NEON Implementation of an Attribute-Based Encryption Scheme

Ana Helena Sánchez and Francisco Rodríguez-Henríquez[⋆]

Computer Science Department, CINVESTAV-IPN
asanchez@computacion.cs.cinvestav.mx, francisco@cs.cinvestav.mx

Abstract. In 2011, Waters presented a ciphertext-policy attribute-based encryption protocol that uses bilinear pairings to provide control access mechanisms, where the set of user's attributes is specified by means of a linear secret sharing scheme. Some of the applications foreseen for this protocol lie in the context of mobile devices such a smartphones and tablets, which in a majority of instances are powered by an ARM processor supporting the NEON vector set of instructions. In this paper we present the design of a software cryptographic library that implements a 127-bit security level attribute-based encryption scheme over mobile devices equipped with a 1.4GHz Exynos 4 Cortex-A9 processor and a developing board that hosts a 1.7 GHz Exynos 5 Cortex-A15 processor. For the latter platform and taking advantage of the inherent parallelism of the NEON vector instructions, our library computes a single optimal pairing over a Barreto-Naehrig curve approximately 2 times faster than the best timings previously reported on ARM platforms at this level of security. Further, using a 6-attribute access formula our library is able to encrypt/decrypt a text/ciphertext in less than 7.5mS and 15.67mS, respectively.

Keywords: Atribute based-encryption, pairing-based protocols, Barreto-Naehrig curves, elliptic curve scalar multiplication, ARM processor.

1 Introduction

It was long assumed that the task of computing a single bilinear pairing was rather expensive, so much so that when assessing the complexity of a given protocol, a designer could safely ignore the computational cost of all the other cryptographic components included in it. Nevertheless, in the last few years we have witnessed a dramatic reduction in the timing required to calculate a single pairing, which has had the side effect that the computation of the other ancillary functions associated to pairing-based protocols have acquired a renewed importance. Some examples of these auxiliary blocks include, fixed/variable point

[⋆] A portion of this work was performed while the author was visiting University of Waterloo.

M. Jacobson et al. (Eds.): ACNS 2013, LNCS 7954, pp. 322–338, 2013.

scalar multiplication for elliptic curves defined over finite fields and their extensions, the projection of arbitrary strings to a random point in those elliptic curves, exponentiation in field extensions, *etc.* Furthermore, as pointed out in [15], several pairing-based protocols admit further optimizations, such as the computation of fixed-argument pairings and products of pairings.

Unfortunately as of today, very few works have analyzed in detail the complexity and overall computational weight of non-pairing cryptographic operations in a given protocol. This lack of research in the implementation of pairing-based protocols is especially acute for mobile platforms such as the ones using ARM processors.

An important number of major IT players such as Apple, Samsung, Sony, to name just a few, have adopted the ARM Cortex family of processors for powering their tablets, smartphones and other mobile devices. A majority of those devices support the vector set of instructions NEON. In spite of their ever increasing popularity, it is only until recently that some research works have studied the implementation of cryptographic primitives over ARM processor platforms.

Among the research papers reporting pairing implementations in the ARM Cortex family of processors are [1] and [10]. In [1], authors propose the idea that affine coordinates could be more attractive than the projective ones when implementing pairings in constrained devices, whereas the software library of [10] reports the current record in the computation of a single asymmetric pairing at the 128, 224 and 320-bit security levels. As for the implementation of pairing-based protocols on mobile devices, the only work that we are aware of is [2], where the authors described the design of an attribute-based encryption scheme able to preserve the confidentiality of the medical electronic records generated within a hospital environment.

In this work we present the design of a software library that implements Waters' attribute-based encryption scheme [16], over a set of mobile device platforms equipped with the latest models of the ARM Cortex family of processors and the vectorized set of instructions NEON. Our library was specifically tailored for computing optimal pairings over Barreto-Naehrig curves at the 127-bit security level. When executed on a developing board that hosts a 1.7 GHz Exynos 5 Cortex-A15 processor, our software computes a single optimal pairing in approximately 5.84M clock cycles, which is about two times less than the estimated cycling count reported in [10] for a single pairing computation over a TI 1.2GHz OMAP 4460 Cortex-A9 processor.

Our library also implements single/multi-pairing computations with fixed/-variable input points, as well as other auxiliary functions associated with most pairing-based protocols such as scalar multiplication and the projection of arbitrary strings to elliptic curve points defined over extension finite fields, among others. In particular, when executed on the Cortex-A15 processor mentioned above and when using an access formula composed of six attributes, our library computes the encryption/decryption primitives of Waters' attributed-based encryption protocol in less than 12.75M clock cycles and 26.64M clock cycles, roughly equivalent to 7.5mS and 15.67mS, respectively.[1]

2 Mathematical Background

Let p be a prime, and let E be an elliptic curve defined over the finite field $\mathbb{F}_p$. Let r be a prime with $r \mid \#E(\mathbb{F}_p)$ and $\gcd(r, p) = 1$. The embedding degree k is defined as the smallest positive integer such that $r \mid (p^k - 1)$. In this paper, only the Barreto-Naehrig (BN) pairing-friendly family of elliptic curves [4] was considered for pairing implementation. All BN curves have embedding degree $k = 12$ and they are defined by the equation $E : y^2 = x^3 + b, b \in \mathbb{F}_p^*$, where the characteristic p of the prime field, the group order r, an the trace of Frobenius t are parametrized as,

$$\begin{aligned} p(z) &= 36z^4 + 36z^3 + 24z^2 + 6z + 1; \\ r(z) &= 36z^4 + 36z^3 + 18z^2 + 6z + 1; \\ t(z) &= 6z^2 + 1, \end{aligned} \tag{1}$$

where $z \in \mathbb{Z}$, is an arbitrary integer known as the BN parameter, such that $p(z)$ and $r(z)$ are prime numbers. BN curves admit a sextic degree twist curve, defined as $\tilde{E}(\mathbb{F}_{p^2}) : Y^2 = X^3 + b/\xi$, where $\xi \in \mathbb{F}_{p^2}$ is neither a square nor a cube in $\mathbb{F}_{p^2}$.

Let $\pi : (x, y) \mapsto (x^p, y^p)$ be the p-th power Frobenius endomorphism. The trace of the Frobenius is defined as $t = p + 1 - \#E(\mathbb{F}_p)$. Let $\mathbb{G}_1 = \{P \in E[r] : \pi(P) = P\} = E(\mathbb{F}_p)[r]$, where $\mathbb{G}_1$ is the 1-eigenspace of π acting on $E[r]$. Let $\Psi : \tilde{E} \to E$ be the associated twisting isomorphism. Let $\tilde{Q} \in \tilde{E}(\mathbb{F}_{p^2})$ be a point of order r; then $Q = \Psi(\tilde{Q}) \notin E(\mathbb{F}_p)$. The group $\mathbb{G}_2 = \langle Q \rangle$ is the p-eigenspace of π acting on $E[r]$. Let $\mathbb{G}_T$ denote the order-r subgroup of $\mathbb{F}_{p^{12}}^*$. The bilinear pairing studied in this paper is defined as the non-degenerate map $\hat{a}_{opt} : \mathbb{G}_2 \times \mathbb{G}_1 \to \mathbb{G}_T$, corresponding to the optimal *ate* pairing given as:

$$\begin{aligned} \hat{a}_{opt} : \mathbb{G}_2 \times \mathbb{G}_1 &\to \mathbb{G}_T \\ (Q, P) &\mapsto [f_{s,Q}(P) \cdot \ell_{(s)Q,\pi(Q)}(P) \cdot \\ &\qquad \ell_{(s)Q+\pi(Q),\pi^2(Q)}(P)]^{(p^{12}-1)/r} \end{aligned} \tag{2}$$

where $f_{s,Q}$ is a Miller function of length $s = 6z + 2$, which is a rational function in $\bar{\mathbb{F}}_p(E)$ with divisor $\mathrm{div}(f_{s,R}) = s[R] - [sR] - (s - 1)[\mathcal{O}]$, while ℓ_{Q_1,Q_2} is the line equation given by the point addition of $Q_1 \in \mathbb{G}_2$ and $Q_2 \in \mathbb{G}_2$. Algorithm 1 computes the optimal pairing as defined in Eq. (2).

