
Private Measurement of Single Events

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What is “single-event measurement”?

- Queries which observe the outcome associated with single events.
- e.g. “Did *source impression* lead to a conversion, or not?”

Attribution Reporting API - event-level reports	Supported
Attribution Reporting API - summary reports	Supported
Interoperable Private Attribution	Supported
Private Click Measurement	Limited support

Goal for this discussion: either

1. **Agree** single-event measurement with differential privacy satisfies our privacy goals, OR
2. **Disagree** and investigate mitigations

This presentation

1. Differential privacy on single events can protect users
2. Noisy, per-event data can be useful
3. “Aggregation” as a boundary is hard to rigorously defend

Context:

- <https://github.com/patcg/docs-and-reports/issues/41>
- <https://github.com/patcg-individual-drafts/ipa/issues/60>

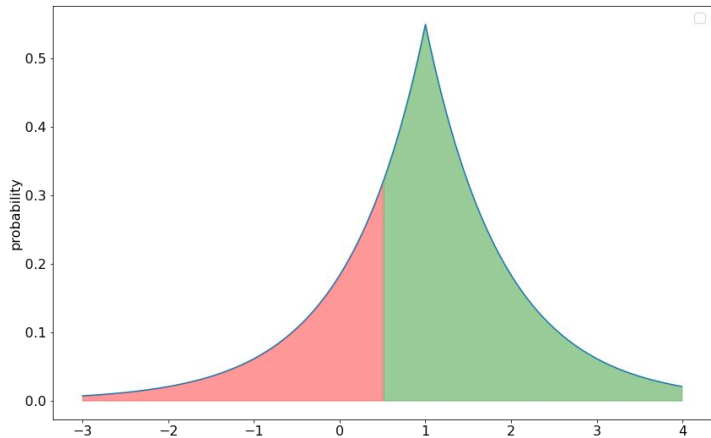
***Differential privacy* on single events can protect users**

Per-event differential privacy

Did *source impression* lead to a conversion, or not? Imagine it did:

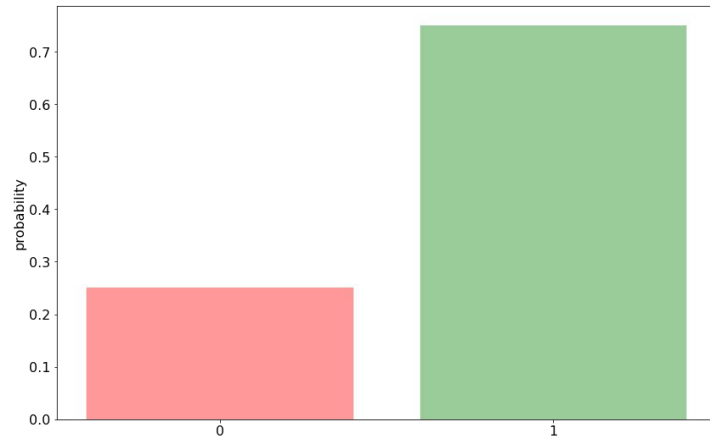
Laplace mechanism

```
return val + laplace(1 / epsilon)
```



Randomized response

```
if random() < 2 / (1 + exp(epsilon)):  
    return choice([0, 1])  
return val
```



