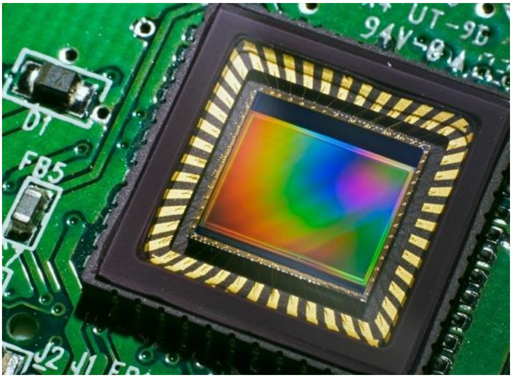
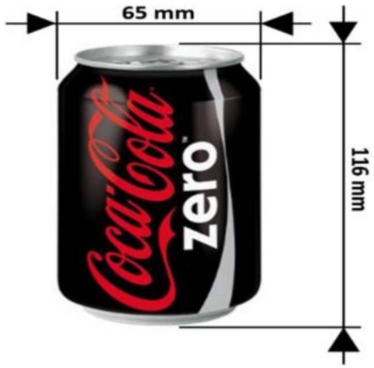
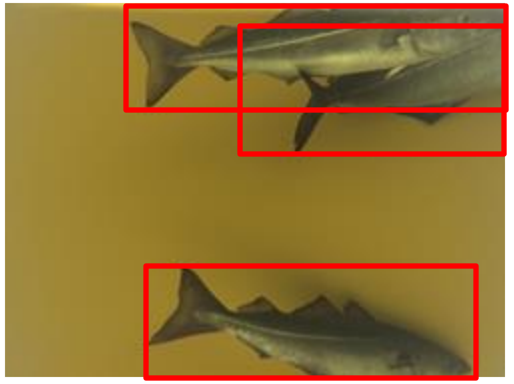
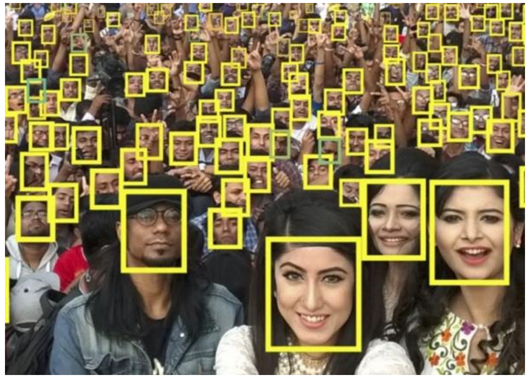
Multiple View Geometry

Image Formation and Camera Modeling









Objectives of this class

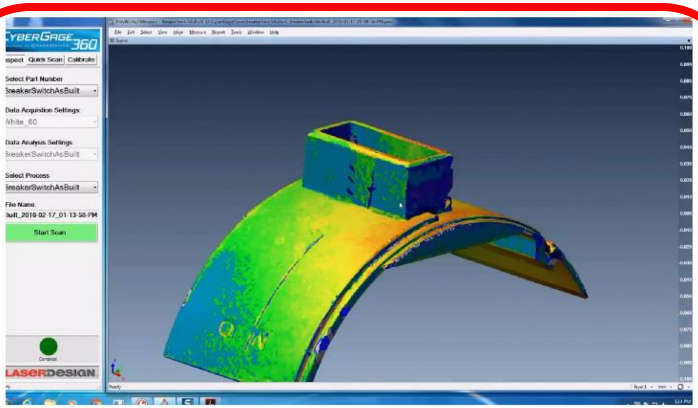
- Obtaining a first overview of how the image is formed in a camera
- Analysing the projection of a 3D scene in the image plane
- Knowing the *pinhole* model
- Understanding the equations that describe the geometry of the image formation process

Contents

Camera Modeling and Calibration

Obtaining 3D information

- Applications



Visual Inspection(LDI Cybergage360)

Industrial Manipulation(Intel RealSense)

Motion Modeling(Vicon + MotionBuilder)

Autonomous Driving(Google car)

Obtaining 3D information

- Applications

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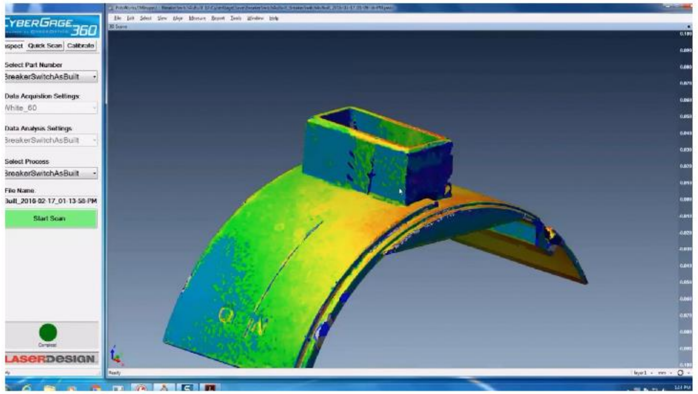

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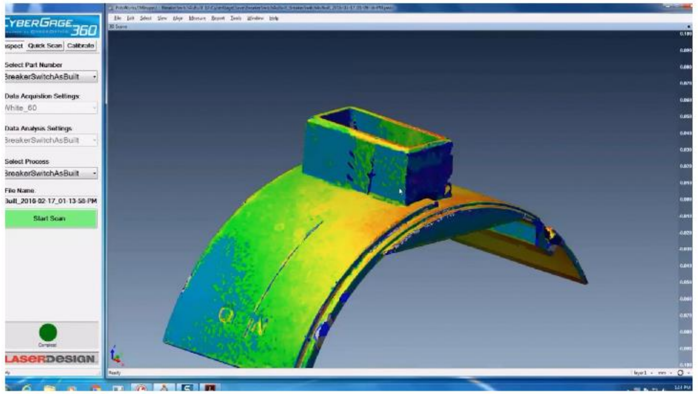

