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LOG-*f* TABLES FOR BETA DECAY*

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An extensive tabulation of log-*f* values is presented for use in the calculation of the comparative half-lives for beta transitions. Values of capture-to-positron ratios are also given. These log-*f*- and ϵ/β^+-values incorporate screening and finite nuclear size corrections. They are tabulated for allowed and first-forbidden unique transitions for Z from 6 to 95 as a function of transition energy. The range of transition energies covered is 0.010 MeV to 10 MeV with energy increments chosen so that intermediate values can be obtained from the tables by means of linear extrapolation. The values for first-forbidden unique transitions are larger than those given in older tables by 1.079, the log of 12. [See Eqs. (19) and (21).]

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CONTENTS

INTRODUCTION

Beta-spectra can be discussed conveniently in terms of shape factors $S_n(Z,W)$ defined so that, for an n^{th}-forbidden β-branch, the probability per unit time that a nucleus will decay via this branch is

$$\lambda_n = \frac{g^2}{2\pi^3}\int_1^{W_0} pW(W_0 - W)^2 S_n(Z,W)dW \quad (1)$$

where

g = weak interaction coupling constant
Z = atomic number of the daughter nucleus
p = momentum of β-particle
$W = \sqrt{p^2+1}$ = total energy of β-particle
W_0 = maximum β-particle energy. In β^--decay $W_0 = Q + 1$, in β^+-decay $W_0 = Q - 1$ where Q is the mass difference between initial and final states of neutral atoms.

For an n^{th}-forbidden β-branch, $\Delta J = n, n+1$; $\Delta\pi = (-1)^n$ with the added provision that ΔJ can be zero for $n = 1$. For a unique transition of forbiddenness n, $\Delta J = n + 1$, $\Delta\pi = (-1)^n$. Equation (1) is written in natural units ($\hbar = m = c = 1$) so that the unit of momentum is mc, the unit of energy is mc^2, and the unit of time is $\hbar/mc^2$.

The Fierz interference term has been omitted from Eq. (1). In an observation of the β-energy distribution, possible correlations between linear and angular momenta give rise to a term $1 \pm b/W$ which multiplies the integrand in Eq. (1). However, on the basis of the most recent experimental evidence concerning the magnitude of b, this quantity is zero or at least negligible (p. 44, 66Sc35).

β-DECAY SHAPE FACTORS

The shape factors $S_n(Z,W)$ defined by Eq. (1) have the form (p. 187, 66Ko30; p. 260, 66Sc35; 65Bu02)

$$S_n(Z,W) = \sum_{k_e,k_\nu=1}^{\infty} [\lambda_k M_L^2 + \lambda_k' m_L^2 - 2\lambda_k'' M_L m_L] \quad (2)$$

where

$$\lambda_k(Z,W) = \frac{g_{-k}^2 + f_k^2}{2p^2}\left[\frac{(2k-1)!!}{(pR)^{k-1}}\right]^2 \qquad (3)$$

and λ', λ'' are other bilinear combinations of the f's and g's. The λ' and λ'' appear only in expressions for non-unique forbidden decays and not in the expressions for allowed and unique forbidden decays encountered in this work. The $M_L(k_e,k_\nu,Z,W)$ and $m_L(k_e,k_\nu,Z,W)$ are combinations of energy-dependent terms with unknown nuclear matrix elements as coefficients: $g_k(Z,W)$ and $f_k(Z,W)$ are the radial components of the continuum electron or positron wavefunction. The index L denotes the orbital angular momentum carried off by the β-particle and neutrino. The indices $k_e(\equiv k)$ and k_ν, referring to the β-particle and neutrino, respectively, are defined by $k = |\kappa|$ where

$$\begin{aligned} \kappa &= l \qquad \text{for } j = l - \tfrac{1}{2} \\ \kappa &= -l-1 \quad \text{for } j = l + \tfrac{1}{2}. \end{aligned} \qquad (4)$$

The functions f and g appearing in Eq. (3) should ideally be evaluated at the location of the transforming nucleon. However, this location depends on the nuclear radial wavefunctions which are not generally known. In the present work, f and g are evaluated at the nuclear radius R [Eq. (30)]. An alternative approach is proposed by Stech and Schulke (64St27), Schulke (64Sc28), Schopper (66Sc35), Behrens and Bühring (68Be03), and Behrens and Janecke (69Be42) in which f and g are evaluated at the origin.

In the case of allowed and unique forbidden transitions, the energy dependence of Eq. (2) can be factored out so that the energy integration in Eq. (1) can be performed.

