Search: overview

Course plan

Search problems

Markov decision processes

Adversarial games

Constraint satisfaction problems

Bayesian networks

Reflex **States** **Variables** **Logic**

"Low-level intelligence" "High-level intelligence"

Machine learning

Beyond reflex

Classifier (reflex-based models):

$x \longrightarrow \boxed{f} \longrightarrow$ single action $y \in \{-1, +1\}$

Search problem (state-based models):

$x \longrightarrow \boxed{f} \longrightarrow$ **action sequence** $(a_1, a_2, a_3, a_4, \dots)$

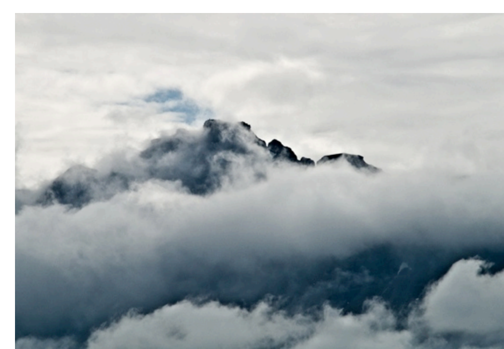
Key: need to consider future consequences of an action!

Problem types

- Fully observable, deterministic
  - single-belief-state problem

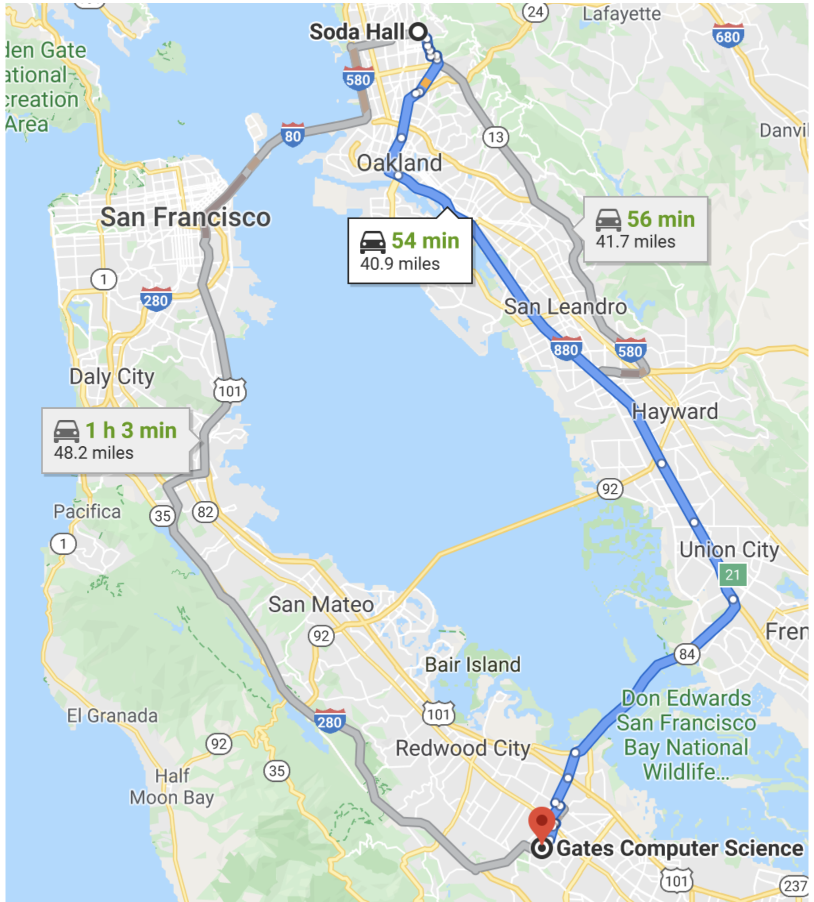
- Non-observable
  - sensorless (conformant) problem

- Partially observable/non-deterministic
  - contingency problem
  - interleave search and execution



- Unknown state space
  - exploration problem
  - execution first

Application: route finding



Objective: shortest? fastest? most scenic?

Actions: go straight, turn left, turn right

Application: robot motion planning

Objective: fastest path

Actions: acceleration and throttle

Application: robot motion planning

Objective: fastest? most energy efficient? safest? most expressive?

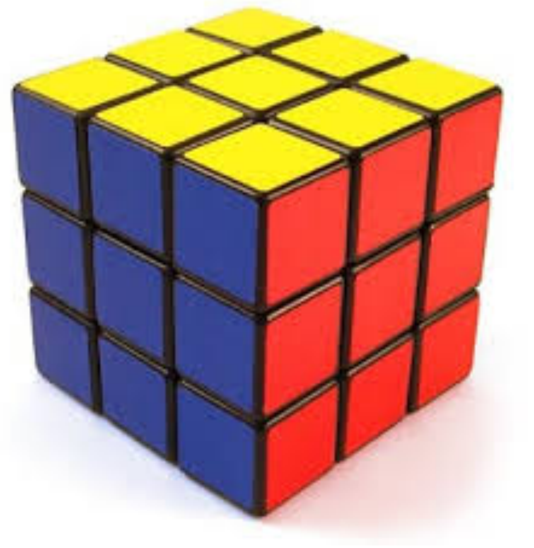
Actions: translate and rotate joints

Application: multi-robot systems

Objective: fastest? most energy efficient?

Actions: acceleration and steering of all robots

Application: solving puzzles



Objective: reach a certain configuration

Actions: move pieces (e.g., `Move12Down`)

Application: machine translation

la maison bleue

$\downarrow$

the blue house

Objective: fluent English and preserves meaning

Actions: append single words (e.g., `the`)

Roadmap

Modeling [redacted]

Modeling Search Problems [redacted]

Algorithms

Tree Search

Dynamic Programming

Uniform Cost Search

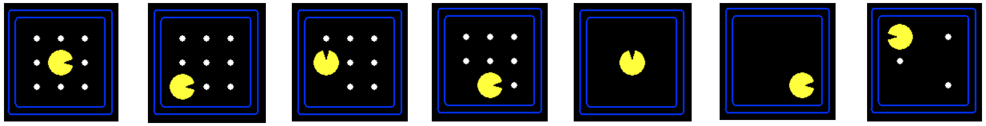
Programming and Correctness of UCS

A*

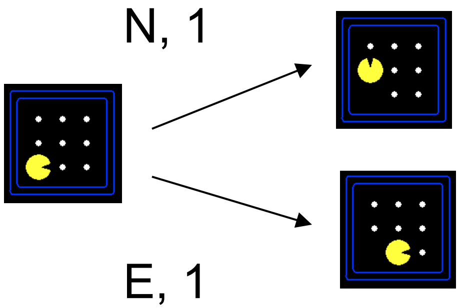
A* Relaxations

Search Problems

- A search problem consists of:
  - A state space

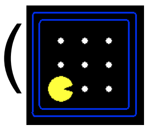
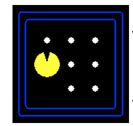
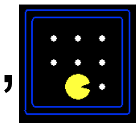
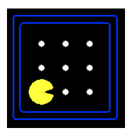
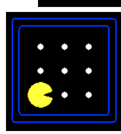
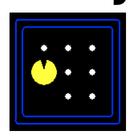
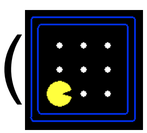
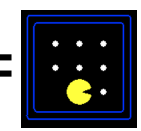
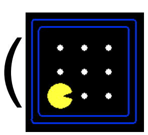
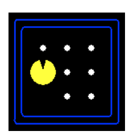
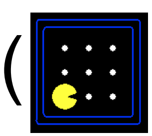
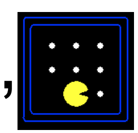


  - A transition model

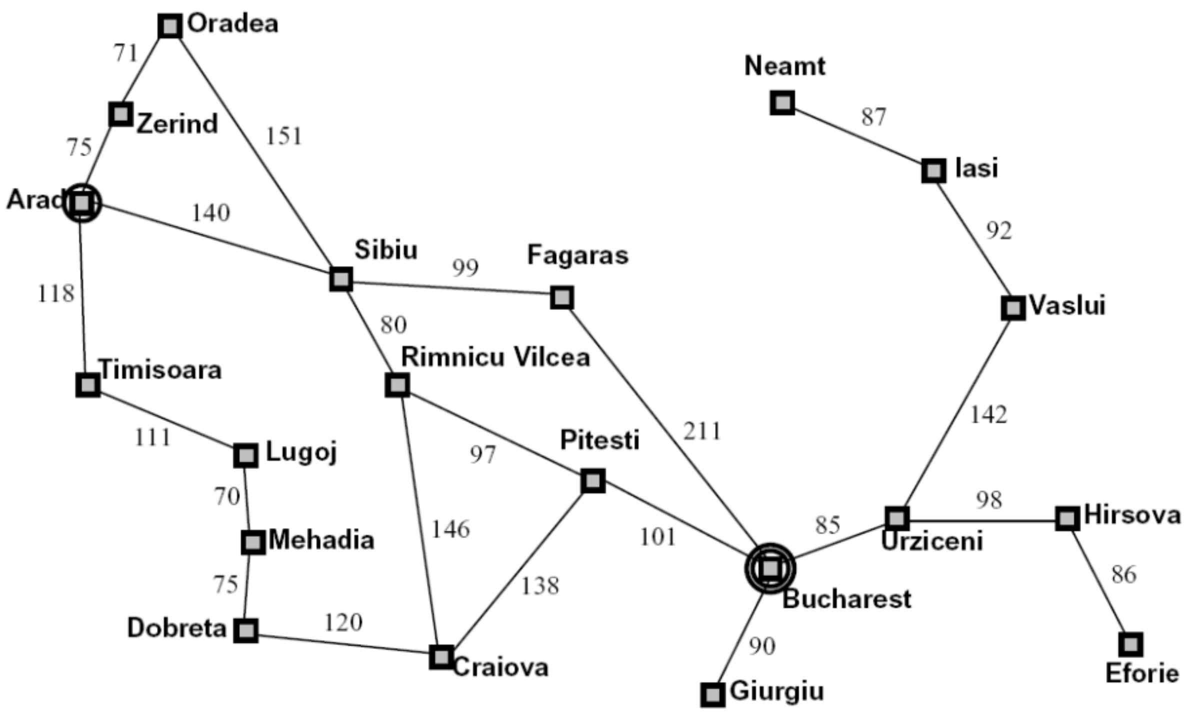


  - A start state, goal test, and path cost function
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Transition Models

- Successor function
  - Successors() = {(N, 1, ), (E, 1, )}
- Actions and Results
  - Actions() = {N, E}
  - Result(, N) =  ; Result(, E) = 
  - Cost(, N, ) = 1; Cost(, E, ) = 1

Example: Romania



- State space:
  - Cities
- Successor function:
  - Go to adj city with cost = dist
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

Farmer Cabbage Goat Wolf

Actions:

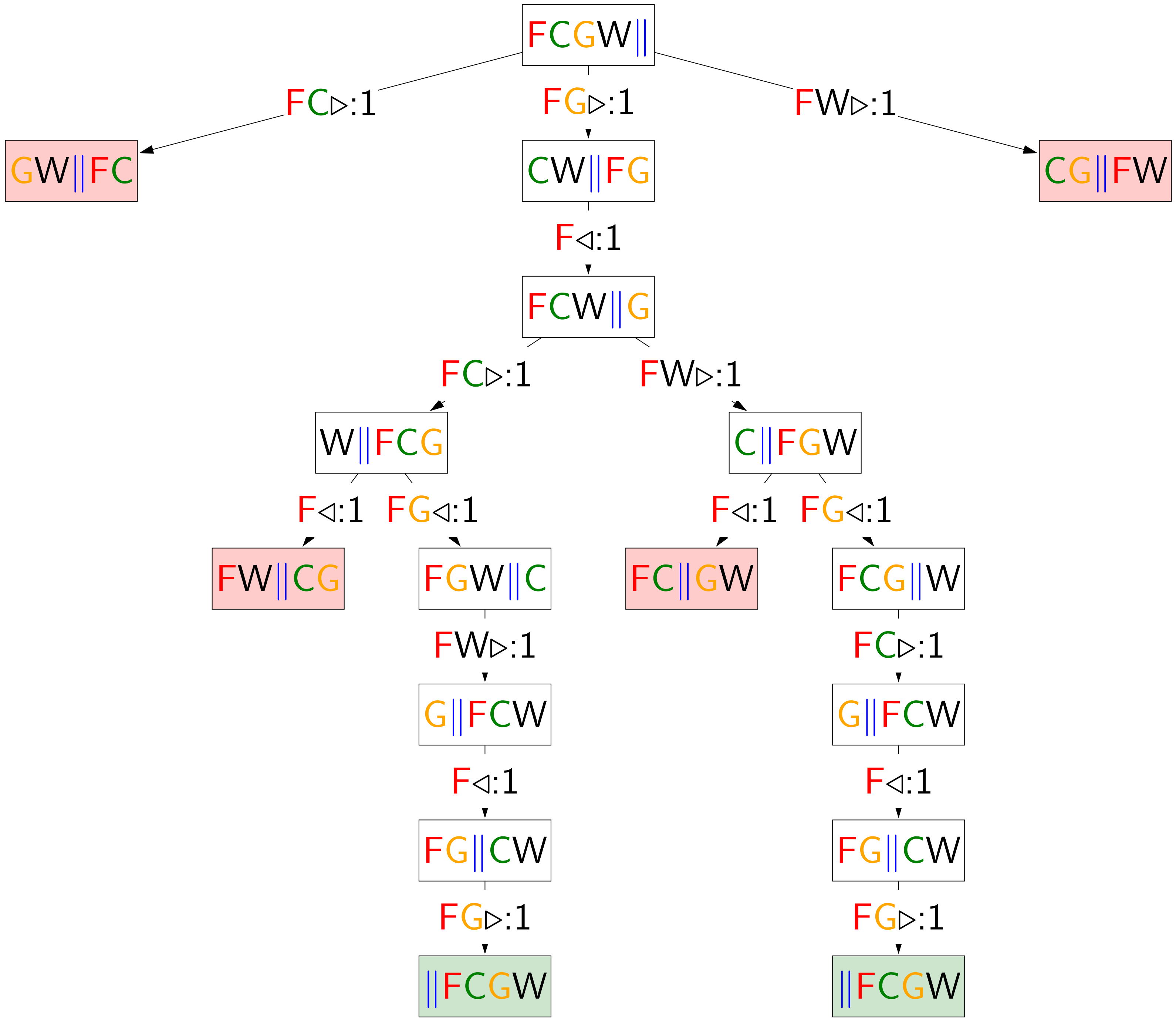
$F\triangleright$ $F\triangleleft$

$FC\triangleright$ $FC\triangleleft$

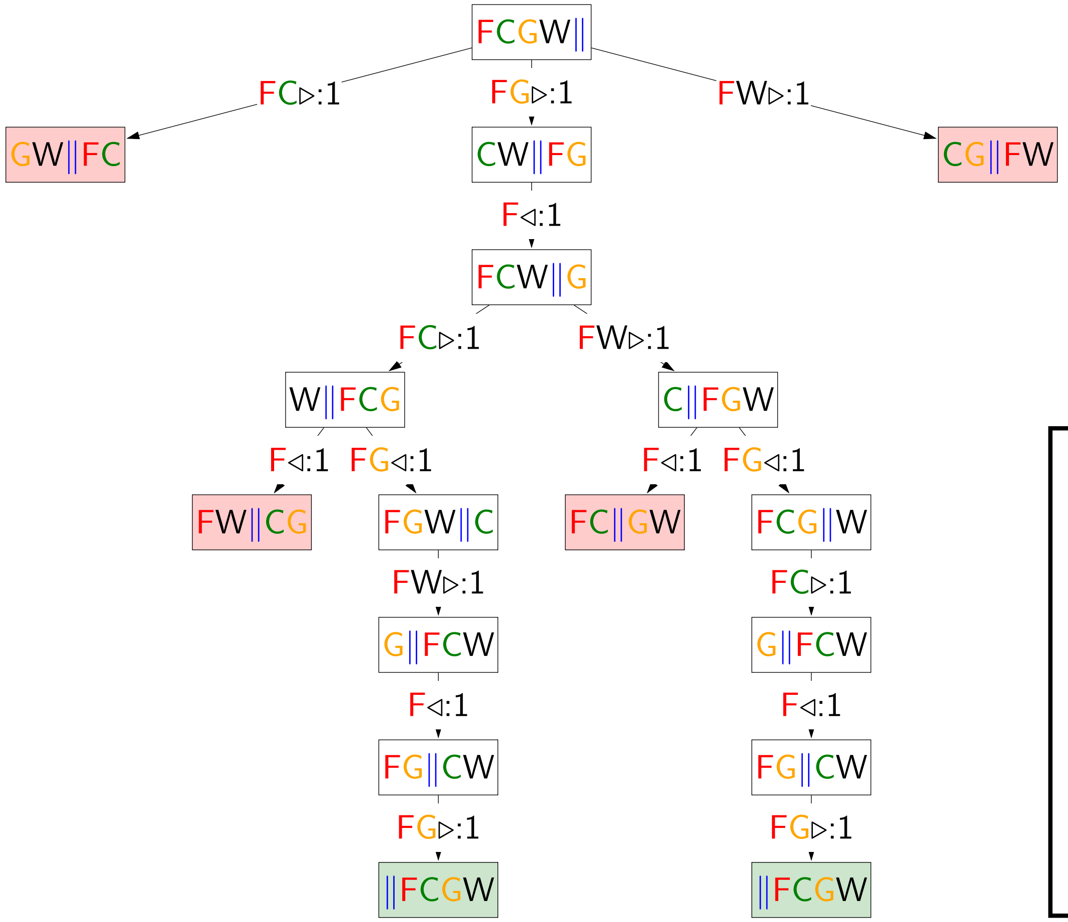
$FG\triangleright$ $FG\triangleleft$

$FW\triangleright$ $FW\triangleleft$

Approach: build a **search tree** ("what if?")


FCGW||
FC▷:1
FG▷:1
FW▷:1
GW||FC
CW||FG
CG||FW
F◁:1
FCW||G
FC▷:1
FW▷:1
W||FCG
C||FGW
F◁:1
FG◁:1
F◁:1
FG◁:1
FW||CG
FGW||C
FC||GW
FCG||W
FW▷:1
FC▷:1
G||FCW
G||FCW
F◁:1
F◁:1
FG||CW
FG||CW
FG▷:1
FG▷:1
||FCGW
||FCGW

Search problem



Definition: search problem

- s_{start}: starting state
- Actions(s): possible actions
- Cost(s, a): action cost
- Succ(s, a): successor
- IsEnd(s): reached end state?

Transportation example

Example: transportation

Street with blocks numbered 1 to n.

Walking from s to $s + 1$ takes 1 minute.

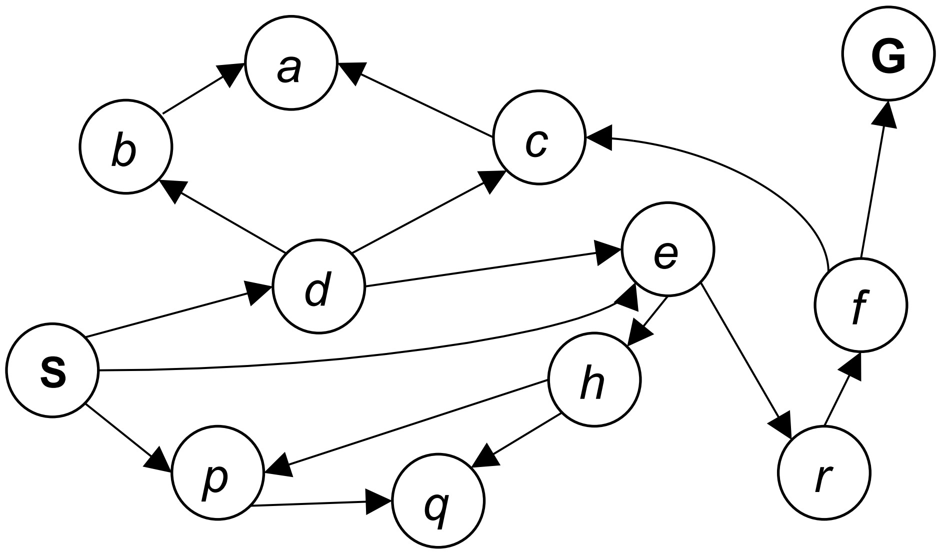
Taking a magic tram from s to $2s$ takes 2 minutes.

How to travel from 1 to n in the least time?

[semi-live solution: `TransportationProblem`]

State Space Graphs

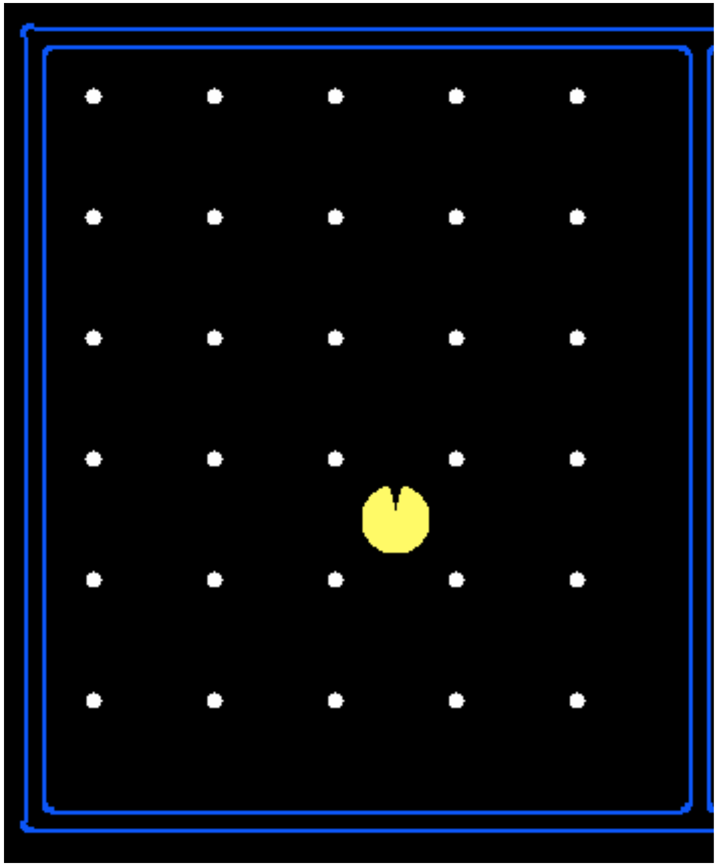
- State space graph: A mathematical representation of a search problem
  - For every search problem, there's a corresponding state space graph
  - The successor function is represented by arcs
- This can be large or infinite, so we won't create it in memory



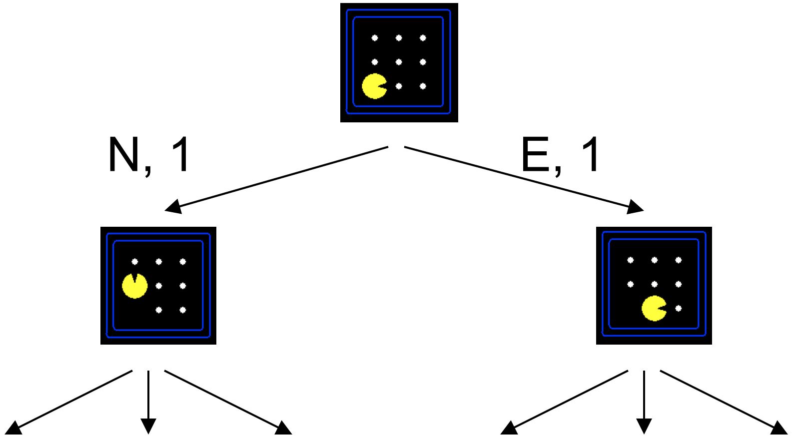
Ridiculously tiny search graph for a tiny search problem