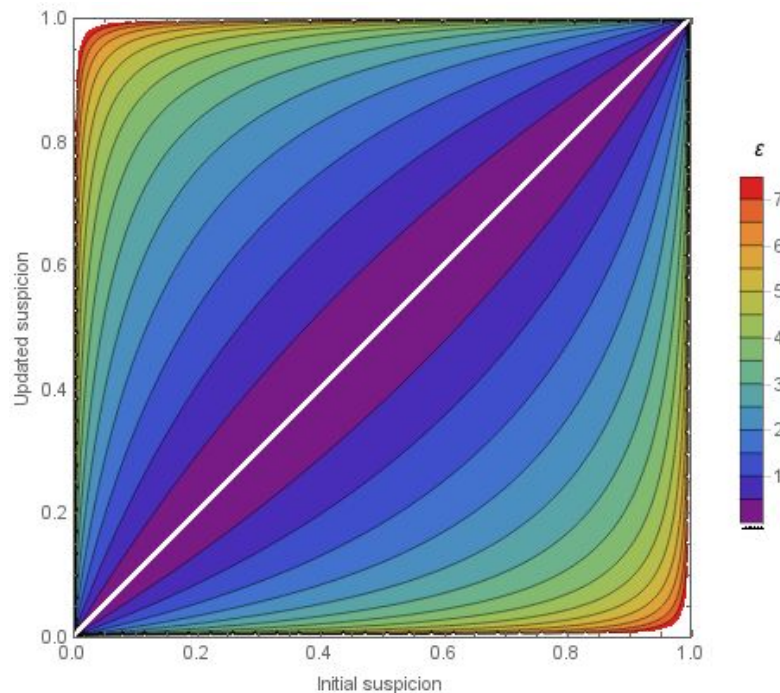
Semantic interpretation of differential privacy

- Attacker has a prior on the user's data
- Privacy mechanism bounds the posterior after looking at the data
- Applies to *any* mechanism satisfying DP
 - Includes mechanisms permitting single event measurement

$\epsilon = \sim 1.1$ bounds a prior of 50% to [25%, 75%]

$\epsilon = \sim 2.2$ bounds a prior of 50% to [10%, 90%]

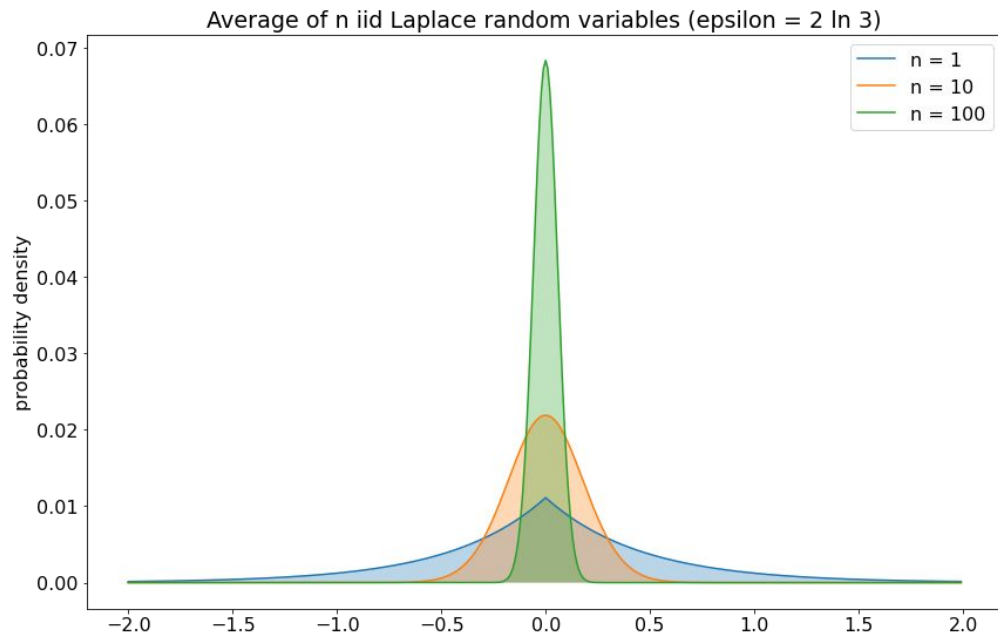
$\epsilon = \sim 2.9$ bounds a prior of 50% to [5%, 95%]