For allowed and unique forbidden transitions, Eq. (2) reduces to (p. 186, 204, 66Ko30)

$$\begin{aligned} S_n(Z,W) = {} & 4\pi C_A^2 R^{2n} \langle \sigma \cdot T_{n+1}^n \rangle^2 \frac{(2n+1)!}{[(2n+1)!!]^2} \\ & \times \sum_{k=1}^{n+1} \frac{\lambda_k(Z,W) p^{2(k-1)} (W_0 - W)^{2(n-k+1)}}{(2k-1)![2(n-k+1)+1]!} \\ & + 4\pi C_V^2 \langle Y_0 \rangle^2 \lambda_1(Z,W) \delta_{n,0}. \end{aligned} \qquad (5)$$

In particular, for allowed transitions

$$\begin{aligned} S_0(Z,W) = {} & 4\pi C_A^2 \langle \sigma \cdot T_1^0 \rangle^2 \lambda_1(Z,W) \\ & + 4\pi C_V^2 \langle Y_0 \rangle^2 \lambda_1 \end{aligned} \qquad (5a)$$

and for first-forbidden unique transitions

$$\begin{aligned} S_1(Z,W) = {} & \frac{4\pi R^2}{9} C_A^2 \langle \sigma \cdot T_2^1 \rangle^2 \\ & \times [\lambda_1 (W_0 - W)^2 + \lambda_2 p^2]. \end{aligned} \qquad (5b)$$

The infinite sum over k_e, k_ν in Eq. (2) has been restricted, in Eqs. (5), by the condition $k_e + k_\nu = n + 2$. This restriction follows from the fact that the matrix elements M_L and m_L in Eq. (2) contain the product $(PR)^{k_e-1}(qR)^{k_\nu-1}$. Since $PR \ll 1$, $qR \ll 1$, only the lowest possible values of $k_e + k_\nu$ will be of importance. For an allowed transition or a unique transition of forbiddenness n, the minimum value of $k_e + k_\nu$ is just $n + 2$ (p. 265, 66Sc35; p. 197, 66Ko30).

In the above expressions, the β-moments $\langle \sigma \cdot T_{n+1}^n \rangle^2$ and $\langle Y_0 \rangle^2$ are reduced matrix elements defined by (p. 179, 66Ko30)

$$\langle \sigma \cdot T_J^L \rangle^2 = \sum |\langle J_f M_f | \sum_a \tau_\pm^a [\sigma^a \cdot T_{JM}^L] | J_i M_i \rangle|^2$$

$$\langle Y_J \rangle^2 = \sum |J_f M_f| \sum_a \tau_\pm^a [Y_{JM}(\hat{\mathbf{r}}^a)] | J_i M_i \rangle|^2$$

where $\tau_\pm$ is the isospin raising and lowering operator

$$\tau_+ |\text{neutron}\rangle = |\text{proton}\rangle, \qquad \tau_- |\text{proton}\rangle = |\text{neutron}\rangle$$

and the sum is taken over all nucleons in the nucleus.

Note that $4\pi \langle Y_0 \rangle^2 = \langle 1 \rangle^2$, where $\langle 1 \rangle^2$ is the usual definition of the Fermi matrix element.

A slightly different definition of the β-moment has been introduced by Warburton et al. (68Wa18, 70Wa11). In terms of the moments defined above, Warburton introduces

$$\langle G_n \rangle^2 = \frac{4\pi}{2n+1} \langle R^n \sigma \cdot T_{n+1} \rangle^2. \qquad (6)$$

In terms of the $\langle G_n \rangle^2$, the shape factors for allowed and unique forbidden decays can be written

$$\begin{aligned} S_n = {} & \frac{(2n+1)(2n+1)!}{[(2n+1)!!]^2} C_A^2 \langle G_n \rangle^2 \\ & \times \sum_{k=1}^{n+1} \frac{\lambda_k(Z,W) p^{2(k-1)} (W_0 - W)^{2(n-k+1)}}{(2k-1)![2(n-k+1)+1]!} \\ & + C_V^2 \langle 1 \rangle^2 \lambda_1(Z,W) \delta_{n,0} \end{aligned} \qquad (7)$$

from which

$$S_0 = \lambda_1 [C_A^2 \langle G_0 \rangle^2 + C_V^2 \langle 1 \rangle^2] \qquad (7a)$$

and

$$S_1 = \frac{1}{3} C_A^2 \langle G_1 \rangle^2 [\lambda_1 (W_0 - W)^2 + \lambda_2 p^2]. \qquad (7b)$$

In the Cartesian coordinate system the shape factors for allowed and first-forbidden unique transitions have the form

$$S_0 = \lambda_1 [C_A^2 |\int \sigma|^2 + C_V^2 |\int 1|^2 \qquad (8)$$

$$S_1 = C_A^2 \frac{(\int Bij)^2}{12} [\lambda_1 (W_0 - W)^2 + \lambda_2 p^2]. \qquad (9)$$

LOG-*ft*

The partial half-life of an n^{th}-forbidden β-branch is given by

$$t = \frac{T_{1/2}}{I} = \frac{\ln 2}{\lambda_n} \tag{10}$$

where λ is defined by Eq. (1), $T_{1/2}$ is the half-life of the parent nucleus, and I is the intensity of the branch per decay of the parent nucleus. We define the "reduced" or "comparative" half-life as (p. 278, 66Sc35)

$$f_n t = \frac{2\pi^3 \ln 2}{g^2 \eta^2} \tag{11}$$

where

$$f_n = \int p W (W_0 - W)^2 \frac{S_n(Z,W)}{\eta^2} dW \tag{12}$$

and η^2 is a combination of matrix elements.