[1] An open source code of our software library is available at `http://sandia.cs.cinvestav.mx/index.php?n=Site.NEONabe`

Algorithm 1. Optimal ate pairing

```
Require: $P \in \mathbb{G}_1$, $Q \in \mathbb{G}_2$
Ensure: $g = \hat{a}_{opt}(Q, P)$
Write $s = 6z+2$ as $s = \sum_{i=0}^{l-1}$, $s_i \in \{-1, 0, 1\}$
$T \leftarrow Q$, $f \leftarrow 1$
for $i = l - 2 \rightarrow 0$ do
    $f \leftarrow f^2 \cdot \ell_{T,T}(P)$, $T \leftarrow 2T$
    if $s_i = 1$ then
        $f \leftarrow f \cdot \ell_{T,Q}(P)$, $T \leftarrow T + Q$
    else if $s_i = -1$ then
        $f \leftarrow f \cdot \ell_{T,-Q}(P)$, $T \leftarrow T - Q$
    end if
end for
$Q_1 \leftarrow \pi(Q)$, $Q_2 \leftarrow \pi^2(Q)$
$f \leftarrow f \cdot \ell_{T,Q_1}(P)$, $T \leftarrow T + Q_1$, $f \leftarrow f \cdot \ell_{T,-Q_2}(P)$, $T \leftarrow T - Q_2$
$g \leftarrow f^{(p^{12}-1)/r}$
return $g$
```

3 Tower Extension Field Arithmetic

Efficient arithmetic over extension finite fields is a necessary requirement in the development of high-performance pairing-based schemes. In this work, we represent $\mathbb{F}_{p^{12}}$ using the same tower extension employed in [3], namely, we first construct a quadratic extension, which is followed by a quadratic and cubic extensions of it and finally by a quadratic one, using the following irreducible binomials,

- $\mathbb{F}_{p^2} = \mathbb{F}_p[u]/(u^2 - \beta)$, where $\beta = -1$
- $\mathbb{F}_{p^4} = \mathbb{F}_{p^2}[V]/(V^2 - \xi)$, where $\xi = u + 1$
- $\mathbb{F}_{p^6} = \mathbb{F}_{p^2}[V]/(V^3 - \xi)$, where $\xi = u + 1$
- $\mathbb{F}_{p^{12}} = \mathbb{F}_{p^6}[W]/(W^2 - V)$ or $\mathbb{F}_{p^4}[T]/(T^3 - V)$

Furthermore, as in [3] we selected $z = -(2^{62} + 2^{55} + 1)$, which using Eq. 1 yields a prime $p \equiv 3 \bmod 4$. This allows for a faster arithmetic over $\mathbb{F}_{p^2}$, since the multiplication by the constant β reduces to a simple subtraction. For the computation of the pairing final exponentiation, cyclotomic subgroup arithmetic $\mathbb{G}_{\Phi_6}(\mathbb{F}_{p^2})$ [9] was extensively used.

Algorithm 2. Montgomery product

```
Require: prime $p$, $p'$, $r = 2^k$ and $\tilde{a}$, $\tilde{b} \in \mathbb{F}_p$
Ensure: $\tilde{c} = \text{MontPr}(\tilde{a}, \tilde{b})$
$t \leftarrow \tilde{a} \cdot \tilde{b}$
$u \leftarrow (t + (t \cdot p' \bmod r) \cdot p)/r$
if $u > p$ then
    return $u - p$
else
    return $u$
end if
```

3.1 Field Multiplication Over $\mathbb{F}_p$

The single most important base field arithmetic operation is modular multiplication, which is defined as $c = a \cdot b \bmod p$, with $a, b, c \in \mathbb{F}_p$. Since in general BN primes are not suitable for fast reductions, this operation was performed via the Montgomery multiplication algorithm. The Montgomery product is defined as, $\tilde{c} = \tilde{a} \cdot \tilde{b} \cdot r^{-1} \bmod p$, where $\tilde{a}, \tilde{b} \in \mathbb{F}_p$ are given as, $\tilde{a} = a \cdot r \bmod p$ and $\tilde{b} = b \cdot r \bmod p$, respectively. This formulation allows trading the costly division by p with divisions by r, where $r = 2^k$ with $k - 1 < |p| < k$. If required, the modular product c, can be easily recovered using $c = \tilde{c} \cdot r^{-1} \bmod p$. Algorithm 2 shows the classical version of the Montgomery product, which expects as input parameter an integer p' that can be precomputed before hand using Bezout's identity for two co-prime integers, namely, $r \cdot r^{-1} - p \cdot p' = 1$.

In our library both, the Separated Operand Scanning (SOS) and the Coarsely Integrated Operand Scanning (CIOS) multi-precision Montgomery product variants as described in [13] were implemented. The SOS method computes first the integer product $t = a \cdot b$, followed by the Montgomery reduction step that calculates u such that $u = (t + m \cdot p)/r$, where $m = t \cdot p' \pmod{r}$. In this case $r = 2^{wn}$, where w is the wordsize of the processor and $n = [(\lfloor log_2\, p \rfloor + 1)/w]$. The CIOS method interleaves the calculations corresponding to the integer product with the ones required for getting u. Since in both methods, the reduction is implemented word by word, then the operation $m = t \cdot p' \bmod r$ can be performed replacing p' by $p'_0 = p' \bmod 2^{\omega}$, which redounds in a more efficient computation. According to [13], the CIOS variant is more efficient than the SOS one. Nevertheless, the later method allows lazy reduction, which was the reason why we implemented both variants.

3.2 Extension Field Arithmetic Computational Cost

Let us denote by (a, m, s, i) and $(\tilde{a}, \tilde{m}, \tilde{s}, \tilde{i})$ the computational cost of the addition, multiplication, squaring and inversion operations over $\mathbb{F}_p$ and $\mathbb{F}_{p^2}$, respectively. The field arithmetic procedures used in this work extensively exploits lazy reduction, which closely resembles the approaches adopted in [3,10]. Let $\tilde{m}_E$, $\tilde{s}_E$ and $\tilde{r}_E$ denote integer multiplication, integer squaring and reduction over $\mathbb{F}_{p^2}$, respectively, where $\tilde{m} = \tilde{m}_E + \tilde{r}_E$ and $\tilde{s} = \tilde{s}_E + \tilde{r}_E$. The rationale behind the costs given in Table 1, can be summarized as follows.

