

Importance of Digital Systems

- Easy to use

- Trend

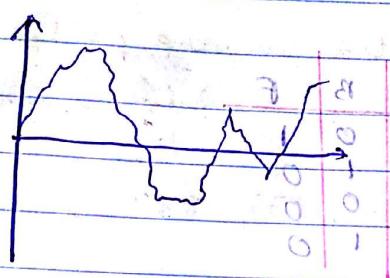
- Small in size

- High Accuracy

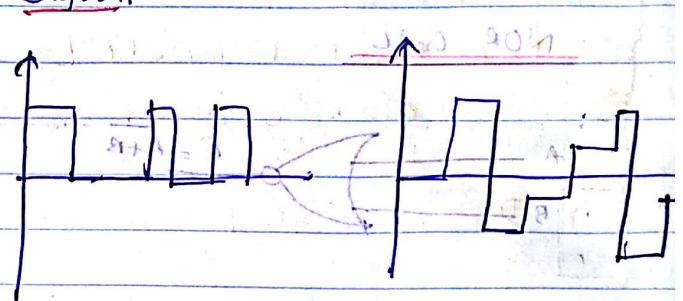
- High speed

- low cost

Analog



Digital



Boolean Algebra

Boolean variables

$$A = 0 \Rightarrow \bar{A}$$

$$A = 1 \Rightarrow A$$

Boolean Operators

(1) NOT

(2) OR

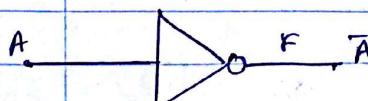
(3) AND

(4) NOR

(5) NAND

(6) EX-OR

Not gate



A	F
0	1
1	0

OR Gate



0

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

And Gate



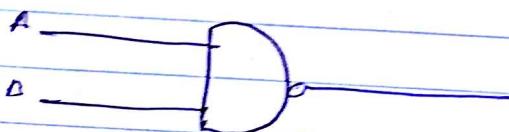
A	B	F
0	0	0
1	0	0
0	1	0
1	1	1

NOR Gate



A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

NAND Gate



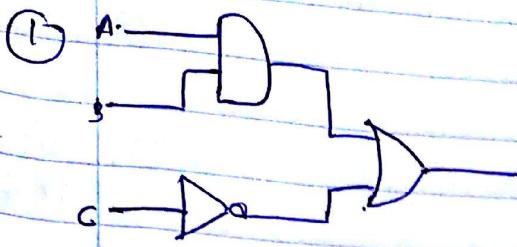
A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

Ex-OR Gate

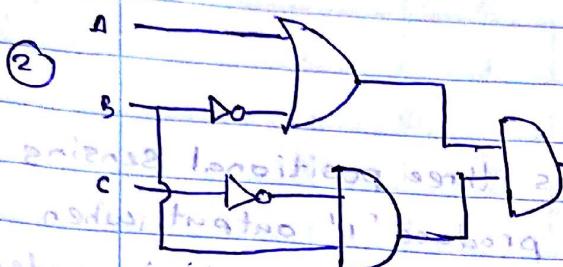
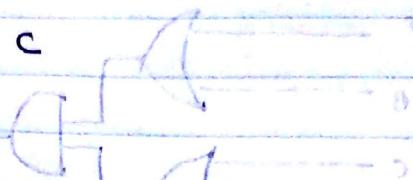


