IPGP main field candidate for IGRF 2020

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This short note provides some information on the way the IPGP candidate to the main field model 2020 for the IGRF-13, has been built. It is organised in three sections on magnetic data, on the modelling process and on the derivation of the IGRF 2020.

1 Magnetic data

1.1 Origin of the data sets

Two types of data are used :

- Hourly mean vector observatory data from all available observatories from 2013.5 to 2019.5 (Macmillan & Olsen 2013)
- Satellite vector data from the SWARM-A satellite for the era spanning 2013.8 to 2019.5

No scalar magnetic field strength data are used. The full data set spans 2013.5 to 2019.5.

1.2 Selection criteria

We distinguish between "high latitude" (HL) data and "mid to low latitude" (ML) data. HL data are at magnetic latitudes $|\theta| > 55°$.

For all satellite data, the following criteria apply :

- ML data are selected for local times between 11.00 pm and 5.00 am, and rejected for sunlit ionosphere.
- Data are selected for positive values of the IMF vertical component: $B^Z_{IMF} > 0$.
- Data are selected for D_{st} values inside $[-30 : 30]\, nT$ and its variation $\dot{D}_{st}$ between $[-100 : 100]\, nT/day$
- one Swarm satellite data is selected per minute at HL , and one every 30 seconds at ML.

For observatory hourly mean data, the following criteria apply :

- Data are selected for local times between 11.00 pm and 5.00 am, and rejected for sunlit ionosphere.
- Data are selected for positive values of the IMF vertical component: $B^Z_{IMF} > 0$.
- Data are selected for D_{st} values inside $[-30 : 10]\, nT$ and its variation $\dot{D}_{st}$ between $[-100 : 100]\, nT/day$

LZH observatory data have been removed from the data set between 2012 and 2019.5, due to anomalous drift of these data.

1.3 Data weights

The variances attributed to each type of data are given in the table 1.3. The inverse of the variances are used to weight the data.
HL data are handled in the usual North, East, Center (NEC) reference frame, whereas ML data are used in a Solar Magnetic (SM) reference frame, reducing this way the correlations between vector data component errors.

Component	Sat. ML	Sat. HL	Obs. ML	Obs. HL
X	$\sigma^2 = 9\text{nT}^2$	$\sigma^2 = 100\text{nT}^2$	$\sigma^2 = 16\text{nT}^2$	$\sigma^2 = 36\text{nT}^2$
Y	$\sigma^2 = 9\text{nT}^2$	$\sigma^2 = 81\text{nT}^2$	$\sigma^2 = 16\text{nT}^2$	$\sigma^2 = 25\text{nT}^2$
Z	$\sigma^2 = 16\text{nT}^2$	$\sigma^2 = 81\text{nT}^2$	$\sigma^2 = 25\text{nT}^2$	$\sigma^2 = 36\text{nT}^2$

Table 1: *Data variances for each data type*

2 Modelling method and model parameterisation

The approach used to build this candidate is an iterative Kalman filter process. It consists in two successive steps : analysis and prediction. Six iterations of the Kalman filter are necessary to cover the full data time span. These are followed ultimately by a smoothing process that does not affect our IGRF candidate and is therefore not presented here.

2.1 Model definition

All sources (except the observatory offset) are described through spherical harmonics. The model includes at each Kalman filter step the following sources :

- Constant main field (up to SH degree 18)
- Constant secular variation (up to SH degree 18)
- Lithospheric field (from SH degree 15 to 30) - A known crustal field (Lesur et al. 2013) is subtracted from the data for degrees 30 to 110.
- Static external field in GSM coordinate system (SH degree 3)
- Static external field in SM coordinate system (SH degree 3)
- D_{st} dependant external field (SH degree 3)
- IMF Y dependant field in SM coordinate system (SH degree 3)
- Internal induced field generated by magnetospheric sources (SH degree 6)
- Internal induced variation field (SH degree 6)
- D_{st} dependant induced field in SM(SH degree 3)
- Observatory offsets (3x200 parameters)

2.2 Prior information

For each analysis Kalman step, the prior information of each model component is provided through the description of a Gaussian distribution characterised by a mean and a covariance matrix. This information is derived from the combination of the outputs of the previous Kalman prediction step with the default prior information setting. At the first Kalman analysis step, only the default prior information is used.

This default setting is such that the component means are null and associated covariance matrices defined as in Holschneider et al. (2016). Such covariance matrices are controlled by two parameters given in table 2.2 for each model component outside the main field and its SV. For these latter two components, this default covariance matrix is estimated from output of numerical dynamo experiments – here the CE dynamo (Aubert et al. 2013).

