Skylight Polarization and Rotation Estimation

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May 30, 2017

Notations

Frames

$\mathcal{W}$	World Frame $\{ENU\}$ (East North Up)
$\mathcal{W}'$	Global Frame of the IMU
$\mathcal{V}$	Vehicle Frame $\{FLU\}$ (Forward Left Up)
$\mathcal{I}$	IMU Frame $\{RLD\}$ (Rear Left Down)
$\mathcal{C}$	Camera Frame $\{RDF\}$ (Right Down Forward)

Rotations

$R_{wv}(t)$	Rotation of the vehicle $\mathcal{W} \rightarrow \mathcal{V}$
$R_{ww'}$	Rotation matrix between the World Frame and the IMU global frame $\mathcal{W} \rightarrow \mathcal{W}'$
$R_{w'i}(t)$	Ground Truth Rotation matrix provided by the IMU, $\mathcal{W}' \rightarrow \mathcal{I}$
R_{vi}	Rotation of the IMU in the vehicle frame, $\mathcal{V} \rightarrow \mathcal{I}$
R_{vc}	Rotation of the camera in the vehicle frame, $\mathcal{V} \rightarrow \mathcal{C}$
R_{cp}	Rotation matrix from the Camera Frame $\mathcal{C}$ to the Pixel Frame $\mathcal{P}$

Vectors

s	Sun position in the world frame $\mathcal{W}$
z	Vector representing the vertical in the world frame $\mathcal{W}$
$v(t)$	Vectors obtained from polarization measurements according to the camera calibration
$w(t)$	Vectors obtained from polarization measurements according to the camera calibration
c	Celestial point (3-vector) in the World Frame $\mathcal{W}$
o, b	3-vectors in the World Frame $\mathcal{W}$
E	Electrical field vector in the World Frame $\mathcal{W}$
E_{obc}	Electrical field vector in the Pixel Frame $\mathcal{P}$

Misc

$\cdot$	dot product
$\wedge$	cross product

- A vector y expressed in the Pixel Frame $\mathcal{P}$ can be expressed in the World Frame $\mathcal{W}$ according to:

$$\begin{aligned} x &= R_{wv} \cdot R_{vc} \cdot R_{cp} \cdot y \\ &= R \cdot R_{cp} \cdot y \end{aligned}$$

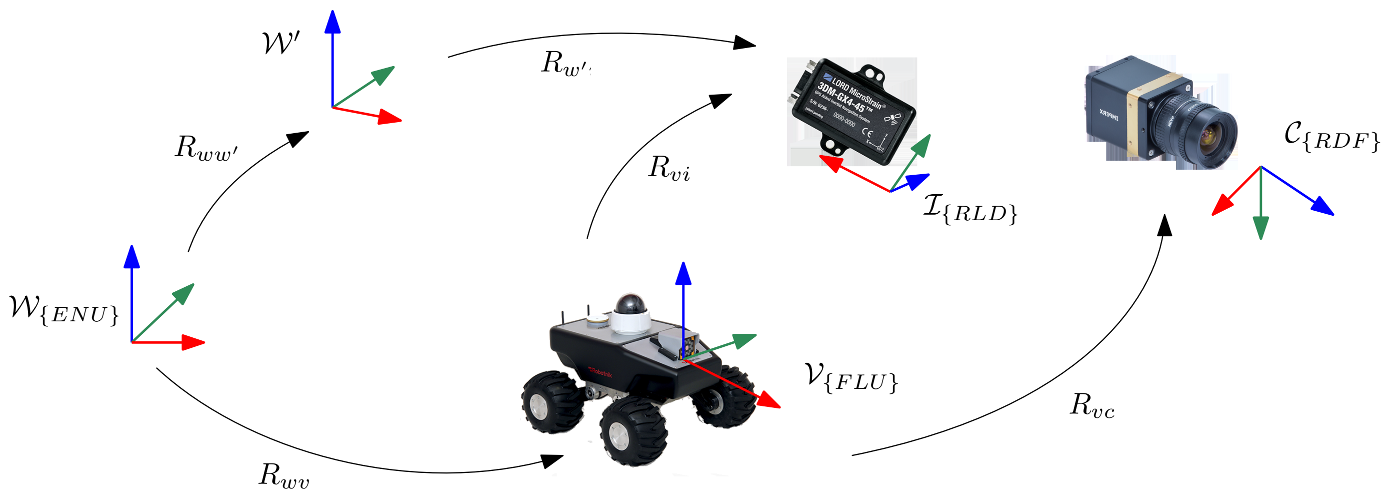

Figure 1: Frame conventions and notation.