The cost of reductions over $\mathbb{F}_{p^2}$ is twice the cost of reduction over $\mathbb{F}_p$, *i.e.*, $\tilde{r}_E = 2r_E$. At a field extension $\mathbb{F}_{p^d}$, $d = 2^i 3^j$, $i, j \in \mathbb{Z}^+$; the product $c = a \cdot b$, $a, b, c \in \mathbb{F}_{p^d}$ can be computed with $3^i 6^j$ integer multiplications and $2^i 3^j$ reductions modulo p (Theorem 1 of [3]). Field inversion was based on the procedure described in [10]. In the case of the quadratic and twelfth field extensions $\mathbb{F}_{p^2}$ and $\mathbb{F}_{p^{12}}$, field squaring was computed using the complex method at a cost of 2 multiplications. The inversion of an element $A = a_0 + a_1 u \in \mathbb{F}_{p^2}$ was obtained through the identity $(a_0 + a_1 u)^{-1} = (a_0 - a_1 u)/(a_0^2 - \beta a_1^2)$. In $\mathbb{F}_{p^4}$ the squaring was implemented at a cost of $3\tilde{s}$. This operation is required for computing squarings in the cyclotomic group $\mathbb{G}_{\Phi_6}(\mathbb{F}_{p^2})$, having a cost of 3 squarings over $\mathbb{F}_{p^4}$. The asymmetric squaring formula for cubic extensions of [5] was used in the field $\mathbb{F}_{p^6}$ at a cost of $2\tilde{m} + 3\tilde{s}$. Inversion in $\mathbb{F}_{p^6}$ has a computational complexity of $9\tilde{m} + 3\tilde{s} + i$ [11]. Notice that m_ξ stands for a multiplication by the constant ξ.

Table 1. Computational cost of the tower extension field arithmetic

Field	Addition	Multiplication	Squaring	Inversion
$\mathbb{F}_{p^2}$	$\tilde{a} = 2a$	$\tilde{m} = 3m_E + 2r_E + 8a + m_\beta$	$\tilde{s} = 2m_E + 2r_E + 3a$	$\tilde{i} = 2m_E + r_E + 2m + 2a + i$
$\mathbb{F}_{p^4}$	$2\tilde{a}$		$3\tilde{s} + m_\xi + 4\tilde{a}$	
$\mathbb{F}_{p^6}$	$3\tilde{a}$	$6\tilde{m}_E + 3\tilde{r}_E + 2m_\xi + 24\tilde{a}$	$2\tilde{m} + 3\tilde{s} + 2m_\xi + 9\tilde{a}$	$9\tilde{m}_E + 3\tilde{s}_E + 7\tilde{r}_E + 4m_\xi + 10\tilde{a} + \tilde{i}$
$\mathbb{F}_{p^{12}}$	$6\tilde{a}$	$18\tilde{m}_E + 6\tilde{r}_E + 7m_\xi + 96\tilde{a}$	$12\tilde{m}_E + 6\tilde{r}_E + 6m_\xi + 63\tilde{a}$	$25\tilde{m}_E + 9\tilde{s}_E + 16\tilde{r}_E + 13m_\xi + 79\tilde{a} + \tilde{i}$
$\mathbb{G}_{\Phi_6}(\mathbb{F}_{p^2})$			$9\tilde{s} + 4m_\xi + 30\tilde{a}$	$3\tilde{a}$

3.3 Field Arithmetic Implementation Using NEON

The performance of a field arithmetic library is strongly influenced by the processor micro-architecture features, the size of the operands and the algorithms and programming techniques associated to them. In our case, the size of the operands is of 254 bits, which conveniently allows the usage of lazy reduction. The word size in the ARM processors is of 32 bits and the processors considered in this work include the NEON vector set of instructions.

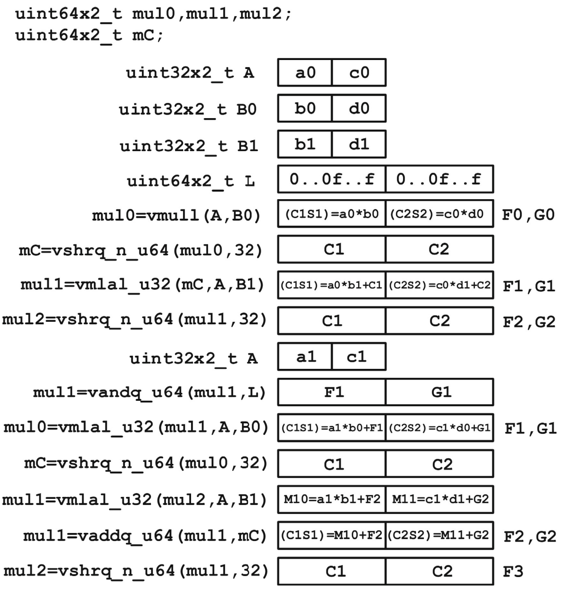
Algorithm 3. Computing double integer product with NEON

```
Require: $a = (a_0, a_1)$, $b = (b_0, b_1)$, $c = (c_0, c_1)$ and $d = (d_0, d_1)$
Ensure: $F = a \cdot b$, $G = c \cdot d$
$F \leftarrow 0$, $G \leftarrow 0$
for $i = 0 \rightarrow 1$ do
    $C_1 \leftarrow 0$, $C_2 \leftarrow 0$
    for $j = 0 \rightarrow 1$ do
        $(C_1, S_1) \leftarrow F_{i+j} + a_j \cdot b_i + C_1$, $(C_2, S_2) \leftarrow G_{i+j} + c_j \cdot d_i + C_2$,
        $F_{i+j} = S_1$, $G_{i+j} = S_2$
    end for
    $F_{i+n} = C_1$, $G_{i+n} = C_2$
end for
return $F, G$
```

NEON is a 128-bit Single Instruction Multiple Data (SIMD) architecture extension for the ARM Cortex family of processors. NEON architecture has 32 registers of 64 bits (*doubleword*), which can be viewed as 16 registers of 128 bits (*quadword*). Our library mostly made use of two intrinsic instructions:

```
uint64x2_t vmull_u32 (uint32x2_t, uint32x2_t);
uint64x2_t vmlal_u32 (uint64x2_t, uint32x2_t, uint32x2_t).
```

The first one performs two 32-bit integer multiplications storing the corresponding result into two 64-bit registers. The second one performs a multiplication that is accumulated with the addition of a 64-bit scalar. Algorithm 3 illustrates the usage of NEON for computing the double integer product $F = a \cdot b$, $G = c \cdot d$. This is the core operation for the SOS Montgomery multiplication variant. Each field element a, b, c, d is represented with two 32-bit words. Figure 1 depicts the NEON dataflow of this algorithm.



```
uint64x2_t mul0,mul1,mul2;
uint64x2_t mC;
              uint32x2_t A   | a0 | c0 |
             uint32x2_t B0   | b0 | d0 |
             uint32x2_t B1   | b1 | d1 |
              uint64x2_t L   | 0..0f..f | 0..0f..f |
           mul0=vmull(A,B0)  | (C1S1)=a0*b0 | (C2S2)=c0*d0 |  F0,G0
     mC=vshrq_n_u64(mul0,32) | C1 | C2 |
   mul1=vmlal_u32(mC,A,B1)   | (C1S1)=a0*b1+C1 | (C2S2)=c0*d1+C2 |  F1,G1
  mul2=vshrq_n_u64(mul1,32)  | C1 | C2 |  F2,G2
              uint32x2_t A   | a1 | c1 |
      mul1=vandq_u64(mul1,L) | F1 | G1 |
  mul0=vmlal_u32(mul1,A,B0)  | (C1S1)=a1*b0+F1 | (C2S2)=c1*d0+G1 |  F1,G1
     mC=vshrq_n_u64(mul0,32) | C1 | C2 |
  mul1=vmlal_u32(mul2,A,B1)  | M10=a1*b1+F2 | M11=c1*d1+G2 |
     mul1=vaddq_u64(mul1,mC) | (C1S1)=M10+F2 | (C2S2)=M11+G2 |  F2,G2
  mul2=vshrq_n_u64(mul1,32)  | C1 | C2 |  F3
```