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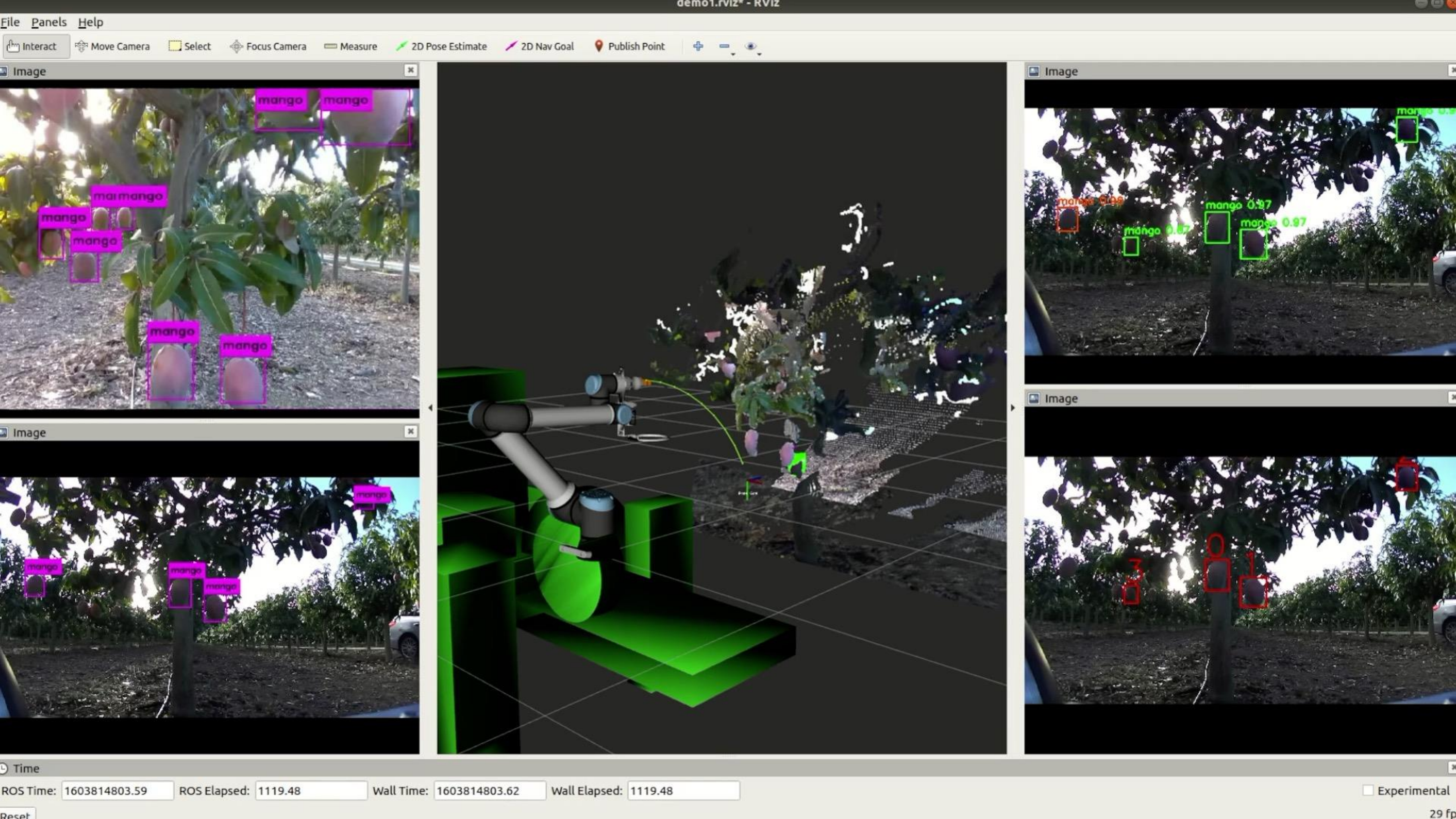
demo1.rviz* - RViz
File
Panels
Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Image
mango
mango
mango
mango
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mango
Image
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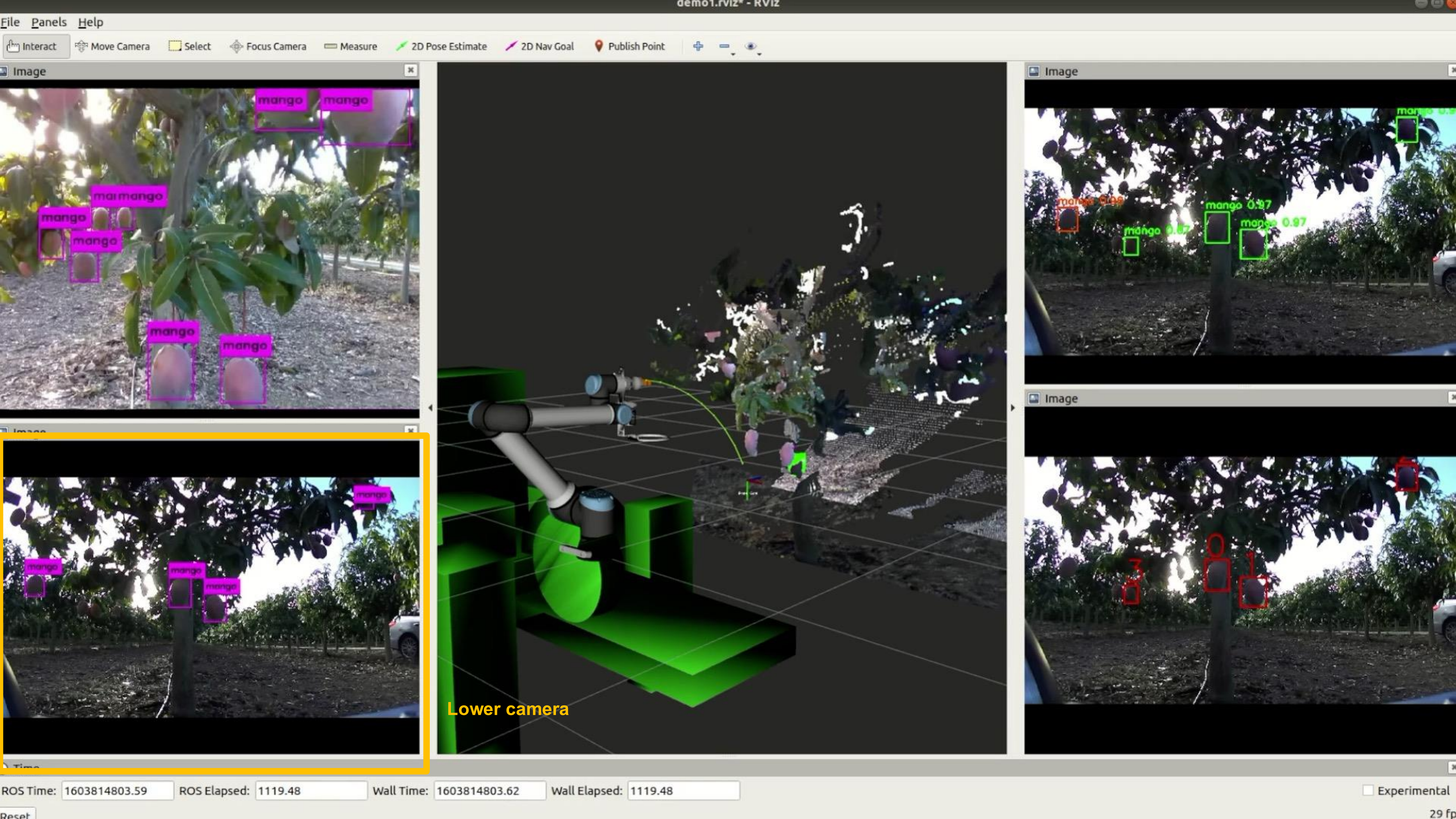
mango
Image
mango 0.97
mango 0.97
Image
0
1
3
Time
ROS Time: 1603814803.59
ROS Elapsed: 1119.48
Wall Time: 1603814803.62
Wall Elapsed: 1119.48
Experimental
Reset


demo1.rviz* - RViz
File
Panels
Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Image
mango
Image
Image
Lower camera
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ROS Elapsed: 1119.48
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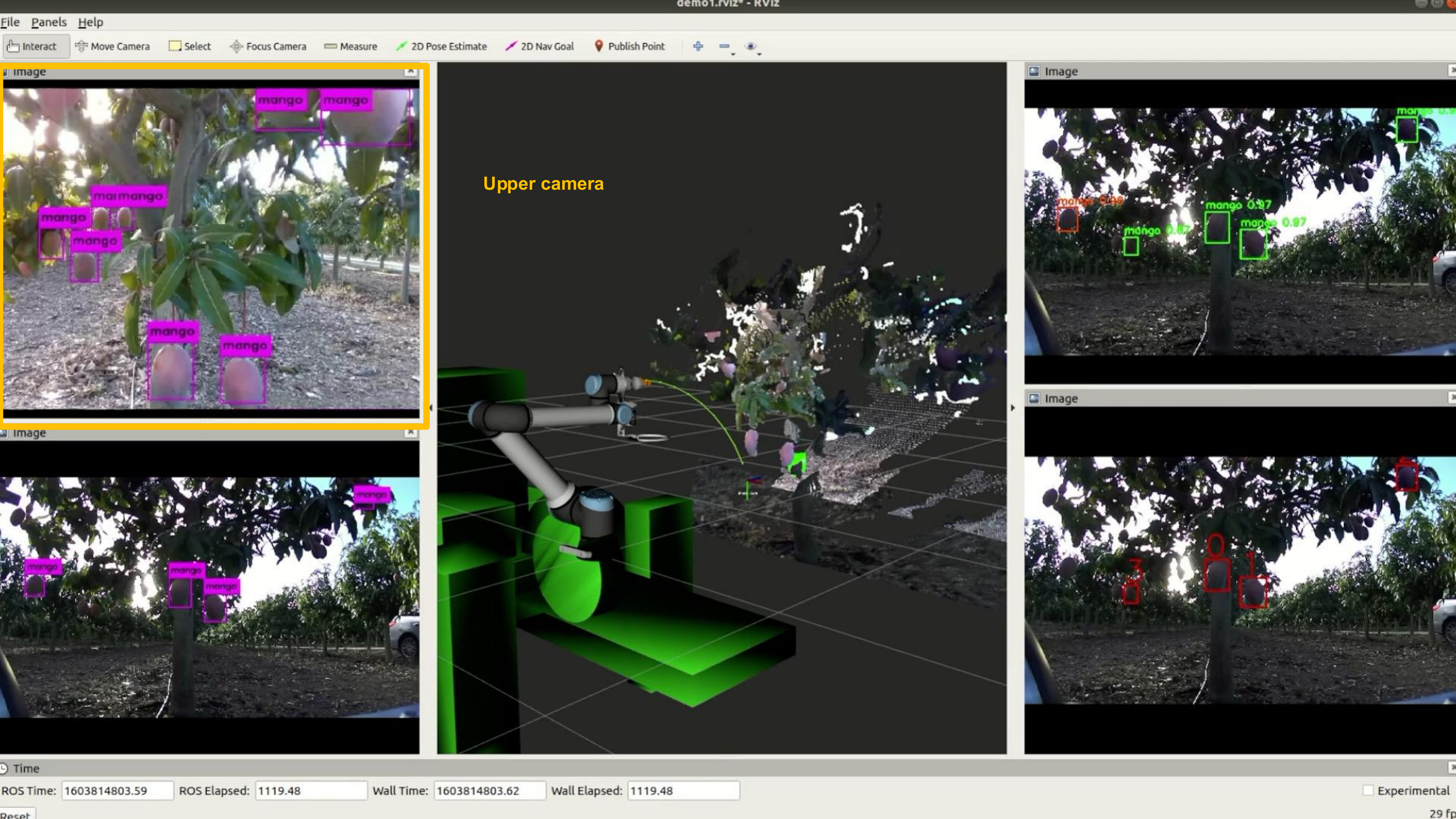
Wall Elapsed: 1119.48
Experimental
Reset
29 fps