- The rotation matrix R_{cp} is defined by:

$$R_{cp} = \begin{bmatrix} \cos\theta_c \cos\phi_c & -\sin\phi_c & \sin\theta_c \cos\phi_c \\ \cos\theta_c \sin\phi_c & \cos\phi_c & \sin\theta_c \sin\phi_c \\ -\sin\theta_c & 0 & \cos\theta_c \end{bmatrix} = R_{z_c}(\phi_c) \cdot R_{y_c}(\theta_c)$$

Relations

We have:

$$R_{wv}(t) = R_{ww'} \cdot R_{w'i}(t) \cdot R_{vi}^t. \tag{1}$$

Let define the rotation matrix R according to:

$$\begin{aligned} R(t) &= R_{wv}(t) \cdot R_{vc}, & (2)\\ &= R_{ww'} \cdot R_{w'i}(t) \cdot R_{vi}^t \cdot R_{vc}. & (3) \end{aligned}$$

1 Polarization by scattering

Using Rayleigh model, the Electric field is orthogonal to the scattering plane defined by vectors s and c. Therefore the normalized Electric field vector E is given by:

$$E = \frac{s \wedge c}{\|s \wedge c\|} \tag{4}$$

During experiments, the Electric field E vector is measured in the Pixel frame $\mathcal{P}$:

$$E_{obc} = \begin{bmatrix} E_o \\ E_b \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix} \tag{5}$$

where α is the measured the angle of polarisation.

From equation (4) and (5) we can write:

$$\begin{cases} \frac{s\wedge c}{\|s\wedge c\|} \cdot o &= E_o = \cos\alpha \\ \frac{s\wedge c}{\|s\wedge c\|} \cdot b &= E_b = \sin\alpha \end{cases}, \tag{6}$$

leading to:

$$\begin{cases} (s \wedge c) \cdot o &= \|s \wedge c\| \cos\alpha \\ (s \wedge c) \cdot b &= \|s \wedge c\| \sin\alpha \end{cases}, \tag{7}$$

