# 2(C)

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①

(4 points) An experiment has five outcomes: I, II, III, IV, and V. If P(1) = 0.08, P(II) = 0.20,
P(III) =0.33, what are the possible values for the probability of outcome V? If outcomes IV and V
are equally likely, what are their probability values?

we know,

$$\sum_{w=0}^{n} P(w) = 1$$

given,

$$P(IV) = P(V)$$

let, $P(IV) = P(V) = w$

So, $P(I) + P(II) + P(III) + P(IV) + P(V) = 1$

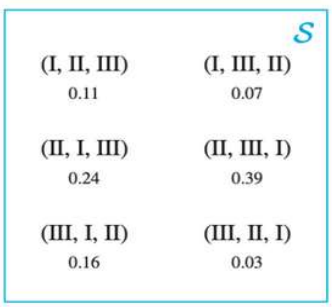
$$0.08 + 0.2 + 0.33 + w + w = 1$$

$$2w = 0.61$$

$$w = 30.5 \text{ (Ans)}$$

2

(4 points) Three types of batteries are being tested, type I, type II, and type III. The outcome (I, II, III) denotes that the battery of type I fails first, the battery of type II next, and the battery of type III lasts the longest. The probabilities of the six outcomes are given in the following figure.




What is the probability that
(a) the type I battery lasts longest?
(b) the type I battery lasts longer than the type II battery?
(c) A type I battery lasts longest conditional on it not failing first.
(d) A type I battery lasts longest conditional on a type II battery failing first

(a) Battery I lasting longest Probability

$$P(I) = P(II, III, I) + P(III, II, I)$$

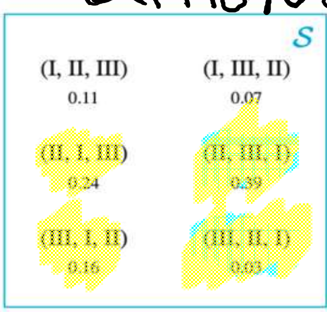
$$= 0.39 + 0.03$$

$$= 0.42$$

(b) Battery I lasts longer than type II

$$P(II) = P(II, I, III) + P(II, III, I) + P(III, II, I)$$
$$= 0.24 + 0.39 + 0.03$$
$$= 0.66$$

(c) I type battery last longest given condition on not failing first.


$S$

| | |
|---|---|
| (I, II, III) 0.11 | (I, III, II) 0.07 |
| (II, I, III) 0.24 | (II, III, I) 0.39 |
| (III, I, II) 0.16 | (III, II, I) 0.03 |

$$P(I_L) = P(II, III, I) + P(III, II, I)$$
$$= 0.39 + 0.03$$
$$= 0.42$$

$$P(I_f) = P(I, II, III) + P(I, III, II)$$
$$= 0.11 + 0.07 = 0.18$$

$$P(I_L \mid I_f) = \frac{P(I_L \cap I_f)}{P(I_f)}$$

$$P(I_f)$$

$$= \frac{P(II, III, I) + P(III, II, I)}{1 - P(I, II, III) - P(I, III, II)}$$

$$= \frac{0.39 + 0.03}{1 - 0.11 - 0.07}$$

$$= \frac{0.42}{0.82}$$

$$= 0.5121$$

(d)

| $S$ | |
|---|---|
| (I, II, III) 0.11 | (I, III, II) 0.07 |
| (II, I, III) 0.24 | (II, III, I) 0.39 |
| (III, I, II) 0.16 | (III, II, I) 0.03 |

$$P(I_L \mid II_f) = \frac{P(I_L \cap II_f)}{P(II_f)}$$

$$= \frac{P(II, III, I)}{P(II, III, I) + P(III, II, I)}$$

$$= \frac{0.39}{}$$

$$= \frac{0.39}{0.03 + 0.39}$$

$$= \frac{13}{14} = 0.928$$

3

3. (3 points) A car repair is either on time or late and either satisfactory or unsatisfactory. If a repair made on time, then there is a probability of 0.85 that it is satisfactory. There is a probability of 0.77 that a repair will be made on time. What is the probability that a repair is made on time and

is satisfactory?