demo1.rviz* - RViz
File
Panels
Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Image
mango
Upper camera
Image
Time
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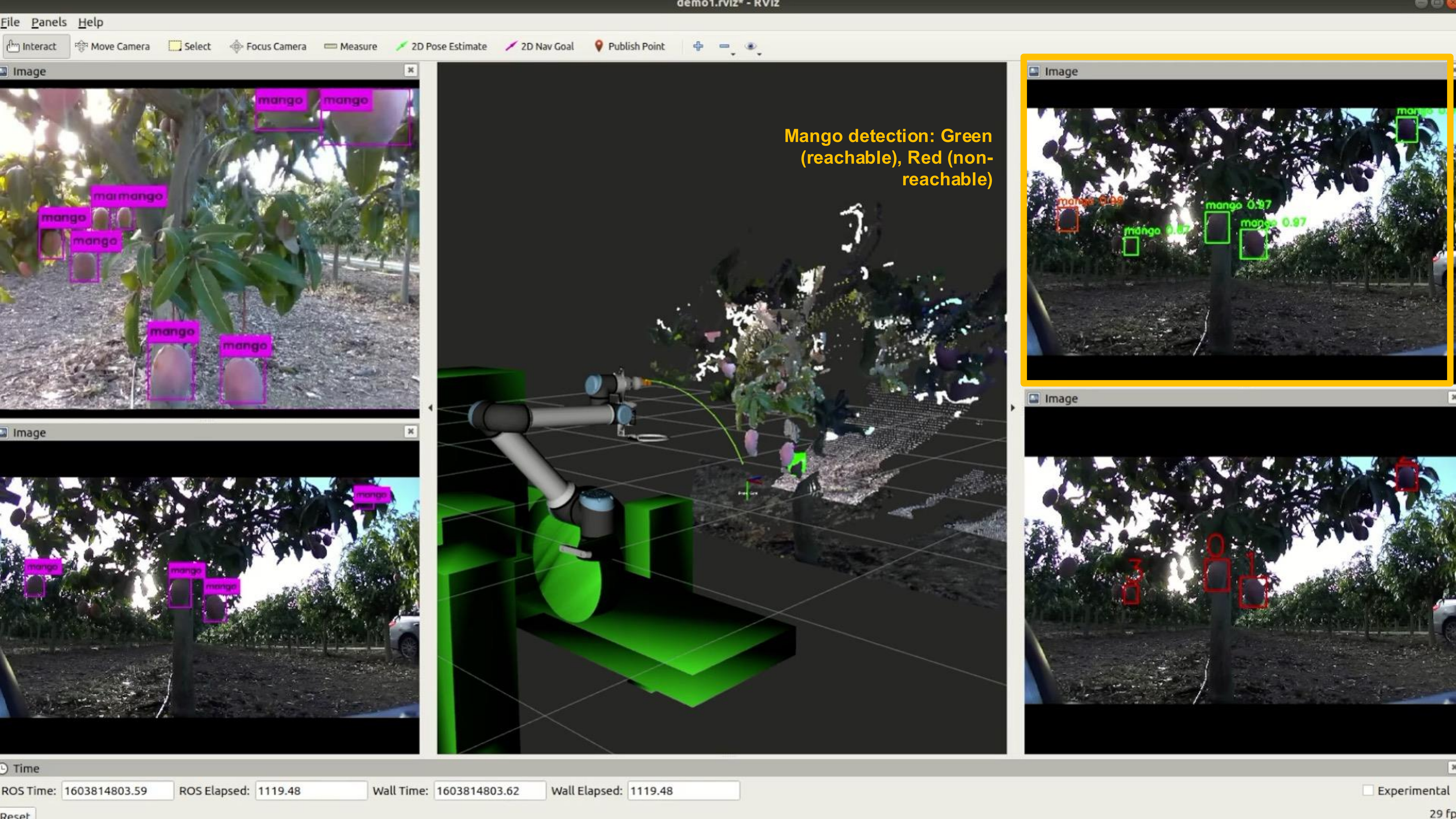
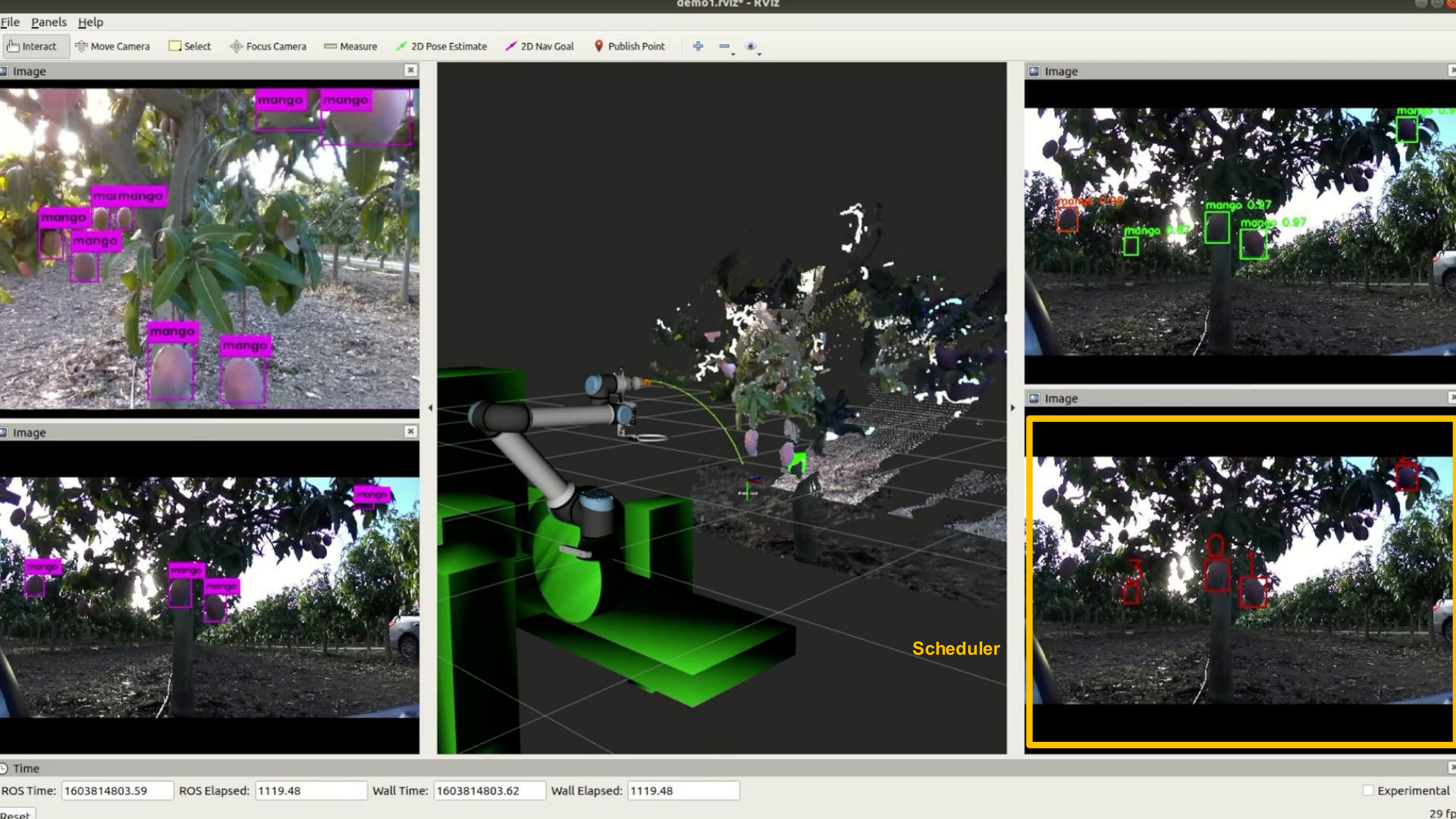
ROS Elapsed: 1119.48
Wall Time: 1603814803.62
Wall Elapsed: 1119.48
Experimental
Reset