Source: <https://desfontain.es/privacy/differential-privacy-in-more-detail.html>

Aggregation is a critical *post-processing* step here

- Take $\epsilon = \sim 2.2$
- $\text{Laplace}(1/\epsilon) \rightarrow \sigma = \sim .64$
- You can guess a single user's value, but in general this won't lead to accurate results
- What if you average N users?
 - Yields $\sigma' = \sigma / \text{sqrt}(N)$
 - $N \geq \sim 150$ yields $\sigma' = \sim .05$

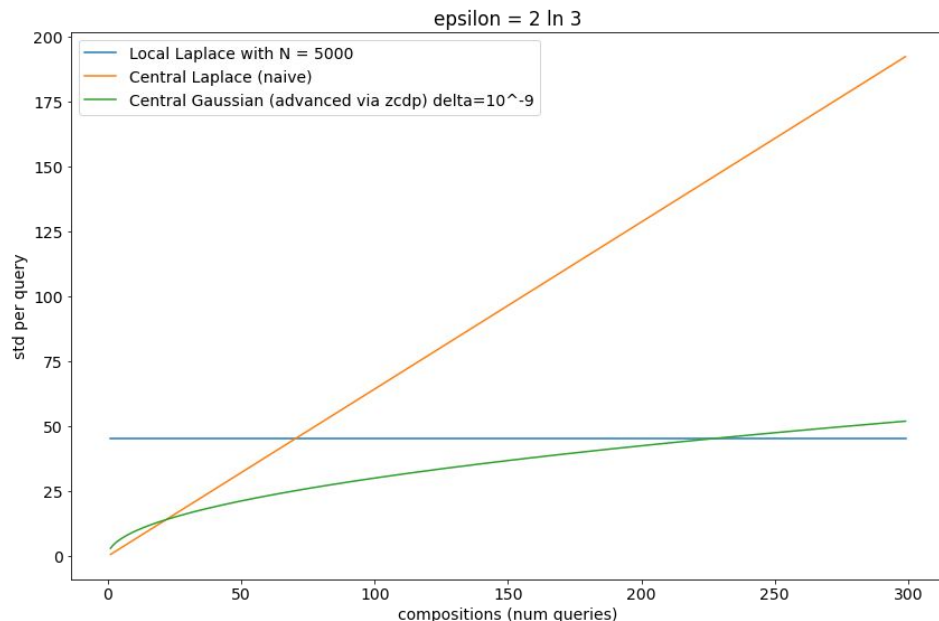


Under high privacy regimes, single-event privacy ~requires aggregation for meaningful utility

Noisy, per-event data can be useful

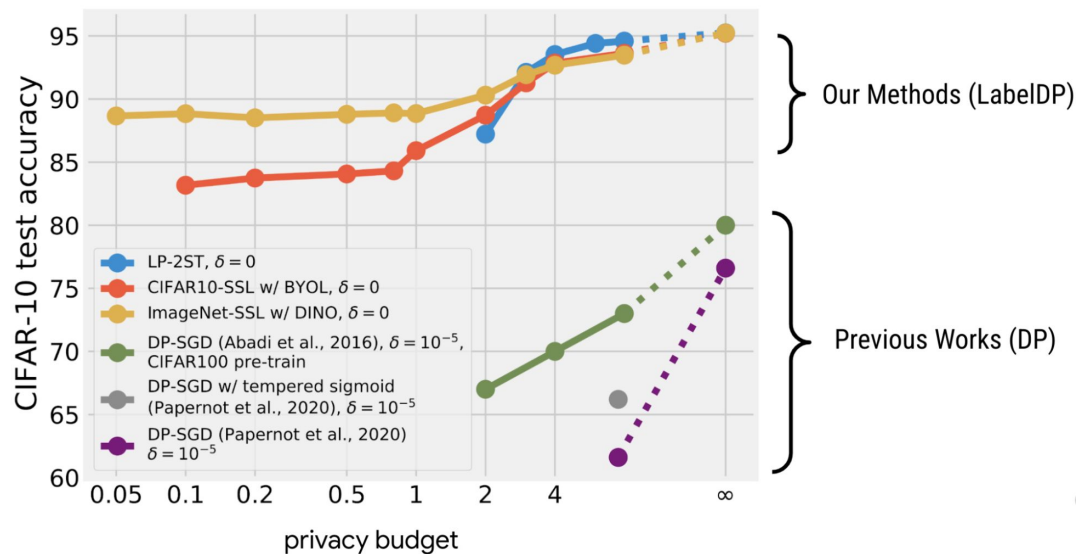
Flexible aggregation via post-processing