Exponential State Space Sizes

- Search Problem:
  Eat all of the food
- Pacman positions:
  10 x 12 = 120
- Food count: 30

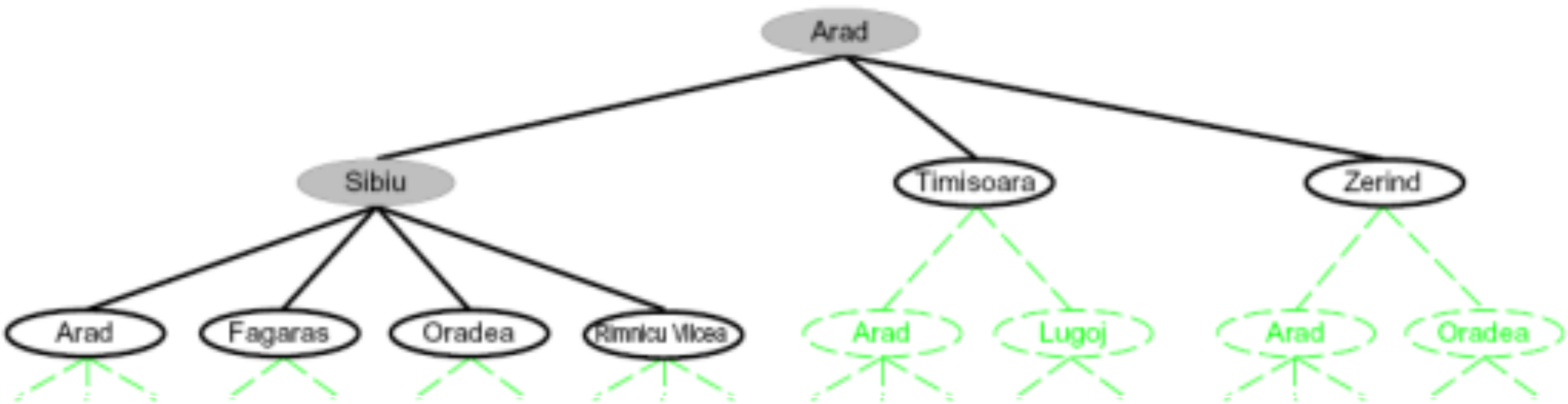


Search Trees



- A search tree:
  - This is a "what if" tree of plans and outcomes
  - Start state at the root node
  - Children correspond to successors
  - Nodes contain states, correspond to **paths** to those states
  - For most problems, we can never actually build the whole tree

Another Search Tree



- Search:
  - Expand out possible plans
  - Maintain a frontier of unexpanded plans
  - Try to expand as few tree nodes as possible

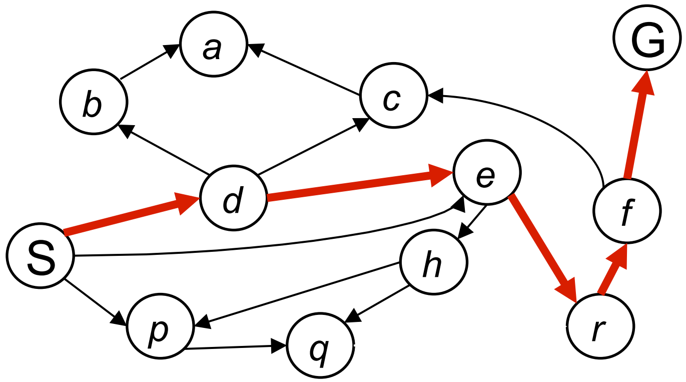
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

Detailed pseudocode is in the book!

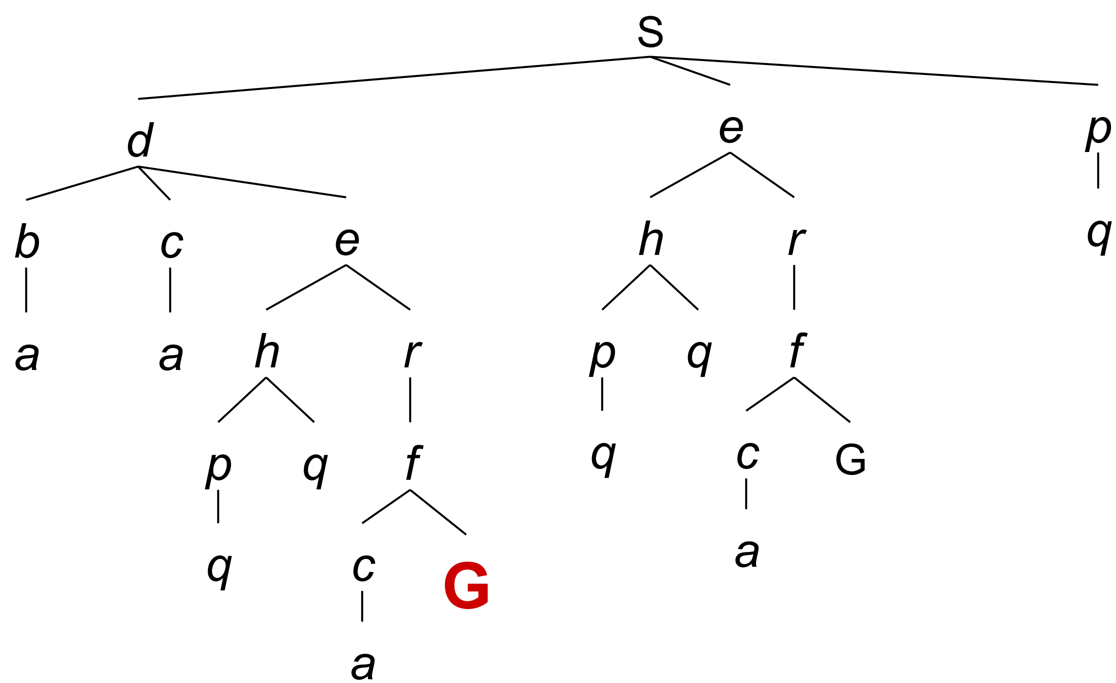
- Important ideas:
  - Frontier (aka fringe)
  - Expansion
  - Exploration strategy
- Main question: which frontier nodes to explore?

State Space vs. Search Tree



Each NODE in in the search tree is an entire PATH in the state space.

We construct both on demand – and we construct as little as possible.

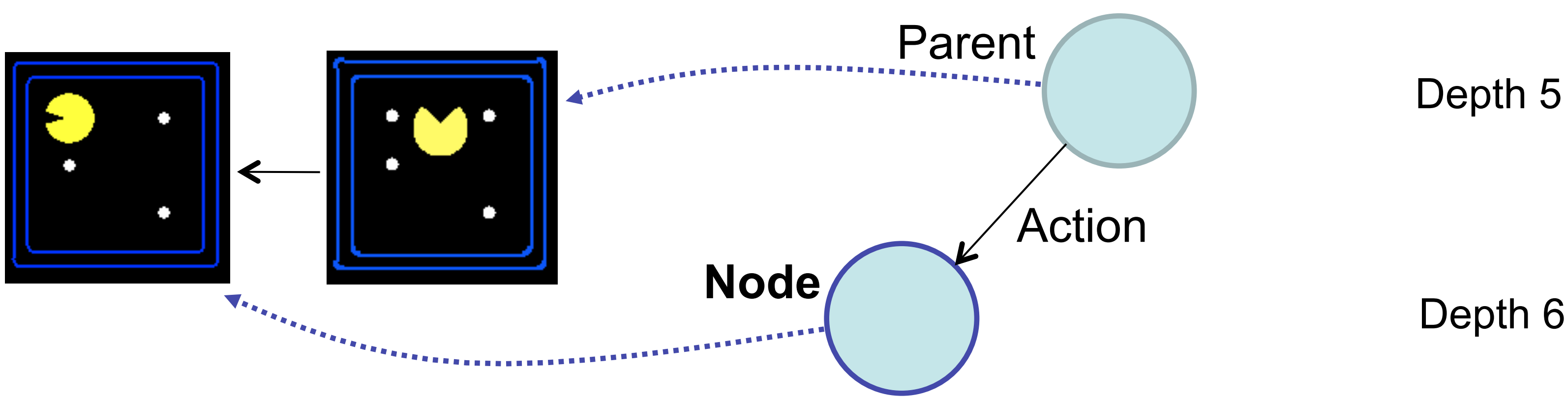


States vs. Nodes

- Nodes in state space graphs are problem states
  - Represent an abstracted state of the world
  - Have successors, can be goal / non-goal, have multiple predecessors
- Nodes in search trees are paths
  - Represent a path (sequence of actions) which results in the node's state
  - Have a problem state and one parent, a path length, (a depth) & a cost
  - The same problem state may be achieved by multiple search tree nodes

State Space Graph

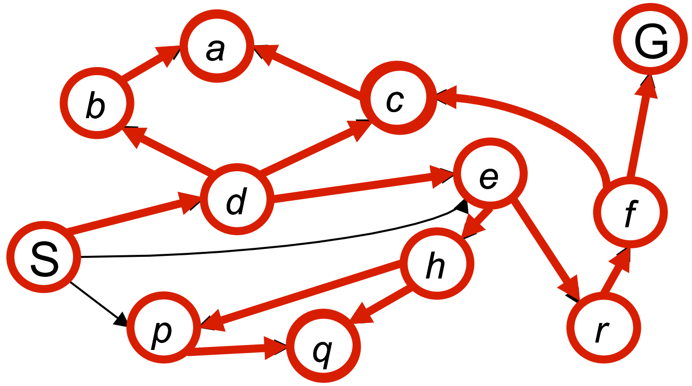
Search Tree

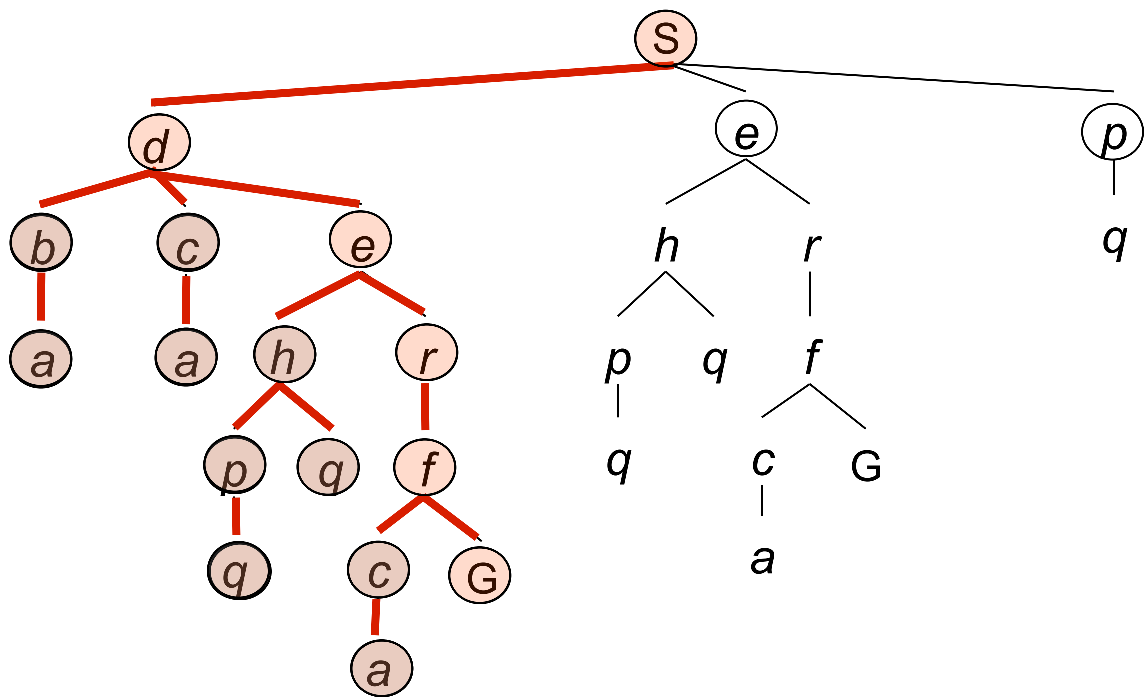


Depth First Search

Strategy: expand deepest node first

Implementation: Frontier is a LIFO stack



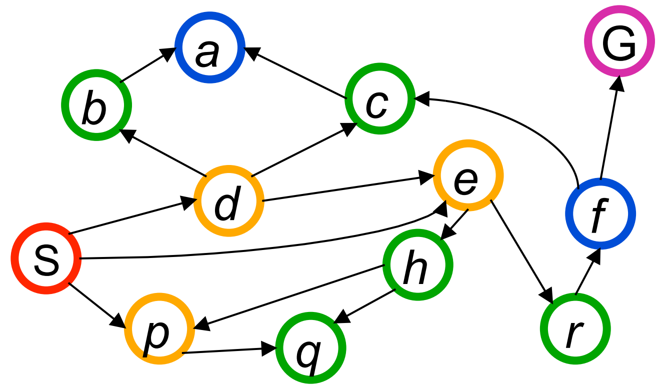


[demo: dfs]