$P(S|O) = 0.85$ = Satisfactory | on time

$P(O) = 0.77$ = on time

$P(S \cap O)$ = on time and satisfactory

we know,

$$P(S|O) = \frac{P(S \cap O)}{P(O)}$$

$$P(S \cap O) = P(S|O) \cdot P(O)$$

$$= 0.85 \times 0.77$$

$$= 0.6545$$

4

4. (4 points) Two fair dice are thrown, one red and one blue. Calculate:
(a) P(red die is 5 | sum of scores is 8)
(b) P(either die is 5 | sum of scores is 8)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |
| 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |
| 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

(a) P(red die 5 | sum 8)

$$P(R_5) = \frac{6}{36} = \frac{1}{6}$$

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |
| 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

$$P(S_8) = \frac{5}{36}$$

$$P(R_5 \cap S_8) = \frac{1}{36}$$

$$P(R_5 | S_8) = \frac{P(R_5 \cap S_8)}{P(S_8)} = \frac{\frac{1}{36}}{\frac{5}{36}}$$

$$= \frac{1}{5}$$

$$= 0.2$$

6

$$P(R_5) = \frac{6}{36} = \frac{1}{6}$$

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |

$$P((R_5 \cup S_8) \cap (B_5 \cup S_8)) = \frac{2}{36}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |
| 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

$$P((R_5 \cup S_8) \cap (B_5 \cup S_8)) = \frac{2}{36}$$

$$= \frac{1}{18}$$

$$P(B_5) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{either die } 5 \mid \text{sum } 8) = \frac{P((R_5 \cup S_8) \cap (B_5 \cup S_8))}{P(S_8)}$$

$$= \frac{\frac{1}{18}}{\frac{5}{36}} = \frac{2}{5}$$

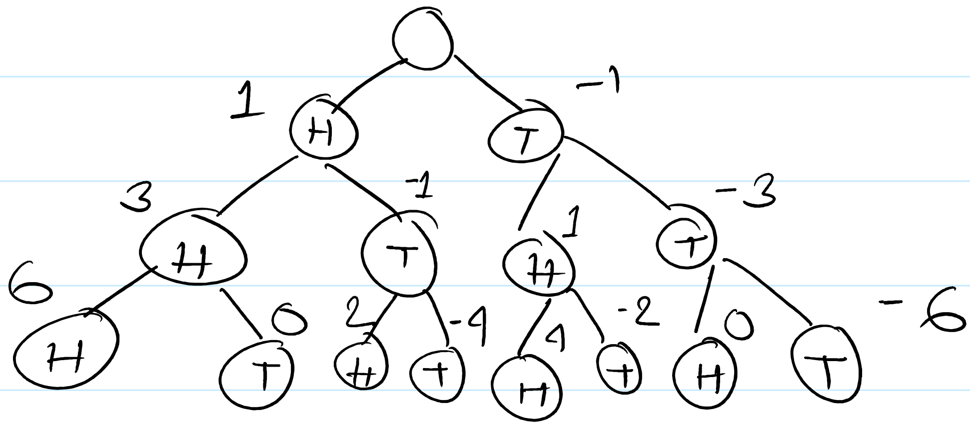
$$= 0.4$$

# 5

5. (4 points) A fair coin is tossed three times. A player wins \$1 if the first toss is a head, but loses
\$1 if the first toss is a tail. Similarly, the player wins \$2 if the second toss is a head, but loses \$2
if the second toss is a tail, and wins or loses S3 according to the result of the third toss. Let the
random variable X be the total winnings after the three tosses (possibly a negative value if losses are
incurred).
(a) Construct the probability mass function.

(b) Construct the cumulative distribution function.
(c) What is the most likely value of the random variable
(d) Plot the probability mass function and cumulative distribution function.

a b




| | $-6$ | $-4$ | $-2$ | $0$ | $2$ | $4$ | $6$ |
|---|---|---|---|---|---|---|---|
| $P(X=x)$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| $F(x)$ | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | $\frac{6}{8}$ | $\frac{7}{8}$ | $\frac{8}{8}$ |

(a) Probability mass function
(b) Cumulative distribution function

(c) as we can see $P(X=0)$ has

the highest probability mass function.