Fig. 1. NEON Implementation of Algorithm 3

Algorithm 4. NEON multiplication over $\mathbb{F}_{p^2}$

```
Require: $A = a_0 + a_1 u, \quad B = b_0 + b_1 u \in \mathbb{F}_{p^2}$
Ensure: $C = A \cdot B \in \mathbb{F}_{p^2}$
$s \leftarrow a_0 + a_1$
$t \leftarrow b_0 + b_1$
$(d_0, d_1) \leftarrow mule_{NEON}(s, t, a_0, b_0)$
$d_2 \leftarrow \mathrm{mul256}(a_1, b_1)$
$d_0 \leftarrow d_0 - d_1 - d_2$
$d_1 \leftarrow d_1 - d_2$
$(c_1, c_0) \leftarrow red_{NEON}(d_0, d_1)$
return $C = c_0 + c_1 u$
```

Because of their ability to perform two multiplications at once, NEON instructions are very useful for accelerating arithmetic computations. However, data loading and storing is in general costly since the NEON registers have to be fed by storing data into consecutive 32-bit ARM registers. Hence, in order to take a real advantage of NEON, load/store instructions should be avoided as much as possible, which is easier to accomplish if the arithmetic algorithms are specified with little data dependency among the multiplier operands. In the case of $\mathbb{F}_{p^2}$ arithmetic, two independent multiplications over $\mathbb{F}_p$ were implemented using NEON as follows. Let us consider $|p| = 254$ bits and define the following three functions: mul_{NEON}, $mule_{NEON}$ and red_{NEON}. The first one performs two independent multiplications in $\mathbb{F}_p$ using the CIOS method, *i.e.* given $a, b, c, d, f, g \in \mathbb{F}_p$, define mul_{NEON} as $(f, g) \leftarrow mul_{NEON}(a, b, c, d)$ where $f = a \cdot b \bmod p$ and $g = c \cdot d \bmod p$. The second function $mule_{NEON}$ performs two integer multiplications: $(F, G) \leftarrow mule_{NEON}(a, b, c, d)$, with $a, b, c, d \in \mathbb{F}_p$, $F = a \cdot b$ and $G = c \cdot d$,

Algorithm 5. NEON Squaring over $\mathbb{F}_{p^2}$

```
Require: $A = a_0 + a_1 u \in \mathbb{F}_{p^2}$
Ensure: $C = A^2 \in \mathbb{F}_{p^2}$
$c_0 \leftarrow a_0 - a_1$
$c_2 \leftarrow a_0 + a_1$
$(c_1, c_0) \leftarrow mul_{NEON}(a_0, a_1, c_0, c_2)$
$c_1 \leftarrow 2c_1$
return $C = c_0 + c_1 u$
```

where $|G| = |F| = 508$ bits. Finally, the third function red_{NEON} implements the Montgomery reduction defined as $(f, g) \leftarrow red_{NEON}(F, G)$, where $f, g \in \mathbb{F}_p$ and $|G| = |F| = 512$ bits. Making use of the aforementioned functions, Algorithms 4 and 5 compute a multiplication and a squaring in $\mathbb{F}_{p^2}$, respectively. Notice that In step 4 of Alg. 4 the function $mul256$ stands for a single integer multiplication.

4 Elliptic Curve Arithmetic

Elliptic curve points were represented using projective Jacobian coordinates, where a BN elliptic curve E is given as $Y^2 = X^3 + BZ^6$. A point $(X_1 : Y_1 : Z_1)$ in E corresponds to the affine point $(X_1/Z_1^2, Y_1/Z_1^3)$, with $Z_1 \neq 0$. The point at infinity $\mathcal{O}$ is represented as $(1, 1, 0)$, whereas the additive inverse of $(X_1 : Y_1 : Z_1)$ is $(X_1 : -Y_1 : Z_1)$.

Algorithm 6. Point doubling with Jacobian coordinates

```
Require: $P = (X_1 : Y_1 : Z_1) \in \mathbb{G}_1$
Ensure: $2P = (X_3 : Y_3 : Z_3) \in \mathbb{G}_1$
$(t_1, t_4) \leftarrow mul_{NEON}(Y_1, Y_1, 3X_1, X_1)$
$(t_2, t_3) \leftarrow mul_{NEON}(4X_1, t_1, 4t_1, 2t_1)$
$(X_3, Z_3) \leftarrow mul_{NEON}(t_4, t_4, 2Y_1, Z_1)$
$X_3 \leftarrow X_3 - 2t_2$
$Y_3 \leftarrow t_4 \cdot (t_2 - X_3) - t_3$
return $(X_3 : Y_3 : Z_3)$
```

Algorithm 7. Mixed point addition

```
Require: $P = (X_1 : Y_1 : Z_1)$ and $Q = (X_2 : Y_2 : 1) \in \mathbb{G}_1$
Ensure: $R = P + Q = (X_3 : Y_3 : Z_3) \in \mathbb{G}_1$
$t_1 \leftarrow Z_1^2$
$(t_2, t_3) \leftarrow mul_{NEON}(Z_1, t_1, X_2, t_1)$
$t_5 \leftarrow t_3 - X_1$
$(t_4, t_7) \leftarrow mul_{NEON}(Y_2, t_2, t_5, t_5)$
$(t_8, t_9) \leftarrow mul_{NEON}(t_7, t_5, t_7, X_1)$
$t_6 \leftarrow t_4 - Y_1$
$(X_3, Z_3) \leftarrow mul_{NEON}(t_6, t_6, Z_1, t_5)$
$X_3 \leftarrow X_3 - (t_8 + 2t_9)$
$(Y_3, t_0) \leftarrow mul_{NEON}(t_6, t_9 - X_3, Y_1, t_8)$
$Y_3 \leftarrow Y_3 - t_0$
return $(X_3 : Y_3 : Z_3)$
```

Point Doubling. Given $P = (X_1 : Y_1 : Z_1)$, the point $2P = (X_3 : Y_3 : Z_3)$ can be calculated with 4 squarings and 3 multiplications according to the next sequence of operations,

$$t_1 \leftarrow Y_1^2,\, t_2 \leftarrow 4X_1 \cdot t_1,\, t_3 \leftarrow 8t_1^2,\, t_4 \leftarrow 3X_1^2,$$
$$X_3 \leftarrow t_4^2 - 2t_2,\, Y_3 \leftarrow t_4 \cdot (t_2 - X_3) - t_3,\, Z_3 \leftarrow 2Y_1 \cdot Z_1$$

Mixed Point Addition. Let $P = (X_1 : Y_1 : Z_1)$ with $Z_1 \neq 0$ and $Q = (X_2 : Y_2 : 1)$, for $P \neq \pm Q$, the addition $R = P + Q = (X_3 : Y_3 : Z_3)$ can be obtained at a cost of 3 squarings and 8 multiplications according to the next sequence,

$$t_1 \leftarrow Z_1^2,\, t_2 \leftarrow Z_1 \cdot t_1,\, t_3 \leftarrow X_2 \cdot t_1,\, t_4 \leftarrow Y_2 \cdot t_2,\, t_5 \leftarrow t_3 - X_1,$$
$$t_6 \leftarrow t_4 - Y_1,\, t_7 \leftarrow t_5^2,\, t_8 \leftarrow t_7 \cdot t_5,\, t_9 \leftarrow X_1 \cdot t_7,$$
$$X_3 \leftarrow t_6^2 - (t_8 + 2t_9),\, Y_3 \leftarrow t_6 \cdot (t_9 - X_3) - Y_1 \cdot t_8,\, Z_3 \leftarrow Z_1 \cdot t_5$$

Taking advantage of the inherent parallelism of the above sequences, a NEON implementation of the point addition and point doubling operations is shown in Algorithms 6 and 7, respectively.

