A2

Friday, April 22, 2022 3:19 PM

1. (6 points) The time in days between breakdowns of a machine is exponentially distributed with $\lambda = 0.25$.
(a) What is the median time between machine breakdowns?
(b) What is the probability that after the machine is repaired it lasts at least a week before failing again?
(c) If the machine has performed satisfactorily for six days, what is the probability that it lasts at least three more days before breaking down?

(a)

we know

$P(x) = 1 - e^{-\lambda t}$ [for median $P(x) = 0.5$]

$\Rightarrow 0.5 = 1 - e^{-\lambda t}$

$\Rightarrow -0.5 = -e^{-\lambda t}$

$\Rightarrow \ln(0.5) = -\lambda t$

$\Rightarrow -0.6931 = -\lambda t$ [$\lambda = 0.25$]

$\Rightarrow t = \frac{0.6931}{0.25} = 2.772$ Days (Ans)

(b)

We know,

$P(x) = 1 - e^{-\lambda t}$

$P(7) = 1 - e^{-0.25 \times 7}$

$= 1 - 0.17377$

$= 0.8262$ (Ans)

(c)

As mechin broke own The
X=2 for next 2 days.
We know,

$P(x) = 1 - e^{\lambda t}$

$P(2) = 1 - e^{-0.25 \times 2}$

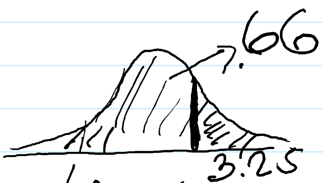
$= 0.606$

2. (10 points) The thicknesses of glass sheets produced by a certain process are normally distributed witha mean of $\mu = 3.20$ mm and a standard deviation of $\sigma = 0.12$ mm.
(a) What is the probability that a glasssheet is thicker than 3.25 mm?
(b) What is the probability that a glasssheet is thinner than 2.75 mm?
(c) What is the value of c for which there is a 98% probability that a glass sheet has a thickness withinthe interval [3.00 — c, 3.00 + c]?
(d) What is the probability that four glass sheets placed one on top of another have a total thicknessgreater than 9.50 mm?
(e) What is the probability that eight glass sheets have an average thickness less than 3.10 mm?

$\sigma = 0.12mm$

$\mu = 3.20mm$

$P(3.25 < x) = P\left(\frac{3.25-3.20}{0.12} < \frac{x-\mu}{\sigma}\right)$



$= P(z \geq 0.4167)$

$(1-0.66095) = 0.33905$ (Ans)

(b)

$P(x < 2.75) = P\left(\frac{x-\mu}{\sigma} < \frac{2.75-3.2}{0.12}\right)$

$= P(z < -3.75)$

$\simeq 0.0001$ (Ans)

(c)

$c = \frac{\sigma}{\sqrt{n}} \cdot z_{\alpha/2}$

$= \frac{.12}{\sqrt{1}} \times z_{0.01}$

$1-\alpha = .98$

$\alpha = 0.02$

$\frac{\alpha}{2} = 0.01$

$= \frac{.12}{\sqrt{1}} \times 2.325$

$= 0.279$ (Ans)

(d)

$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$

0.12 ... 0.06

$\sqrt{n}$

$$= \frac{0.12}{\sqrt{4}} = 0.06$$

per sheet, $\bar{x} = \frac{9.50}{4} = 2.375\text{ mm}$

$P(2.375 < x)$

$P\left(\frac{2.375 - 3.2}{0.06} < \frac{x - \mu}{\sigma}\right)$

$P(-13.75 < z)$

$= 1$

(e)

$\bar{x} = 3.1\text{ mm}$

$$\sigma_{\bar{x}} = \frac{0.12}{\sqrt{8}}$$

$$= 0.0424$$

$$P(x < 3.10) = P\left(\frac{x - \mu}{\sigma} < \frac{3.10 - 3.20}{0.0424}\right)$$

$$= P(z < -2.36)$$

$$= 0.0091$$

3. (4 points) Suppose that X ~ N(μ, σ2) and that P(X ≤ 10) = .65 and P(X ≤ 0) = .45 Find the values of μ and σ2.