demo1.rviz* - RViz
File Panels Help
Interact Move Camera Select Focus Camera Measure 2D Pose Estimate 2D Nav Goal Publish Point
Image
mango
Image
Image
Mango detection: Green (reachable), Red (non-reachable)
Image
Time
ROS Time: 1603814803.59 ROS Elapsed: 1119.48 Wall Time: 1603814803.62 Wall Elapsed: 1119.48
Experimental
Reset



demo1.rviz* - RViz
File
Panels
Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Image
mango
Image
Image
Image
Scheduler
Time
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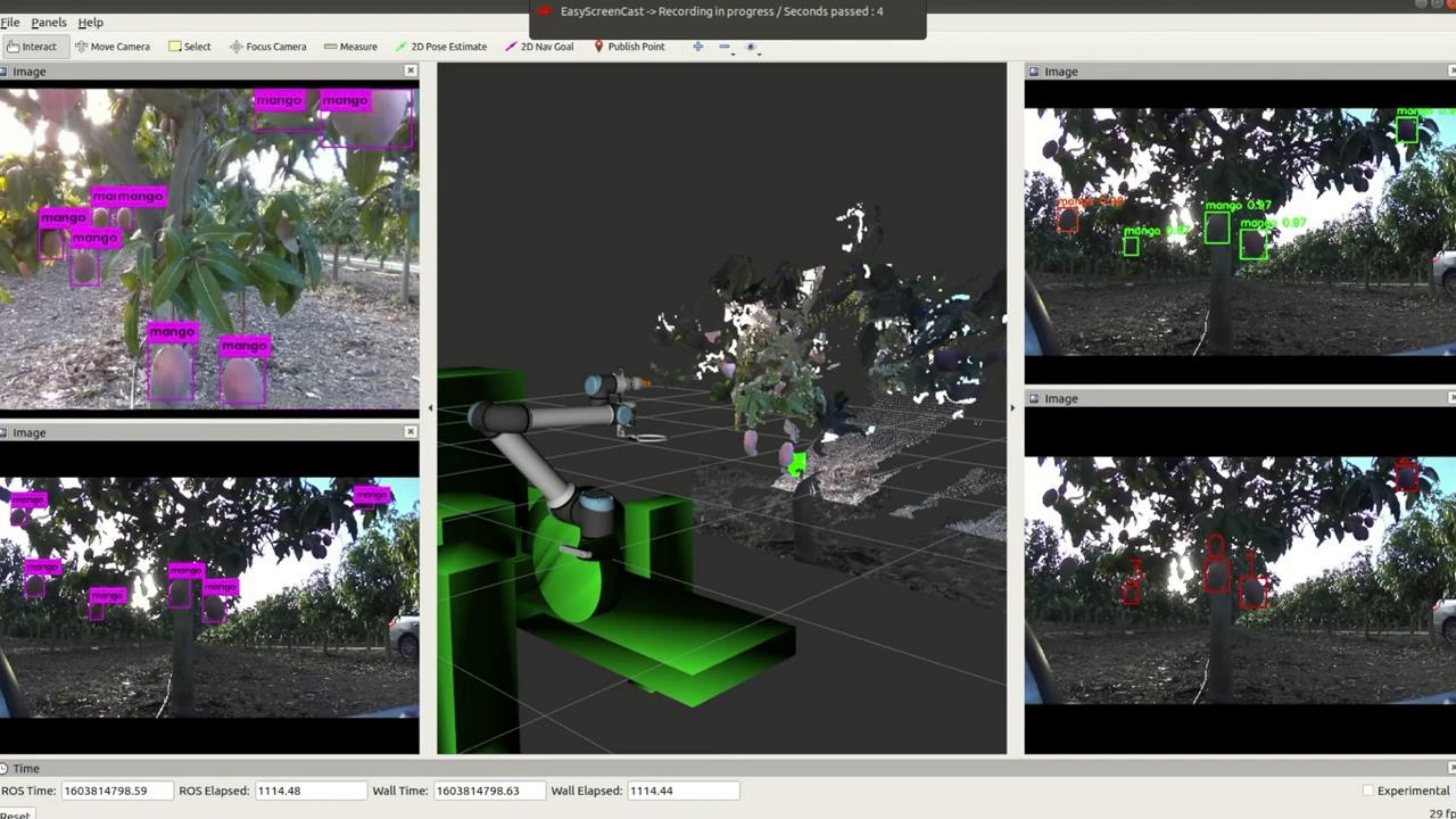
Wall Time: 1603814803.62
Wall Elapsed: 1119.48
Experimental
Reset


EasyScreenCast -> Recording in progress / Seconds passed : 4
File
Panels
Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Image
mango
mango
mango
mango
mango
mango
mango
Image
Image
mango 0.97
mango 0.97
Image
Time
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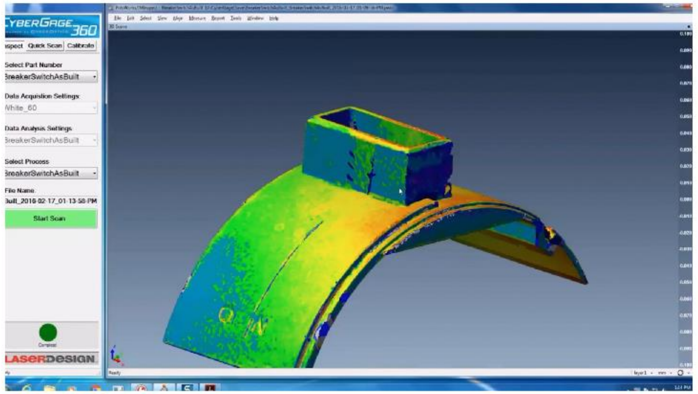
ROS Elapsed: 1114.48
Wall Time: 1603814798.63
Wall Elapsed: 1114.44
Experimental
Reset

Obtaining 3D information

- Applications



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vectra
CANFIELD

Obtaining 3D information

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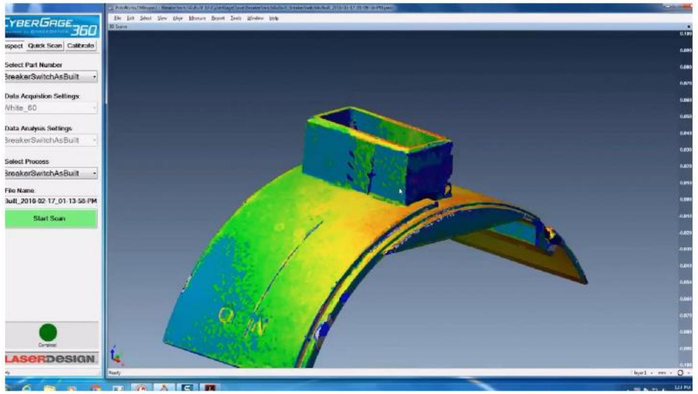