For allowed transitions, η^2 is taken to be S_0/λ_1 as defined by Eq. (7a) so that

$$f_0 t = \frac{2\pi^3 \ln 2}{g^2 [C_A^2 \langle G_0 \rangle^2 + C_V^2 \langle 1 \rangle^2]} \tag{13}$$

with

$$f_0 = \int p W (W_0 - W)^2 \lambda_1(Z,W) dW. \tag{14}$$

For unique forbidden transitions, following Konopinski (p. 204, 66Ko30) and Warburton (68Wa18), η^2 is taken to be [from Eq. (7)]

$$\frac{(2n+1)}{[(2n+1)!!]^2} C_A^2 \langle G_n \rangle^2 \tag{15}$$

so that

$$f_n t = \frac{2\pi^3 \ln 2 [(2n+1)!!]^2}{g^2 (2n+1) C_A^2 \langle G_n \rangle^2} \tag{16}$$

and

$$f_n = (2n+1)! \int p W (W_0 - W)^2 \times \sum_{k=1}^{n+1} \frac{\lambda_k p^{2(k-1)} (W_0 - W)^{2(n-k+1)}}{(2k-1)![2(n-k+1)+1]!} dW. \tag{17}$$

Thus, for first-forbidden unique transitions,

$$f_1 t = \frac{6\pi^3 \ln 2}{g^2 C_A^2 \langle G_1 \rangle^2} \tag{18}$$

with

$$f_1 = \int p W (W_0 - W)^2 [\lambda_1 (W_0 - W)^2 + \lambda_2 p^2] dW. \tag{19}$$

It is customary to quote *ft*-values in seconds rather than in the natural units used above. If we adopt $gC_V = 1.406 \times 10^{-49}$ erg-cm^3 (p. 52, 66Sc35), $|C_A/C_V| = 1.24$ (71Ch45), and note that, from Eq. (6), $\langle G_n \rangle$ has the dimensions of (length)^n, then Eq. (16) reduces to

$$f_n t = \frac{4043 \lambda\!\!\!^{-2n} [(2n+1)!!]^2}{(2n+1) \langle G_n \rangle^2} \text{ sec}$$

where $\lambda\!\!\!^{-}$, the Compton wavelength of the electron, has the value 3.861×10^{-11} cm.

Because of the large spread in experimental *ft*-values, it is customary to consider the quantity log-*ft* rather than *ft* itself.

In many older works in which the Cartesian coordinate system was adopted, Eqs. (18) and (19) were written

$$f_1 t = \frac{2\pi^3 \ln 2}{g^2 C_A^2 (\int B_{ij})^2} \tag{20}$$

$$f_1 = \frac{1}{12} \int p W (W_0 - W)^2 [\lambda_1 (W_0 - W)^2 + \lambda_2 p^2] dW, \tag{21}$$

respectively. Note that the log-f_1 values given in the present tables are larger than those of Eq. (21) by the amount $\log 12 = 1.079$.

CONTINUUM RADIAL WAVEFUNCTIONS

As shown by Eqs. (2), (3), and (12), calculation of the quantity f_n requires knowledge of the radial components of the continuum electron and positron wavefunctions, f_κ, g_κ. For a central potential $V(r)$, these components are solutions of the coupled set of differential equations (p. 159, 61Ro33),

$$\left[\frac{d}{dr} + \frac{(\kappa+1)}{r}\right] g_\kappa - [W + 1 \pm V(r)] f_\kappa = 0$$
$$\left[\frac{d}{dr} - \frac{(\kappa-1)}{r}\right] f_\kappa + [W - 1 \pm V(r)] g_\kappa = 0. \tag{22}$$

where κ is defined by Eq. (4). The upper sign for $V(r)$ applies for positrons and the lower sign for electrons. Ideally, $V(r)$ should include the effects of the extended nuclear charge distribution and of the screening by orbital electrons. However, no solution of Eq. (22) with such a potential is known (65Bu02). In the present work, solutions of Eq. (22) for a point-nucleus unscreened Coulomb field are used. Finite nuclear size and screening effects are treated as corrections.

Point-Nucleus Unscreened Radial Wavefunctions

With $V(r)$ given by $-Ze^2/r$, Eq. (22) has the solution (p. 194, 61Ro33)

$$\begin{Bmatrix} f_\kappa \\ g_\kappa \end{Bmatrix} = \frac{(1 \mp W)^{1/2} (2pr)^{\gamma_\kappa} e^{\pi\nu/2} |\Gamma(\gamma_\kappa + i\nu)|}{2rW^{1/2}\Gamma(2\gamma_\kappa + 1)} \times [e^{-ipr+i\eta} (\gamma_\kappa + i\nu) \times F(\gamma_\kappa + 1 + i\nu, 2\gamma_\kappa + 1; 2ipr) \mp c.c.] \tag{23}$$

where

$\gamma_\kappa = (\kappa^2 - \alpha^2 Z^2)^{1/2}$

$y = \pm \alpha ZW/p$ $\quad > 0$ for negatron emission, < 0 for positron emission

α = fine structure constant = 1/137.0360

$$e^{2i\eta} = \frac{-(\kappa - iy/W)}{\gamma_\kappa + iy}$$

Γ = gamma function

F = confluent hypergeometric function

$c.c.$ = complex conjugate.