Breadth First Search

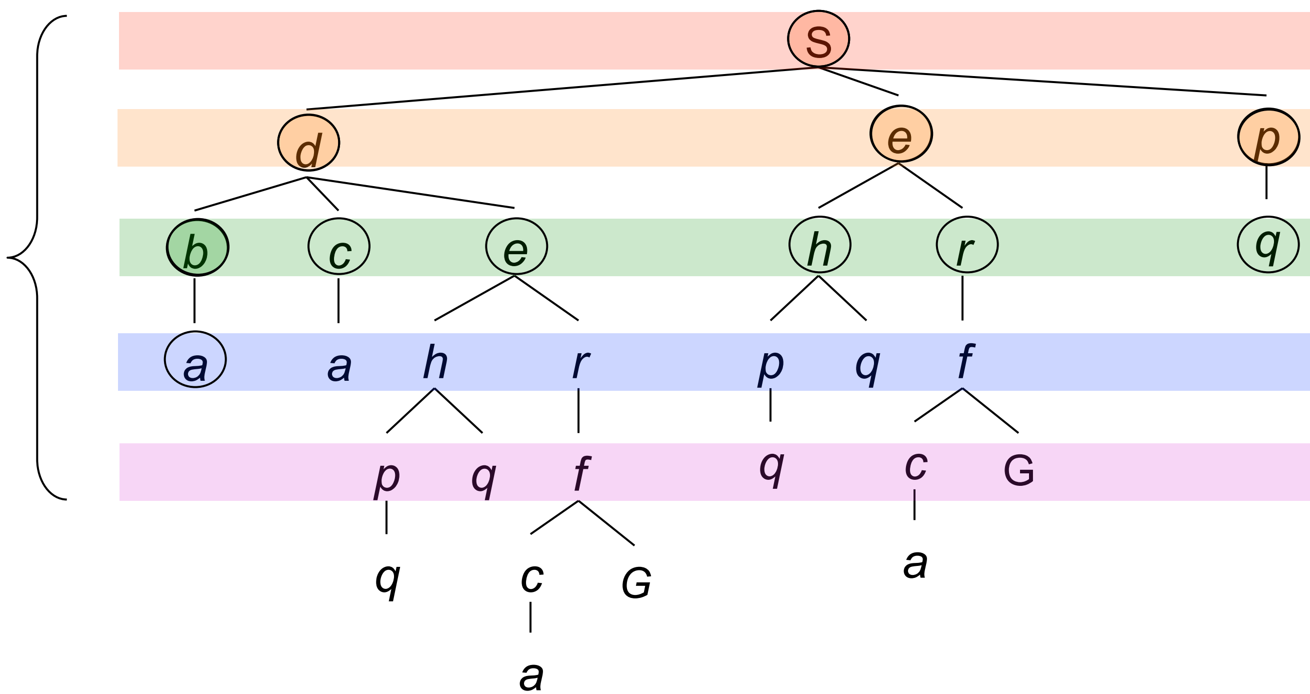
Strategy: expand shallowest node first

Implementation: Fringe is a FIFO queue



Search

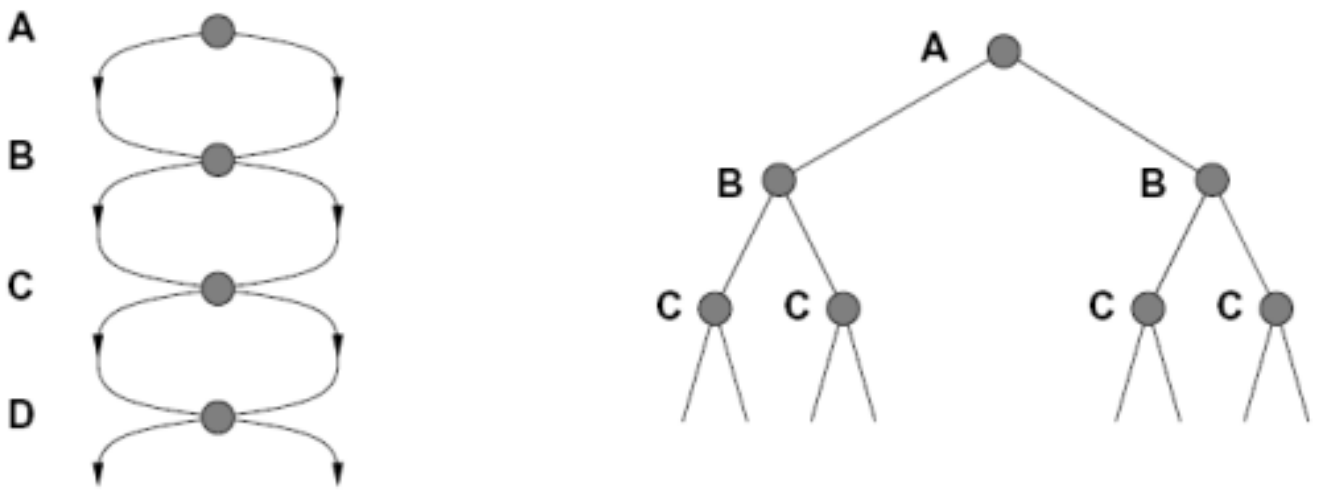
Tiers



[demo: bfs]

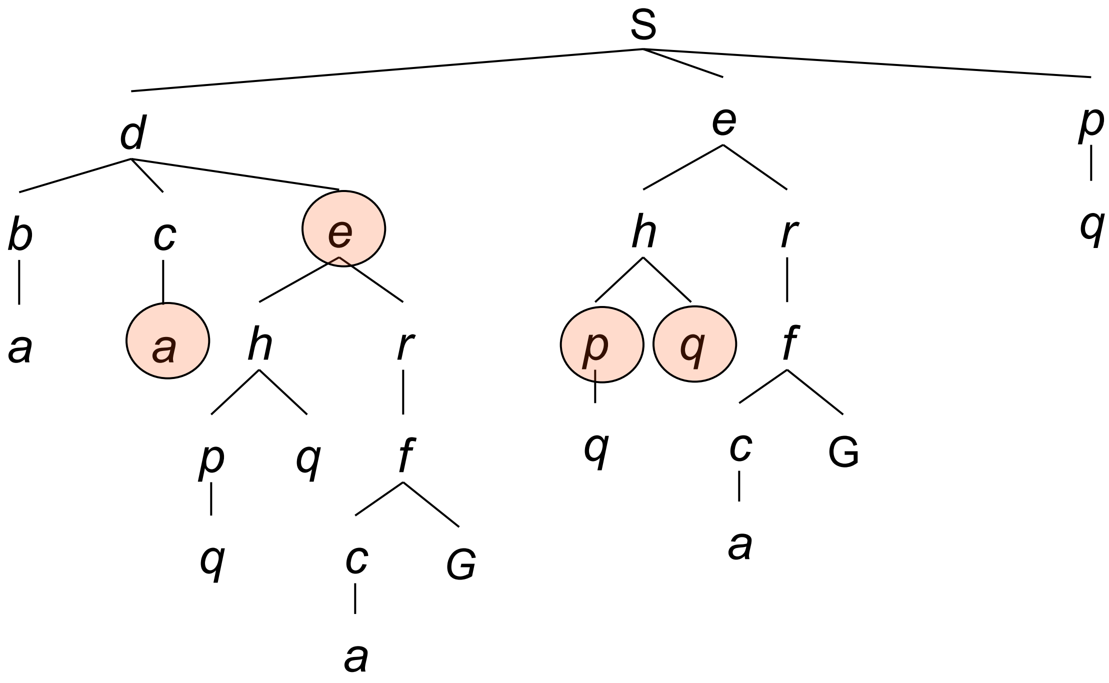
Santayana's Warning

- *"Those who cannot remember the past are condemned to repeat it."* – George Santayana
- Failure to detect repeated states can cause exponentially more work (why?)



Graph Search

- In BFS, for example, we shouldn’t bother expanding the circled nodes (why?)



Graph Search

- Very simple fix: never expand a state twice

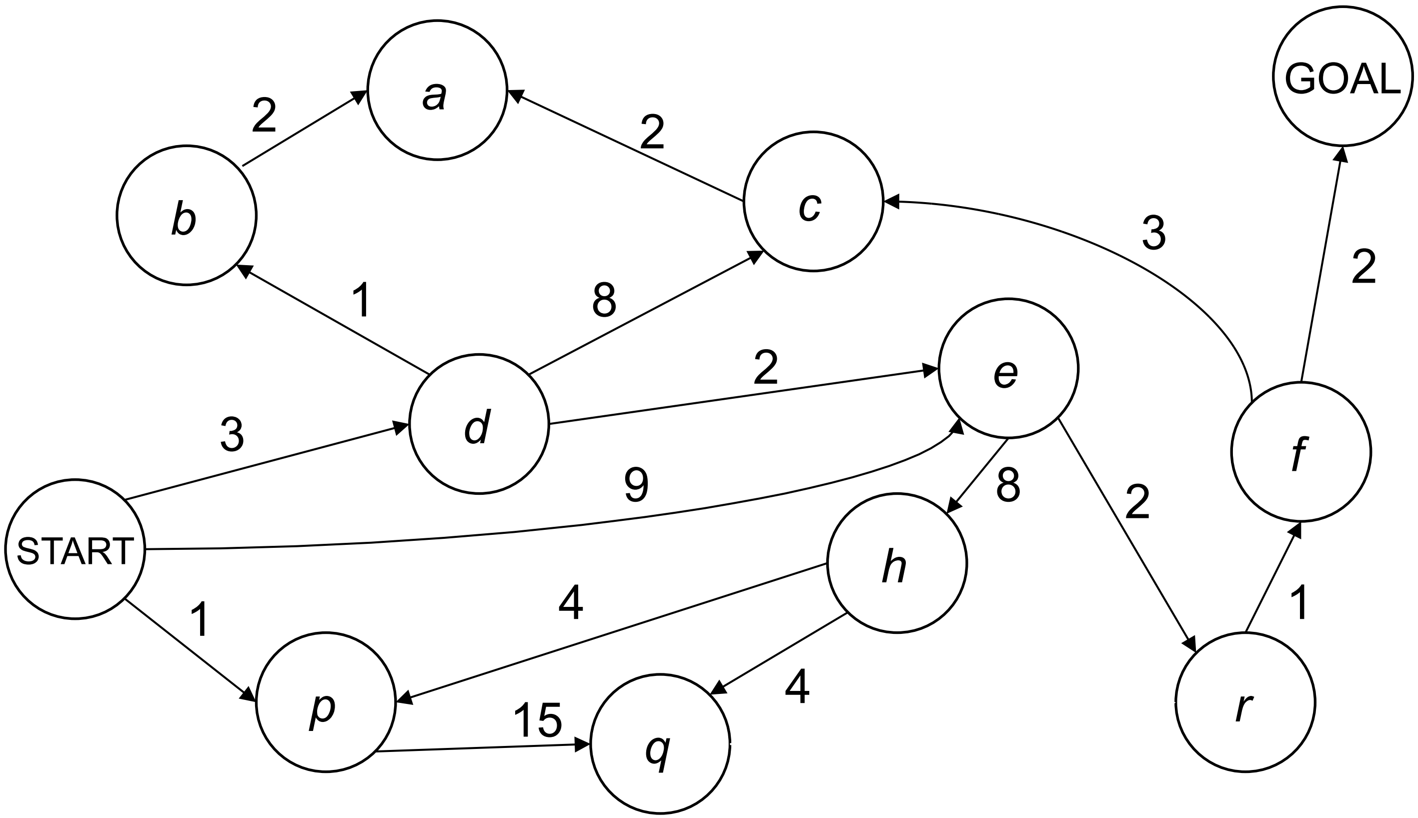
function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  initialize the explored set to be empty
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    add the node to the explored set
    expand the chosen node, adding the resulting nodes to the frontier
      only if not in the frontier or explored set

- Can this wreck completeness? Lowest cost?