(d)

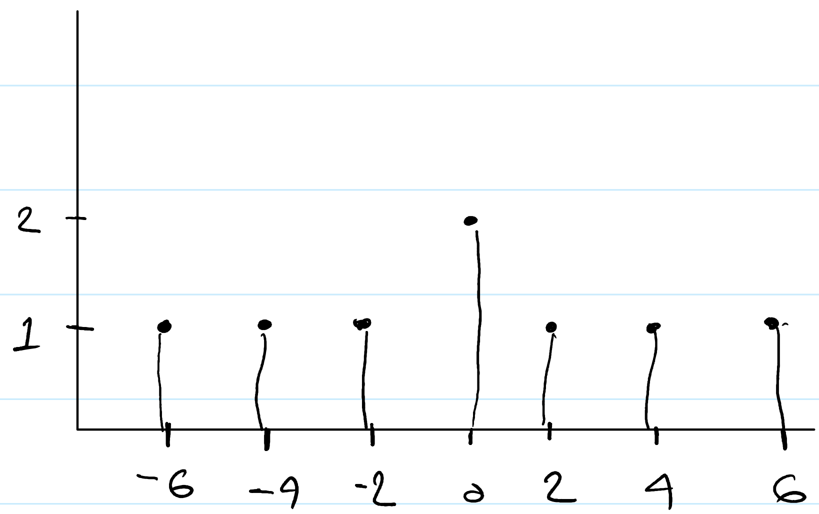



fig - Probabity mass function

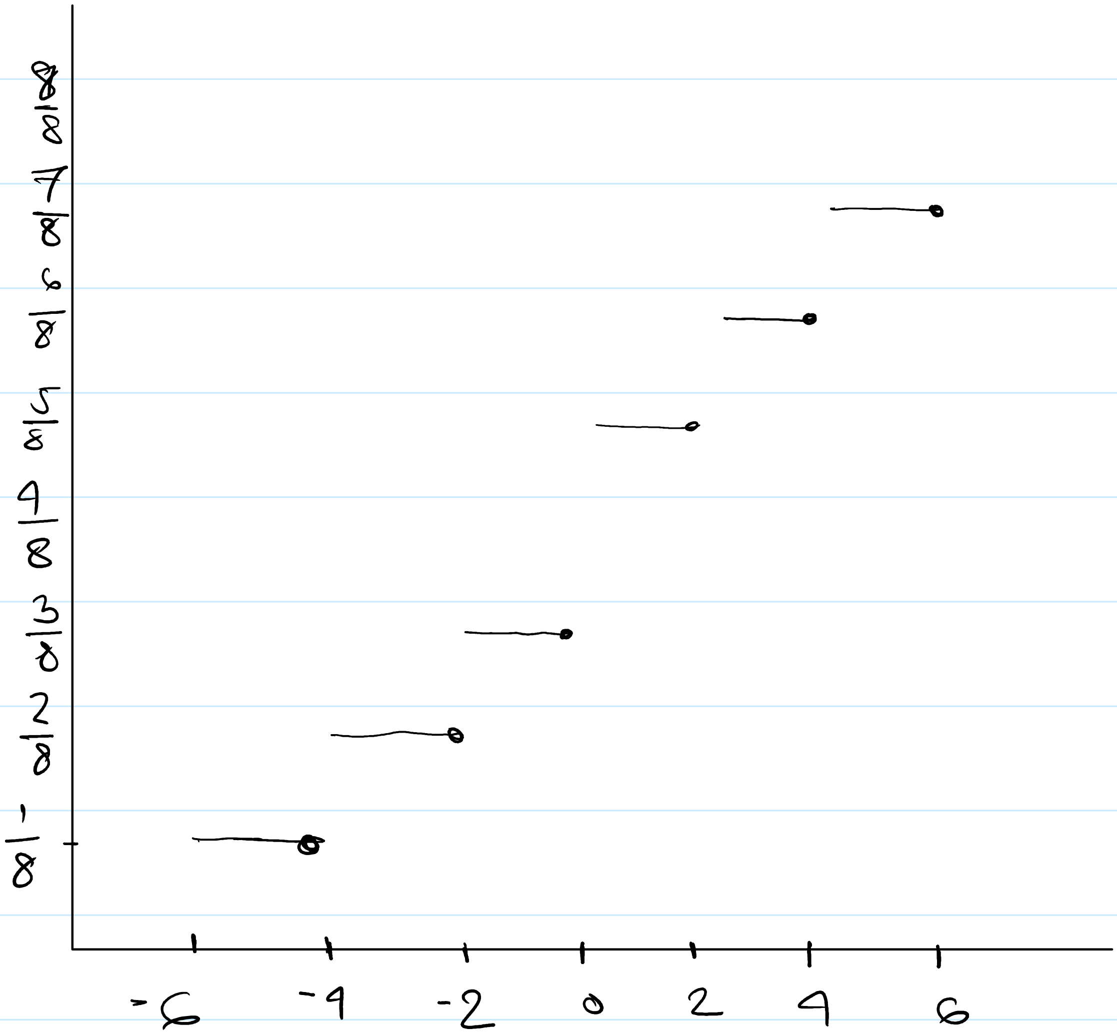


fig- Accumulative mass function

6

6. (5 points) The resistance X of an electrical component has a probability density function f(t) =

Ax(130-x^2) for resistance values in the range 10<r<11.
(a) Calculate the value of A.
(b) Calculate the cumulative distribution function.
(c) What is the probability that the electrical component has a resistance between 10.25 and 10.5?
(d) What is the median value of the resistance?
(e) What is the 80th percentile of the resistance?

(a) given

$$f(x) = Ax(130-x^2)$$

we know, Probability Denenety function

$$F(x) = \int_{10}^{11} Ax(130-x^2) = 1$$

$$= A\int_{10}^{11} x(130-x^2) = 1$$

$$\Rightarrow A\int_{10}^{11} 130x - x^3 = 1$$

$$\Rightarrow A\left[\frac{130x^2}{2} - \frac{x^4}{4}\right]_{10}^{11} = 1$$

4

$$\Rightarrow A = \frac{4}{819}$$

(b) the cumulative distribution function

$$F(x) = \int_{10}^{x} f(x)\, dy$$

$$= \int_{10}^{y} \frac{4x}{819} (130 - x^2)\, dy$$

$$= \frac{4}{819} \int_{10}^{x} 130x - x^3\, dy$$

$$= \frac{4}{819} \left[ 65x^2 - \frac{x^4}{4} \right]_{10}^{x} dx$$

$$= \frac{4}{819} \left( 65x^2 - \frac{x^4}{4} - 4000 \right)$$