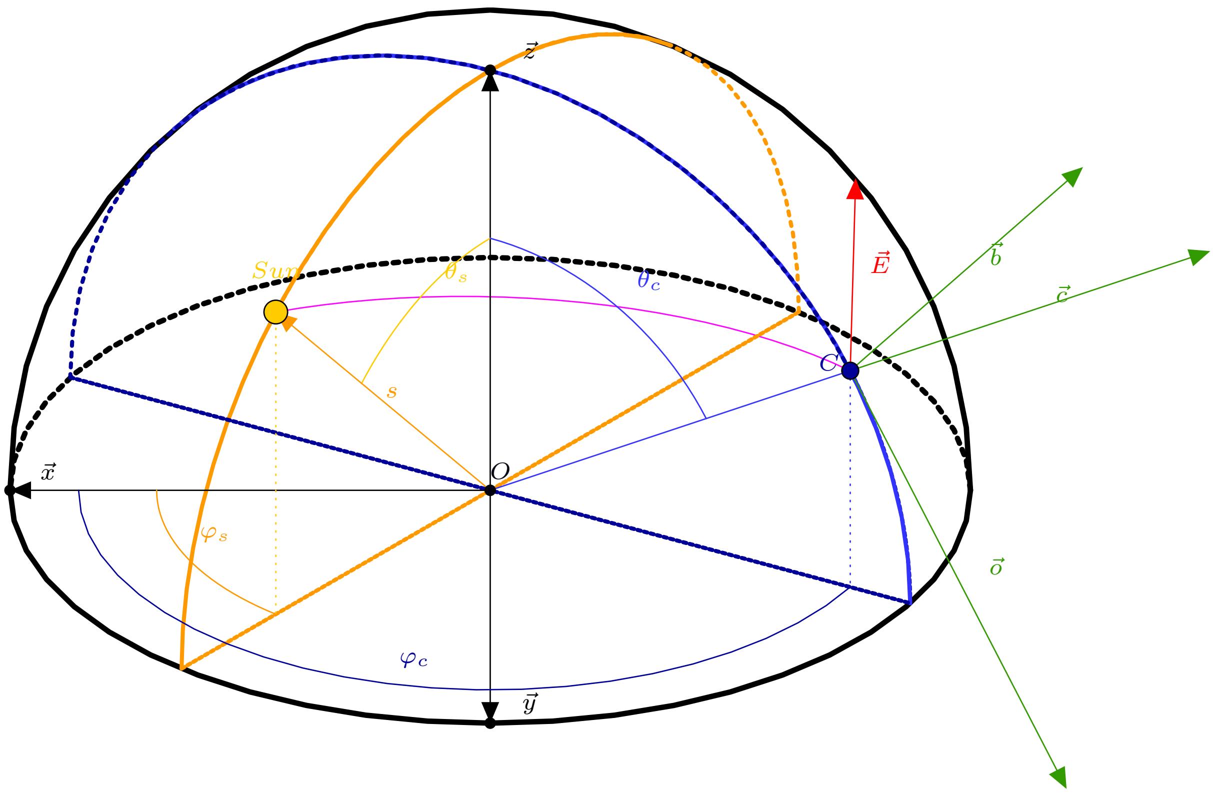

Figure 2: Skylight polarization.

and since$\|s \wedge c\| = \|s\| * \|c\| * \sin\gamma = \sin\gamma$ (with γ the angular distance between the observed celestial point and the sun) we can write:

$$\begin{cases} (s \wedge c) \cdot o & = \sin\gamma\cos\alpha \\ (s \wedge c) \cdot b & = \sin\gamma\sin\alpha \end{cases}, \tag{8}$$

The previous system can be rewritten by applying the following rules:

$$\begin{aligned} (s \wedge c) \cdot o & = (s \wedge (o \wedge b)) \cdot o \\ & = ((s \cdot b)\, o - (s \cdot o)\, b) \cdot o \ , \\ & = s \cdot b \end{aligned}$$

and

$$\begin{aligned} (s \wedge c) \cdot b & = (s \wedge (o \wedge b)) \cdot b \\ & = ((s \cdot b)\, o - (s \cdot o)\, b) \cdot b \ . \\ & = -s \cdot o \end{aligned}$$

We finally have:

$$\begin{cases} s \cdot b & = \sin\gamma\cos\alpha \\ s \cdot o & = -\sin\gamma\sin\alpha \end{cases} \tag{9}$$

The measured degree of polarization is given by [pomozi2001]:

$$\rho = \rho_{max} * \frac{1 - \cos^2\gamma}{1 + \cos^2\gamma}. \tag{10}$$

Consequently, by inverting equation (10) we have:

$$\cos\gamma = s \cdot c = \pm\sqrt{\frac{1 - \rho'}{1 + \rho'}}, \tag{11}$$

with $\rho' = \frac{\rho}{\rho_{max}}$.

Expressing the sun vector in the Pixel Frame $\mathcal{P}$ gives a direct relation between the measured polarization parameters (angle of polarization α and degree of polarization ρ directly related to γ) and the sun position and the celestial point:

$$R^t \cdot s = R_{cp} \begin{bmatrix} -\sin\gamma \sin\alpha \\ \sin\gamma \cos\alpha \\ \cos\gamma \end{bmatrix} = v, \tag{12}$$

where v is a vector obtained from polarization measurements according to the selected pixel.

2 Polarization by reflection with an horizontal surface

The Electric field is orthogonal to the incidence plane defined by vectors c and $z = [0, 0, 1]$. Therefore the normalized Electric field vector E is given by:

$$E = \frac{z \wedge c}{\|z \wedge c\|} \tag{13}$$

Applying the same blabla as in section 1 we obtain:

$$R^t \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = R_{cp} \cdot \begin{bmatrix} -\sin\gamma \sin\alpha \\ \sin\gamma \cos\alpha \\ \cos\gamma \end{bmatrix} = w. \tag{14}$$

where α and δ are respectively the measured angle of polarization and the angular distance between c and z. γ is also equal to the angle of incidence (assuming a specular reflection) and is linked to the measured degree of polarization and the refractive index of the media using Fresnel properties.