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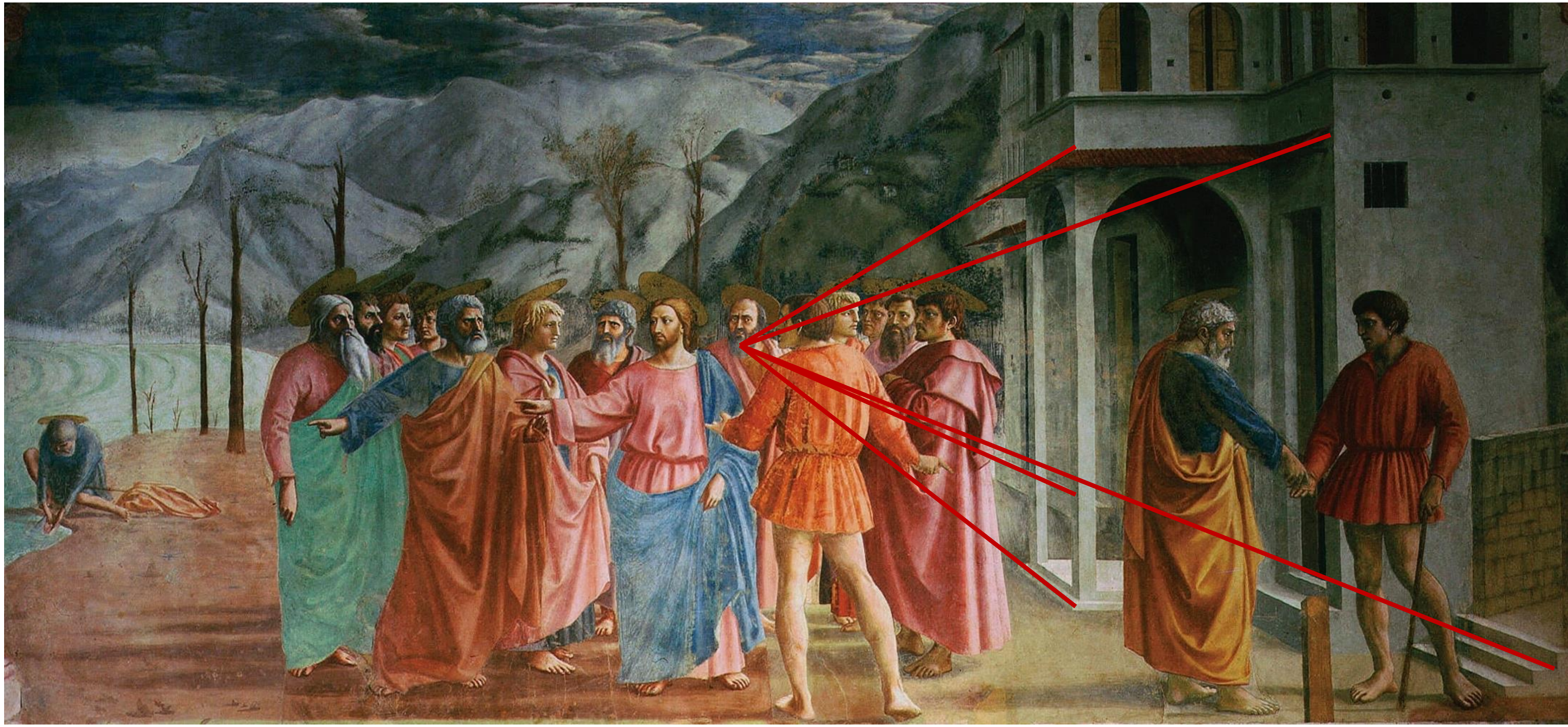
Renaissance: first perspective paintings

Year 1415: Fillipo Brunelleshi

Year 1426: Masaccio

Il Pagamento del tributo. Capella Brancacci, Florence



https://it.wikipedia.org/wiki/Pagamento_del_tributo#/media/File:Masaccio7.jpg

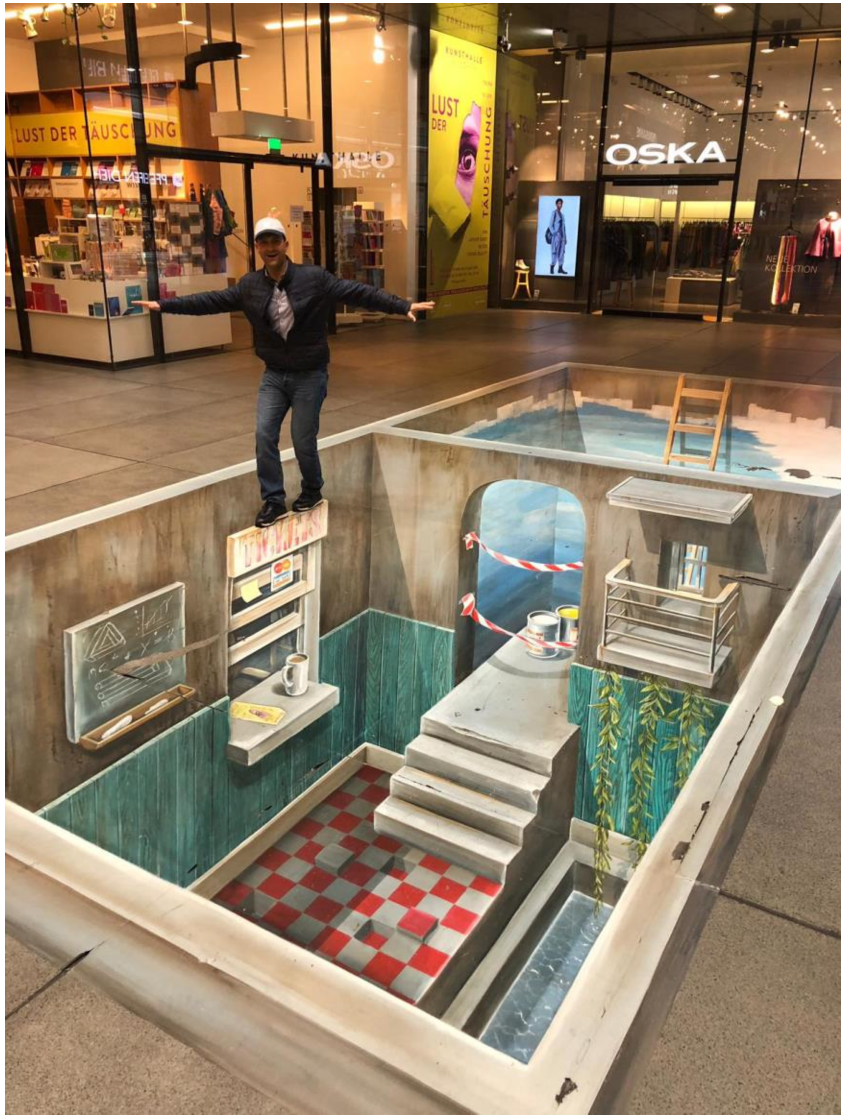
Renaissance: first perspective paintings

Year 1470: Piero della Francesca

"The Ideal City"