For β^--decay with small W, Eq. (23) has the form (for $Z \neq 0$)

$$\begin{Bmatrix} f_\kappa \\ g_\kappa \end{Bmatrix} = \frac{2\sqrt{2\pi}\, p^{1/2}(2r)^{\gamma_\kappa - 1}(\alpha Z)^{\gamma_\kappa - 1/2}}{\Gamma(2\gamma_\kappa + 1)} \begin{Bmatrix} \frac{\alpha^2 Z^2}{2}\left(1 - \frac{4\alpha Zr}{2\gamma_\kappa + 1}\right) \\ \left[-\kappa(\gamma_\kappa - \kappa) - \frac{\alpha^2 Z^2}{2}\right]\left[1 - \frac{4\alpha Zr}{2\gamma_\kappa + 1}\right] - \frac{2\alpha Zr(\gamma_\kappa - \kappa)}{2\gamma_\kappa + 1} \end{Bmatrix}^{1/2}. \tag{24}$$

Screening Correction

The effect on the β-spectrum of the screening of the nuclear Coulomb field by the electron cloud is treated here according to the method suggested by Rose (36Ro06). The screening approximation of Rose (see also 54G69) consists of the following replacements in Eq. (3)

$$\begin{aligned} g^2_{-k}(Z,W) &\longrightarrow \frac{pW'}{p'W} g^2_{-k}(Z,W') \\ f^2_k(Z,W) &\longrightarrow \frac{pW'}{p'W} f^2_k(Z,W') \end{aligned} \tag{25}$$

where $W' = W + V_0$ for β^+-decay, $W' = W - V_0$ for β^--decay, $p' = \sqrt{(W')^2 - 1}$, and $V_0(Z)$ is the difference between the point-nucleus Coulomb potential energy and the exact potential energy of the emitted β-particle evaluated at the nuclear surface R. Equation 3 then becomes

$$\begin{aligned} \lambda_k(Z,W) &= \left(\frac{p'W'}{pW}\right)\left[\frac{g^2_{-k}(Z,W') + f^2_k(Z,W')}{2p'^2}\right] \\ &\times \left[\frac{(2k-1)!!}{(pR)^{k-1}}\right]^2 = \left(\frac{p'W'}{pW}\right)\left(\frac{p'}{p}\right)^{2k-2} \lambda_k(Z,W'). \end{aligned} \tag{26}$$

The substitutions indicated by Eq. (25) are valid in β^--decay only for $W > 1 + V_0$ since for $W < 1 + V_0$, p' is imaginary. However, for small p, as shown by Eq. (24), f^2 and g^2 are proportional to p so that $g^2(Z,W')/p'$ and $f^2(Z,W')/p'$ are independent of p'. In this work, the forms of f and g given in Eq. (24) are used for values of p corresponding to $W < 1 + V_0$. For β^+-decay, Eq. (24) should be multiplied by $\exp(-2\pi\alpha Z/p)$ so that for p corresponding to $W < 1 + V_0$, f and g are already ≈ 0.

From a study of screening corrections based on solutions to the Schrödinger and Klein-Gordon equations, Durand (64Du09) has shown that the WKB screening method should be correct for $p^2 \gg W \times V_0$.

A comparison of λ_1(screened)/λ_1(unscreened), calculated with the WKB approximation, with that calculated by Behrens and Janecke (69Be42) (see OTHER WORK), shows that for low-energy, high-Z positrons the WKB method greatly overestimates the screening correction (see also 66Ma57). For negatrons, the differences between the two methods of calculation are negligible ($\lesssim$0.1%).

In the present work in order to treat the screening for low-energy positrons, a p-dependent screening potential of the form

$$V_0(Z,p) = V_0(Z)\exp(-a/p - b/p^2) \tag{27}$$

is introduced. The parameters $a(Z)$ and $b(Z)$ are chosen so that the ratio λ_1(screened)/λ_1(unscreened) given by the present program reproduces the values given by (69Be42) for $p \geq 0.2\, mc^2$ [$E(\beta^+) \geq 0.010$ MeV]. The parameters a and b so determined are

$$\begin{aligned} a &= -0.102 + 0.238 \times 10^{-2}\, Z \\ &\quad + 0.101 \times 10^{-4} Z^2 - 0.111 \times 10^{-6}\, Z^3 \\ b &= 0.0156 - 0.360 \times 10^{-4} Z \\ &\quad - 0.383 \times 10^{-5}\, Z^2 + 0.242 \times 10^{-7}\, Z^3. \end{aligned}$$

In the present work, values of $V_0(Z)$ are taken from Garrett and Bhalla (67Ga16). A less extensive tabulation of V_0 vs. Z is given by Matese and Johnson (66Ma57). The two sets of values differ slightly at high-Z. The effect of this difference on log-f is shown in Table B.

To illustrate the effect of the screening correction, some values of log-f with and without screening are given in Table A.

Finite Nuclear Size Correction

The effect on the β-spectrum of the extended charge distribution of the nucleus has been estimated

by Rose and Holmes (51R33 and 57R21). These authors express their results in terms of correction factors which multiply the quantities λ, λ', and λ'' of Eq. (2). In the present work we need the correction to λ. We write

$$\lambda_k \longrightarrow \lambda_k(1 + \Delta\lambda_k) = \bar{\lambda}_k. \tag{28}$$

From graphs of $\Delta\lambda_k$ as a function of Z and W given by the above authors, the following approximate expressions for $\Delta\lambda_k$ have been obtained.

$$\begin{aligned}
\Delta\lambda_k &= 0(<1\%) && \text{for } k > 1,\\
&= 0(<1\%) && \text{for } Z \leq 50(\beta^-),\ Z \leq 80(\beta^+).\\
\Delta\lambda_1(\beta^-) &= (Z-50)[-25\times10^{-4} - 4\times10^{-6}W\times(Z-50)] && \text{for } Z > 50.\\
\Delta\lambda_1(\beta^+) &= (Z-80)[-17\times10^{-5}\times W + 6.3\times10^{-4}/W - 8.8\times10^{-3}/W^2] && \text{for } Z > 80.
\end{aligned} \tag{29}$$

Examples of the effect on log-f of the finite nuclear size correction are shown in Table A.