A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

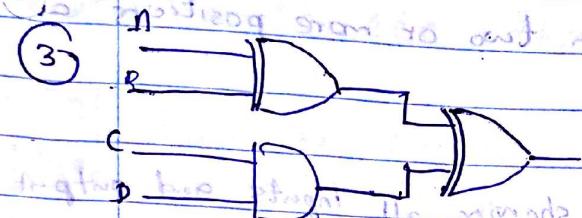
বিনামূলক গুণাত্মক অসমুক্তির ক্ষেত্রে



$$AB + C$$

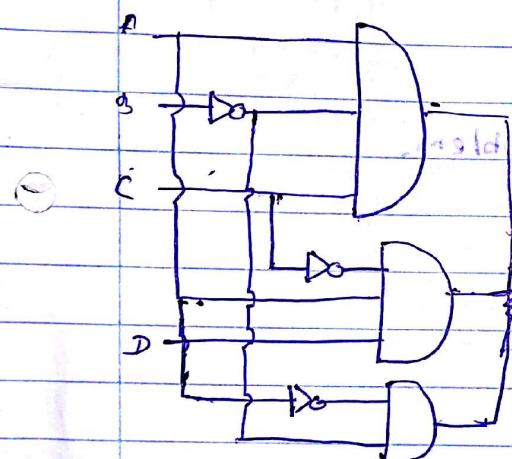
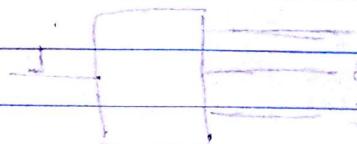


$$(C \cdot B)(B + A)$$



$$(A \oplus B) \oplus C \cdot D$$

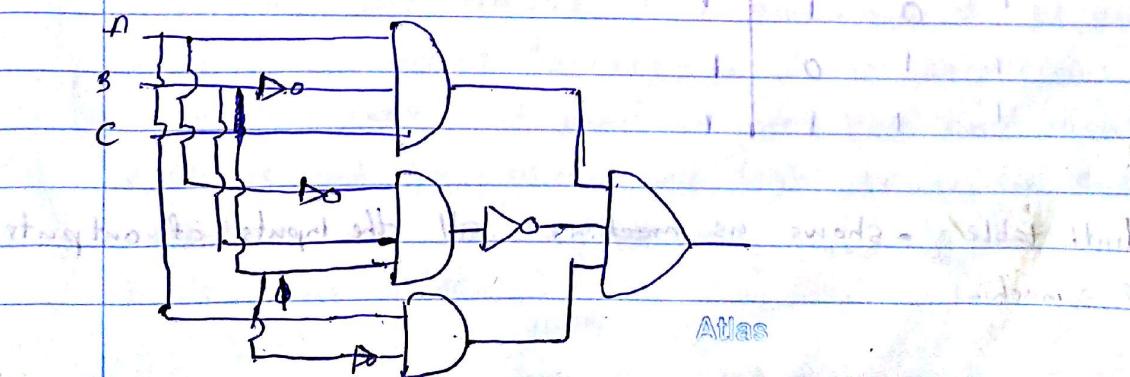
④ $F = A\bar{B}C + A\bar{C}D + \bar{A}\bar{B}$



A	B	C	G
0	0	0	0
0	1	0	0
0	0	1	0
1	1	1	0

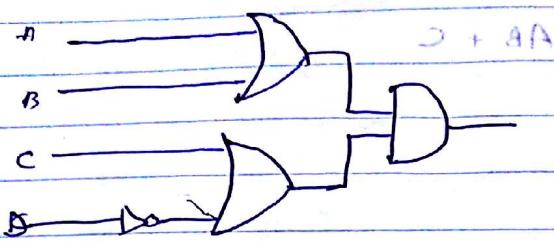
⑤ $G = A\bar{B}C + \bar{A}\bar{B}C + A\bar{C}$

A	B	C	G
1	0	1	1
1	1	0	1
0	1	0	0
0	0	1	0



Atlas

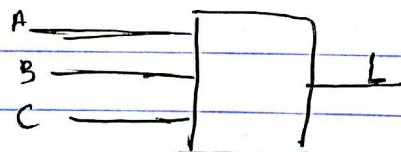
$$⑥ L = (A+B)(C+\bar{D})$$



An Electrical control system uses three positional sensing devices, A, B, C, each of which produce '1' output when the position is detected. Design a digital circuit in order to produce an output L, when two or more positions are detected.

① Step

Draw a block diagram showing all inputs and outputs.



② Step

Prepare a truth table for the problem.

A	B	C	L
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table shows us machine
of a machine

all the inputs of output

Now we have converted our problem to a table.

