

Hamiltonian Mechanics unter besonderer Berücksichtigung der höheren Lehranstalten

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Abstract. Breast cancer is the second most common cancer and the leading cause of cancer death among women. Medical imaging has become an indispensable tool for its diagnosis and follow up. During the last decade, the medical community has promoted to incorporate Ultra-Sound (US) screening as part of the standard routine. The main reason for using US imaging is its capability to differentiate benign from malignant masses, when compared to other imaging techniques. The increasing usage of US imaging encourages the development of Computer Aided Diagnosis (CAD) systems applied to Breast Ultra-Sound (BUS) images. However accurate delineations of the lesions and structures of the breast are essential for CAD systems in order to extract information needed to perform diagnosis.

This article proposes a highly modular and flexible framework for segmenting lesions and tissues present in BUS images. The proposal takes advantage of optimization strategies using super-pixels and high-level descriptors, which are analogous to the visual cues used by radiologists. Qualitative and quantitative results are provided stating a performance within the range of the state-of-the-art.

Keywords: Breast Ultra-Sound, BI-RADS lexicon, Optimization based Segmentation, Machine-Learning based Segmentation, Graph-Cuts

1 Introduction

Breast cancer is the second most common cancer (1.4 million cases per year, 10.9% of diagnosed cancers) after lung cancer, followed by colorectal, stomach, prostate and liver cancers. In terms of mortality, breast cancer is the fifth most common cause of cancer death. However, it is ranked as the leading cause of cancer death among females in both western countries and economically developing countries [14].

Medical imaging plays an important role in breast cancer mortality reduction, contributing to its early detection through screening for diagnosis, image-guided biopsy, treatment follow-up and suchlike procedures [22]. Although Digital Mammography (DM) remains the reference imaging modality for breast cancer screening, Ultra-Sound (US) imaging has proven to be a successful adjunct image modality [22]. The main advantage of US imaging, opposed to other image modalities, lies in the discriminative power that US offers to visually differentiate benign from malignant solid lesions [23]. In this manner, US screening contributes to reduce the amount of unnecessary biopsies [3], which is estimated to be between 65 ~ 85% of the prescribed biopsies [26], in favour of a less traumatic short-term screening follow-up using Breast Ultra-Sound (BUS) images. As the standard for assessing this BUS images, the American College of Radiology (ACR) proposes the Breast Imaging-Reporting and Data System (BI-RADS) lexicon for BUS images [19]. This US BI-RADS lexicon is a set of standard markers that characterizes the lesions encoding the visual cues found in BUS images and facilitates their analysis. Further details regarding the US BI-RADS lexicon descriptors proposed by the ACR, can be found in this document in section 2.1, where visual cues of BUS images and breast structures are discussed to define feature descriptors.

All these facts show the interest in the medical community for incorporating US screening as part of the standard procedure in breast screening programs [19], which encourages the development of Computer Aided Diagnosis (CAD) systems using US to be applied to breast cancer diagnosis. Building CAD systems based on the clinical tools already in use (*i.e.* US BI-RADS lexicon) is not straight forward. Shortcomings like the location and explicit delineation of the lesions need to be addressed, since those tasks are intrinsically carried out by the radiologists during their visual assessment of the images to infer the lexicon representation of the lesions. Therefore, developing accurate segmentation methodologies for breast lesions and structures are crucial in order to develop CAD systems that can take advantage of the already validated clinical tools.

This article proposes a highly modular and flexible framework for segmenting lesions and tissues present in BUS images. The proposal takes advantage of an energy-based strategy to perform segmentations based on discrete optimizations using super-pixels and a set of novel features analogous to the elements encoded by the US BI-RADS lexicon [19].

2 Description of the segmentation methodology

Optimization methodologies offer a standardized manner to approach segmentation by minimizing an application-driven cost function [4]. Figure 1 illustrates a generic representation of the segmentation strategy here adopted to delineate breast tissues or lesions in US images. The overall segmentation can be seen as a three-steps strategy: (1) a mapping of the image into a discrete set of elements \mathcal{S} , (2) the optimization stage which is formulated as a *metric labelling* problem,

and (3) a re-mapping the labels obtained from the previous stage to produce the final delineation.