https://www.nytimes.com/2012/05/09/arts/09iht-conway09.html

Perspective Projection



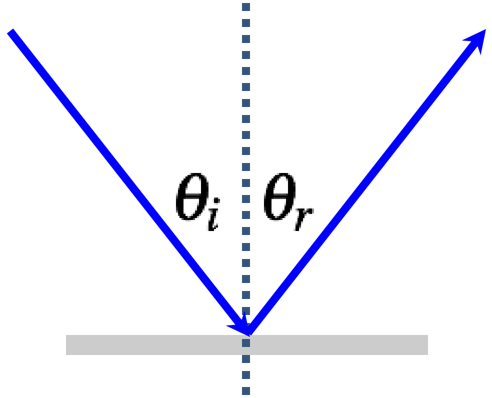
Perspective Projection

http://www.julianbeever.net/

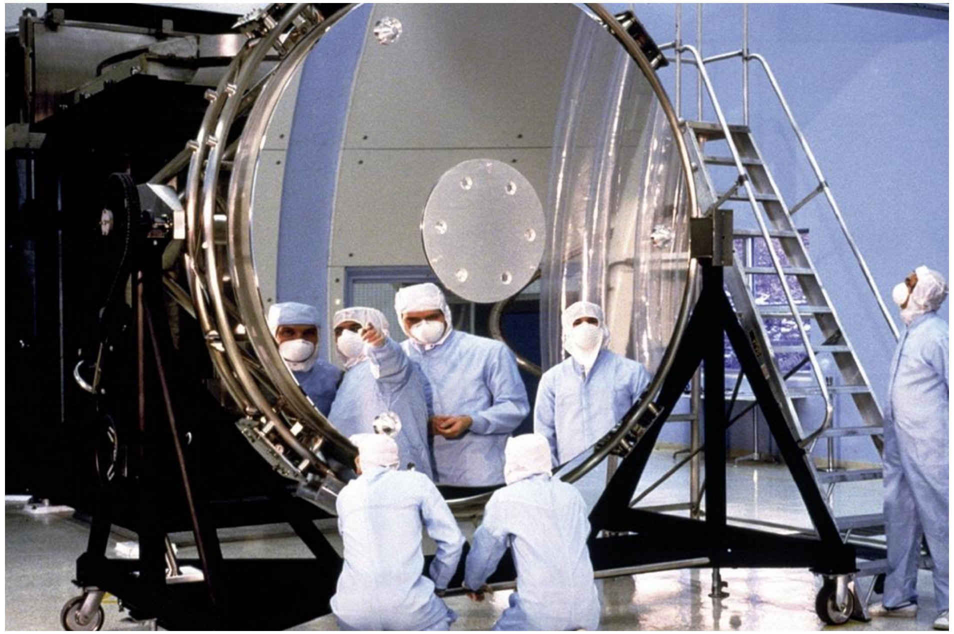
Perspective Projection

http://www.julianbeever.net/

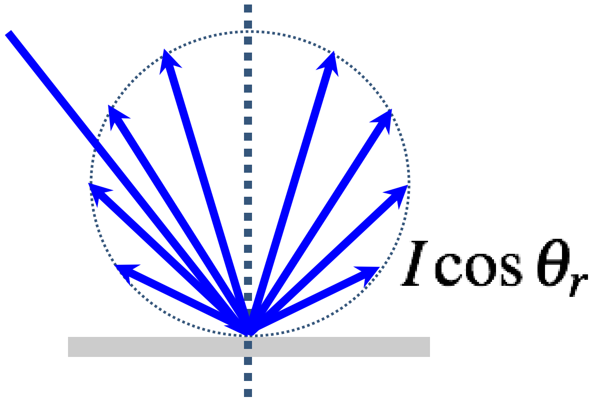
Light Reflection



- **Specular reflection**
  - **angle of incidence equals angle of reflection**



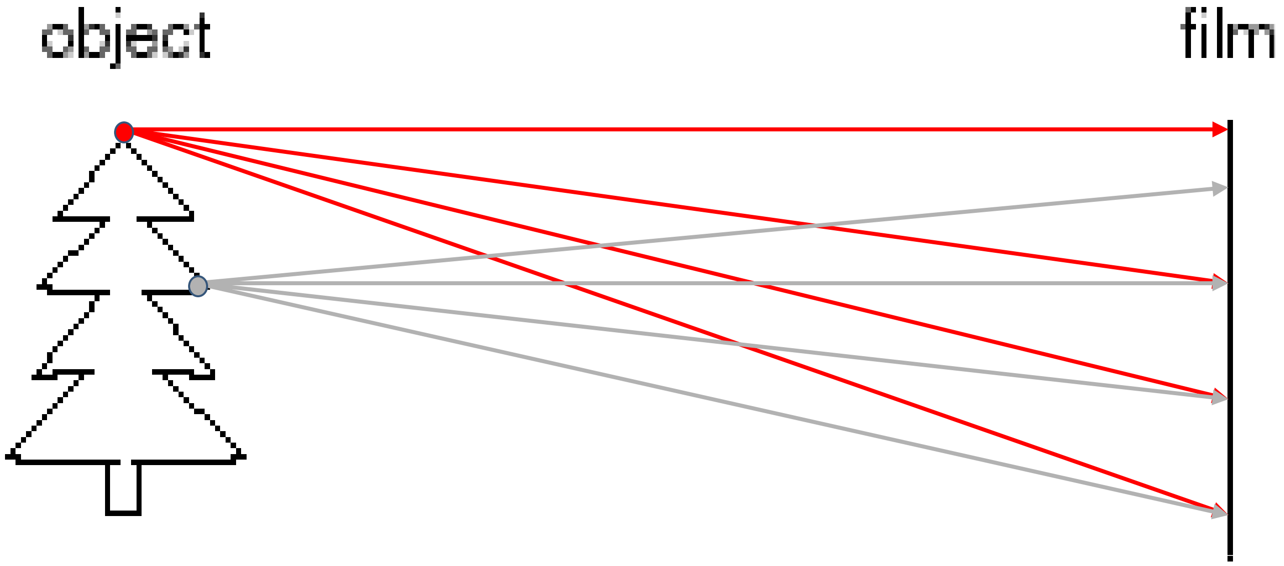
Light Reflection



- **Lambertian reflection**
  - **diffuse/matte surface**
  - **brightness invariant to observer's angle of view**

Johann Heinrich Lambert 1728-1777

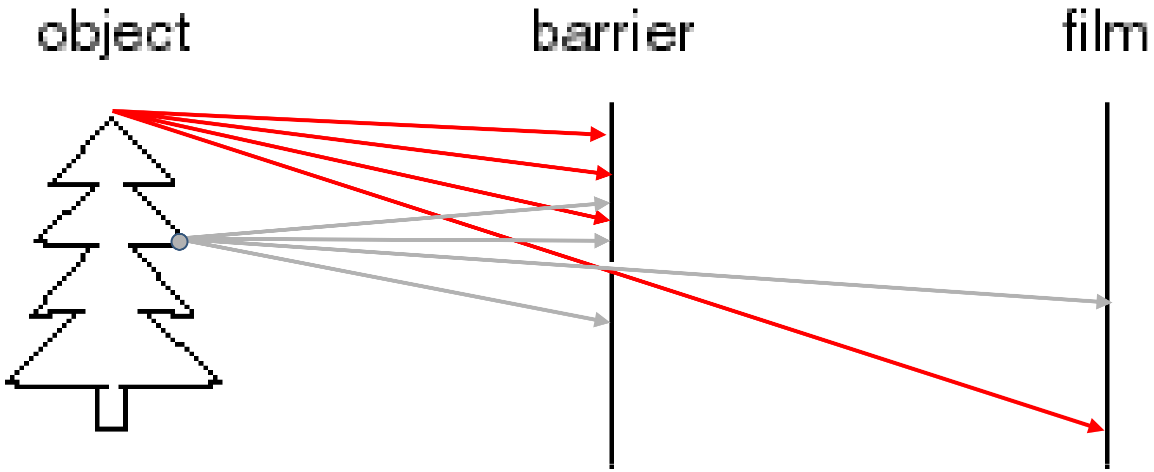
Pinhole Camera



- We place a photosensitive film in front of the object. Would this produce a reasonable image?
- What should be do?