Nuclear Radius

In the present work, the following radius formula, due to Elton (61E13), was used.

$$\begin{aligned}
R &= (1.123\,A^{1/3} - 0.941\,A^{-1/3})\times10^{-13}\,\text{cm}\\
&= (0.002908\,A^{1/3} - 0.002437\,A^{-1/3})\hbar/mc.
\end{aligned} \tag{30}$$

To illustrate the small effect on log-f due to a change in nuclear radius, some log-f's calculated with $R = 1.0\,A^{1/3}\times10^{-13}$ cm and $1.5\,A^{1/3}\times10^{-13}$ are shown in Table A. In the main tables the mass number used for a given Z was obtained from the following functions, which fit most decays to within a few mass units.

$$A = 1.82 + 1.90\,Z + 0.01271\,Z^2 - 0.00006\,Z^3 \qquad \beta^-\text{-decay.}$$

$$A = -1.9 + 1.96\,Z + 0.0079\,Z^2 - 0.00002\,Z^3 \qquad \beta^+\text{-decay.}$$

The effect on log-f of a 10% change in A for fixed Z is shown in Table C.

Electromagnetic Corrections

Electromagnetic corrections to the decay rate and spectrum shape in β-decay have been discussed by several authors (see, for example, 68Ha38, 63Du14, and 66Fr15). Examples of this type of correction are the interaction of the emitted β-particle with the decaying nucleon and the emission of internal bremsstrahlung during the decay. Internal bremsstrahlung is also present in electron-capture decay. Since the electromagnetic corrections have been evaluated for only a few $0^+ \longrightarrow 0^+$ superallowed β-decays, this type of correction is not included in the present work.

DECAY-RATE FUNCTIONS FOR $\beta^\pm$-DECAY

Expressions for the decay-rate function f [defined by Eq. (12)] including the effects of electron screening and finite nuclear size can be obtained from the preceding sections.

For allowed $\beta^\pm$-decay, from Eqs. (3), (23), (25), (28), and (14), one obtains

$$f_0^\pm(Z,W_0) = \int_1^{W_0} p'W'(W_0 - W)^2\bar{\lambda}_1^\pm(Z,W')dW. \tag{31}$$

For first-forbidden unique $\beta^\pm$-decay, from Eqs. (3), (23), (25), (28), and (19), one obtains

$$\begin{aligned}
f_1^\pm(Z,W_0) = \int_1^{W_0} & p'W'(W_0 - W)^2\\
&\times[(W_0 - W)^2\bar{\lambda}_1^\pm(Z,W')\\
&+ p'^2\lambda_2(Z,W')]dW.
\end{aligned} \tag{32}$$

All quantities in Eq. (23), that is, gamma functions, hypergeometric functions, etc., were computed to an accuracy of 1 part in 10^6.

DECAY-RATE FUNCTIONS FOR ELECTRON CAPTURE

The comparative half-life for electron capture can be written in a form analogous to that for β-decay [Eqs. (13) and (18)]. For an allowed transition (60Bo38)

$$f_0^\xi t = \frac{2\pi^3\ln 2}{g^2[C_V^2\langle 1\rangle^2 + C_A^2\langle G_0\rangle^2]}$$

with

$$\begin{aligned}
f_0^\xi = \frac{\pi}{2}\Big[& q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1} + q_{L_2}^2 f_{L_2}^2 B_{L_2}\\
&+ \sum_i q_i^2 g_i^2 + \sum_j q_j^2 f_j^2\Big].
\end{aligned} \tag{33}$$

For a first-forbidden unique transition,

$$f_1^\xi t = \frac{6\pi^3\ln 2}{g^2 C_A^2\langle G_1\rangle^2}$$

with

$$f_1^{\xi} = \frac{\pi}{2}\Big[q_K^4 g_K^2 B_K + q_{L_1}^4 g_{L_1}^2 B_{L_1} + q_{L_2}^4 g_{L_2}^2 B_{L_2} + \sum_i q_i^4 g_i^2 + \sum_j q_j^4 f_j^2 + \frac{9}{R^2}\Big(q_{L_3}^2 g_{L_3}^2 B_{L_3} + \sum_m q_m^2 g_m^2\Big)\Big]. \quad (34)$$

In the above expressions

$g(Z), f(Z)$ = radial components of the bound-state electron wavefunction evaluated at the nuclear surface

Z = atomic number of the parent nucleus

B_K, B_{L_1}, etc. = exchange and overlap corrections (63Ba21 and 63Ba72)

$q = Q(\epsilon) - E_B$ = neutrino energy. $Q(\epsilon)[= E(\epsilon)]$ is the atomic mass difference between initial and final states, and E_B is the electron binding energy in the daughter nucleus

$i = M_1, N_1, \ldots$

$j = M_2, N_2, \ldots$

$m = M_3, N_3, \ldots$

The indices K, L_1, etc., denote the atomic subshell from which the electron is captured.