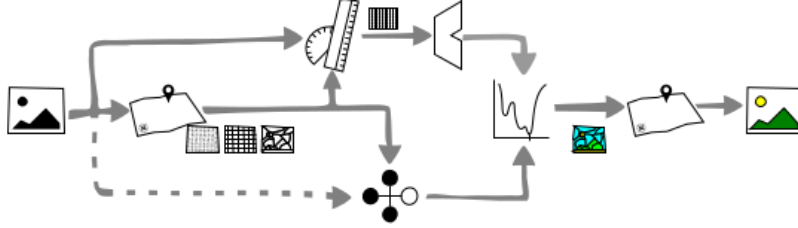


Fig. 1: Conceptual block representation of the segmentation methodology

In order to formulate the segmentation like a metric labelling problem, the image is conceived as a discrete set of elements \mathcal{S} that need to be labelled using a label l from the labelling set \mathcal{L} (i.e. $\mathcal{L} = \{\text{lesion}, \overline{\text{lesion}}\}$ or $\mathcal{L} = \{\text{lungs}, \text{fat}, \dots, \text{lesion}\}$). Let \mathcal{W} be all the possible labelling configurations of the set \mathcal{S} , given \mathcal{L} . Let $U(\cdot)$ be a cost function encoding the goodness of the labelling configuration $\omega \in \mathcal{W}$ based on the appearance of the elements in \mathcal{S} , their inner relation and some designing constraints. Then, the desired segmentation $\hat{\omega}$ corresponds to the labelling configuration that minimizes this cost function, as described in Equation (1).

$$\hat{\omega} = \arg \min_{\omega} U(\omega) \quad (1)$$

It is worth to mention here that not all the minimization strategies are applicable or adequate to find $\hat{\omega}$. The convenience of a particular minimization strategy is determined by the nature of $U(\cdot)$ and \mathcal{W} (see section 2.3).

This goodness measure $U(\cdot)$ must be defined to take into account the appearance of the target region, its relation with other regions and other designing constraints. Equation (2) describes this cost function as the combination of two independent costs that need to be simultaneously minimized as a whole.

$$U(\omega) = \sum_{s \in \mathcal{S}} D_s(\omega_s) + \sum_s \sum_{r \in \mathcal{N}_s} V_{s,r}(\omega_s, \omega_r) \quad (2)$$

Where, the left hand side of the expression integrates the so-called *data* term, while the right hand side integrates the *pairwise* term, which is also referred as the *smoothing* term. Both terms are shaped by \mathcal{S} and evaluated in the labelling space \mathcal{W} . Figure 2 uses the problem of delineating the tissues present in a BUS image to represent the working principles of the data and pairwise terms in eq. (2).

In general, \mathcal{S} can be any discrete set representing the image (i.e. pixels, overlapping or non overlapping windows, etc.). For this work \mathcal{S} is chosen to be a super-pixels representation of the image. Super-pixels can be seen as the output

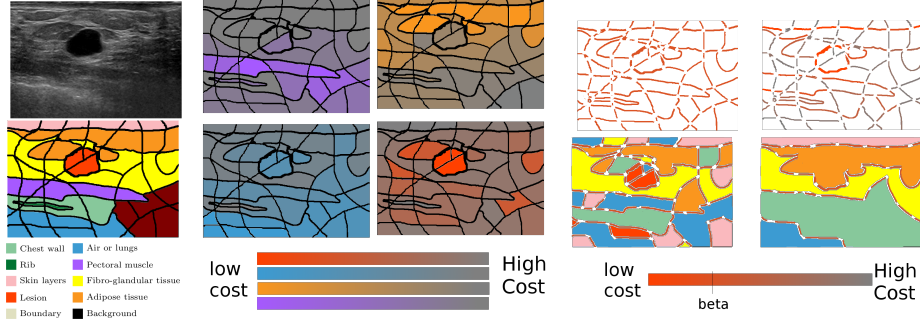


Fig. 2: Methodology: (a) Problem definition, (b) Data term, (c) Pairwise term.

of an over-segmentation process or as a set of pixel collections that are contiguous and coherent with respect to a given metric. Either way, super-pixels are no overlapped irregular groups of similar connected pixels [1]. Section 2 illustrates one such representation \mathcal{S} , applied to a BUS image example. The super-pixels are coloured according to the image’s Ground Truth (GT). For the rest of this work, \mathcal{S} is considered to be the super-pixels resulting from an over-segmentation of the image using Quick-shift.