Slide by Steve Seitz

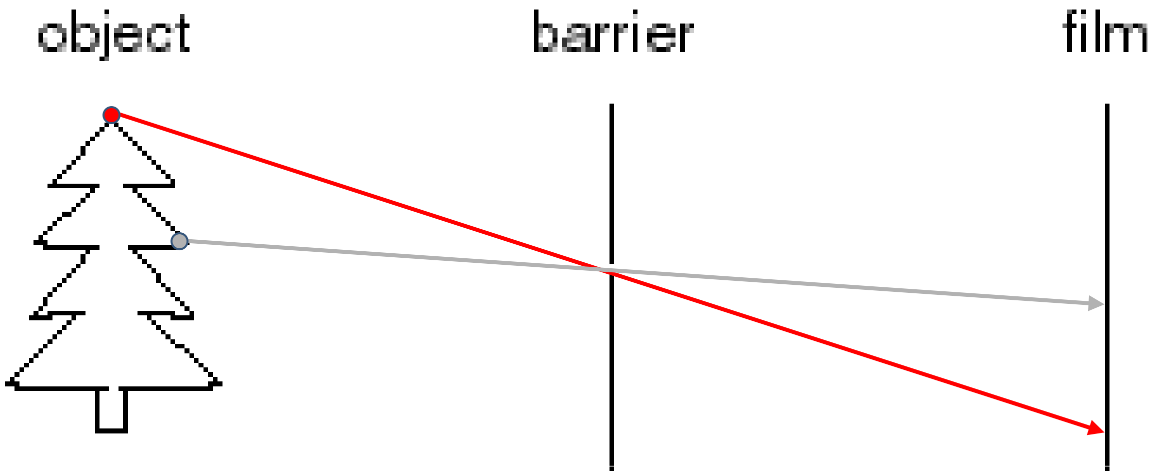
Pinhole Camera



Pinhole camera

- Add a barrier with a small pinhole

Pinhole Camera



- Pinhole Model
- Camera obscura

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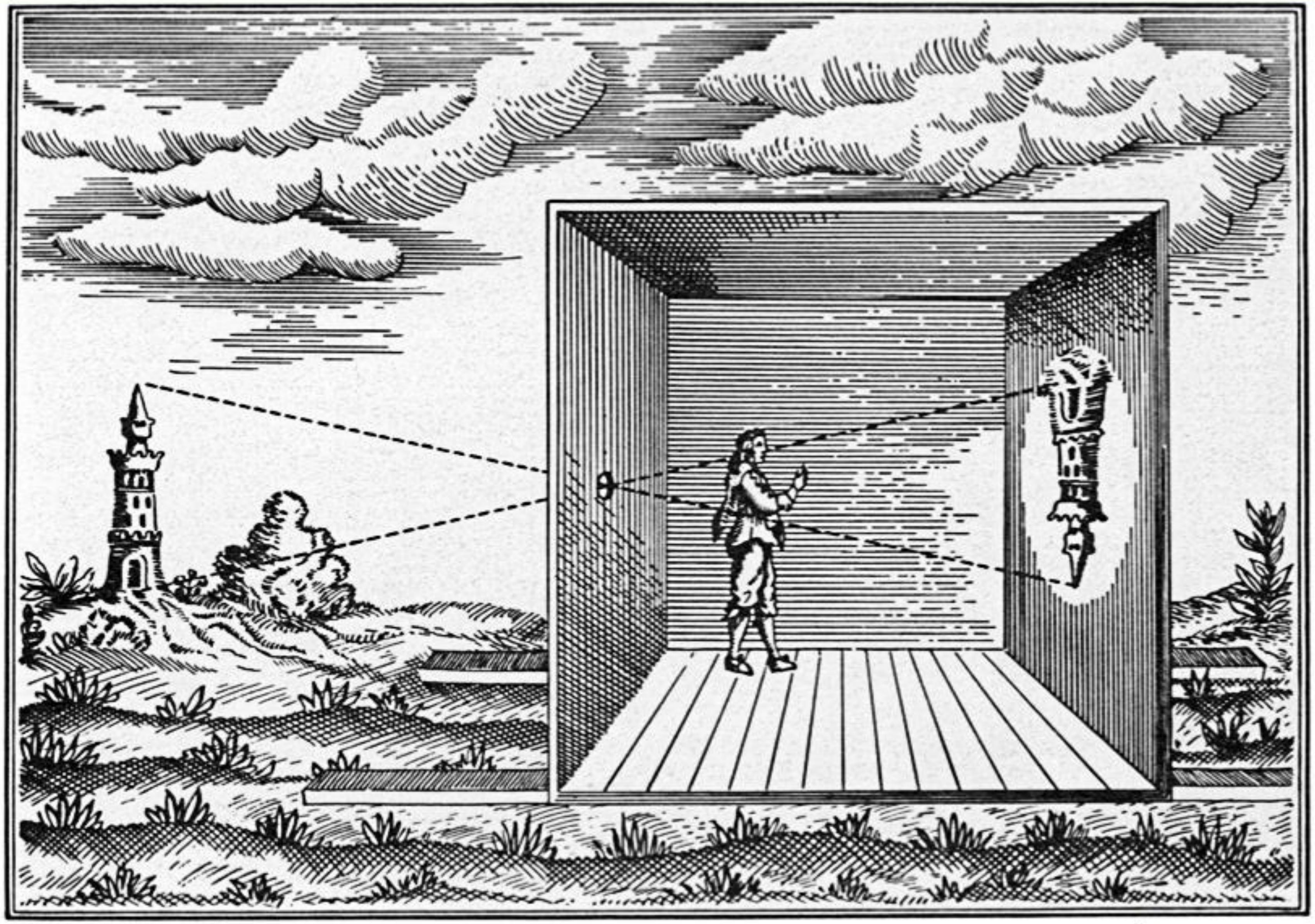
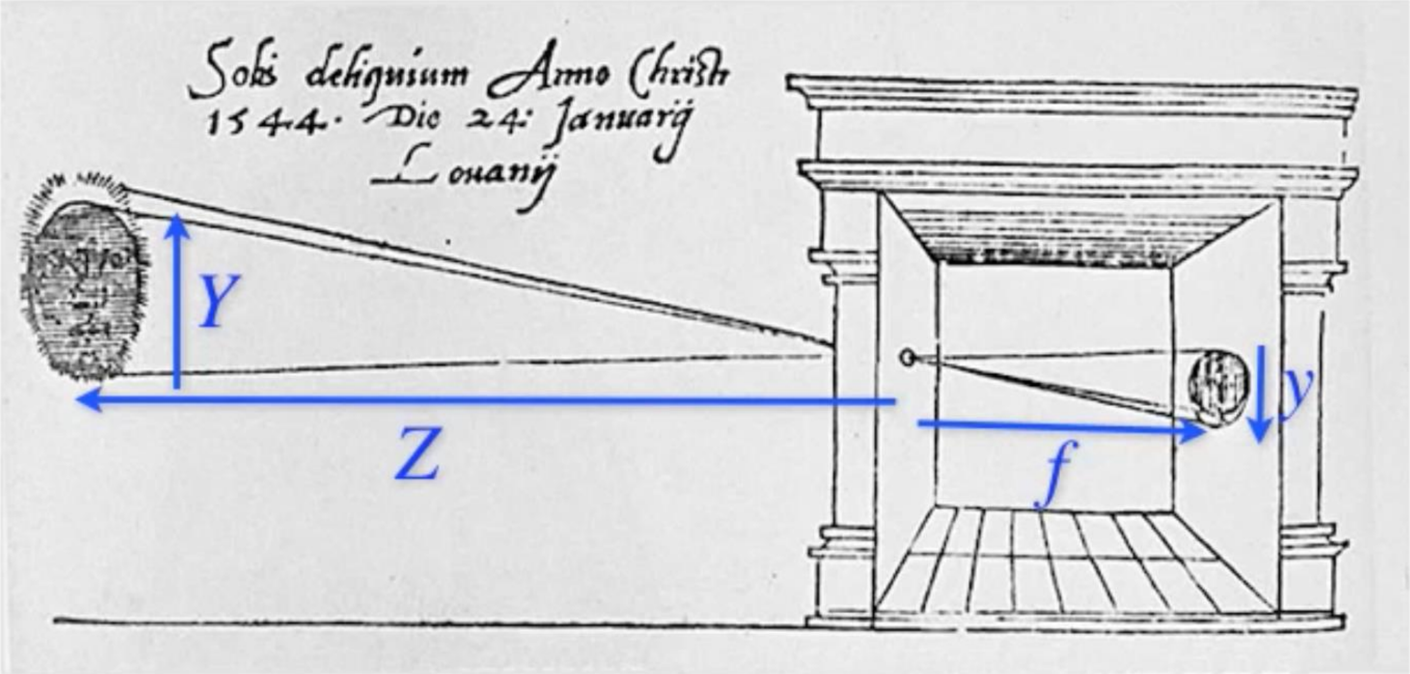
Pinhole Camera



Pinhole Camera

Abelardo Morell: https://www.youtube.com/watch?v=48cQLsOmbWc

Pinhole Camera: equations



$$\frac{Y}{Z} = \frac{y}{f}$$

- Image formation is the mapping of scene points to the image plane

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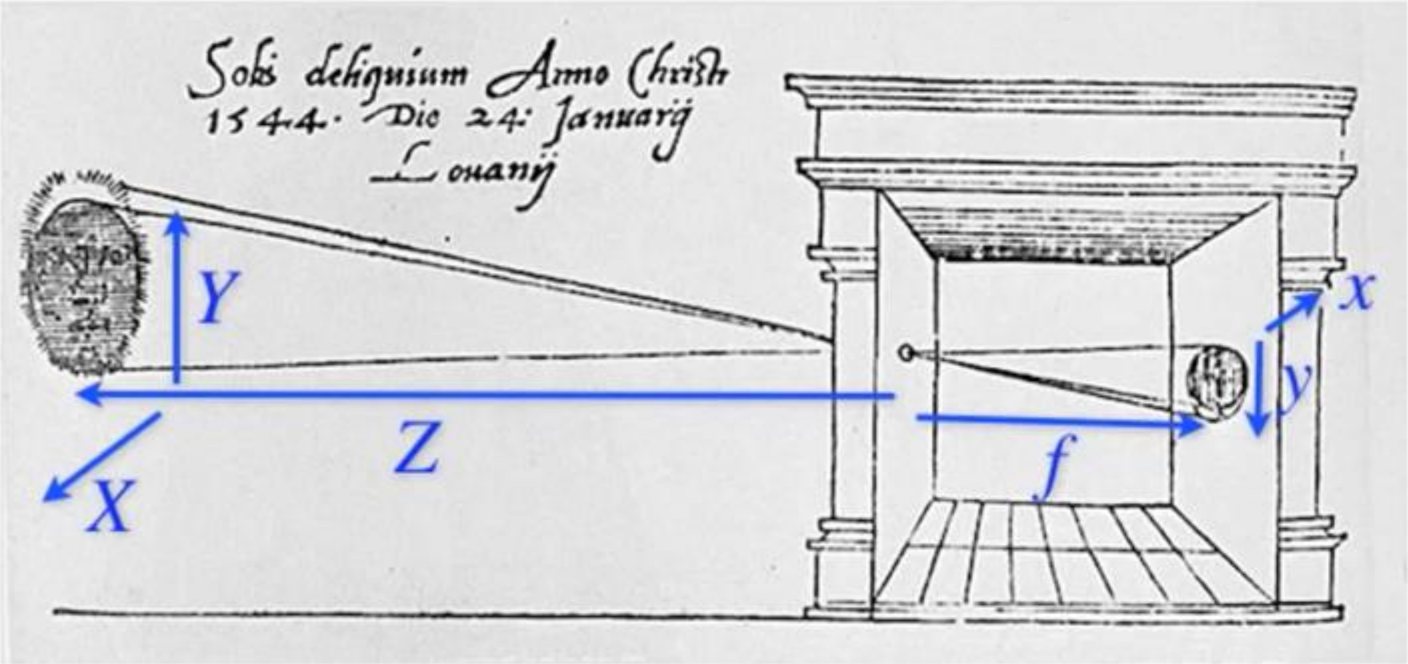
Pinhole Camera: equations