4.1 Efficient Techniques for Computing Scalar Multiplication

The elliptic curve scalar multiplication operation computes the multiple $R = [\ell]P$, with $\ell \in \mathbb{Z}_r$, $P, R \in E(\mathbb{F}_p)$, which corresponds to the point resulting of adding P to itself ℓ times. The average cost of computing $[\ell]P$ by a random $n-$bit scalar ℓ using the customary double-and-add method is of about, $nD + \frac{n}{2}A$, where A is the cost of a point addition, and D is the cost of a point doubling.

The customary method to speed up this operation reduces the Hamming weight of the scalar ℓ by representing it in its non-adjacent form (NAF). The technique can be easily extended to the w-NAF representation, namely, $\ell = \sum_{i=0}^{n-1} \ell_i 2^i$, $|\ell_i| \leq 2^{w-1}$, and at most one of any w consecutive digits is non-zero, with $\ell_{n-1} \neq 0$, where the length n is at most one bit larger than the bitsize of the scalar n and the resulting Hamming weight is approximately $1/(w+1)$. The estimated cost of the scalar multiplication reduces to, $nD + \frac{n}{w+1}A$, plus the cost of the precomputation of the multiples $P_i = [i]P$, for $i \in [1, 3, \ldots, 2^{w-1} - 1]$. Due to this exponential penalty, in most applications a rather conservative value of $w \in [3, 5]$ is selected.

When the point P is fixed, some form of the comb method is usually preferred. Given an w-window size and a known-point P, one can pre-compute for all of the possible bit strings $(a_{w-1}, \ldots, a_0)$ the following 2^w multiples of P: $[a_{w-1}, \ldots, a_2, a_1, a0]P = a_{w-1}2^{(w-1)d}P + \ldots + a_2 2^{2d}P + a_1 2^d P + a_0 P$. Then, the scalar ℓ is scanned column-wise by recoding it into d blocks each one with a bit length of w bits, where $d = |r|/w$. The computational and storage costs of the comb method is of $d(A + D)$, and a look-up table of 2^w points, respectively. Notice that the storage cost can be reduced to a half by using a signed representation of the scalar ℓ. A further speedup in the computation of the scalar multiplication can be achieved if there exists an efficient-computable endomorphism ψ over $E/\mathbb{F}_p$ such that $\psi(P) = \lambda P$ [8]. In the case of BN curves, given a cube root of unity $\beta \in \mathbb{F}_p$, one has that the mapping $\psi : E_1 \to E_1$ defined as, $(x, y) \to (\beta x, y)$ and $\mathcal{O} \to \mathcal{O}$, is an endomorphism over $\mathbb{F}_p$ with a characteristic polynomial given as $\lambda^2 + \lambda \equiv -1 \bmod r$, $\lambda = 36z^4 - 1$. The scalar ℓ can be rewritten as $\ell \equiv \ell_0 + \ell_1\lambda \bmod r$, where $|\ell_i| < |\sqrt{r}|$, which allows to compute the scalar multiplication as, $[\ell]P = [\ell_0]P + [\ell_1]\psi(P)$, at an approximate cost of $\left[D + (2^{w-2} - 1)A\right] + \left[\frac{n}{w+1}A + \frac{n}{2}D\right]$.

In the case that the scalar multiplication in $\mathbb{G}_2$, *i.e.* the computation of the multiple $S = [\ell]Q$, with $\ell \in \mathbb{Z}_r$, $Q, S \in E(\mathbb{F}_{p^2})$, is of interest, one can take advantage of the Frobenius endomorphism to extend the two-dimensional GLV method to a four dimension version using the GS approach [7]. Let E be a BN elliptic curve over $\mathbb{F}_p$ with embedding degree $k = 12$ and let $\tilde{E}(\mathbb{F}_{p^2})$ be the sixth degree twist of E. Let π_p be the Frobenius operator in E, then $\psi = \phi^{-1}\pi_p\phi$ is an endomorphism on $\tilde{E}$ such that $\psi : \tilde{E}(\mathbb{F}_{p^2}) \to \tilde{E}(\mathbb{F}_{p^2})$. Then for $Q \in \tilde{E}(\mathbb{F}_{p^2})$, it holds that $\psi^k(Q) = Q$, $\psi(Q) = pQ$, and ψ satisfies $\psi^4 - \psi^2 + 1 = 0$. Since $p \equiv t - 1 \bmod r$, the scalar ℓ can be decomposed as $\ell = \ell_0 + \ell_1\lambda + \ell_2\lambda^2 + \ell_3\lambda^3$ with $\lambda = t - 1$ and $|\ell_i| \approx |r|/4$, which allows to compute the scalar multiplication in $\mathbb{G}_2$, as, $[\ell]Q = [\ell_0]Q + [\ell_1]\psi(Q) + [\ell_2]\psi^2(Q) + [\ell_3]\psi^3(Q)$, at an approximate cost of $\left[D + (2^{w-2} - 1)A\right] + \left[\frac{n}{w+1}A + \frac{n}{4}D\right]$.

Likewise, in the case of the exponentiation in $\mathbb{G}_T$, the operation f^e with $f \in \mathbb{G}_T, e \in \mathbb{Z}_r$ can then be accomplished by rewriting the exponent e in base p as, $e = e_0 + e_1 \cdot p + e_2 \cdot p^2 + e_3 \cdot p^3$, with $|e_i| \approx |r|/4$, followed by the computation, $f^e = f^{e_0} \cdot f^{e_1^p} \cdot f^{e_2^{p^2}} \cdot f^{e_3^{p^3}}$. Notice that the Frobenius mapping e^{p^i} for $i = 0, \ldots, 3$, has a negligible computational cost. Notice also that the identity $f^p = f^\lambda = f^{6x^2}$, holds.

5 Bilinear Pairing Arithmetic

In this section we briefly describe the Miller's loop and final exponentiation computations as well as the algorithm utilized to perform multi-pairing computations.

Miller Loop. The main operations of Algorithm 1 are the evaluation of the tangent line $\ell_{T,T}$ and the doubling of the point T; as well as the secant line evaluation and the computation of the point addition $T + P$. The most efficient way to perform above operations is through the usage of standard projective coordinates, where the projective point $(X_1 : Y_1 : Z_1)$ in the elliptic curve E corresponds to the affine point $(X_1/Z_1, Y_1/Z_1)$. Given the curve $\tilde{E}/\mathbb{F}_{p^2}$ defined as $\tilde{E} : y^2 = x^3 + b'$ whose projective form is $Y^2Z = X^3 + bZ^3$, one can calculate $2T = (X_3 : Y_3 : Z_3) \in E'(\mathbb{F}_{p^2})$ using the formulas [3]:

$$\begin{aligned}
X_3 &= \tfrac{X_1Y_1}{2}(Y_1^2 - 9b'Z_1^2)\\
Y_3 &= \left[\tfrac{1}{2}(Y_1^2 + 9b'Z_1^2)\right]^2 - 27b'^2Z_1^4\\
Z_3 &= 2Y_1^3Z_1
\end{aligned}$$

whereas the line $\ell_{T,T}$ evaluated on $P = (x_P, y_P) \in E(\mathbb{F}_p)$ is given as,

$$\ell_{T,T}(P) = -2Y_1Z_1y_P + 3X_1^2x_Pw + (3b'Z_1^2 - Y_1^2)w^3 \in \mathbb{F}_{p^{12}}$$