Bear in mind that given an unseen BUS image, the ultimate goal is to represent the image as a set of super-pixels and infer the appropriated labelling for each of them. This goal requires to define: a data term, a pairwise term, and a proper minimization methodology.

2.1 The data term

Given a label configuration $\omega \in \mathcal{W}$, the data term penalizes the labelling of a particular image element or site ($\omega_s = l$) based on the data associated to s . In this manner, $D_s(\omega_s = l_{\checkmark}) \ll D_s(\omega_s = l_{\times})$. Section 2 illustrates the data cost associated to some arbitrary labelling configurations to clarify the desired effect (or behaviour) of this data term (section 2 shows the GT of each site s). Notice that the labelling configurations ω used in section 2 have the particularity that all sites share the same label, $\omega \in \{\omega_s = l, \forall s \in \mathcal{S}\}$.

Designing an obscure heuristic to comply with the desired behaviour of $D(\cdot)$ out of the box, is rather a complicated task. Therefore, an easier and cleaner approach is to take advantage of Machine Learning (ML) techniques to design this data cost in a systematic manner based on a training stage. The idea is to generate a data model for each label (or class) in \mathcal{L} from training samples, and let $D(\cdot)$ be a distance or goodness measure reflecting the likelihood for s to belong to class l . ML offers a systematic but flexible approach to customize $D(\cdot)$. Different features to represent the data, custom construction of the data models by using different classifiers, training techniques or including arbitrary

constraints; can be used to achieve the desired data term without changing the overall scheme.

Despite details regarding the construction $D(\cdot)$ are out of the scope of this report, the rest of this section 2.1 summarizes this process. The usage of ML as part of the proposed framework to determine $D(\cdot)$ is represented at the upper side of the diagram in fig. 1, which can be divided into two blocks: (a) the features to represent the samples, and (b) the tools to encode $D(\cdot)$ based on the features and the training.

BUS features to build the data term Figure 3 is a three-parts illustration to graphically summarize the visual cues that can be found in BUS images and their incorporation to the data term. The structures of the breast and their rendering when using a hand-held 2D US probe are shown in section 2.1. Section 2.1 illustrates the lexicon proposed by the ACR [19]. Whereas section 2.1 relates the visual cues to the following features:

Appearance Based on the multi-labelled GT, a Median Absolute Deviation (MAD) histogram model for every tissue label is built. The Appearance feature is computed as the χ^2 distance between histogram of s and the models.

Atlas Based on the multi-labelled GT an atlas is build to encode the label likelihood based on the location of s .

Brightness Takes an intensity descriptor of s (*i.e.*: mean, median, mode) and compares it with some intensity markers of the set \mathcal{S} such as the minimum intensity value, the maximum, its mean, etc.

Self-Invariant Feature Transform (SIFT)-Back-of-Features (BoF) s is represented as the occurrences of a SIFT dictionary of 36 words [18].

In order to incorporate multi-resolution, each super-pixel is group with its adjacent super-pixels such that $s' = \{s \cup \mathcal{N}_s\}$, the features are recalculated using s' and concatenated to the original feature descriptor of s . This operation can be repeated several times.

Notice that not all the visual cues compiled by the lexicon are suitable to build the data term. Mass shape, orientation and margin cues do not characterize an individual super-pixel but a group of contiguous super-pixels sharing the same label. Characterising super-pixels in those terms imply that super-pixels should be image elements of the same order as the lesions. This is not the case, since the bottom-line is to aggregate small regions to form the segmentation. The lesion boundary cue falls at the other extreme. To encode this visual cue as data term feature implies to use smaller super-pixels in order to be fully contained within the halo, limiting the discriminatory power of other features that need larger regions to build higher-level descriptors.