(3)
Step

Write Output Boolean eqn for the output

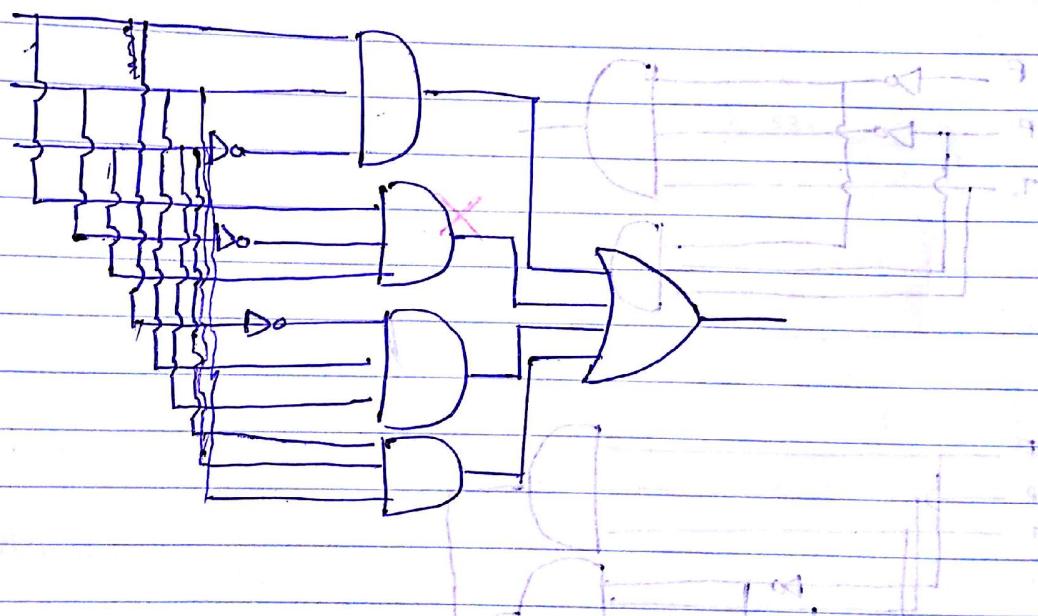
A	B	C	L	J	F	T	Q	R
0	1	1	1	1	0	0	0	1
1	0	1	1	1	0	0	1	0
1	1	0	1	1	0	1	1	1
1	1	1	1	1	1	0	1	0

$$L = ABC + A\bar{B}C + \bar{A}BC \quad \text{ABC } 1 \times 0 \quad 1 \quad 1 \quad 1$$

$$1 \times 0 \quad 0 \quad 0 \quad 0$$

(4)
Step

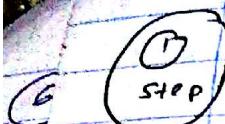
Draw the Circuit using basic logic gates



(5)

The step in a manufacturing process depends on the flow rate F, pressure P and temperature T of the material, safety conditions require that an alarm A be sounded before the process become dangerous. Danger conditions occur when flow rate and pressure are low and also when pressure and temperature are high. Write the Boolean equations for sounding the alarm and implement it using basic logic gates