In the same way, given the points $T = (X_1, Y_1, Z_1), Q = (X_2, Y_2, 1) \in E'(\mathbb{F}_{p^2})$ and $P = (x_P, y_P) \in E(\mathbb{F}_p)$ one can calculate the point addition $R = T + Q = (X_3, Y_3, Z_3)$ and $\ell_{T,Q}(P)$ as [3]:

$$\ell_{T,Q}(P) = \lambda y_P - \theta x_P w + (\theta X_2 - \lambda Y_2) w^3,$$

$$\begin{aligned} X_3 &= \lambda(\lambda^3 + Z_1\theta^2 - 2X_1\lambda^2) \\ Y_3 &= \theta(3X_1\lambda^2 - \lambda^3 - Z_1\theta^2) - Y_1\lambda^3 \\ Z_3 &= Z_1\lambda^3 \end{aligned}$$

where $\theta = Y_1 - Y_2 Z_1$ and $\lambda = X_1 - X_2 Z_1$.

Another important aspect of the Miller's algorithm is the multiplication of the Miller variable f by the line (either tangent or secant) evaluation. However, the evaluation of the lines $\ell_{Q,Q}$ and $\ell_{T,Q}$ produce a sparse element in the group $\mathbb{F}^*_{p^{12}}$ with half of its coefficients having a zero value, which motivates the idea that any product of $f \in \mathbb{F}_{p^{12}}$ with $\ell_{Q,Q}$ or $\ell_{T,Q}$ should be performed using a procedure specially tailored for computing sparse multiplications.

Final Exponentiation. The exponent $e = (p^k - 1)/r$ in the BN final exponentiation can be broken into two parts as,

$$(p^{12} - 1)/r = [(p^{12} - 1)/\Phi_{12}(p)] \cdot [\Phi_{12}(p)/r],$$

where $\Phi_{12}(p) = p^4 - p^2 + 1$ denotes the twelfth cyclotomic polynomial evaluated in p. Computing the map $f \mapsto f^{(p^{12}-1)/\Phi_{12}(p)}$ is relatively inexpensive, costing only a few multiplications, inversions, and inexpensive p-th exponentiations in $\mathbb{F}_{p^{12}}$. Raising to the power $d = \Phi_{12}(p)/r = (p^4 - p^2 + 1)/r$ is considered more difficult. This part was computed using a multiple d' of d where $r \nmid d$ as discussed in [6], which allowed a lower number of operations. Using the BN parameter z, the exponentiation of $g^{d'(z)}$ requires the calculation of the following addition chain,

$$f^z \mapsto f^{2z} \mapsto f^{4z} \mapsto f^{6z} \mapsto f^{6z^2} \mapsto f^{12z^2} \mapsto f^{12z^3},$$

which requires 3 exponentiations by z, 3 squarings an one multiplication over $\mathbb{F}_{p^{12}}$. Finally, given the variables $a = g^{12z^3} \cdot g^{6z^2} \cdot g^{6z}$ and $b = a \cdot (g^{2z})^{-1}$, the exponentiation of $g^{d'(z)}$ is calculated as follows:

$$g^{d'(z)} = \left[a \cdot g^{6z^2} \cdot g\right] \cdot [b]^p \cdot [a]^{p^2} \cdot \left[b \cdot g^{-1}\right]^{p^3} \in \mathbb{F}^{\times}_{p^{12}}$$

Since $g \in \mathbb{G}_{\Phi_6}(\mathbb{F}_{p^2})$, the cost of $g^{d'(z)}$ is 3 Frobenius operators, 3 exponentiations by z, 10 multiplications in $\mathbb{F}_{p^{12}}$ and 3 squarings in $\mathbb{G}_{\Phi_6}(\mathbb{F}_{p^2})$. It should be noted that we use the Karabina compressed squaring formulas [12] for performing the exponentiation-by-z step.

Multipairing. Products of pairings are computations required in Waters' attribute-based protocol. For this operation, one can make use of the pairing bilinear property to group pairings sharing one of the input parameters. If all the pairings share a common input point, then one can exchange n pairing products by $n-1$ point additions and a single pairing using the identity,

$$\prod_{i=0}^{n-1} e(Q, P_i) = e(Q, \sum_{i=0}^{n-1} P_i),$$

If a product of pairings is still needed, and the previous method was already exploited, there is still room for obtaining significant speedups. For instance, one can compute this product by performing a multi-pairing (or simultaneous product of pairings), by exploiting the well-known techniques employed in the multi-exponentiation setting. In essence, in a multipairing computation not only the costly final exponentiation step can be shared, but also, one can share both the Miller variable f, and the squaring computations performed in the step 4 of Algorithm 1. A further performance improvement can be achieved if the point Q in $\mathbb{G}_2$ is known in advance, since in this case one can pre-compute some of the operations involved in the line evaluations. In particular, the cost of computing the line evaluated at the point $P \in \mathbb{G}_1$ given as, $\ell_{\cdot,\cdot}(P) = l_0 y_P + l_1 x_P w + l_2 w^3$, reduces to two scalar multiplications since l_0, l_1, l_2, can be precomputed offline.

6 Attribute Based-Encryption

Attribute-Based Encryption (ABE) is a relatively new encryption scheme where an identity is seen as a set of attributes. In this scheme a user can access to some resources only if she had a set of privileges (called attributes) satisfying a control access policy previously defined. The policy is described through a boolean formula, which can be represented by an access structure and can be implemented using a linear secret-sharing scheme (LSSS) [14, 16]. The LSSS structure is described by the pair (M, ρ), where $M \in \mathbb{F}_r$ is an $u \times t$ matrix, where u, t are the number of required attributes and the access policy threshold, respectively; whereas ρ is a label function that according to the policy links each row of the matrix M to an attribute. For the sake of efficiency and as Scott did in [15], we reformulate the protocol from its original symmetric setting to an asymmetric one where some scheme parameters are conveniently defined in $\mathbb{G}_1$ whereas others are in $\mathbb{G}_2$. The ABE scheme is made up of four algorithms [16]: Setup, Encrypt, Key Generation and Decrypt, as described next.

Setup. This algorithm takes as input the security parameter λ and the set of U attributes. The security parameter becomes the main criterion to select the groups $\mathbb{G}_1$ and $\mathbb{G}_2$ of order r and the generators $P \in \mathbb{G}_1$ and $Q \in \mathbb{G}_2$. The points $H_1, \ldots, H_U \in \mathbb{G}_1$ are generated from the attribute universe and two random integers $a, \alpha \in \mathbb{F}_r$, are chosen at random. The public key is published as, $\text{PK} = \{P, Q, e(Q, P)^\alpha, [a]P, H_1, \ldots, H_U\}$. Additionally, the authority establishes $\text{MSK} = [\alpha]P$ as her master secret key. This algorithm has a cost of one pairing, two scalar multiplications and U MapToPoint functions. Notice that it is assumed that the elements P, Q, $[a]P$ and $e(Q, P)^\alpha$ are all known in advance. On the contrary, the points $H_1, \ldots H_U$ were not considered as fixed points since the attribute universe has a variable length, a fact that is also reflected in the storage cost.