3 Estimating γ

Assuming that we are only measuring the angle of polarization α in scattering effects, we have to estimate γ to get the vector v defined in eq(12). Note that the same method can be appplied in the case of the reflection to get the vector w.

The equation (12) is valid for two different celestial points c_1 and c_2:

$$\begin{cases} R^t \cdot s = R_{cp_1} \cdot \begin{bmatrix} -\sin\gamma_1 \sin\alpha_1 \\ \sin\gamma_1 \cos\alpha_1 \\ \cos\gamma_1 \end{bmatrix} \\ R^t \cdot s = R_{cp_2} \cdot \begin{bmatrix} -\sin\gamma_2 \sin\alpha_2 \\ \sin\gamma_2 \cos\alpha_2 \\ \cos\gamma_2 \end{bmatrix} \end{cases} \tag{15}$$

so we can write:

$$R^t \cdot s = R_{cp_1} \cdot \begin{bmatrix} -\sin\gamma_1 \sin\alpha_1 \\ \sin\gamma_1 \cos\alpha_1 \\ \cos\gamma_1 \end{bmatrix} = R_{cp_2} \cdot \begin{bmatrix} -\sin\gamma_2 \sin\alpha_2 \\ \sin\gamma_2 \cos\alpha_2 \\ \cos\gamma_2 \end{bmatrix} \tag{16}$$

The equation(16) can be rewritten according to:

$$R_{cp_1} \cdot \begin{bmatrix} \cos\alpha_1 & -\sin\alpha_1 & 0 \\ \sin\alpha_1 & \cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \sin\gamma_1 \\ \cos\gamma_1 \end{bmatrix} = R_{cp_2} \cdot \begin{bmatrix} \cos\alpha_2 & -\sin\alpha_2 & 0 \\ \sin\alpha_2 & \cos\alpha_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \sin\gamma_2 \\ \cos\gamma_2 \end{bmatrix}$$

leading to:

$$M_1 \cdot \begin{bmatrix} 0 \\ \sin\gamma_1 \\ \cos\gamma_1 \end{bmatrix} = M_2 \cdot \begin{bmatrix} 0 \\ \sin\gamma_2 \\ \cos\gamma_2 \end{bmatrix} \tag{17}$$

Since the angles of polarization α_1, α_2 are known for the 2 points, M_1 and M_2 are known and γ_1, γ_2 can be found by solving (17). It can be rewritten according to:

$$M_2^t \cdot M_1 \begin{bmatrix} 0 \\ \sin\gamma_1 \\ \cos\gamma_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\gamma_2 \\ \cos\gamma_2 \end{bmatrix}.$$

If we denote the matrix M defined such that $M = M_2^t \cdot M_1$, we have :

$$\begin{cases} \gamma_1 = -\arctan \frac{M_{02}}{M_{01}} \\ \gamma_2 = -\arctan \frac{M_{20}}{M_{10}} \end{cases} \tag{18}$$

The determination of vectors v or w impose to have α and γ defined respectively π and 2π modulus or defined respectively 2π and π modulus. Without loss of generality, it can be imposed that γ (that represents the angle of scattering or the angle of reflection) is determined in the range $[0, \pi]$ and α is determined in the range $[-\frac{\pi}{2}, \frac{3\pi}{2}]$. Consequently, eq(17) must be solved considering the four possible cases :

$$(\alpha_1, \alpha_2)\,,\ (\alpha_1, \alpha_2 + \pi)\,,\ (\alpha_1 + \pi, \alpha_2)\,,\ (\alpha_1 + \pi, \alpha_2 + \pi)\,.$$

To compute v or w, this can be reduced to two solutions:

$$(\alpha_1, \gamma_1) \text{ and } (\alpha_1 + \pi, -\gamma_1)\,. \tag{19}$$

4 Rotation estimation

4.1 Absolute rotation

Assumptions

- Sun position s is known
- From 2 measurements of the angle of polarization from the blue sky, v can be estimated satisfying:

$$s = R(t) \cdot v(t) \tag{20}$$

- From 2 measurements of the angle of polarization from reflection by horizontal surface (area of water), w can be estimated satisfying:

$$z = R(t) \cdot w(t) \tag{21}$$

Derivation

Using the cross product between (20) and (21) we have the third equation:

$$s \wedge z = R(t) \cdot v(t) \wedge w(t), \tag{22}$$

leading to the following system:

$$\begin{cases} [s, z, s \wedge z] & = R(t) \cdot [v(t), w(t), v(t) \wedge w(t)] \\ & = R_{wv}(t) \cdot R_{vc} \cdot [v(t), w(t), v(t) \wedge w(t)] \end{cases}. \tag{23}$$

Solving equation (23) enables to get $R_{wv}(t)$ if the sun position s is known.