In this manner the B.Echo-texture, is mainly encoded by the SIFT-BoF but also through the Appearance feature since elements would have the same model. Acoustic Posterior is mainly captured by the Brightness feature, but the Atlas also brings crucial information to compensate for intensity inhomogeneities

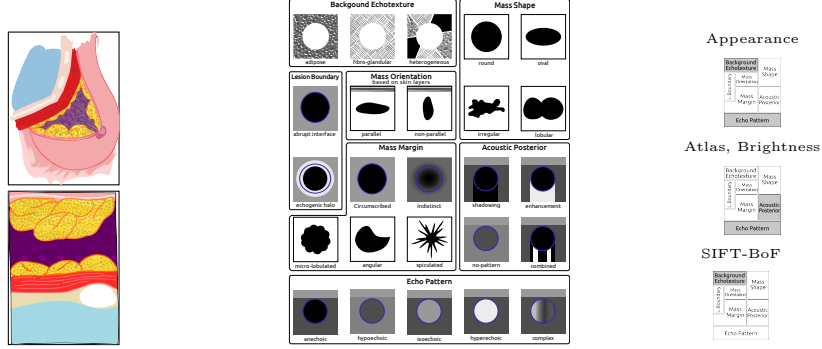


Fig. 3: Visual reference: (a) breast structures, (b) US BI-RADS lexicon, (c) encoded visual cues.

caused by signal attenuation present in the most posterior parts of the image. The Echo Pattern cue, encodes the echogenicity of a region with respect to the adipose tissue. This can be found at the anterior part of the image with its echogenicity close to the middle of the spectrum. Appearance, Brightness and the Atlas account for such information.

The data term construction For the work here reported, a Radial Basis Function (RBF)-Support Vector Machines (SVM) classifier is used to determine the data model during the training stage. During the testing stage, $D_s(\omega_s = l)$ corresponds to the distance between the testing samples and the model associated to l as the SVM classification reward.

2.2 The pairwise (or smoothing) term

The pairwise term represents the cost associated to ω_s taking into account the labels of its neighbour sites, ω_r , $r \in \mathcal{N}_s$. This term models a Markov Random Fields (MRFs) or a Conditional Random Fields (CRFs). The typical form of this term, given in eq. (3), is called homogenization which acts as a regularization factor favouring configurations that have a coherent labelling.

$$V_{s,r}(\omega_s, \omega_r) = \begin{cases} \beta, & \text{if } \omega_s \neq \omega_r \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Section 2 offers a visual interpretation of this cost. The more fragmented is the segmentation ω , the higher the overall pairwise term; since every boundary brings a penalization β to the total cost $U(\omega)$. In this manner the regularization term can be seen as a post-processing or denoising stage since that some sites will flip their labelling if the cost of fragmenting the regions is larger than the cost of adopting their neighbour's label.

More sophisticated smoothing terms, where boundaries have different penalization based on site relations in \mathcal{S} and image information as illustrated in section 2, are also naturally handled by the proposed framework.

2.3 Searching the best labelling configuration

Once defined $U(\omega)$ so that the cost for a particular labelling configuration ω can be computed, the problem of finding $\hat{\omega}$ corresponding to the global minimum of the space \mathcal{W} of all possible labelling configurations needs to be faced. Graph-Cuts (GC) when applicable allows to rapidly find a strong local minima guaranteeing that no other minimum with lower energies can be found [6]. GC is applicable if, and only if, the pairwise term favours coherent labelling configurations and penalizes labelling configurations where neighbours labels differs; such is our case, given in eq. (3).

3 Method evaluation and comparison

A 16 BUS images dataset with accompanying multi-label GT delineating all the structures present in the images has been used to evaluate the proposed methodology for lesion segmentation application. Every image in the dataset presents a single lesion with variable extension. The size of the lesions ranges from under 1/100 to over 1/5 of the image. The dataset composed of cysts, Fibro-Adenomas (FAs), Ductal Infiltrating Carcinomas (DICs) and Infiltrating Lobular Carcinomas (ILCs).

Figure 4 shows qualitative results, whereas the quantitative results from the best configuration are reported as a table in ???. Notice that for the proposed framework, the performance in terms of Area Overlap (AOV) is limited by the capacity of the super-pixels to snap the desired boundary. Figure 4 shows how the delineation resulting from a proper labeling of the super-pixels differs from the GT.