Encrypt. The algorithm for encryption takes as input the public key PK, the message $\mathcal{M}$ to be encrypted, and the LSSS access structure (M, ρ), where $M \in \mathbb{F}_r$ is an $u \times t$ matrix as described above. The algorithm starts by randomly selecting a column vector $v = (s, y_2, \ldots, y_t)^T \in \mathbb{F}_r^n$ that will be used to securely share the secret exponent s. For $i = 1$ to u, it calculates $\lambda_i = M_i \cdot v$ where M_i is the $1 \times t$ vector corresponding to the i-th row of M. The scalars $r_1, \ldots, r_u \in \mathbb{F}_r$ are also randomly chosen. Then, the cipher CT is published as follows:

$$\mathrm{CT} = \begin{cases} C = \mathcal{M}e(Q,P)^{\alpha s}, C' = [s]Q, \\ (C_1 = [\lambda_1]([a]P) - [r_1]H_{\rho(1)}, D_1 = [r_1]Q), \\ \vdots \\ (C_u = [\lambda_u]([a]P) - [r_u]H_{\rho(u)}, D_u = [r_u]Q), \end{cases}$$

which is sent along with the LSSS access structure (M, ρ).

The comb method was applied to compute scalar multiplications involving the fixed points Q, P and $[a]P$. In the same way, one can apply a variation of this method to obtain the powering of $e(Q, P)^\alpha$ by the exponent s. The GLV method was used to compute scalar multiplications with the points $H_i \in \mathbb{G}_1$. Hence the cost of encryption is one multiplication and one fixed exponentiation in $\mathbb{G}_T$, u fixed point multiplications in $\mathbb{G}_1$, $u + 1$ fixed point multiplications in $\mathbb{G}_2$ and u point multiplication in $\mathbb{G}_1$.

Key Generation. This algorithm takes as input the master secret key MSK = αP and a set of attributes S. First the algorithm selects a random number $t \in \mathbb{F}_r$, then it generates the attribute-based private key as follows,

$$\mathrm{SK} = \{\, K = [\alpha]P + [t]([a]P),\, L = [t]Q,\, \forall x \in S\ K_x = [t]H_x \,\}$$

Let N be the number of attributes on S, since the points aP and Q are known, the cost of this algorithm is one fixed point multiplication in $\mathbb{G}_1$, one fixed point in $\mathbb{G}_2$ and N point multiplications in $\mathbb{G}_1$. For the first two scalar multiplications the *comb* method was used, and the GLV method for the rest.

Decrypt. This algorithm takes as inputs the cipher CT with the access structure (M, ρ) and the private key SK for a set S. Suppose S satisfies the access structure and define $I \subset \{1, 2, \ldots, u\}$ as $I = \{i : \rho(i) \in S\}$. Let $\{\omega_i \in \mathbb{Z}\}_{i \in I}$ be the set of constants such that if λ_i is a valid share of a secret s according to M, then $\sum_{i \in I} \omega_i \lambda_i = \Delta s$ with $\Delta \in \mathbb{F}_r$. The decryption algorithm first computes:

$$\left(e(L, \sum_{i \in I} [\omega_i] C_i) \prod_{i \in I} e(D_i, [\omega_i] K_{\rho(i)}) \right)^{\frac{1}{\Delta}} / e(C', K) = e(P, Q)^{-\alpha s}, \qquad (3)$$

Followed by the multiplication of this value by C as defined in Eq. (3). If S satisfies the access structure this should recover the message $\mathcal{M}$.

The variable Δ guarantees low size constants ω_i, i.e., $\mid \omega_i \mid < 64$ bits, which allows us to perform the scalar multiplications involving these constants using a w-NAF method. We called this operation *short scalar multiplication.* Let $N < u$ be the number of elements of I, then the computational cost of Eq. (3) is of $2N$ short multiplications in $\mathbb{G}_1$, an $N + 2$ multipairing computation, N point additions in $\mathbb{G}_1$ and one exponentiation in $\mathbb{G}_T$ which can be computed using the GS method. Also, since $L \in \mathbb{G}_2$ is a known point, its lines evaluations were precomputed.

Table 2. Clock cycle comparison for Single pairing computations

Work	**Processor**	10^3 **clock cycles for** 254 **bits**						
		$\tilde{a}$	$\tilde{m}$	$\tilde{s}$	$\tilde{i}$	ML	FE	Pairing
[1]	Tegra 2[a]	1.42	8.18	5.20	26.61	26,320	24,690	51,010
[10]	Apple A5[b]	0.25	3.48	2.88	19.19	8,338	5,483	13,821
	TI OMAP[c]	0.16	3.37	2.53	16.86	8,231	5,258	13,489
	(ASM)	0.12	2.95	2.48	16.60	7,376	4,510	11,886
This Work	Tegra 2[a]	0.17	3.41	2.41	39.25	8,313	5,269	13,582
	Exynos[d]	0.17	3.42	2.41	39.21	8,348	4,607	13,618
	(NEON)	0.16	2.29	2.00	60.37	5,758	3,794	9,477
	Exynos[e]	0.14	1.36	0.86	29.01	3,388	2,353	5,838

a. NVidia Tegra 2 (ARM v7) Cortex-A9 a 1.0 GHz (C)

b. iPad 2 (ARM v7) Apple A5 Cortex-A9 a 1.0 GHz (C)

c. Galaxy Nexus (ARM v7) TI OMAP 4460 Cortex-A9 a 1.2 GHz (Two versions: C and ASM)

d. Galaxy Note (ARM v7) Exynos 4 Cortex-A9 a 1.4 GHz (Two versions: C and NEON)

e. Arndaleboard (ARM v7) Exynos 5 Cortex-A15 a 1.7 GHz (NEON)

7 Implementation Results

This section presents the main implementation results classified into three subsections: bilinear pairings, scalar multiplication and the ABE scheme timings.

7.1 Pairing Timings

Let us recall that $(\tilde{a}, \tilde{m}, \tilde{s}, \tilde{i})$ denote the computational cost of the addition, multiplication, squaring and inversion operations over $\mathbb{F}_{p^2}$. These field arithmetic operations are used to perform a single pairing computation, a task that as it was described in section 5, can be split into two main parts: the Miller Loop (ML) and the Final Exponentiation (FE).

Using above definitions, Table 2 presents a comparison against the works [1] and [10] In [1] a pairing library that employs affine coordinates was presented, whereas [10] reports an assembler optimized pairing library using standard projective coordinates.

7.2 Costs of the Scalar Multiplication and Field Exponentiation

Table 3 shows the timings obtained for the computation of scalar multiplication in the groups $\mathbb{G}_1$ and $\mathbb{G}_2$, and the field exponentiation in the group $\mathbb{G}_T$. The computation of the scalar multiplication using the w-NAF approach was only utilized for small 64-bit scalars, with $w = 3$. The comb method was the choice for computing fixed point scalar multiplication with a window size of $w = 8$. We stress that the w-NAF was used in combination with both the GLV and GS methods.[2]

Table 3. Scalar mult. and exponentiation timings (in 10^3 clock cycles)

Processor	$\mathbb{G}_1$ Mult.			$\mathbb{G}_2$ Mult.			$\mathbb{G}_T$ Exp.		
	w-NAF	GLV	Comb	w-NAF	GS	Comb	w-NAF	GS	Comb
Tegra 2	779	1977	626	2059	4190	1745	2643	5998	2727
Exynos 4	785	1973	627	2096	4189	1742	2633	4777	2155
(NEON)	676	1698	556	1493	2933	1214	1827	4102	1863
Exynos 5	337	822	251	797	1571	636	1125	2522	1121

Table 4. ABE scheme with 6 attributes (Timings in 10^3 clock cycles)

Processor	**Key Generation**	**Encryption**	**Decryption** ($\Delta = 1$)	**Decryption** ($\Delta > 1$)
Tegra 2	18,340	31,830	63,870	74,140
Exynos 4	18,270	29,480	63,810	73,930
Exynos 4 (NEON)	15,333	24,167	43,980	50,808
Exynos 5	7,617	12,748	26,638	31,161

7.3 Attribute-Based Encryption Costs

We could not compare our ABE scheme timings against [2], because this work only implements the decryption algorithm and it does not present the exact timings. Table 4 reports the timings obtained when a 6-attribute policy is employed in the three main primitives of the ABE protocol, namely, key generation, encryption and decryption that were discussed in section 6. Note that for the decryption algorithm we present the cases when $\Delta = 1$ and $\Delta > 1$ (see Eq. 3).