The bound-state radial wavefunctions and the correction factors B_K, B_{L_1}, etc., were obtained by use of the relativistic self-consistent-field computer code developed at the Oak Ridge National Laboratory (66Ne10, 68Tu03, 69Tu02, and 71Lu14). The wavefunctions calculated by this code are solutions of the Dirac equation with a Hartree self-consistent potential. Electron exchange is included in the Slater approximation (51S97). The nuclear potential adopted is that appropriate to a finite-size nucleus with charge distribution

$$\rho(r) = \rho_0\left(1 + exp\frac{r - R}{a}\right)^{-1},$$

where R is the "half-density" radius and $a = S/4$ (ln 3) where S is the surface thickness. The parameters R and S, as taken from Elton (61El3), are

$$R = (0.002908\, A^{1/3} - 0.002437\, A^{-1/3})\ \hbar/mc$$
$$= (1.123\, A^{1/3} - 0.941\, A^{-1/3}) \times 10^{-13}\ \text{cm}$$

and

$$S = 0.0065\ \hbar/mc = 2.5 \times 10^{-13}\ \text{cm} \quad \text{for } A \geq 16$$
$$= 0.0040\ \hbar/mc = 1.9 \times 10^{-13}\ \text{cm} \quad \text{for } A < 16.$$

Values of the K-shell density, the density ratios L_1/K, L_2/L_1, L_3/L_1, and $(M_1 + M_2 + N_1 + N_2 + \ldots)/L_1$, as well as the correction factors, B_K, B_{L_1}, etc., have been tabulated in Appendix III of "Radioactive Atoms" (70MaBl).

OTHER WORK

$\beta^{\pm}$-Decay

The functions λ_k [defined by Eq. (3)] appearing in the definition of the decay-rate functions, f_0 and f_1, can be written

$$\lambda_k = F_0 L_{k-1}\left[\frac{(2k-1)!!}{p^{k-1}}\right]^2$$

where

$$F_0(Z,W) = 4(2pR)^{-2(1-\gamma_1)} e^{\pi\nu}\, \frac{|\Gamma(\gamma_1 + iy)|^2}{[\Gamma(2\gamma_1 + 1)]^2}$$

and

$$L_{k-1} = \frac{(g_{-k}^2 + f_k^2)}{2p^2 F_0 R^{2(k-1)}}.$$

In particular, the Fermi function is given by

$$\lambda_1 = F = F_0 L_0.$$

The quantities L_{k-1} were originally introduced by Konopinski and Uhlenbeck (41K09; see also 66Ko30). The functions F, F_0, and L_{k-1} have been tabulated by a number of authors. However, with the exception of the Behrens and Janecke work mentioned below, no extensive tabulation exists in which the effects of both finite size and screening are adequately treated.

Values of $F = F_0 L_0$ and $(p/W) \times F_0 L_0$ for an unscreened point-nucleus have been tabulated in the NBS Tables (52F39) and by Dismuke et al. (52Di36), respectively. Corrections to these values due to screening and finite nuclear size can be applied in the manner discussed in the present work. In the above tabulations, the radial wavefunctions f_1 and g_{-1} were replaced by their leading terms in an expansion in powers of pr. In this approximation the function L_0 reduces to the simple form

$$L_0 = \frac{1 + \gamma_1}{2}$$

where

$$\gamma_1 = \sqrt{1 - \alpha^2 Z^2}.$$

This additional approximation can be corrected for by use of the finite de Broglie wavelength correction factors calculated by Rose and Perry (53R36) and by Morita (59M127). In addition to the above tabulations, exact values of L_0 and L_1 (exact in that higher powers of pr are included) as well as other bilinear combinations of the f's and g's have been calculated by Rose, Perry, and Dismuke (53R37).

Dzhelepov and Zyrianova (56Dz63) calculated values of F_0, L_0, and L_1 which include corrections for

finite size and for screening. The screening correction employed was that of Reitz (50Re16), obtained by numerical integration of the Dirac equation. The results of Reitz, however, have since been shown to be in error for medium and high β-momenta (66Ma57, 64Du09, and 64Br39). Tables of F_0 and of f_1, g_{-1}, f_2, g_{-2}, which include the effects of finite size but not of screening have been published by Bhalla and Rose (60Bh05, 61Bh02, and 64Bh02). The values of these authors, however, are apparently in error for positrons with high momenta (68Be03 and 67Hu10).

An exact numerical integration of the Dirac equation in which both screening and finite size were taken into account has been performed by Bühring (65Bu02). For a few Z-values, Bühring tabulates F_0, L_0, and L_1 as well as certain other bilinear combinations of radial densities. Using a method similar to that of Bühring, Behrens and Janecke (69Be42) have published an extensive tabulation of values of F_0, L_0, and L_1 as well as graphs of the integrated decay-rate function f. These authors (Behrens and Janecke), however, define the β-decay shape factors in terms of electron radial densities evaluated at the origin rather than at the nuclear radius. Thus, their tables are not directly comparable to other published works, including the present one. Values given by 69Be42 of the quantity λ_1(screened)/λ_1 (unscreened), however, differ by less than a few percent from values given by the conventional formalism (65Bu02 and 66Ma57) so that the screening correction of 69Be42 could be used to modify the low-p, high-Z positron screening correction used in the present work. (See text under Screening Correction.)