No	Components	Radius	Scaling
1	Lithosphere	6280 km	2.7 10^{-2}
2	External GSM	16 10^3 km	5.4 10^3
3	External SM	6.9 10^3 km	3.56
4	Dependent D_{st}	16 10^3 km	5.4
5	Dependent IMF Y	6.9 10^3 km	1 10^{-1}
6	Internal Induced	2200 km	1. 10^{-3}
7	Internal Induced variation	2200 km	4. 10^{-3}
8	Internal Induced D_{st}	2537 km	1
9	Observatory offsets	-	1 10^3

Table 2: *Control parameters for covariance matrices as defined in Holschneider et al. (2016). Note that the settings for components 6 and 7 are such that these contributions remain weak.*

Between each analysis step all model components, outside the main field, are predicted assuming AR1 stochastic processes with given time scales. These are set to 11years for the SV, infinity for the lithosphere and the observatory offsets. Other components are assumed to have time scales shorter than the analysis time interval.

2.3 Analysis step

The analysis deal with one year of data for each step that are collected in a vector $\mathbf{d}$. These data are linearly related to the model parameters collected in the vector $\mathbf{m}$:

$$\mathbf{d} = \mathbf{A}\mathbf{m} + \mathbf{e} \tag{1}$$

where $\mathbf{e}$ is the vector of errors. The models parameters are estimated by fitting the data through a robust re-weighted iterative least squares process, using Huber-weights. The solution of the least-squares process is:

$$\mathbf{m} = \tilde{\mathbf{m}} + \left(\mathbf{A}^t\mathbf{C}_d^{-1}\mathbf{A} + \mathbf{C}_{\tilde{m}}^{-1}\right)^{-1}\mathbf{A}^t\mathbf{C}_d^{-1}\left(\mathbf{d} - \mathbf{A}\tilde{\mathbf{m}}\right) \tag{2}$$

where $\tilde{\mathbf{m}}$ is the known magnetic field (– i.e. the prior) and $\mathbf{C}_{\tilde{m}}$ is its model covariance matrix. $\mathbf{C}_d$ is the covariance matrix of the data errors. The posterior covariance matrix of the model $\mathbf{m}$ is estimated using:

$$\mathbf{C}_m = \left(\mathbf{A}^t\mathbf{C}_d^{-1}\mathbf{A} + \mathbf{C}_{\tilde{m}}^{-1}\right)^{-1} \tag{3}$$

2.4 Prediction step

The prediction step for the main field is a simple linear extrapolation using the estimated SV. All other components are predicted through an AR1 stochastic processes. The corresponding equations are given below.

Assuming there is a known magnetic field model $\mathbf{m}_k$ and its associated covariance matrix $\mathbf{C}_{m_k}$ at time t_k. We consider the time step $\delta t = t_{k+1} - t_k$ that is set for the derivation of the candidate model to 1 year. Then at time t_{k+1} a prediction of the model $\tilde{\mathbf{m}}_{k+1}$ and its covariance matrix $\mathbf{C}_{\tilde{m}_{k+1}}$ are obtained through:

$$\begin{aligned} \tilde{\mathbf{m}}_{k+1} &= \mathbf{P}_k\, \mathbf{m}_k \\ \mathbf{C}_{\tilde{m}_{k+1}} &= \mathbf{P}_k\, \mathbf{C}_{m_k} \mathbf{P}_k^t + (\mathbf{I}_d - \mathbf{D})\, \mathbf{C}_m^0\, (\mathbf{I}_d - \mathbf{D})^t \end{aligned} \tag{4}$$

where $\mathbf{I}_d$ is the identity matrix. The operator $\mathbf{P}_k$ predicts the evolution of the magnetic field model. This operator is the summation of two operators:

1. The operator that linearly extrapolates the main field.
2. A diagonal operator $\mathbf{D}$ which diagonal elements are $(1 - \frac{\delta t}{\tau})$, where τ is the time scale of the corresponding model component.

$\mathbf{C}_m^0$ is the default covariance matrix of the model.

3 Estimation of the IGRF 2020

The last field model derived through the iterative Kalman filter process has been built over the time interval 2018.5-2019.5. It includes a snapshot of the main field and a co-estimated secular variation. The candidate main field for 2020 is therefore estimated by linear extrapolation of this model. Posterior covariance matrices that are outputs of the Kalman filter process, are also extrapolated to 2020. Their diagonal elements give the formal variances of the model parameters. The model candidate IGRF_2020 is therefore provided together with the square-root of diagonal covariance matrix elements as uncertainty estimates.

References

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