Laws of Propositions

(01) Idempotent Law

$p \vee p \equiv p \qquad p \wedge p \equiv p$

(02) Associative Law

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(03) Commutative Law

$(p \vee q) \equiv (q \vee p)$

$(p \wedge q) \equiv (q \wedge p)$

(04) Distributive Law

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(05) Identity Law

$p \wedge T \equiv p$

$p \vee F \equiv p$

$$\boxed{\begin{array}{l} p \rightarrow q \equiv \neg q \rightarrow \neg p \\ \neg(p \rightarrow q) \equiv p \wedge (\neg q) \\ (p \leftrightarrow \neg q) \equiv (q \leftrightarrow \neg p) \\ p \rightarrow q \equiv \neg p \vee q \end{array}}$$

(06) Domination Law

$p \vee T \equiv T$

$p \wedge F \equiv F$

(07) Complement Law

$p \wedge \neg p \equiv F$

$p \vee \neg p \equiv T$

(08) Double Negation Law

$\neg(\neg p) \equiv p$

(09) Absorption Law

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

(10) Contrapositive Law

$(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$

(11) De Morgan's Law

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Laws of Arguments

(01) Law of Detachment

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

If p implies q is true and p is true, then q is true

(02) Law of Syllogism

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \end{array}$$

If p implies q is true and q implies r is true, then p implies r is true

Boolean Algebra

Definition

A Boolean Algebra is a set B, together with two operations, generally denoted + and ·, such that for all a and b in B both a+b and a·b are in B and the following properties hold;

$\forall a, b \in B$

$(a+b) \in B \qquad (a \cdot b) \in B$

(1) Commutative Law $\forall a, b \in B$

$a+b = b+a$

$a \cdot b = b \cdot a$

(2) Associative Law $\forall a, b, c \in B$

$a+(b+c) = (a+b)+c$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(3) Distributive Law $\forall a, b, c \in B$

$a+(b \cdot c) = (a+b) \cdot (a+c)$

$a \cdot (b+c) = (a \cdot b)+(b \cdot c)$

(4) Identity Law $\forall a \in B$

$a+0 = a \qquad a \cdot 1 = a$

(5) Complement Law $a \in B, \bar{a} \in B$

$a+\bar{a} = 1$

$a \cdot \bar{a} = 0$

Properties of Boolean Algebra

(01) Uniqueness of Complement Law

$\forall a, x \in B$:

if $a+x=1$

$a \cdot x = 0$ then,

$x = \bar{a}$

(02) Double Complement Law

$\forall a \in B$

$\bar{\bar{a}} = a$

(03) De Morgan's Law

$\forall a, b \in B$

$\overline{a+b} = \bar{a} \cdot \bar{b}$

$\overline{a \cdot b} = \bar{a} + \bar{b}$

(04) Absorption Law

$\forall a, b \in B$

$(a+b) \cdot a = a$

$(a \cdot b) + a = a$

(05) Complement Law

$\bar{0} = 1$

$\bar{1} = 0$

(06) Boundedness Law

$\forall a \in B$

$a+1 = 1$

$a \cdot 0 = 0$

(07) Idempotent Law

$a+a = a$

$a \cdot a = a$