ϵ-Decay

Prior to 1968, the most widely used tabulations of the electron densities needed for the calculation of f^{ϵ} [Eqs. (23) and (24)] were those of Band et al. (56B168 and 58B11) and of Brysk and Rose (55B43 and 58B13).

The calculations of Band et al. were based on point-nucleus wavefunctions for the Thomas-Fermi-Dirac statistical atomic model with screening constants taken from Metropolis and Reitz (51M79) and from Thomas (54T40). The calculations of Brysk and Rose were based on relativistic point-nucleus Coulomb radial wavefunctions corrected for screening on the basis of the statistical atomic model. In both these calculations, corrections were made for the effects of finite nuclear size.

Several other calculations of the electron radial wavefunctions have recently become available. Suslov (68Su08) has obtained K- and L-shell densities based on calculations with the nonrelativistic self-consistent-field potential of Herman and Skillman (63He12) (for $Z < 72$) and a similar relativistic potential of Liberman et al. (65Li14) ($72 \leq Z \leq 90$). Electron exchange is included in the Slater approximation (51S97). Suslov incorporates finite nuclear size effects by treating the nucleus as a uniformly charged sphere with a sharp surface.

Behrens and Janecke (69Be42) have calculated K-, L-, and M-shell densities by solving the Dirac equation for the bound electrons with Hartree-Fock potential ($Z \leq 36$) and Thomas-Fermi-Dirac potentials ($Z > 36$). Electron exchange and finite nuclear size effects are included in the same manner as by Suslov.

ϵ/β^+-Values

Early calculations of ϵ_K/β^+-values were carried out by Feenberg and Trigg (50Fe20), Zweifel (49Zw04), and Dzhelepov and Zyrianova (56Dz63). Feenberg and Trigg calculated values of ϵ_K/β^+ for a pure Coulomb field. Zweifel's calculation included screening corrections as given by Reitz (49Re18). Neither set of calculations included any correction for finite nuclear size. The results of Zweifel have also been presented in tabular and graphical form in *Nuclear Spectroscopy Tables* by Wapstra et al. (59W11) and *Table of Isotopes* by Lederer et al. (67LeHo). The calculations of Dzhelepov and Zyrianova were based on their β-decay f-functions and the electron densities of Band.

A more recent calculation of ϵ_K/β^+-values has been published by Zyrianova and Suslov (68Zy01). These calculations incorporate the bound-state wavefunctions of Suslov described above. No details are given concerning the positron part of the calculation.

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AUXILIARY TABLES

Table A. Effect on Log-f of Screening, Finite Nuclear Size, and Choice of Nuclear Radius

Transition			β^--Decay Log-f_0^-				
Z	A	$E(\beta^-)$	From Present Program[a]	No Screening[b]	No Finite Size[b]	$R = 1.0A^{1/3}$ $\times 10^{-13}$ cm[b]	$R = 1.5A^{1/3}$ $\times 10^{-13}$ cm[b]
10	20	0.05	−3.755	−3.750	−3.755	−3.755	−3.756
		0.50	−0.366	−0.364	−0.366	−0.366	−0.367
		5.00	3.776	3.777	3.776	3.776	3.774
50	120	0.05	−2.776	−2.765	−2.776	−2.771	−2.797
		0.50	0.389	0.399	0.389	0.395	0.368
		5.00	4.304	4.306	4.304	4.310	4.276
90	230	0.05	−1.857	−1.836	−1.809	−1.837	−1.929
		0.50	1.269	1.286	1.319	1.290	1.197
		5.00	4.929	4.934	4.990	4.954	4.843
			β^--Decay Log-(f_1^-/f_0^-)				
10	20	0.05	−1.208	−1.187	−1.208	−1.207	−1.207
		0.50	0.109	0.110	0.109	0.108	0.109
		5.00	1.812	1.812	1.812	1.812	1.813
50	120	0.05	−0.918	−0.854	−0.918	−0.919	−0.911
		0.50	0.063	0.074	0.063	0.061	0.069
		5.00	1.798	1.799	1.798	1.796	1.805
90	230	0.05	−0.780	−0.699	−0.795	−0.785	−0.768
		0.50	0.011	0.031	−0.012	0.006	0.031
		5.00	1.800	1.803	1.776	1.795	1.821

[a] Includes screening, finite nuclear size, and Elton nuclear radius parameters as described in the text
[b] From present program except for change indicated in heading

