MDPs: modeling

Dice game

Example: dice game

For each round $r = 1, 2, \ldots$

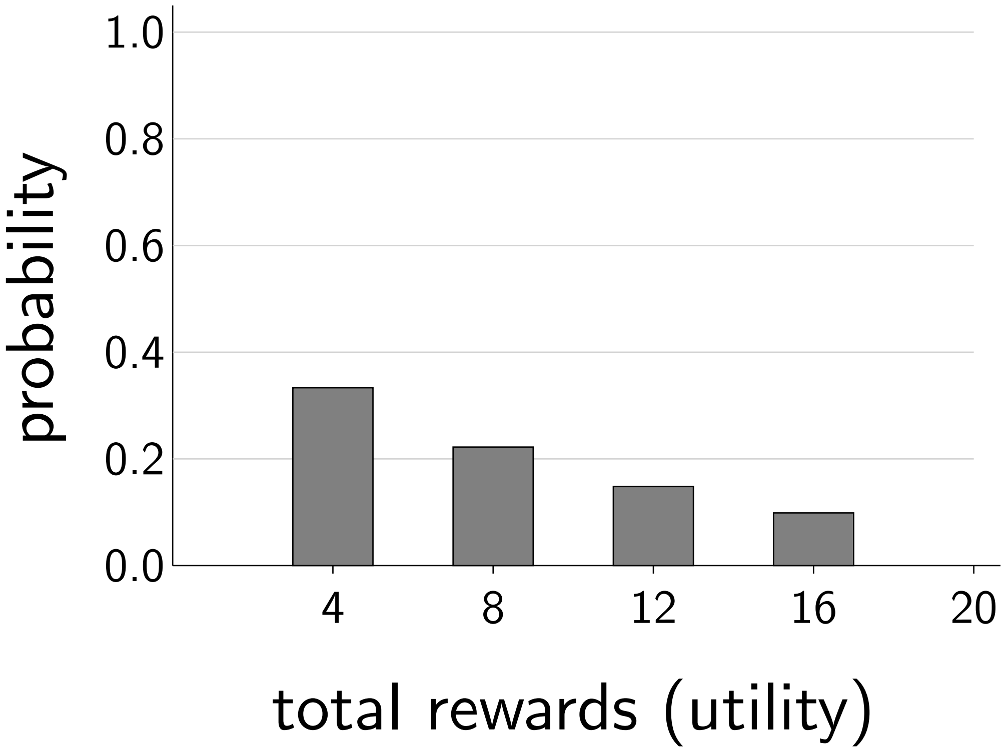
- You choose stay or quit.
- If quit, you get $\$10$ and we end the game.
- If stay, you get $\$4$ and then I roll a 6-sided dice.
  - If the dice results in 1 or 2, we end the game.
  - Otherwise, continue to the next round.

Start Stay Quit

Dice: Rewards: 0

Rewards

If follow policy "stay":