$\overline{FPT} \neq \overline{F} \cdot \overline{P} \cdot \overline{T}$



F P T L

0	0	1	1
---	---	---	---

0	1	0	0
---	---	---	---

1	0	0	0
---	---	---	---

1	1	0	0
---	---	---	---

0	1	1	1
---	---	---	---

1	0	1	0
---	---	---	---

1	1	1	0
---	---	---	---

0	0	0	0
---	---	---	---

$\leftarrow \overline{FPT}$

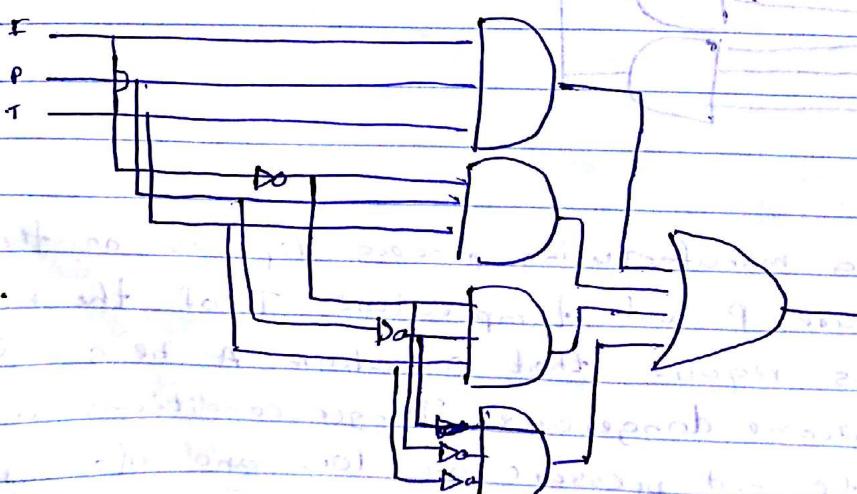
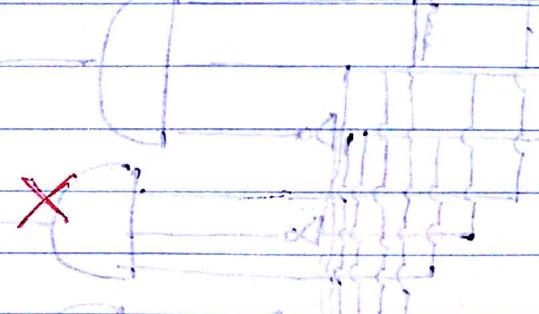
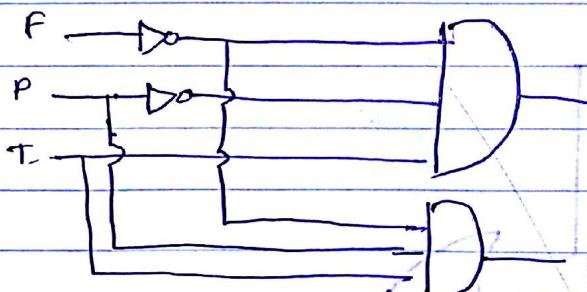
$\leftarrow \overline{FPT}$

$\leftarrow \overline{FPT}$



$$L = \overline{FPT} + \overline{FPT}$$

$$L = \overline{F} \cdot \overline{P} \cdot \overline{T} + \overline{F} \cdot \overline{P} \cdot T + \overline{F} \cdot P \cdot \overline{T} + \overline{F} \cdot P \cdot T$$