$P(x \leq 10) = 0.65$

$P\left(\frac{x-\mu}{\sigma} < \frac{10-\mu}{\sigma}\right) = 0.65$

$$P\left(\frac{x-\gamma}{6} < \frac{10-\gamma}{6}\right) = 0.65$$

$$P\left(Z < \frac{10-\gamma}{6}\right) = 0.65$$

$$\frac{10-\gamma}{6} = 0.385$$

$$10-\gamma = 0.385 \times 6$$

$$10 = 0.3856 \cdot 6 + \gamma \quad \text{— (1)}$$

$$P(x<0) = .45$$

$$P\left(\frac{x-\gamma}{6} < \frac{0-\gamma}{6}\right) = .45$$

$$\frac{-\gamma}{6} = -0.125$$

$$0 = 0.1256 - \gamma \quad \text{— (11)}$$

Solving (1) and (11)

$$6 = 19.585$$

$$\gamma = 2.4481$$

$$6^2 = 383.56$$

(4) The resistance in milliohms of 1 meter of copper cable at a certain temperature is normally distributed with mean μ = 24.8 and variance σ2 = 1.3. What is the interquartile range?

$$P\left(\frac{x-24.8}{\sqrt{1.3}} < z\right) = 0.25$$

$$\frac{x-24.8}{\sqrt{1.3}} = -.665$$

$$x = -0.758 + 24.8$$

$$x_{Q_1} = 24.041\cancel{8}$$

$$P\left(\frac{x-24.8}{\sqrt{1.3}} < z\right) = 0.75$$

$$P\left(\frac{x-24.8}{\sqrt{1.3}} < z\right) = 0.75$$

$$\frac{x-24.8}{\sqrt{1.3}} = 0.665$$

$$x_{Q_3} = 25.558$$

$$Q_3 - Q_1 = 25.558 - 24.0418$$
$$= 1.5164$$

Following data show percent shrinkage on drying of 30 clay specimens produced the following data: » 18.2, 20.8, 16.4, 16.6, 17.4, 19.3, 20.5, 21.2, 19.6, 18.0, 21.2, 19.4, 18.7, 24.0, 23.6, 18.5, 19.0, 20.4, 20.6, 20.8, 23.1, 15.4, 18.2, 17.6, 17.5, 19.3, 17.6, 21.4, 14.8, 15.8
(a) Compute sample mean, median, and standard deviation.
(b) Draw a boxplot.
(c) Group the data into class interval of size 2 percent. Draw a histogram from the resulting grouped data.

(a)

14.8, 15.4, 15.8, 16.4, 16.6, 17.4, 17.5, 17.6, 17.6, 18.0, 18.2,18.2, 18.5, 18.7, 19.0, 19.3, 19.3, 19.4, 19.6,20.4, 20.5, 20.6, 20.8, 20.8, 21.2, 21.2, 21.4, 23.1, 23.6, 24.0

$$\text{mean} = \frac{\sum_{i=0}^{n} x}{n}$$

$$= \frac{574.9}{30}$$

$$= 19.1633 \text{ (Ans)}$$

$$\text{median} = \frac{19 + 19.3}{2} = 19.15$$

$$\text{standard divion} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

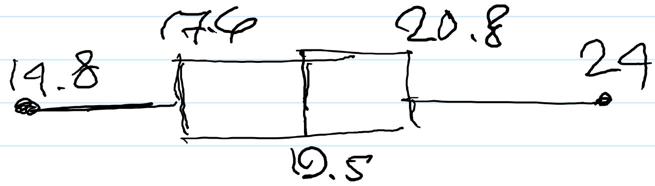
$$= \sqrt{\frac{156.269}{30}}$$

$$= \sqrt{5.21}$$

$= \sqrt{5.21}$

$= 2.2832$

(b)



(c)

class	
14.8 - 16.7	5
16.8 - 18.7	9
18.8 - 20.7	8
20.8 - 22.7	5
22.8 - 24.7	3

$\frac{24 - 14.8}{2} = 4.6$

≈ 5