Graph Search Hints

- Graph search is almost always better than tree search (when not?)
- Implement explored as a dict or set
- Implement frontier as priority Q *and* set

Costs on Actions



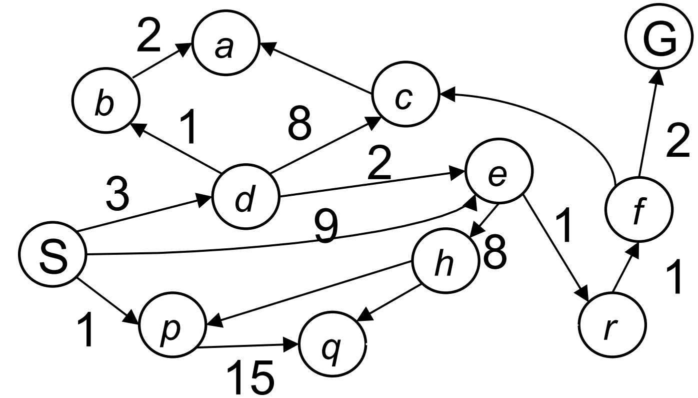
Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

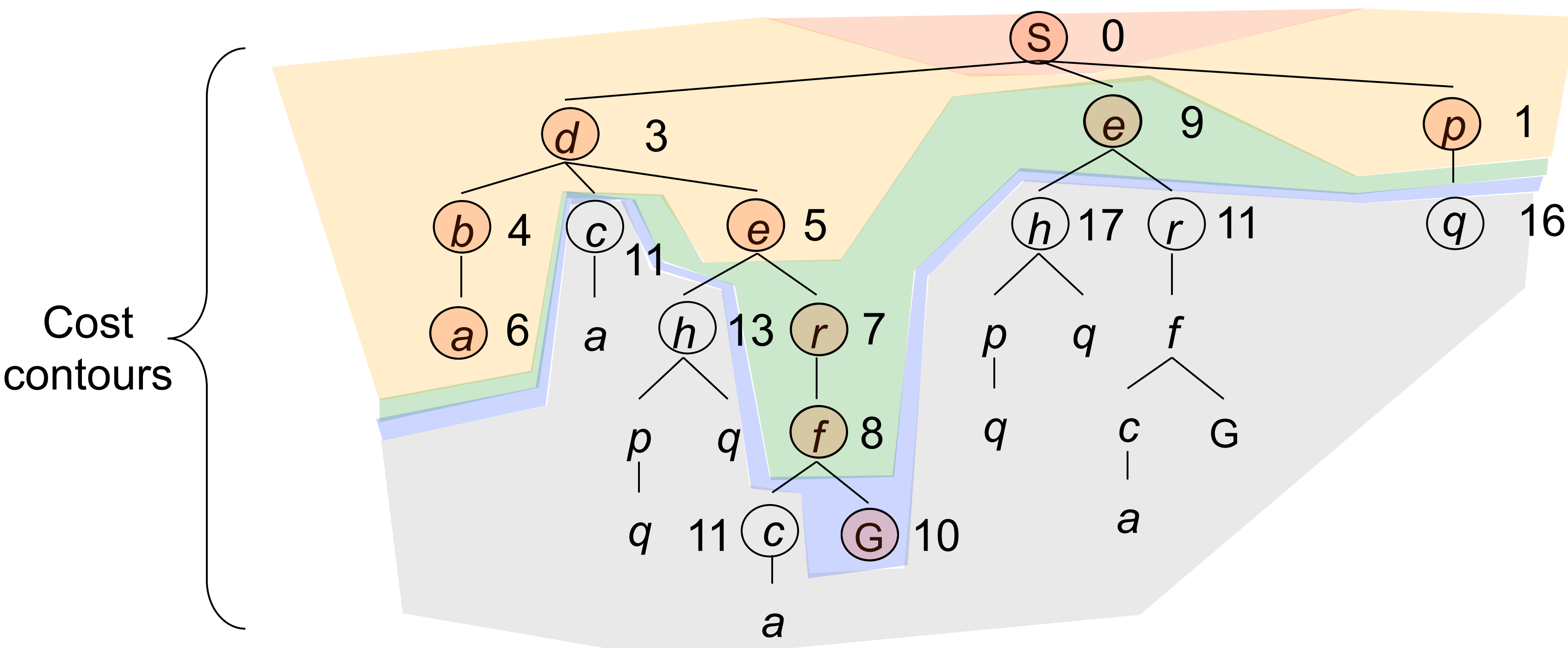
We will quickly cover an algorithm which does find the least-cost path.

Uniform Cost Search

Expand cheapest node first:

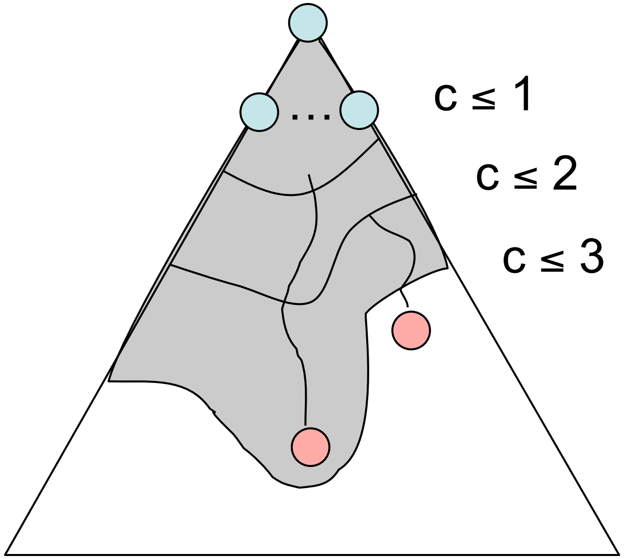
Frontier is a priority queue

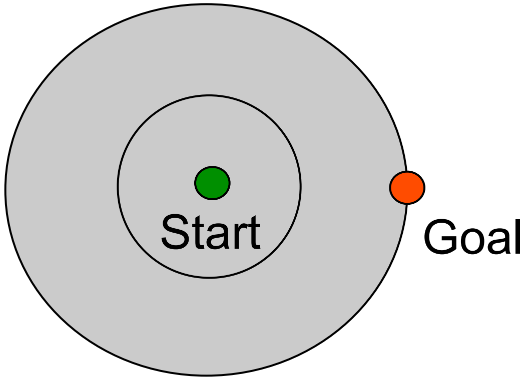




Uniform Cost Issues

- Remember: explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location

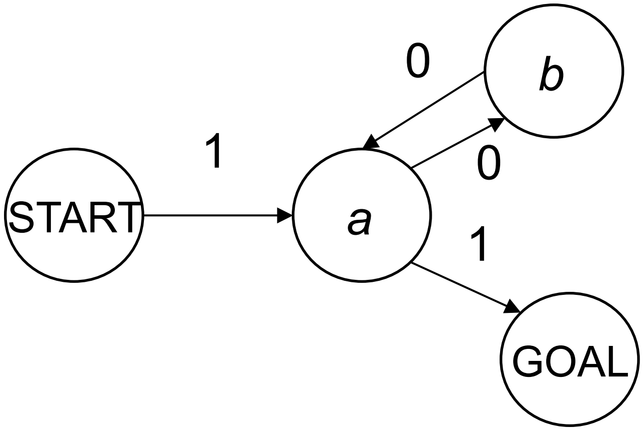




[demos: ucs, ucs2]

Uniform Cost Search

- What will UCS do for this graph?



- What does this mean for completeness?