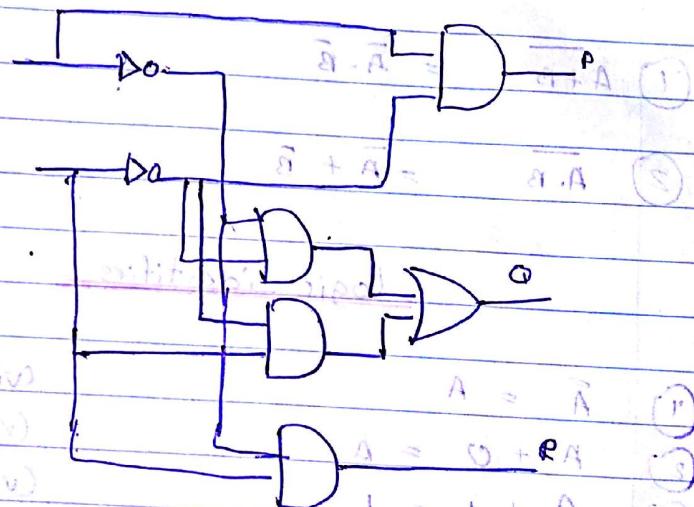


A	B	$(B+A)+A$	P	$A > B$
0	0	$0+0+0 = 0$	Q	$A = B$
0	1	$0+1+0 = 1$	R	$A < B$
1	0	$1+0+1 = 0$		
1	1	$1+1+1 = 1$		

$$P = AB$$

$$Q = \bar{A}\bar{B} + A\bar{B}$$

$$R = \bar{A}B$$



$$O = 0 \cdot A \quad (iv)$$

$$A = 1 \cdot A \quad (iv)$$

$$A = A \cdot A \quad (iv)$$

Commutative law

$$A = \bar{A} \quad (v)$$

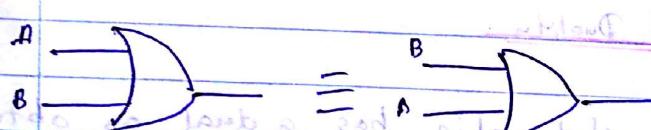
$$A = 0 + RA \quad (vi)$$

$$1 = 1 + A \quad (vii)$$

$$A = A' + A \quad (viii)$$

$$1 = \bar{A} + A \quad (ix)$$

① $A + B = B + A$

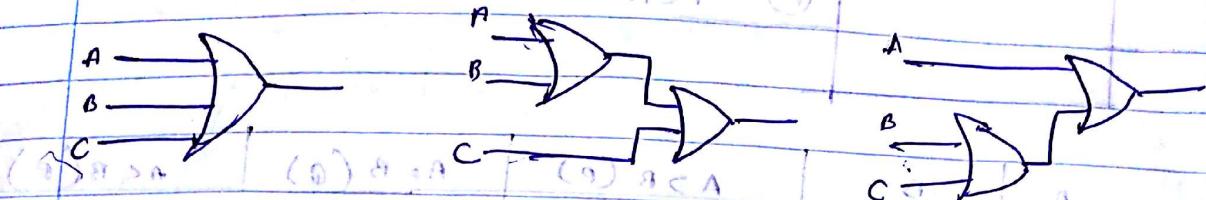


② $A \cdot B = B \cdot A$



Associate law

$$\textcircled{1} \quad A + B + C = (A + B) + C = A + (B + C)$$



Distributive law

$$A(B + C) = AB + BC$$

0	0	0	0
1	0	0	1
0	1	0	1

De morgan's Theory

$$\textcircled{1} \quad A + \bar{B} = \bar{A} \cdot \bar{B}$$

$$\bar{A} \cdot \bar{B} = 0$$

$$\textcircled{2} \quad \bar{A} \cdot \bar{B} = \bar{A} + \bar{B}$$

$$\bar{A} + \bar{B} = 1$$

Logic identities

$$\textcircled{1} \quad \bar{\bar{A}} = A$$

$$\text{(vi)} \quad A \cdot 0 = 0$$

$$\textcircled{2} \quad A + 0 = A$$

$$\text{(vii)} \quad A \cdot 1 = A$$

$$\textcircled{3} \quad A + 1 = 1$$

$$\text{(viii)} \quad A \cdot A = A$$

$$\textcircled{4} \quad A + A = A$$

$$\text{(ix)} \quad A \cdot \bar{A} = 0$$

$$\textcircled{5} \quad A + \bar{A} = 1$$

Principal of Duality

$$A + B = A \cdot A \quad (1)$$

Any theorem, law or relationship has a dual ~~ep~~ obtain by replacing every occurrence "f" by "o", "o" by "f", "1" by "0" and "0" by "1".

