MDPs: value iteration

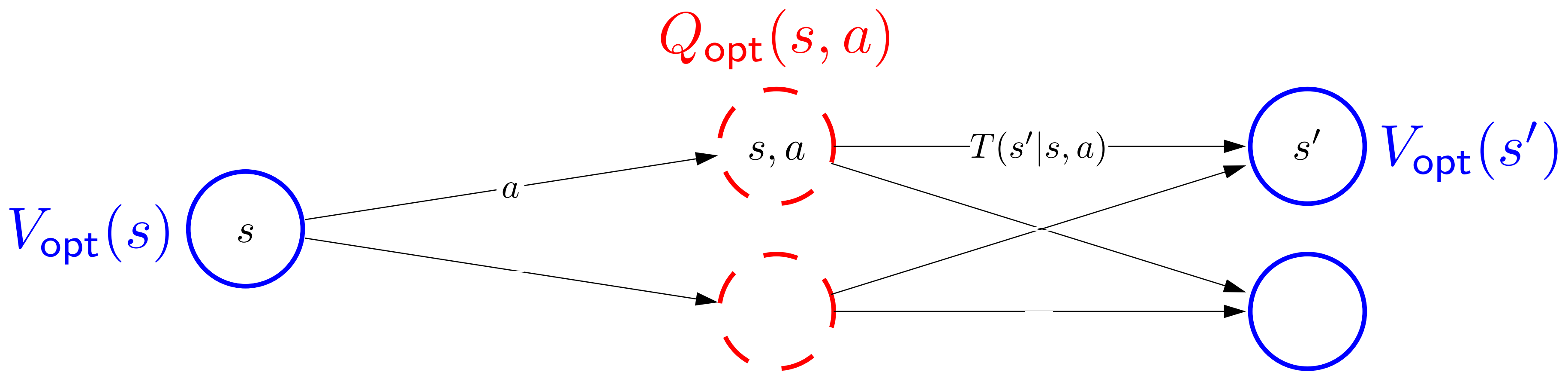
Optimal value and policy

Goal: try to get directly at maximum expected utility

Definition: optimal value

The **optimal value** $V_{\text{opt}}(s)$ is the maximum value attained by any policy.

Optimal values and Q-values



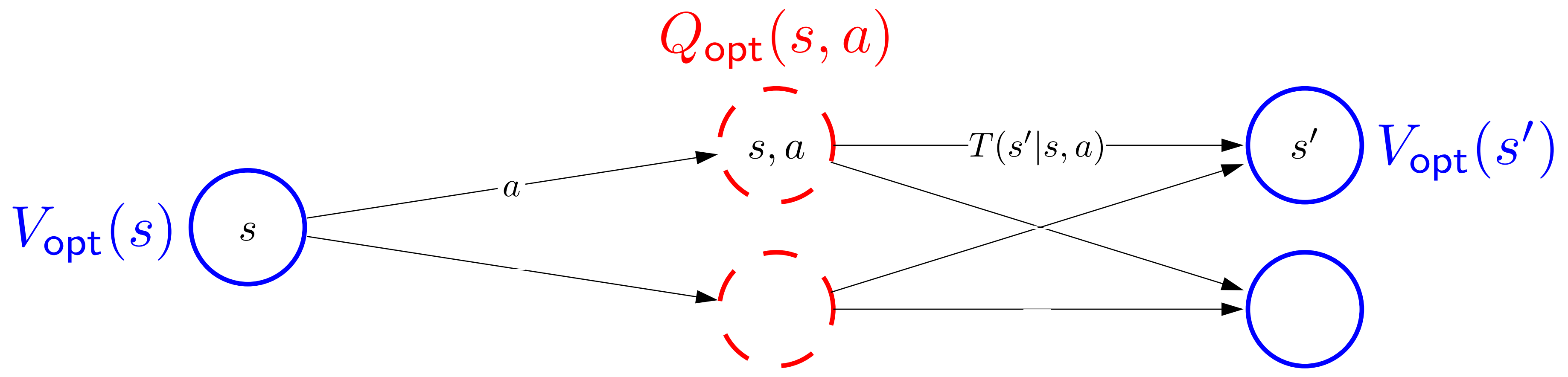
Optimal value if take action a in state s:

$$Q_{\text{opt}}(s,a) = \sum_{s'} T(s,a,s')[\text{Reward}(s,a,s') + \gamma V_{\text{opt}}(s')].$$

Optimal value from state s:

$$V_{\text{opt}}(s) = \begin{cases} 0 & \text{if IsEnd}(s) \\ \max_{a \in \text{Actions}(s)} Q_{\text{opt}}(s,a) & \text{otherwise.} \end{cases}$$

Optimal policies



Given Q_{opt}, read off the optimal policy:

$$\pi_{\text{opt}}(s) = \arg \max_{a \in \text{Actions}(s)} Q_{\text{opt}}(s,a)$$

Value iteration

Algorithm: value iteration [Bellman, 1957]

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Initialize $V_{\text{opt}}^{(0)}(s) \leftarrow 0$ for all states $s$.
For iteration $t = 1, \dots, t_{\text{VI}}$:
    For each state $s$:
        $V_{\text{opt}}^{(t)}(s) \leftarrow \max_{a \in \text{Actions}(s)} \underbrace{\sum_{s'} T(s,a,s')[\text{Reward}(s,a,s') + \gamma V_{\text{opt}}^{(t-1)}(s')]}_{Q_{\text{opt}}^{(t-1)}(s,a)}$
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Time: $O(t_{\text{VI}} S A S')$

[semi-live solution]

Value iteration: dice game

s	end	in	
$V_{\text{opt}}^{(t)}$	0.00	12.00	$(t = 100$ iterations$)$
$\pi_{\text{opt}}(s)$	-	stay	

Convergence

Theorem: convergence

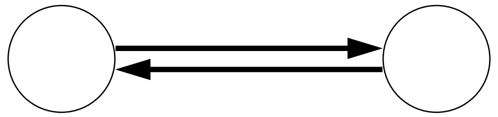
Suppose either

- discount $\gamma < 1$, or
- MDP graph is acyclic.

Then value iteration converges to the correct answer.

Example: non-convergence

discount $\gamma = 1$, zero rewards



Summary of algorithms

- Policy evaluation: (MDP, π) $\rightarrow V_\pi$

- Value iteration: MDP $\rightarrow (Q_{\text{opt}}, \pi_{\text{opt}})$

Unifying idea

Algorithms:

- Search DP computes FutureCost(s)
- Policy evaluation computes policy value $V_\pi(s)$
- Value iteration computes optimal value $V_{\text{opt}}(s)$

Recipe:

- Write down recurrence (e.g., $V_\pi(s) = \cdots V_\pi(s') \cdots$)
- Turn into iterative algorithm (replace mathematical equality with assignment operator)