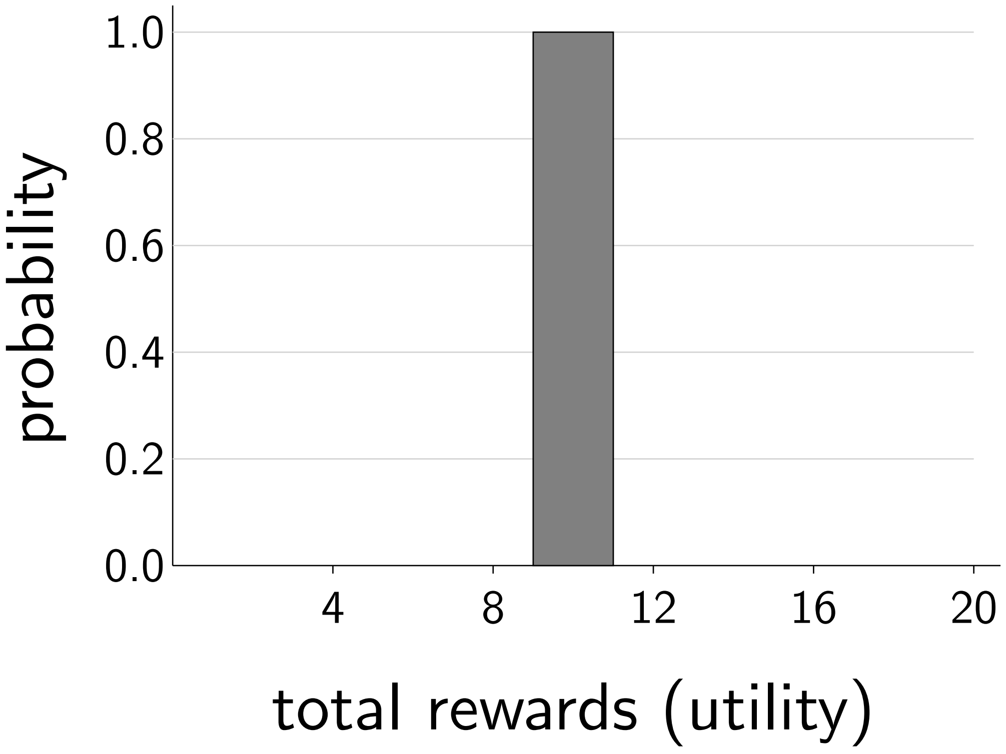


Expected utility:

$$\frac{1}{3}(4) + \frac{2}{3} \cdot \frac{1}{3}(8) + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}(12) + \cdots = 12$$

Rewards

If follow policy "quit":



Expected utility:

$$1(10) = 10$$

MDP for dice game

Example: dice game

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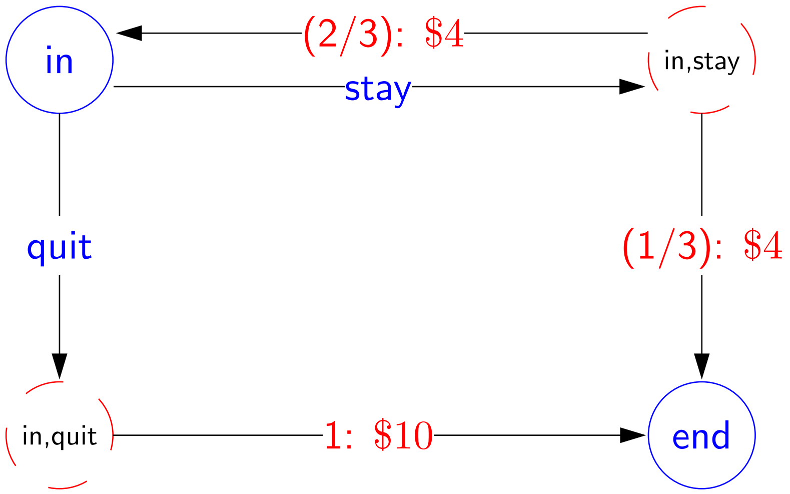

Markov decision process

Definition: Markov decision process

States: the set of states

$s_{\text{start}} \in$ States: starting state

Actions(s): possible actions from state s

$T(s, a, s')$: probability of s' if take action a in state s

Reward(s, a, s'): reward for the transition (s, a, s')

IsEnd(s): whether at end of game

$0 \leq \gamma \leq 1$: discount factor (default: 1)

Search problems

Definition: search problem

States: the set of states

$s_{\text{start}} \in$ States: starting state

Actions(s): possible actions from state s

Succ(s, a): where we end up if take action a in state s

Cost(s, a): cost for taking action a in state s

IsEnd(s): whether at end

- Succ$(s, a) \Rightarrow T(s, a, s')$
- Cost$(s, a) \Rightarrow$ Reward(s, a, s')

Transitions

Definition: transition probabilities

The **transition probabilities** $T(s, a, s')$ specify the probability of ending up in state s' if taken action a in state s.

Example: transition probabilities

s	a	s'	$T(s, a, s')$
in	quit	end	1
in	stay	in	$2/3$
in	stay	end	$1/3$

Probabilities sum to one

Example: transition probabilities

s	a	s'	$T(s, a, s')$
in	quit	end	1
in	stay	in	$2/3$
in	stay	end	$1/3$

For each state s and action a:

$$\sum_{s' \in \text{States}} T(s, a, s') = 1$$

Successors: s' such that $T(s, a, s') > 0$

Transportation example

Example: transportation

Street with blocks numbered 1 to n.

Walking from s to $s+1$ takes 1 minute.

Taking a magic tram from s to $2s$ takes 2 minutes.

How to travel from 1 to n in the least time?

Tram fails with probability 0.5.

[semi-live solution]

What is a solution?

Search problem: path (sequence of actions)

MDP:

Definition: policy

A **policy** π is a mapping from each state $s \in$ States to an action $a \in$ Actions(s).

Example: volcano crossing

s	$\pi(s)$
(1,1)	S
(2,1)	E
(3,1)	N
...	...