$$A + 0 = A$$

↓

$$A \cdot 1 = A$$

$$A \cdot 0 = 0 \quad \text{and also } A + B = \bar{A} \cdot \bar{B}$$

↓

$$A + 1 = 1$$

↓

$$\bar{A} \cdot \bar{B} \vee = \bar{A} + \bar{B}$$

Boolean minimization

$$AA + A\bar{A} = \bar{A}A = 0$$

$$A = \bar{F}\bar{B}\bar{T} + F\bar{P}\bar{T} + \bar{F}P\bar{T} + FP\bar{T}$$

$$= \bar{F}\bar{P}(\bar{T} + T) + PT(\bar{F} + F)$$

$$= \bar{F}\bar{P} + PT$$

3 - NOT

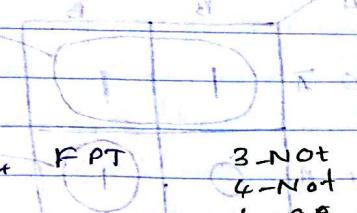
4 - NOT

1 - OR

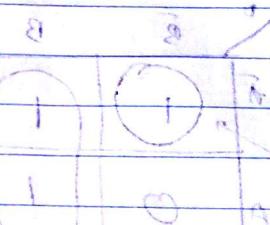
2 - NOT

2 - NOT

1 - OR



$$AA + \bar{A} = 1$$



① Using Boolean Algebra

② Using Karnaugh Map

$$AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$$

$$AB(\bar{C} + C) + C(A\bar{B} + B\bar{A})$$

$$AB + C(A\bar{B} + B\bar{A})$$

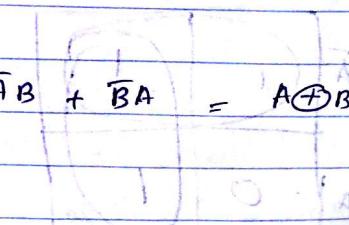
$$A \oplus B \rightarrow AB$$

$$A = A + A$$

(minimized)

Special

$$\begin{array}{|c|c|c|} \hline A & B & F = A \oplus B \\ \hline 0 & 0 & 0 \\ \hline 1 & 0 & 1 \leftarrow * \bar{A} = F = \bar{A}B + \bar{B}A = A \oplus B \\ \hline 0 & 1 & 1 \leftarrow * \\ \hline 1 & 1 & 0 \bar{A} + \bar{B}A = 1 \\ \hline \end{array}$$

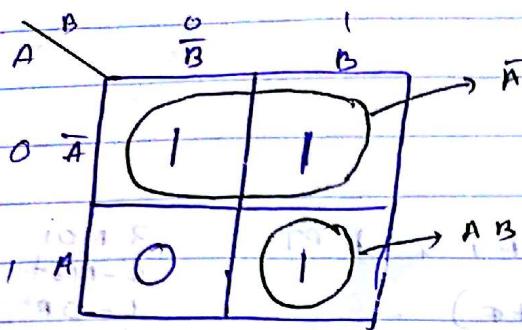


A	B	F
0	0	1
1	0	0
0	1	0
1	1	1

$$F = \bar{A}\bar{B} + AB = \bar{A} \oplus B$$

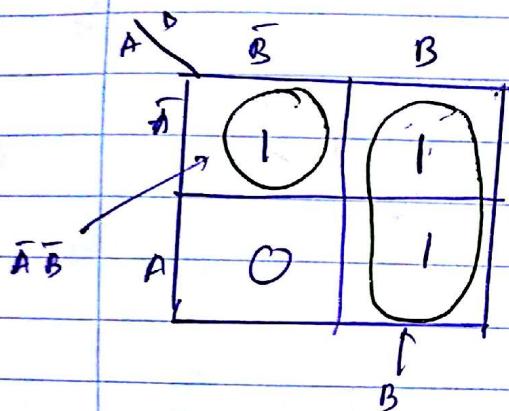
Atlas

K' map method



$$F = \bar{A}\bar{B} + \bar{A}B + AB$$

$$F = \bar{A} + AB$$



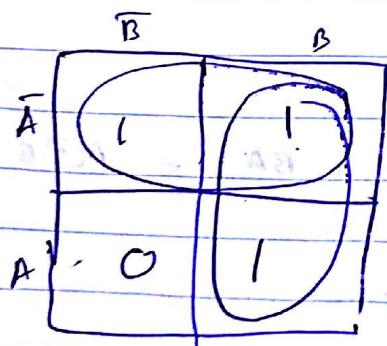
$$F = B + \bar{A}\bar{B}$$

$$(A\bar{B} + B\bar{A})2^3 + (\bar{A}\bar{B} + B\bar{A})2^2 + (\bar{A}B + AB)2^1 + (AB + B\bar{A})2^0$$