- Privacy is already “built-in”
 - Arbitrary aggregate slices
 - Avoids “regretful” queries
- Build complex mechanisms outside of the privacy mechanism
 - Allows us to satisfy use-cases before building custom algorithms for them
- May allow “data sharing” use-cases without industry standardization on breakdown keys
 - Think: multiple ad-tech measurers



Private optimization via Label DP

- Label DP
 - Differentially private optimization where *only the label* is private
 - Label = #conversions, \$\$, etc associated with an impression
- Ghazi et al ([NeurIPS 2021](#), [ICLR 2023](#))
 - “restricted k-ary randomized response”
 - State of the art performance in private learning
 - Continuing to explore future innovations in this setting
- Meta research
 - Malek et al ([NeurIPS 2021](#))
 - [Yuan et al](#) (preprint)



Test accuracy with LabelDP vs. traditional DP learning
on an image dataset

Source: <https://ai.googleblog.com/2022/05/deep-learning-with-label-differential.html>

“Aggregation” as a boundary is hard to rigorously defend

k-anonymity style mitigations

Remove outputs:

- whose inputs to a particular bucket $< k_1$
- whose output buckets $< k_2$

Problems:

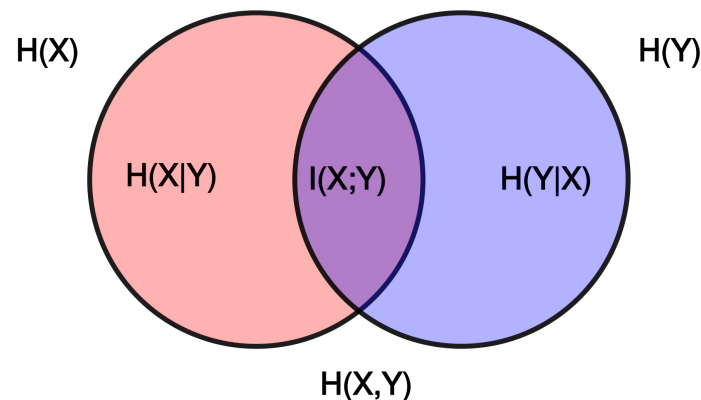
- Adversaries that injects fake events
- Breaks with composition, auxiliary data
 - Overlapping queries
 - Difference attacks
- Protection may rely on distributional assumptions unless backstopped by DP

Campaign	Num impressions ($k_1 < 150$ removed)	Num conversions ($k_2 < 30$ removed)
Campaign1	1004	40
Campaign2	120	31
Campaign3	304	12
Campaign4	13000	1000

*k-anon enforcement **only weakly protects** against measuring single events*

Maximum information gain / channel capacity

- X = encoded message sent through the API
- Y = API output
- Goal of the adversary: maximize mutual information $I(X; Y)$
 - Over all possible encodings \rightarrow *channel capacity*
 - Measured in B “bits”
 - Can observe 2^B distinct events
 - Encompasses both noise and data granularity
- Robust against composition
- No assumptions on adversary in general
- Amplified with DP



Info gain enforcement **only weakly protects** against measuring single events (but it is a robust privacy definition to prevent scaled attacks across many users).

This presentation: in conclusion

1. Differential privacy on single events can protect users
2. Noisy, per-event data can be useful
3. “Aggregation” as a boundary is hard to rigorously defend