Solving ambiguities

For each measurment we have in fact two candidates as described in eq(19) providing 2 vectors v_1, v_2 and 2 vectors w_1, w_2.

Since the angle γ in $w_1$and w_2 represents the angle of reflection must be less than 90° and it imposes only one solution regarding w.

(NEXT TO BE WRITTEN)

4.2 Relative rotation

We can apply formula (23) at 2 different times t_1 and t_2:

$$\begin{cases} [s, z, s \wedge z] & = R_{wv}(t_1) \cdot R_{vc} \cdot [v(t_1), w(t_1), v(t_1) \wedge w(t_1)] \\ [s, z, s \wedge z] & = R_{wv}(t_2) \cdot R_{vc} \cdot [v(t_2), w(t_2), v(t_2) \wedge w(t_2)] \end{cases}.$$

To simplify the representation let rewrite it according to:

$$\begin{cases} [s, z, s \wedge z] & = R_{wv1} \cdot R_{vc} \cdot [v_1, w_1, v_1 \wedge w_1] \\ [s, z, s \wedge z] & = R_{wv2} \cdot R_{vc} \cdot [v_2, w_2, v_2 \wedge w_2] \end{cases}.$$

So we obtain:

$$\begin{cases} [s, z, s \wedge z] & = R_{wv1} \cdot R_{vc} \cdot [v_1, w_1, v_1 \wedge w_1] \\ R_{wv2} & = [s, z, s \wedge z] \cdot [v_2, w_2, v_2 \wedge w_2]^{-1} \cdot R_{vc}^t. \end{cases}$$

And finally:

$$R_{wv2} = R_{wv1} \cdot R_{vc} \cdot [v_1, w_1, v_1 \wedge w_1] \cdot [v_2, w_2, v_2 \wedge w_2]^{-1} \cdot R_{vc}^t. \tag{24}$$

The relative rotation $R_{v1v2} = R_{wv1}^T \cdot R_{wv2}$ is therefore equal to:

$$R_{v1v2} = R_{vc} \cdot [v_1, w_1, v_1 \wedge w_1] \cdot [v_2, w_2, v_2 \wedge w_2]^{-1} \cdot R_{vc}^t \tag{25}$$

A ROS coordinate Frame conventions

Defined in REP 103, all coordinate frames should follow these conventions.

Chirality

All systems are right handed. This means they comply with the right hand rule.

Axis Orientation

In relation to a body the standard is:

- x Forward
- y Left
- z Up

For short-range Cartesian representations of geographic locations, use the east north up [5] (ENU) convention:

- X East
- Y North
- Z Up

To avoid precision problems with large float32 values, it is recommended to choose a nearby origin such as your system's starting position.

Suffix Frames

In the case of cameras, there is often a second frame defined with a "_optical" suffix. This uses a slightly different convention:

- x Right
- y Down
- z Forward

For outdoor systems where it is desireable to work under the north east down (NED) convention, define an appropriately transformed secondary frame with the "_ned" suffix:

- X North
- Y East
- Z Down

Rotation Representation

There are many ways to represent rotations. The preferred order is listed below, along with rationale.

1. quaternion
   - Compact representation
   - No singularities
2. rotation matrix
   - No singularities
3. fixed axis roll, pitch, yaw about X, Y, Z axes respectively
   - No ambiguity on order
   - Used for angular velocities
4. euler angles yaw, pitch, and roll about Z, Y, X axes respectively
   - Euler angles are generally discouraged due to having 24 'valid' conventions with different domains using different conventions by default.

By the right hand rule, the yaw component of orientation increases as the child frame rotates counter-clockwise, and for geographic poses, yaw is zero when pointing east.

This requires special mention only because it differs from a traditional compass bearing, which is zero when pointing north and increments clockwise. Hardware drivers should make the appropriate transformations before publishing standard ROS messages.