Special $A + A = A$

$$A + A + A = A$$

In ~~order~~ ~~order~~ Or gate we can add one ~~thing~~ thing many times



$$F = \bar{A} + B$$

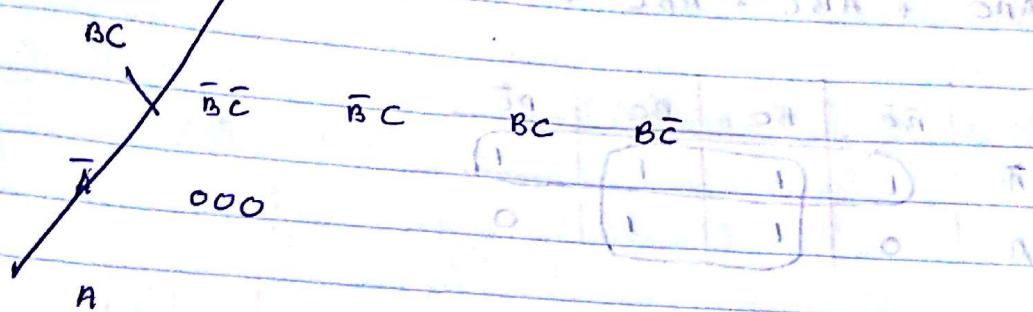
$$F = \bar{A}\bar{B} + \bar{A}B + AB + \bar{A}B$$

order add
order add
order add
order add

$$F = A\bar{A} + A\bar{B} + AB + \bar{A}B$$

Allas

3-variable



- One term can cover many times, If they tell minimization
- Don't cover many times, If they don't tell to do minimization

3 - variables

$$\textcircled{1} \quad F = \bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + ABC + A\bar{B}\bar{C}$$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	1		
A	1	1	1	1

without minimization

$$F = \bar{B} + ABC$$

minimization

$$F = \bar{B} + AC$$

$$\textcircled{2} \quad F = A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	0	0	1
A	1	0	1	1

Consider this diagram as a cylinder

$$(3) F = \bar{A}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C + A\bar{B}C + ABC$$

	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
\bar{A}	1	1	1	1
A	0	1	1	0

$$F = C + \bar{A}$$

$$(4) H = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C$$

abundance of 1-bit flash path AP permit more AND logic

	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC
\bar{A}	1	0	1	0
A	0	1	1	0

- Cannot minimize
- Algebra method

$$\begin{aligned} H &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C \\ &= \bar{A}(\bar{B}\bar{C} + BC) + A(\bar{B}\bar{C} + \bar{B}C) \\ &= \bar{A}(B \oplus C) + A(B \oplus C) \end{aligned}$$

4-variables

$$(5) F = \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + \bar{A}\bar{B}CD + ABC\bar{D}$$

$$+ A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1	0	0	0
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	1	1	0	0

Atlas

$$f = BD + \bar{C}\bar{D} + A\bar{B}\bar{C}$$

(2) $f = \bar{A}BCD + ABCD + AB\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	0	1	1	0
$A\bar{B}$	0	1	1	0
AB	1	0	0	1

$$F = BD + \bar{B}\bar{D}$$

(3) $F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B C + B\bar{C} + A\bar{B} C\bar{D} + A B C D$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	0
$\bar{A}B$	1	1	1	1
$A\bar{B}$	1	1	1	0
AB	0	0	0	1

$$F = \bar{B}\bar{C} + BD + \bar{A}BC + \bar{A}\bar{C}\bar{D}$$

past paper

A logic circuit has four inputs A, B, C and D has two outputs R and S simplify the output functions and S using the karnaugh map and draw the logic circuits for the simplified functions

$$D = AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + ACD + \bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D}$$

CD	00	01	10	11
CD	1	0	0	1
$\bar{C}\bar{D}$	0	0	1	0
$\bar{A}\bar{B}$	0	0	1	0
AB	1	1	1	0
AB	1	1	1	0

$$R = \bar{A}B\bar{C}A + \bar{A}\bar{B}A\bar{C} + \bar{A}B\bar{C} + \bar{A}B\bar{C}A + \bar{A}\bar{B}A\bar{C}$$

$$S = A\bar{C} + CD$$

edges could be connected

larger the group simpler the logic

look for groups of 2, 4, 8

minimize using

CD	00	01	10	11
CD	1	0	1	0
$\bar{C}\bar{D}$	0	0	1	0
$\bar{A}\bar{B}$	0	0	1	0

$$\bar{A}B\bar{C}A + \bar{A}\bar{B}A\bar{C} + \bar{A}B\bar{C} + \bar{A}B\bar{C}A + \bar{A}\bar{B}A\bar{C}$$