AUXILIARY TABLES

Table A. Continued

Transition			β^+-Decay Log-f_0^+				
Z	A	$E(\beta^+)$	From Present Program[a]	No Screening[b]	No Finite Size[b]	$R = 1.0A^{1/3}$ $\times 10^{-13}$ cm[b]	$R = 1.5A^{1/3}$ $\times 10^{-13}$ cm[b]
10	20	0.05	−4.643	−4.671	−4.643	−4.643	−4.644
		0.50	−0.700	−0.703	−0.700	−0.700	−0.701
		5.00	3.575	3.574	3.575	3.575	3.575
50	120	0.05	−6.117	−6.447	−6.117	−6.112	−6.134
		0.50	−1.152	−1.181	−1.152	−1.148	−1.169
		5.00	3.319	3.316	3.319	3.323	3.307
90	230	0.05	−7.088	−7.874	−7.057	−7.070	−7.150
		0.50	−1.330	−1.396	−1.313	−1.311	−1.390
		5.00	3.236	3.229	3.241	3.251	3.187
			β^+-Decay Log-(f_1^+/f_0^+)				
10	20	0.05	−1.028	−1.038	−1.028	−1.028	−1.027
		0.50	0.130	0.129	0.130	0.130	0.130
		5.00	1.810	1.810	1.810	1.810	1.810
50	120	0.05	−0.694	−0.724	−0.694	−0.696	−0.685
		0.50	0.129	0.120	0.129	0.127	0.135
		5.00	1.783	1.781	1.783	1.782	1.785
90	230	0.05	−0.614	−0.646	−0.645	−0.624	−0.580
		0.50	0.040	0.023	0.028	0.032	0.065
		5.00.	1.735	1.733	1.733	1.732	1.745

[a] Includes screening, finite nuclear size, and Elton nuclear radius parameters as described in the text
[b] From present program except for change indicated in heading

AUXILIARY TABLES

Table B. Effect on Log-f of Choice of Screening Potential

β^--Decay

Transition			Log-f_0^-		Log-(f_1^-/f_0^-)	
Z	A	$E(\beta^-)$	V_0 $(MJ)^a$	V_0 $(GB)^b$	V_0 $(MJ)^a$	V_0 $(GB)^b$
70	170	0.05	−2.348	−2.347	−0.800	−0.800
		0.50	0.798	0.799	0.043	0.045
		5.00	4.600	4.600	1.797	1.798
80	200	0.05	−2.115	−2.112	−0.777	−0.776
		0.50	1.022	1.024	0.028	0.031
		5.00	4.759	4.760	1.798	1.799
90	230	0.05	−1.861	−1.857	−0.780	−0.780
		0.50	1.266	1.269	0.008	0.011
		5.00	4.928	4.929	1.800	1.800

β^+-Decay

Transition			Log-(f_0^ϵ/f_0^+)		Log-$[(f_1^\epsilon/f_1^+)/(f_0^\epsilon/f_0^+)]$	
Z	A	$E(\beta^+)$	V_0 $(MJ)^a$	V_0 $(GB)^b$	V_0 $(MJ)^a$	V_0 $(GB)^b$
70	170	0.05	7.786	7.826	1.230	1.233
		0.50	2.758	2.762	0.822	0.824
		5.00	−0.551	−0.551	0.375	0.375
80	200	0.05	8.286	8.364	1.211	1.216
		0.50	3.108	3.116	0.840	0.842
		5.00	−0.207	−0.206	0.384	0.385
90	230	0.05	8.768	8.894	1.179	1.186
		0.50	3.450	3.463	0.854	0.857
		5.00	0.131	0.132	0.393	0.394

[a]Matese, Johnson screening — V_0 = 13.4, 16.9, 20.7 at Z = 70, 80, 90
[b]Garrett, Bhalla screening used in present program — V_0 = 12.3, 14.8, 17.3 at Z = 70, 80, 90

AUXILIARY TABLES

Table C. Effect on Log-f of Variation with A for Fixed Z

β^--Decay

Transition			Log-f_0^-			Log-(f_1^-/f_0^-)		
Z	A	$E(\beta^-)$	$A-10\%$	A	$A+10\%$	$A-10\%$	A	$A+10\%$
10	20	0.05	−3.754	−3.755	−3.755	−1.209	−1.209	−1.209
		0.50	−0.365	−0.366	−0.366	0.108	0.108	0.108
		5.00	3.776	3.776	3.776	1.812	1.812	1.812
50	121	0.05	−2.774	−2.776	−2.778	−0.918	−0.917	−0.917
		0.50	0.392	0.389	0.387	0.062	0.063	0.064
		5.00	4.306	4.303	4.301	1.797	1.798	1.799
90	232	0.05	−1.850	−1.858	−1.865	−2.632	−2.639	−2.645
		0.50	1.277	1.268	1.261	0.009	0.012	0.014
		5.00	4.938	4.928	4.920	1.798	1.801	1.803

β^+-Decay

Transition			Log-f_0^+			Log-(f_1^+/f_0^+)		
Z	A	$E(\beta^+)$	$A-10\%$	A	$A+10\%$	$A-10\%$	A	$A+10\%$
10	18	0.05	−4.643	−4.643	−4.643	−1.028	−1.028	−1.028
		0.50	−0.700	−0.700	−0.700	.130	.130	.130
		5.00	3.575	3.575	3.575	1.810	1.810	1.810
50	113	0.05	−6.114	−6.116	−6.117	− .696	− .695	−0.694
		0.50	−1.149	−1.151	−1.153	.127	.128	.129
		5.00	3.322	3.320	3.319	1.782	1.782	1.783
90	224	0.05	−7.079	−7.087	−7.093	− .619	− .615	− .611
		0.50	−1.321	−1.328	−1.334	.036	.039	.042
		5.00	3.242	3.237	3.232	1.734	1.735	1.736