$$= \frac{20}{63}w^2 - \frac{w^4}{819} - \frac{16000}{819}$$

(c) $$P(10.25 \leq w \leq 10.5) = \int_{10.25}^{10.5} \frac{4w(130-w^2)}{819}\,dy$$

$$= \frac{20}{63}(10.5)^2 - \frac{10.5^4}{819} - \frac{16000}{819} - \frac{20}{63}(10.25)^2$$

$$+ \frac{10.25^4}{819} + \frac{16000}{819}$$

$$= 0.28305$$

(d) $$0.5 = \frac{20}{63}w^2 - \frac{w^4}{819} - \frac{16000}{819}$$

or, $$\frac{w^4}{819} - \frac{20}{63}w^2 + \frac{16000}{819} + 0.5 = 0$$

or, $x = \pm 12.33$, $\pm 10.38$,

as $10 \leq x \leq 11$

median $X = 10.38$

(e) 80th percentile,

$$0.8 = \frac{x^4}{819} - \frac{20x^2}{63} + \frac{16000}{819}$$

$x = \pm 12.07$ $\pm 10.69$

as $10 \leq x \leq 11$

80th percentile, $x = 10.69$

7

7. (10 points) The random variable X measures the concentration of ethanol in a chemical solution, and the random variable ¥ measures the acidity of the solution. They have a joint probability density function f(r, y) = A(20- -2y) for 0 S5 and 0<y<5, and f(z, y) = 0 elsewhere.
(a) What is the value of A?
(b) What is P(1 X<2,2<Y< 3)?
(c) Construct margial probability density function of X and Y.
(d) Are the ethanol concentration and the acidity independent?
(e) What are the expectation and the variance of the acidity?