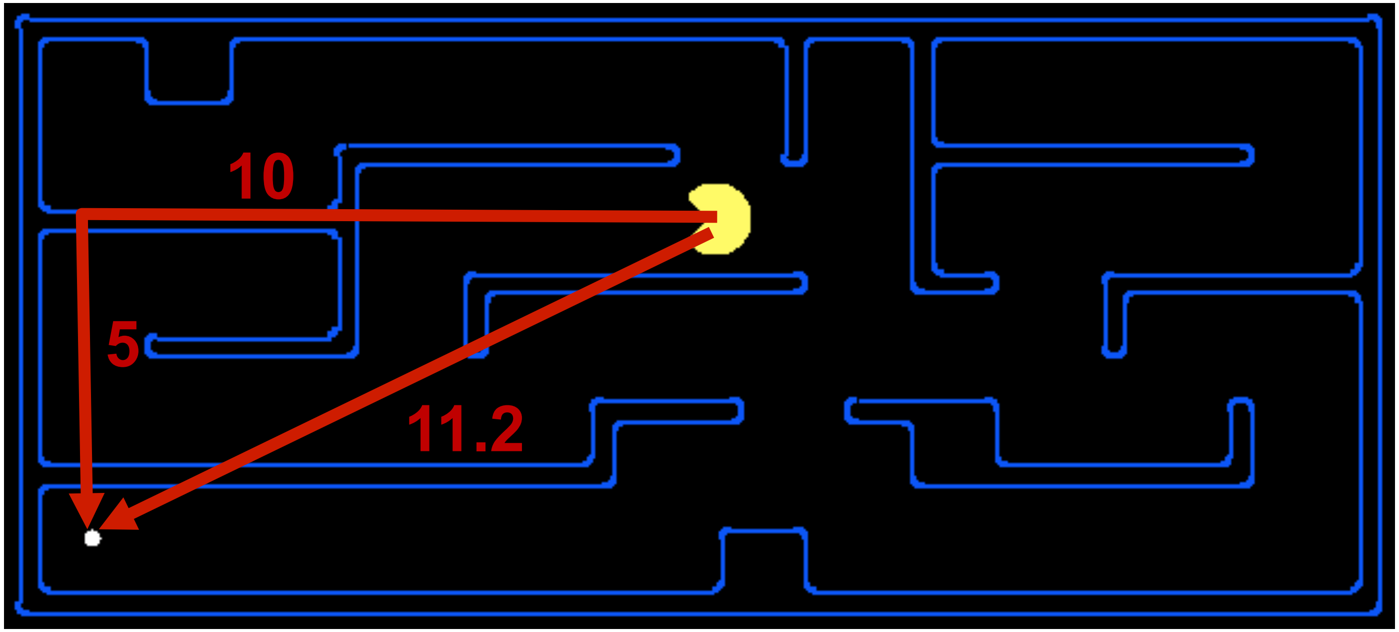
AI Lesson

To do more,

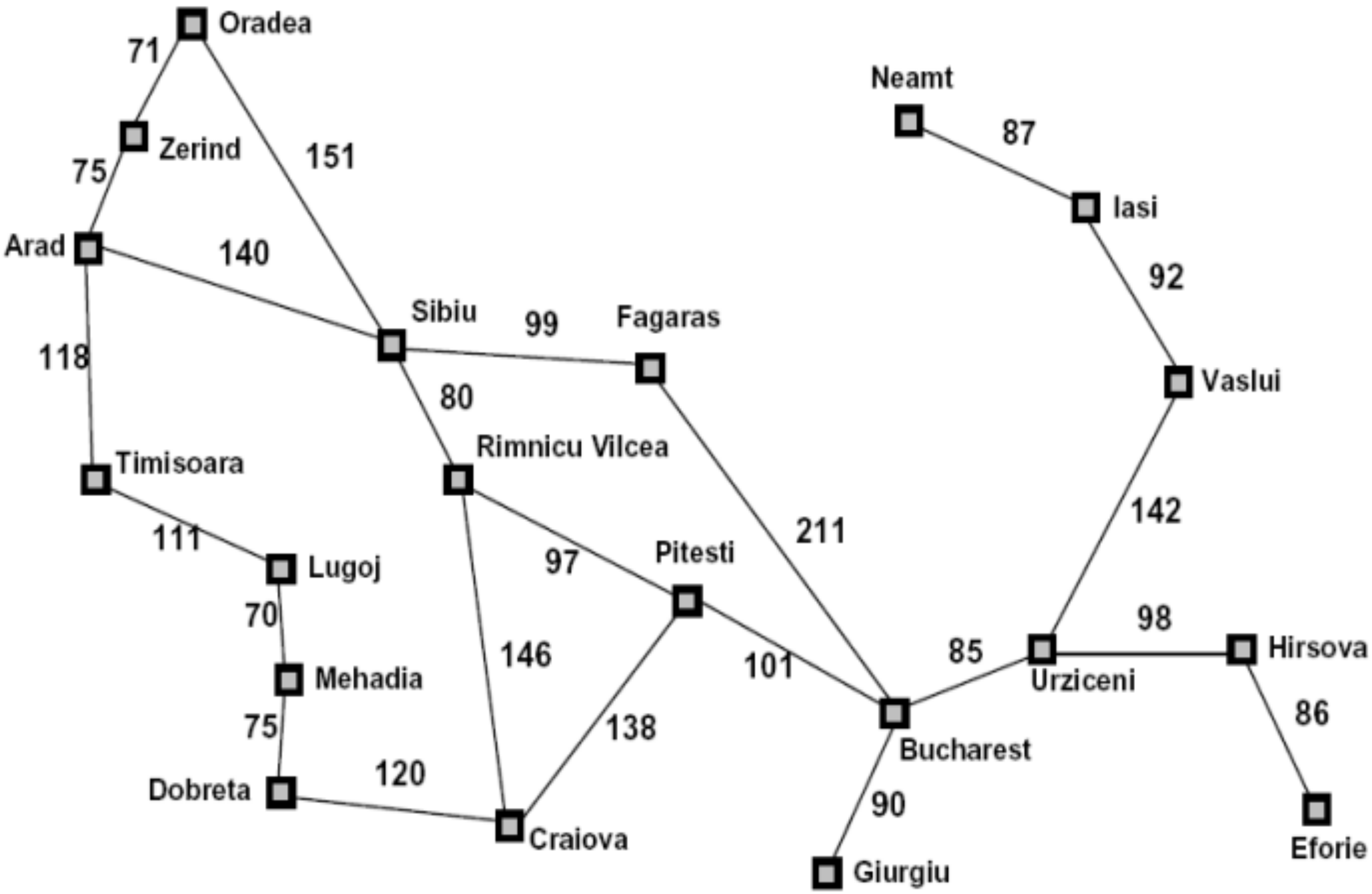
Know more

Search Heuristics

- Any *estimate* of how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance



Heuristics

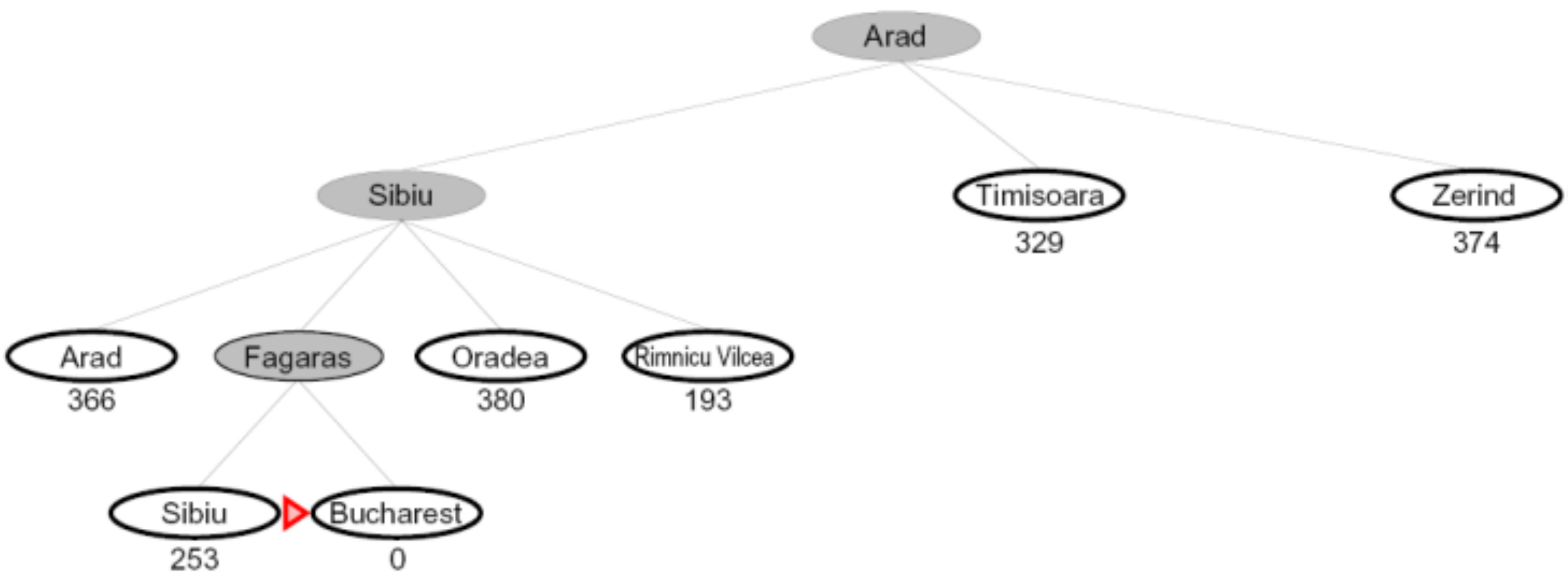


Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Best First Search

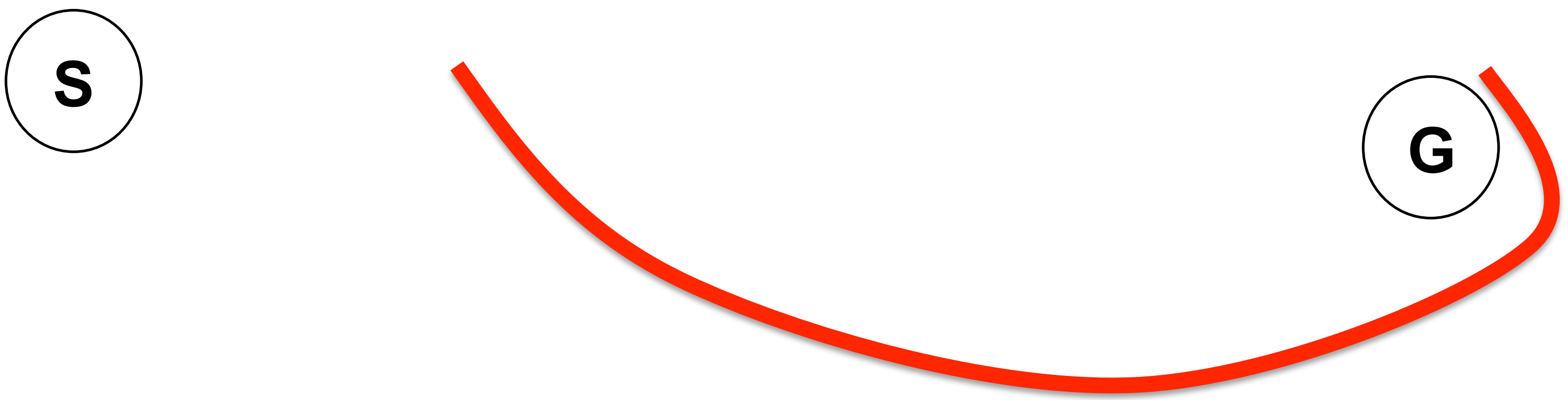
- Expand the node that *seems* closest to goal…



- What can go wrong?

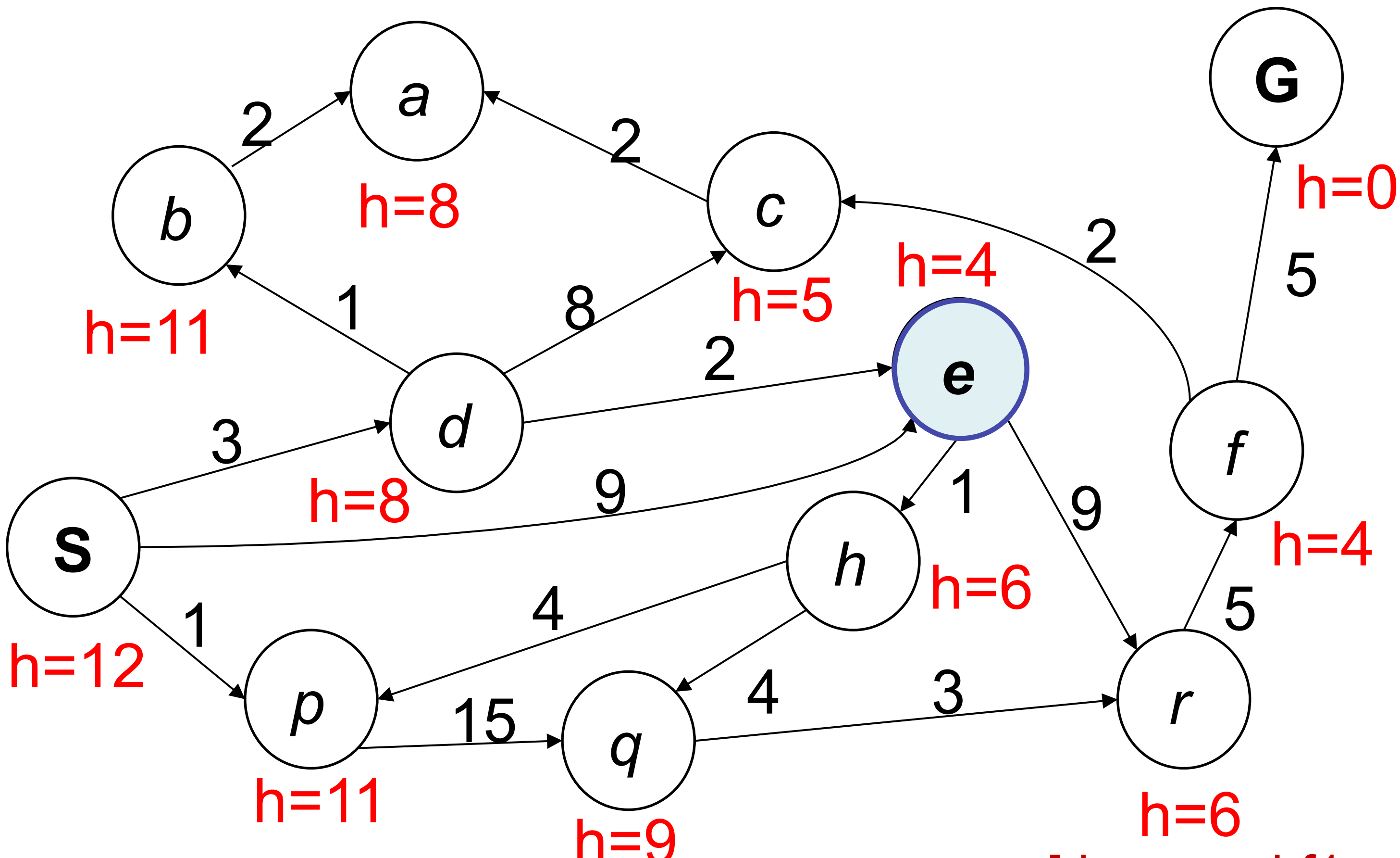
[demos: gbf1, gbf2]

