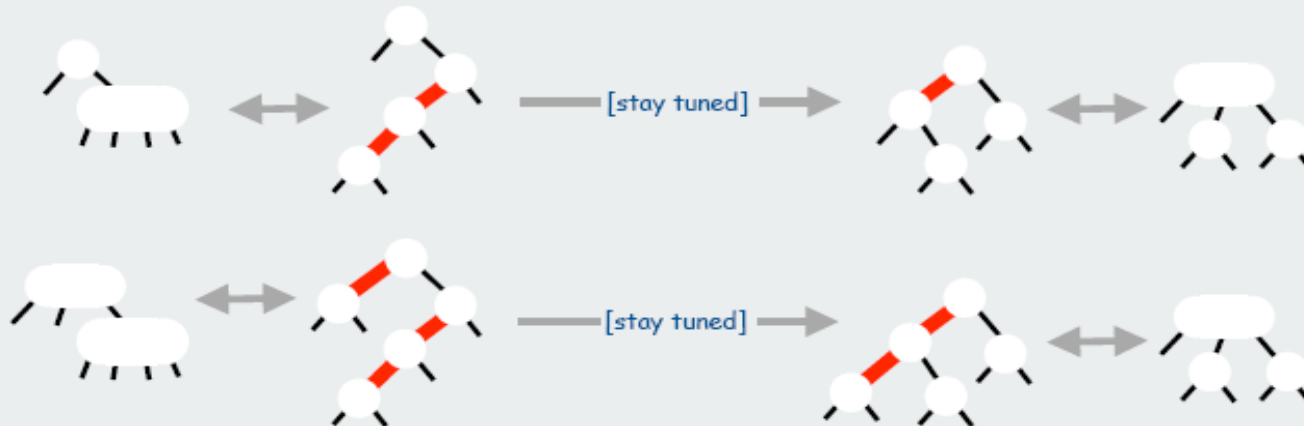
Insert implementation

Basic idea: **maintain 1-1 correspondence with 2-3-4 trees**

1. If key found on recursive search reset value, as usual
2. If key not found **insert a new red node at the bottom**



3. **Split 4-nodes** on the way DOWN the tree.



Complexity

implementation	guarantee			average case			ordered iteration?
	search	insert	delete	search	insert	delete	
unordered array	N	N	N	N/2	N/2	N/2	no
ordered array	lg N	N	N	lg N	N/2	N/2	yes
unordered list	N	N	N	N/2	N	N/2	no
ordered list	N	N	N	N/2	N/2	N/2	yes
BST	N	N	N	1.38 lg N	1.38 lg N	?	yes
randomized BST	7 lg N	7 lg N	7 lg N	1.38 lg N	1.38 lg N	1.38 lg N	yes
2-3-4 tree	c lg N	c lg N		c lg N	c lg N		yes
red-black tree	3 lg N	3 lg N	3 lg N	lg N	lg N	lg N	yes

Libfdr

- Libfdr is a library which contains an implementation for generic red-black trees in C
- Download and compile instructions at <http://www.cs.utk.edu/~plank/plank/classes/cs360/360/notes/Libfdr/>

Jval datatype

- A big union to represent a generic data type

```
typedef union {  
    int i;  
    long l;  
    float f;  
    double d;  
    void *v;  
    char *s;  
    char c;  
    unsigned char uc;  
    short sh;  
    unsigned short ush;  
    unsigned int ui;  
    int iarray[2];  
    float farray[2];  
    char carray[8];  
    unsigned char uarray[8];  
} Jval;
```


Jval usage

- Use Jval to store an integer
Jval j;
j.i = 4;
- Jval.h defines a whole bunch of prototypes for
`constructor functions.'

```
extern Jval new_jval_i(int);  
extern Jval new_jval_f(float);  
extern Jval new_jval_d(double);  
extern Jval new_jval_v(void *);  
extern Jval new_jval_s(char *);
```

Example:

```
Jval j = new_jval_i(4);
```

JRB datatype

- JRB is defined as a pointer to a node of the
typedef struct jrb_node {
 unsigned char red;
 unsigned char internal;
 unsigned char left;
 unsigned char roothed;
 struct jrb_node *flink;
 struct jrb_node *blink;
 struct jrb_node *parent;
 Jval key;
 Jval val;
} *JRB;

JRB API (1)

- Make a new tree
 - JRB make_jrb();
- Insert a new node to a tree
 - JRB jrb_insert_str(JRB tree, char *key, Jval val);
 - JRB jrb_insert_int(JRB tree, int ikey, Jval val);
 - JRB jrb_insert_dbl(JRB tree, double dkey, Jval val);
 - JRB jrb_insert_gen(JRB tree, Jval key, Jval val, int (*func)(Jval,Jval));
- Find a node via key
 - JRB jrb_find_str(JRB root, char *key);
 - JRB jrb_find_int(JRB root, int ikey);
 - JRB jrb_find_dbl(JRB root, double dkey);
 - JRB jrb_find_gen(JRB root, Jval, int (*func)(Jval, Jval));

JRB API (2)

- Free a node (but not the key or val)
 - `void jrb_delete_node(JRB node);`
- Free all the tree
 - `void jrb_free_tree(JRB root);`
- Navigation in the tree
 - `#define jrb_first(n) (n->flink)`
 - `#define jrb_last(n) (n->blink)`
 - `#define jrb_next(n) (n->flink)`
 - `#define jrb_prev(n) (n->blink)`
 - `#define jrb_empty(t) (t->flink == t)`
 - `#define jrb_nil(t) (t)`
 - `#define jrb_traverse(ptr, lst) \`
`for(ptr = jrb_first(lst); ptr != jrb_nil(lst); ptr = jrb_next(ptr))`

Quiz 1

- Try to compile and run some example programs (using libfdr) given at

<http://www.cs.utk.edu/~plank/plank/classes/cs360/360/notes/JRB/>

Quiz 2

- Use libfdr to write the phone book program (add, delete, modify phone numbers). The phone book should be stored in a file.
- NB: In the JRB, the insert function always creates a new node even if the key exists already in the tree.
 - You should check the existence of a record before insert it in the tree

Instruction

- Create a phone book
 - `JRB book = make_jrb();`
- Insert a new entry
 - `jrb_insert_str(book, strdup(name), new_jval_l(number));`
 - You must allocate memory to store the name for the new node's key. This memory should to be free when we delete all the key.
- Navigation
 - `jrb_traverse(node, book)`
`/* code to do something on node */`

Solution

- `phonebook_jrb.c`