a

$$f(x,y) = A(20 - x - 2y)\,dx\,dy$$

$$F(x,y) = \int_0^5\int_0^5 A(20 - x - 2y)\,dx\,dy$$

$$= A\int_0^5\int_0^5 20 - x - 2y \,dx\,dy$$

we know joint probilty of f(x,y) in

$$F(x,y) = 1$$

$f(x, y) = 1$

$$= A \int_0^5 \left. \frac{-1}{2}x^2 - 2xy + 20x \right]_0^5 dy = 1$$

$$= A \int_0^5 \left( \frac{-25}{2} - 10y - 100 \right) dy = 1$$

$$= A \int_0^5 \frac{175}{2} - 10y) \, dy = 1$$

$$= A \left[ \frac{175}{2} y - \frac{10y^2}{2} \right]_0^5 = 1$$

$$= A \frac{625}{2} = 1$$

$$\Rightarrow A = \frac{2}{625}$$

② 

$$P(1 \leq x \leq 2,\ 2 \leq Y \leq 3) = \int_1^2 \int_2^3 \frac{2}{625}(20 - x - 2y) \, dy \, dx$$

$$= \frac{2}{\ } \int^2 \left[ 20y - xy - y^2 \right]^3 dx$$

$$= \frac{2}{625}\int_1 \left[20y - xy - y^2\right]_2^{3} dx$$

$$= \frac{2}{625}\int_1^2 (60 - 40 - 3x + 2x - 9 + 4)\, dx$$

$$= \frac{2}{625}\int_1^2 20 - x - 5 \; dx$$

$$= \frac{2}{625}\left[20x - \frac{x^2}{2} - 5x\right]_1^2$$

$$= \frac{2}{625}\left(40 - 20 - \frac{4}{2} + \frac{1}{2} - 10 + 5\right)$$

$$= \frac{\cancel{2}}{625} \cdot \frac{27}{\cancel{2}} = \frac{27}{625}$$

$$= 0.0432$$

(c)

Marginal probability density function of $x$:

$x$:

$$f_x(x) = \int_0^5 A(20-x-2y)\,dy$$

$$= A\left[20y - xy - y^2\right]_0^5$$

$$= A(100 - 0 - 5x + 0 - 25 - 0)$$

$$= \frac{2}{625}(75 - 5x)$$

$y$:

$$f_y(y) = \int_0^5 f(x,y)\,dx$$

$$= \int_0^5 A(20-x-2y)\,dx$$

$$= \frac{2}{625}\left[20x - \frac{x^2}{2} - 2xy\right]_0^5$$

$$= \frac{2}{625}\left(100 - 0 - \frac{25}{2} + 0 - 10y - 0\right)$$

$$= \frac{2}{625}\left(\frac{175}{2} - 10y\right)$$

(d)

ethonal are independent if

$$f_x(x) \cdot f_y(y) = f(x,y)$$

$$\frac{2}{625} \cdot \frac{2}{625}\left(75-5x\right)\left(\frac{175}{2}-10y\right) = \frac{2}{625}(20-x-2y)$$

$$\frac{2^2}{625^2}\left(75 \cdot \frac{175}{2} - 750y - \frac{1750x}{2} - 100xy\right) = \frac{2}{625}(20-x-2y)$$

$$LHS \neq RHS$$

So they are not independent

(e)