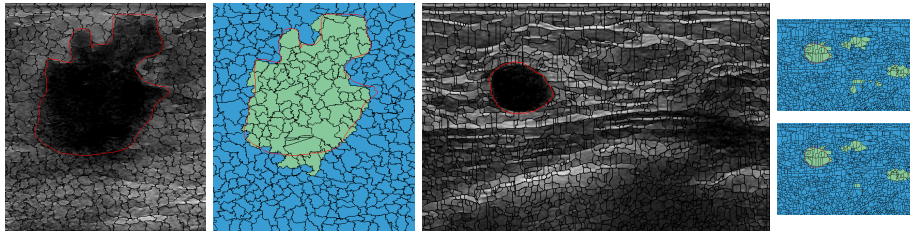


Fig. 4: Qualitative results. (a) Example 1: original image, super-pixels' delineations and GT. (b) Differences between GT and the delineation resulting from super-pixels' boundary. (c) Ex. 2. (d) weak $V(\cdot, \cdot)$ (e) strong $V(\cdot, \cdot)$

Due to the lack of publicly available data (and code) there is no manner to perform a methodology comparison further than compiling the results reported

in the literature. Figure 5 resumes some of the methodologies found in the literature and translates the reported results into AOV for comparison purposes. The iconography used in section 3 relates methodology stages, technology used and if the stages have been treated separately or as a single stage.

Section 3 arranges the information for direct visual comparison. An extra element is also represented in section 3 as blue swatch delimited by two blue dashed lines. The boundaries of this swatch correspond to performance of expert radiologists in terms of AOV based on an inter- and intra-observer experiment carried out by Pons et al. [20].

When comparing the results it is clear the inconvenience of unexciting public data, since several of the results outperform the manual delineations studied in [20]. It can also be seen that the category tested in larger datasets is ML, whereas Active Contour Model (ACM) lead to better segmentations since the lesion boundary is easier to model in ACM compared to ML based techniques.

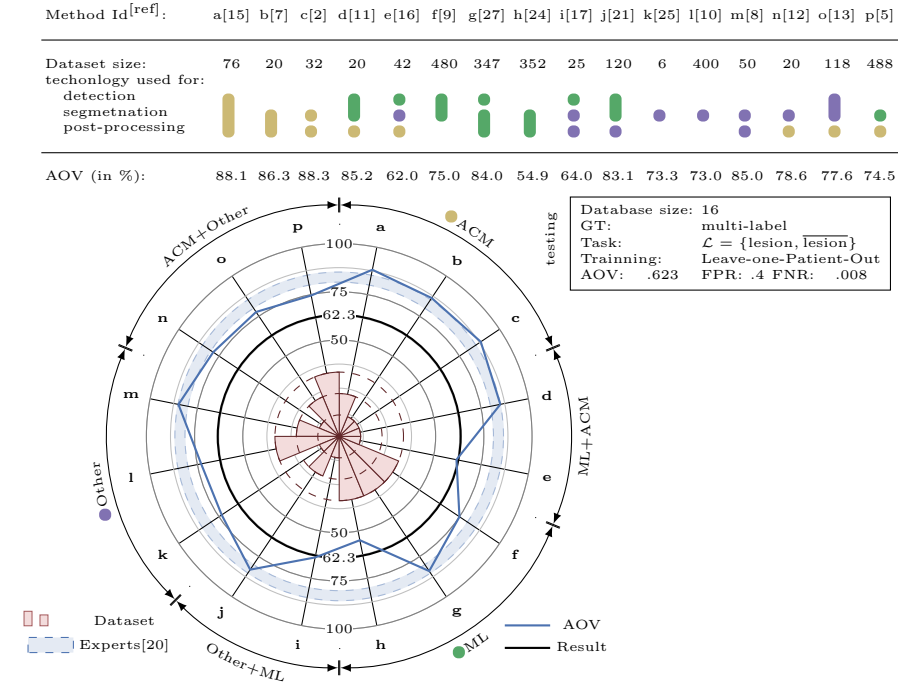


Fig. 5: Quantitative results compilation and comparison

4 Conclusions

This work presents a segmentation strategy to delineate lesions in BUS images using an optimization framework that takes advantage of all the facilities avail-

able when using ML techniques. Despite the limitation that the final segmentation is subject to the super-pixels' boundaries, the AOV results here reported are similar to those reported by other methodologies in the literature. A higher AOV result can be achieved by deforming the delineation resulting from the proposed framework using this to initialize a second post-processing step based on ACM. In this manner the contour constraints could be applied to achieve a more natural delineation.

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