8 Conclusion

We presented a cryptographic library that implements Waters' attribute encryption scheme in mobile devices operated with ARM processors. The main primitives developed were bilinear pairings and scalar multiplications in different flavors. Our library uses four different scalar multiplications according to the

[2] A description of the w-NAF GLS and GS methods for computing scalar multiplications was given in subsection 4.1.

group, scalar size and the type of the point (either fixed or variable), providing a 127 bits security level and achieving record timings for the computation of a single bilinear pairing at this level of security when implemented on the Exynos-5 Cortex-A15 processor.

A key factor that helps us to achieve faster timings than previously reported works was the usage of the NEON technology that allows a better exploitation of the inherent parallelism present in several field and elliptic curve arithmetic operations. It is illustrative to analyze the Exynos-4 scalar multiplication timings shown in Table 3. where NEON produces savings of about 14%, 30% and 15% in the computations over the $\mathbb{G}_1, \mathbb{G}_2$ and $\mathbb{G}_T$ groups, respectively. Notice that the significant better performance of NEON in the computations over $\mathbb{G}_2$ are a consequence of the rich parallelism extracted for the field squaring and multiplication over $\mathbb{F}_{p^2}$ as it was explained in Section 4.

Another interesting aspect to remark is the performance comparison of our work against [1] for the single pairing computation at the 127 bit security level. As shown in Table 2, *without using* NEON, the two libraries perform essentially the same when implemented in the Tegra 2 and Apple A5 processors, respectively. However, taking advantage of NEON, our library outperforms the library in [1] by approximately 20% when implemented in the Exynos 4 and TI OMAP processors, respectively. Moreover, when implemented in the Exynos 5 processor, our library is a bit more than two times faster than the software in [1]. We conclude that the Cortex A-15 micro-architecture and its improved NEON unit, provide a significantly better performance for cryptographic application implementations.

Acknowledgments. We wish to thank Peter Schwabe for explaining us how to perform accurate clock cycle counts on ARM processors and for giving us feedback on the first draft of the paper and Armando Faz-Hernández for benchmarking our software in the Exynos 5 Cortex-A15 processor. We also thank Alfred Menezes for commenting on the earlier draft. The second author acknowledges partial support from CONACyT project 132073

References

1. Acar, T., Lauter, K., Naehrig, M., Shumow, D.: Affine pairings on ARM. In: Abdalla, M., Lange, T. (eds.) Pairing 2012. LNCS, vol. 7708, pp. 203–209. Springer, Heidelberg (2013)
2. Akinyele, J.A., Lehmann, C., Green, M., Pagano, M., Peterson, Z., Rubin, A.: Self-Protecting Electronic Medical Records Using Attribute-Based Encryption. In: Bhattacharya, A., Dasgupta, P., Enck, W. (eds.) The 1st ACM Workshop on Security and Privacy in Smartphones and Mobile Devices SPSM 2011, pp. 75–86. ACM (2010)
3. Aranha, D.F., Karabina, K., Longa, P., Gebotys, C.H., López, J.: Faster Explicit Formulas for Computing Pairings over Ordinary Curves. In: Paterson, K.G. (ed.) EUROCRYPT 2011. LNCS, vol. 6632, pp. 48–68. Springer, Heidelberg (2011)

4. Barreto, P.S.L.M., Naehrig, M.: Pairing-friendly elliptic curves of prime order. In: Preneel, B., Tavares, S. (eds.) SAC 2005. LNCS, vol. 3897, pp. 319–331. Springer, Heidelberg (2006)
5. Chung, J., Hasan, M.A.: Asymmetric Squaring Formulas. In: Kornerup, P., Muller, J.-M. (eds.) Proceedings of the 18th IEEE Symposium on Computer Arithmetic, pp. 113–122. IEEE Computer Society (2007)
6. Fuentes-Castañeda, L., Knapp, E., Rodríguez-Henríquez, F.: Faster hashing to $\mathbb{G}_2$. In: Miri, A., Vaudenay, S. (eds.) SAC 2011. LNCS, vol. 7118, pp. 412–430. Springer, Heidelberg (2012)
7. Galbraith, S.D., Scott, M.: Exponentiation in Pairing-Friendly Groups Using Homomorphisms. In: Galbraith, S.D., Paterson, K.G. (eds.) Pairing 2008. LNCS, vol. 5209, pp. 211–224. Springer, Heidelberg (2008)
8. Gallant, R.P., Lambert, R.J., Vanstone, S.A.: Faster Point Multiplication on Elliptic Curves with Efficient Endomorphisms. In: Kilian, J. (ed.) CRYPTO 2001. LNCS, vol. 2139, pp. 190–200. Springer, Heidelberg (2001)
9. Granger, R., Scott, M.: Faster squaring in the cyclotomic subgroup of sixth degree extensions. In: Nguyen, P.Q., Pointcheval, D. (eds.) PKC 2010. LNCS, vol. 6056, pp. 209–223. Springer, Heidelberg (2010)
10. Grewal, G., Azarderakhsh, R., Longa, P., Hu, S., Jao, D.: Efficient implementation of bilinear pairings on ARM processors. In: Knudsen, L.R., Wu, H. (eds.) SAC 2012. LNCS, vol. 7707, pp. 149–165. Springer, Heidelberg (2013)
11. Hankerson, D., Menezes, A., Scott, M.: Software implementation of pairings (Chapter 12). In: Joye, M., Neven, G. (eds.) Identity-based Cryptography. Cryptology and Information Security, vol. 2, pp. 188–206. IOS Press (2009)
12. Karabina, K.: Squaring in cyclotomic subgroups. Math. Comput. 82(281) (2013)
13. Koc, C.K., Acar, T., Kaliski Jr., B.S.: Analyzing and Comparing Montgomery Multiplication Algorithms. IEEE Micro 16(3), 26–33 (1996)
14. Liu, Z., Cao, Z.: On efficiently transferring the linear secret-sharing scheme matrix in ciphertext-policy attribute-based encryption. IACR Cryptology ePrint Archive, 2010:374 (2010)
15. Scott, M.: On the Efficient Implementation of Pairing-Based Protocols. In: Chen, L. (ed.) IMACC 2011. LNCS, vol. 7089, pp. 296–308. Springer, Heidelberg (2011)
16. Waters, B.: Ciphertext-policy attribute-based encryption: An expressive, efficient, and provably secure realization. In: Catalano, D., Fazio, N., Gennaro, R., Nicolosi, A. (eds.) PKC 2011. LNCS, vol. 6571, pp. 53–70. Springer, Heidelberg (2011)