Greedy goes wrong



Best First / Greedy Search

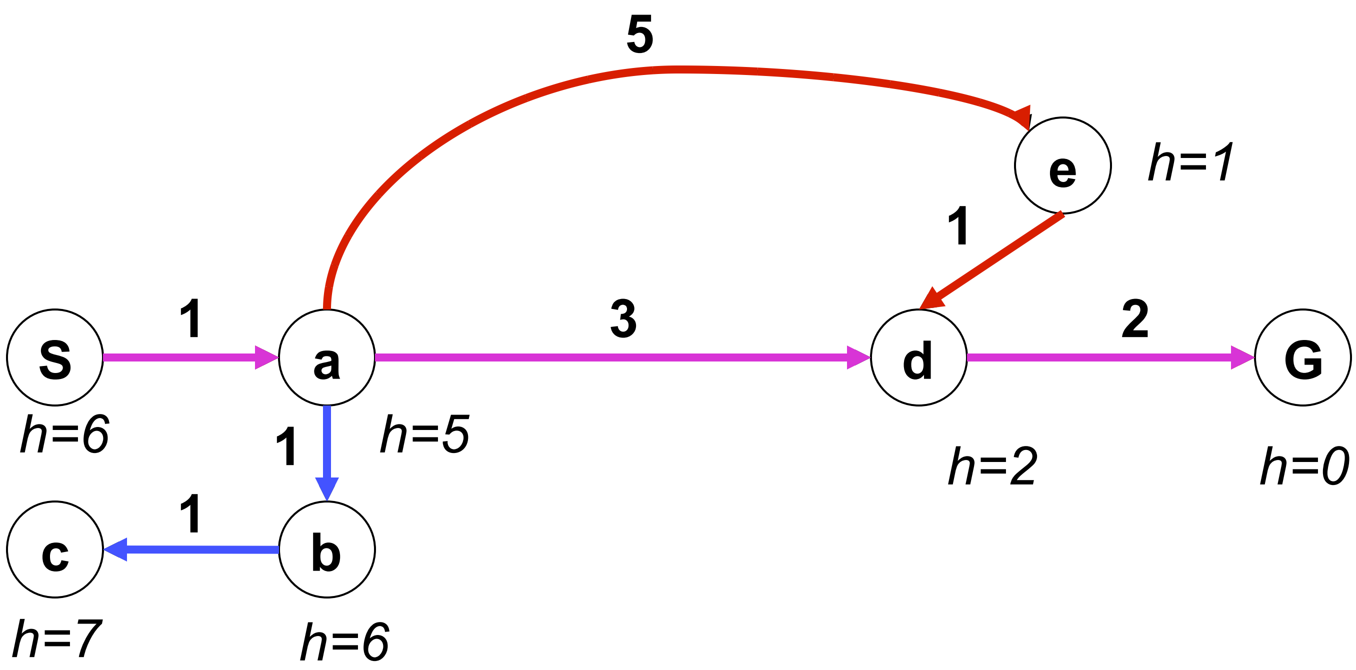
- Strategy: expand the closest node to the goal



[demos: gbf1, gbf2]

Combining UCS and Greedy

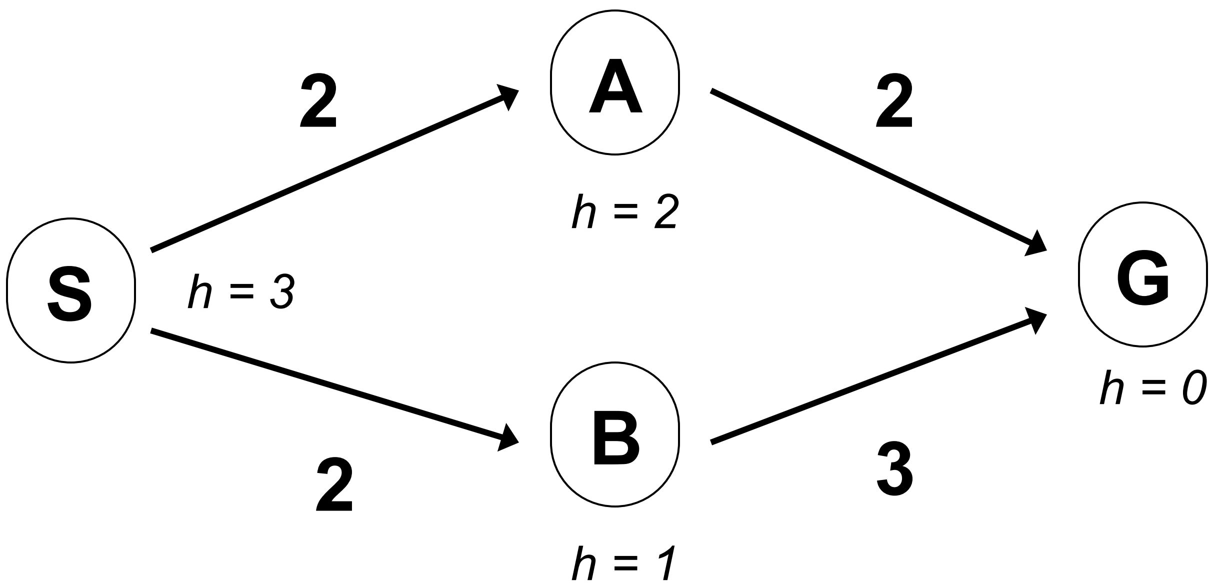
- Uniform-cost orders by path cost, or *backward cost* **g(n)**
- Best-first orders by distance to goal, or *forward cost* **h(n)**



- A* Search orders by the sum: **f(n) = g(n) + h(n)**

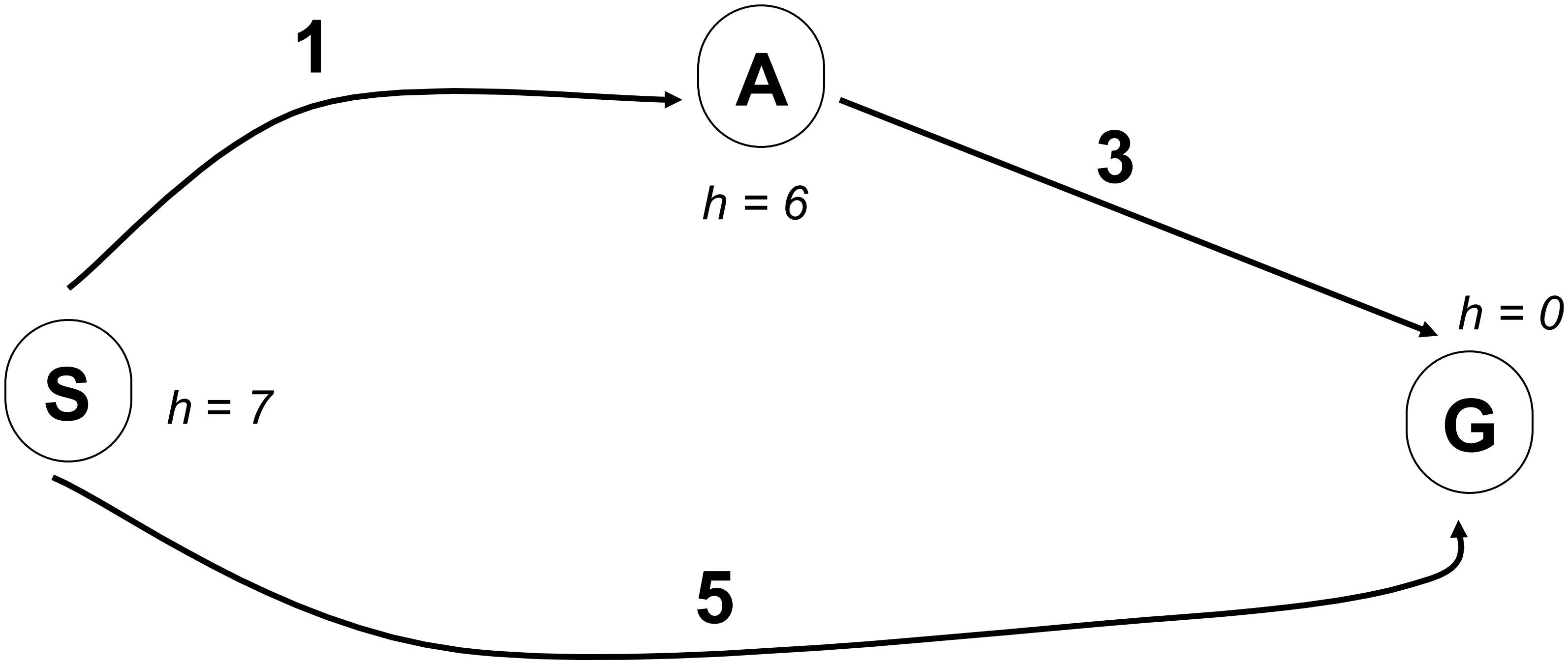
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad path cost (5) < estimate good path cost (1+6)
- We need estimates (h=7) to be less than actual (5) costs!

Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$h(n) \leq h^*(n)$$

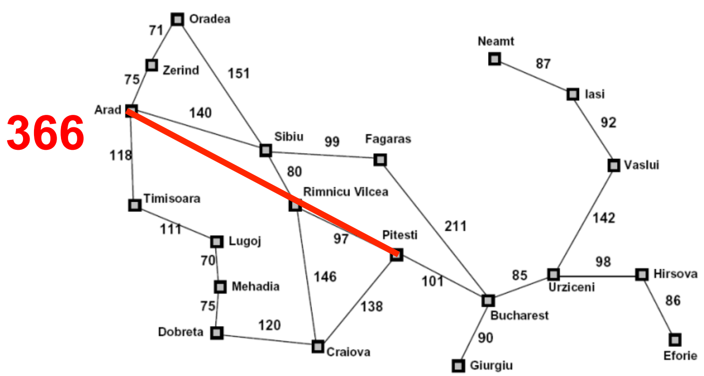
where $h^*(n)$ is the true cost to a nearest goal

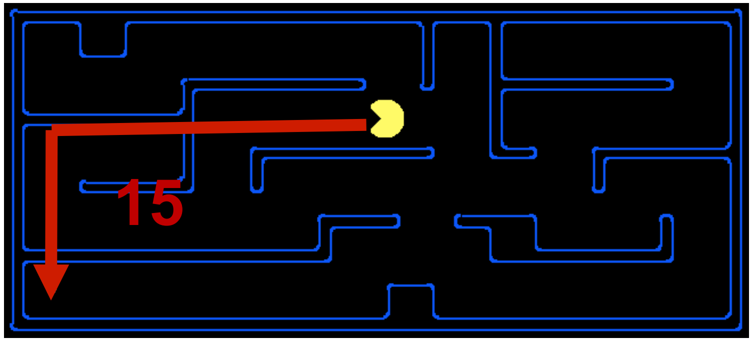
Never overestimate!