$E(X) = \text{Exeeption} = \int_a^b f_x(x)\,x\,dx$

$= \int_0^5 A(20 - x - 2y)\,x\,dx$

$= \int_0^5 \frac{2}{625}(75 - 10x).x\,dy$

$= \frac{2}{625}\left[\frac{75x^2}{2} - \frac{5x^3}{3}\right]_0^5$

$= \frac{2}{625} \times \frac{4375}{6}$

$= \frac{7}{3}$

variance $E(x^2) - (E(x))^2$

$E(x^2) = \int_0^5 x^2 f_x(x)\,dx$

$= A\int_0^5 x^2(75 - 5x)\,dx$

$= \frac{2}{625} \times \left[\frac{75x^3}{3} - \frac{5x^4}{4}\right]_0^5$

$$= \frac{2}{625} \times \left[ \frac{45u^3}{3} - \frac{5u^4}{4} \right]_0$$

$$= \frac{2}{625} \cdot \frac{9375}{4} = \frac{15}{2}$$

$$= 7.5$$

Variance $V(u) = E(u^2) - (E(u))^2$

$$= 7.5 - \left(\frac{7}{3}\right)^2$$

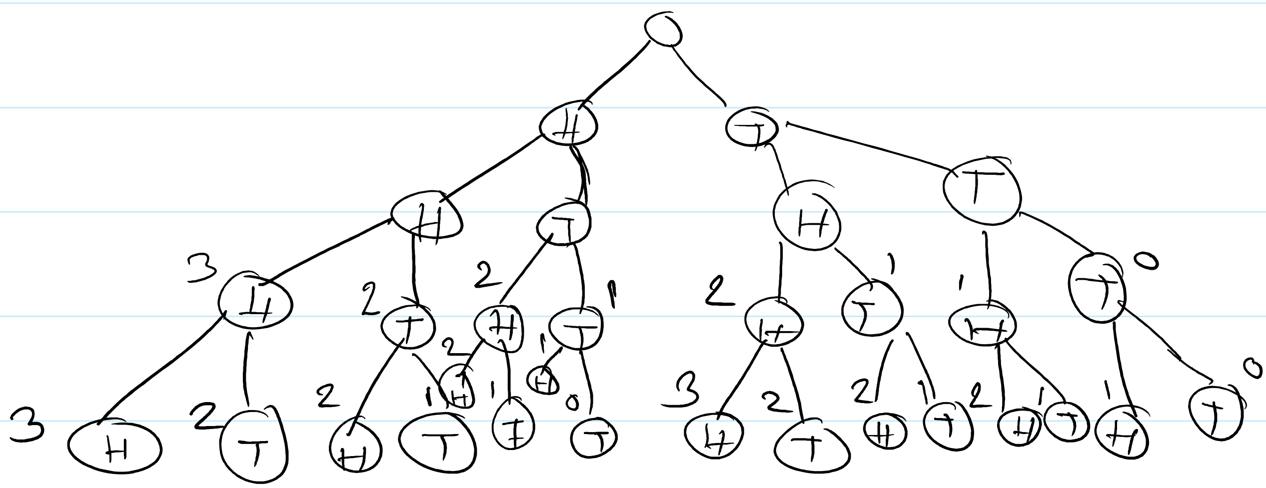
$$= \frac{38}{18}$$

$$= 2.0\dot{5} \text{ (Ans)}$$

8

8. (6 points) A fair coin is tossed four times, and the random variable X is the number of heads in the first three tosses and the random variable Y is the number of heads in the last three tosses. (a) What is the joint probability mass function of X and Y? (b) What are the marginal probability mass functions of X and Y? (c) What are the expectations and variances of the random variables X and Y?

(a) $x = \{0, 1, 2, 3\}$ = Head$_3$



$$0 = \frac{1}{2^3} = \frac{1}{8}$$

$$1 = \frac{{}^3C_1}{2^3} = \frac{3}{8}$$

$$2 = \frac{{}^3C_2}{2^3} = \frac{3}{8}$$

$$3 = \frac{{}^3C_3}{2^3} = \frac{1}{8}$$

$x, y$ 0 1 1 1 2 1 3

| $x, y$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(x)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |
| $P(y)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

| $y \backslash x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | $\frac{1}{8}$ | | | |
| 1 | $\frac{9}{16}$ | $\frac{3}{8}$ | | |
| 2 | $\frac{9}{16}$ | | $\frac{3}{8}$ | |
| 3 | | | | $\frac{1}{8}$ |