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available



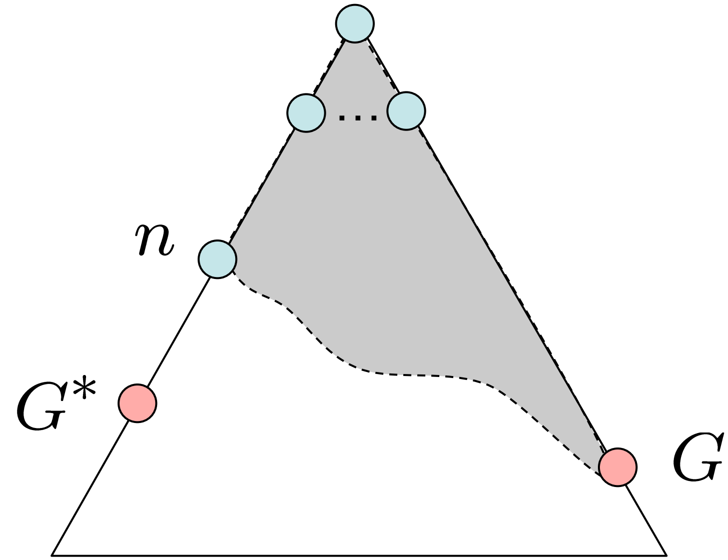


- Inadmissible heuristics are often useful too (why?)

Optimality of A*: Blocking

Notation:

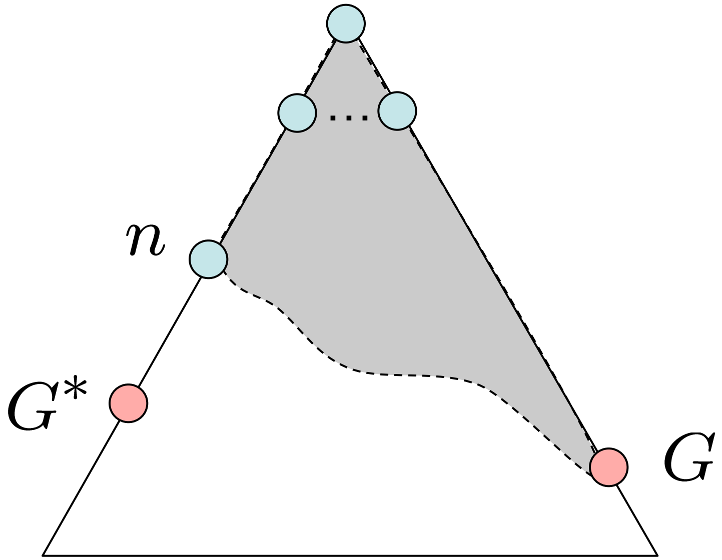
- g(n) = cost to node n
- h(n) = estimated cost from n to the nearest goal (heuristic)
- f(n) = g(n) + h(n) = estimated total cost via n
- G*: a lowest cost goal node
- G: another goal node



Optimality of A*: Blocking

Proof:

- What could go wrong?
- We'd have to have to pop a suboptimal goal G off the frontier before G*
- This can't happen:
  - Imagine a suboptimal goal G is on the queue
  - Some node *n* which is a subpath of G* must also be on the frontier (why?)
  - *n* will be popped before G



$$f(n) = g(n) + h(n)$$

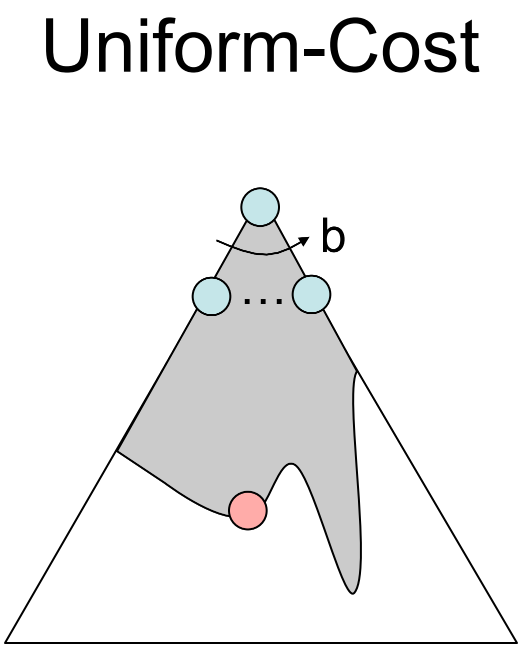
$$g(n) + h(n) \leq g(G^*)$$

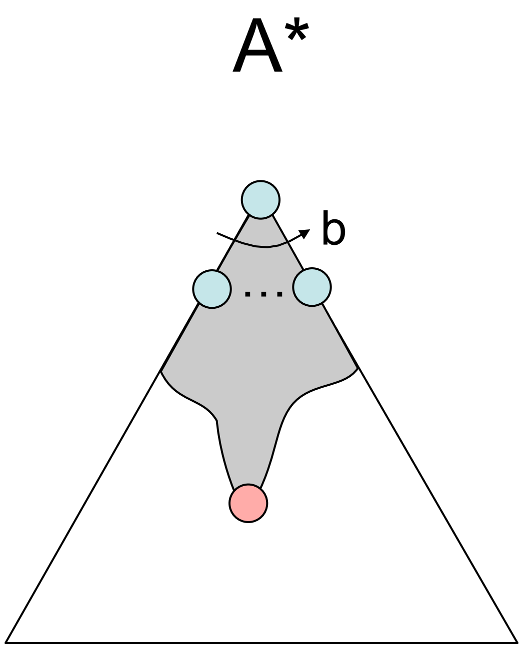
$$g(G^*) < g(G)$$

$$g(G) = f(G)$$

$$f(n) < f(G)$$

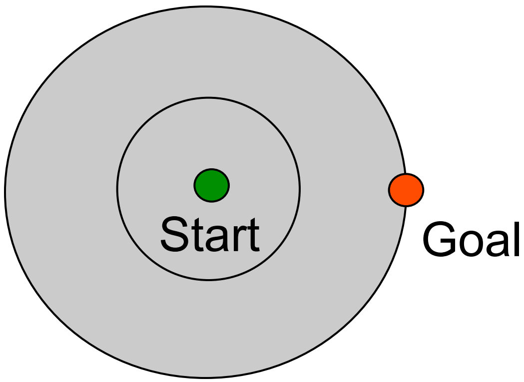
Properties of A*



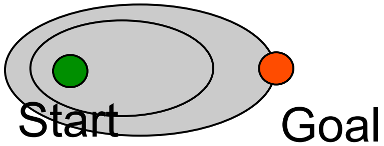


UCS vs A* Contours

- Uniform-cost expanded in all directions

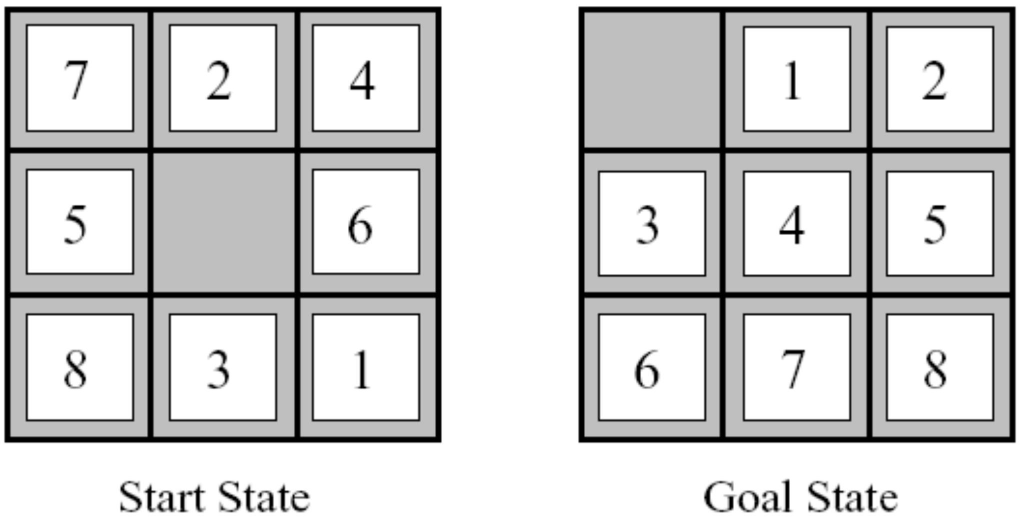


- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



[demos: conu, cona]

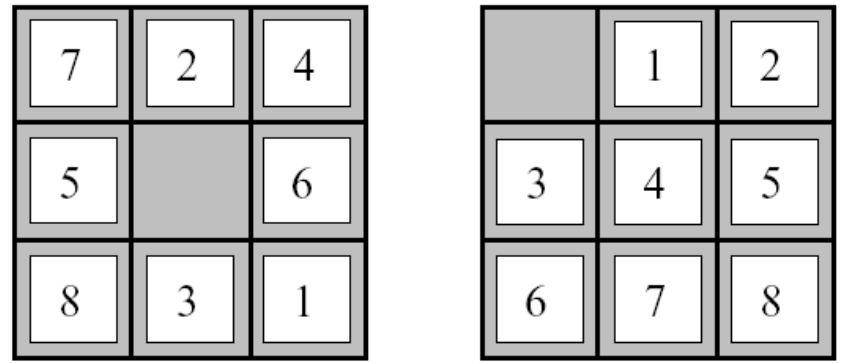
Example: 8 Puzzle



- What are the states?
- How many states?
- What are the actions?
- What states can I reach from the start state?
- What should the costs be?

8 Puzzle

- Heuristic: Number tiles misplaced
- Why is it admissible?



Start State Goal State

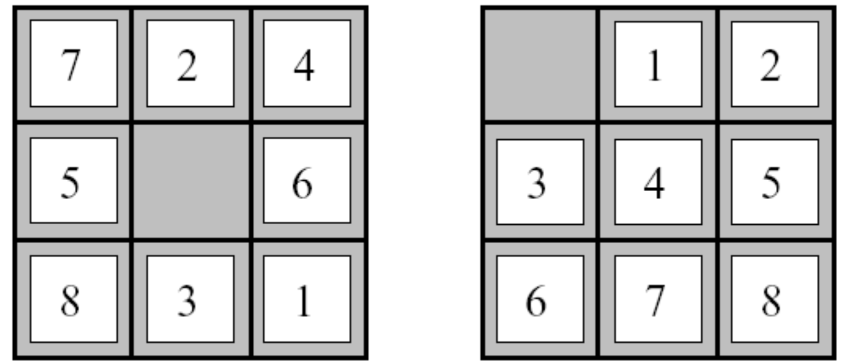
- h(start) =
- 8
- This is a relaxed-problem heuristic:

	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

Move **A** to **B** if ~~adjacent(**A**,**B**) and empty(**B**)~~

8 Puzzle

- What if we had an easier 8-puzzle where any tile could slide one step at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why admissible?



Start State　　Goal State

- h(start) =
- 3 + 1 + 2 + … = 18
- Relaxed problem:

	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

Move **A** to **B** if adjacent(**A**,**B**) ~~and empty(**B**)~~

Trivial Heuristics, Dominance

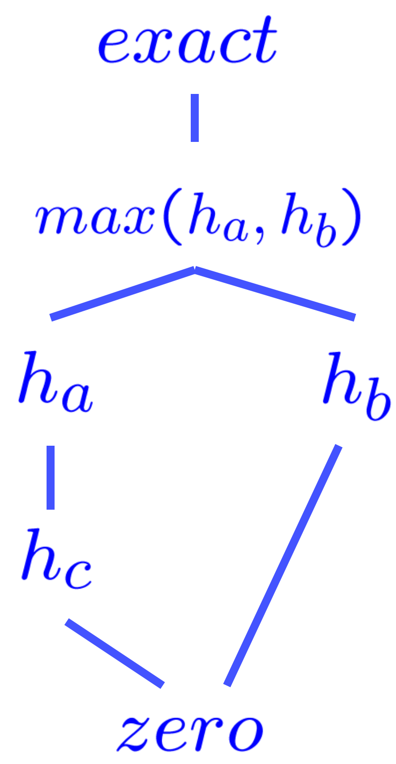
- Dominance: $h_a \geq h_c$ if

  $$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

  $$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic



Other A* Applications

- Path finding / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

Summary: A*

- A* uses both backward costs, **g(n)**, and (estimates of) forward costs, **h(n)**
- A* is optimal with admissible heuristics
- Heuristic design is key: often use relaxed problems
- A* is not the final word in search algorithms (but it does get the final word for today)