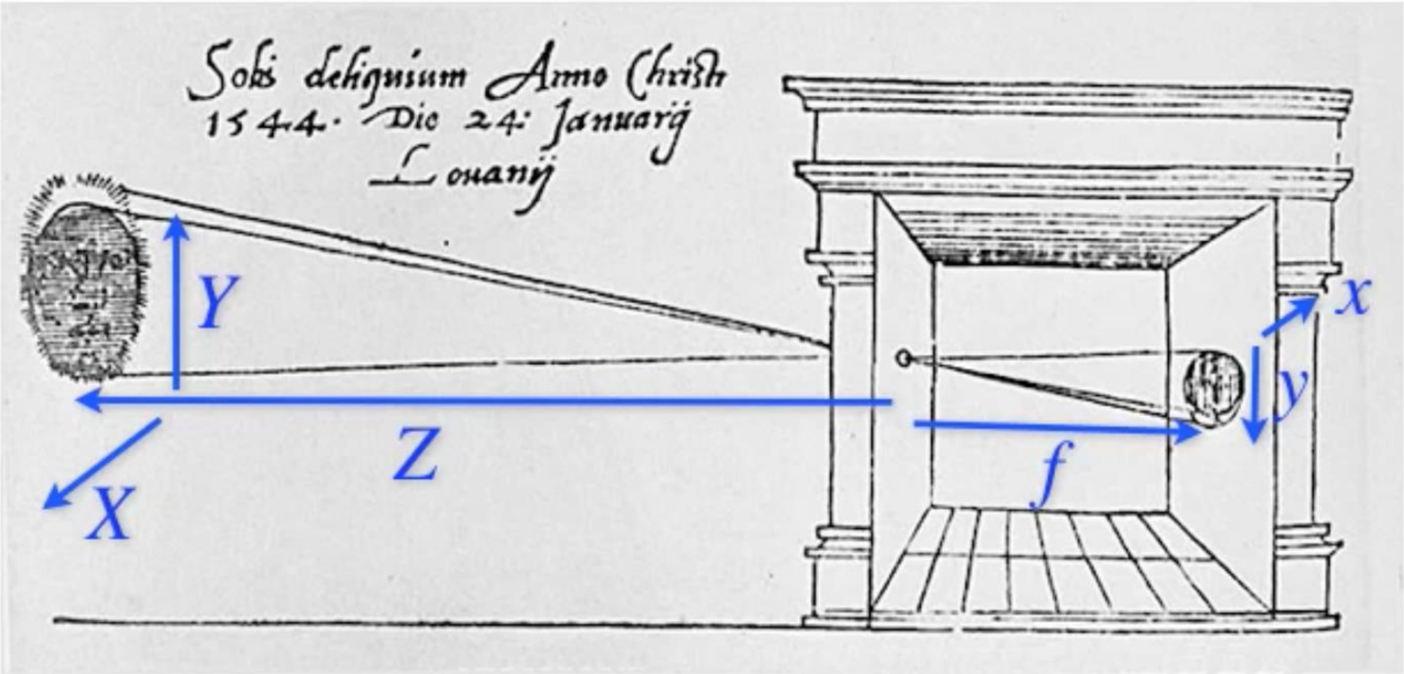
$$\frac{Y}{Z} = \frac{y}{f}$$

$$\frac{X}{Z} = \frac{x}{f}$$

- Image formation is the mapping of scene points to the image plane

Slide by Steve Seitz

Pinhole Camera: equations



$$\frac{Y}{Z} = \frac{y}{f}$$

$$\frac{X}{Z} = \frac{x}{f}$$

$$x = \frac{fX}{Z}, y = \frac{fY}{Z}$$

$$(X,Y,Z) \mapsto (x,y)$$

$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

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Forced Perspective

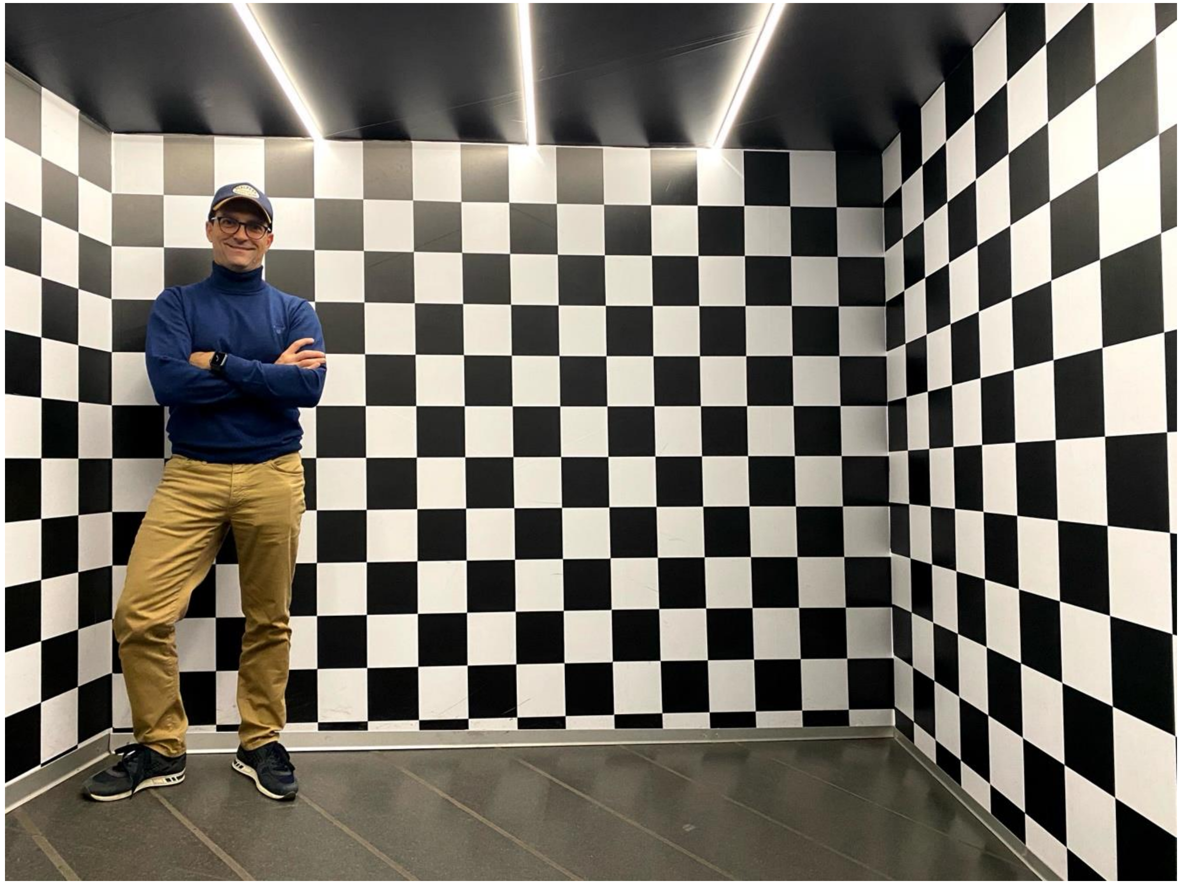
2013

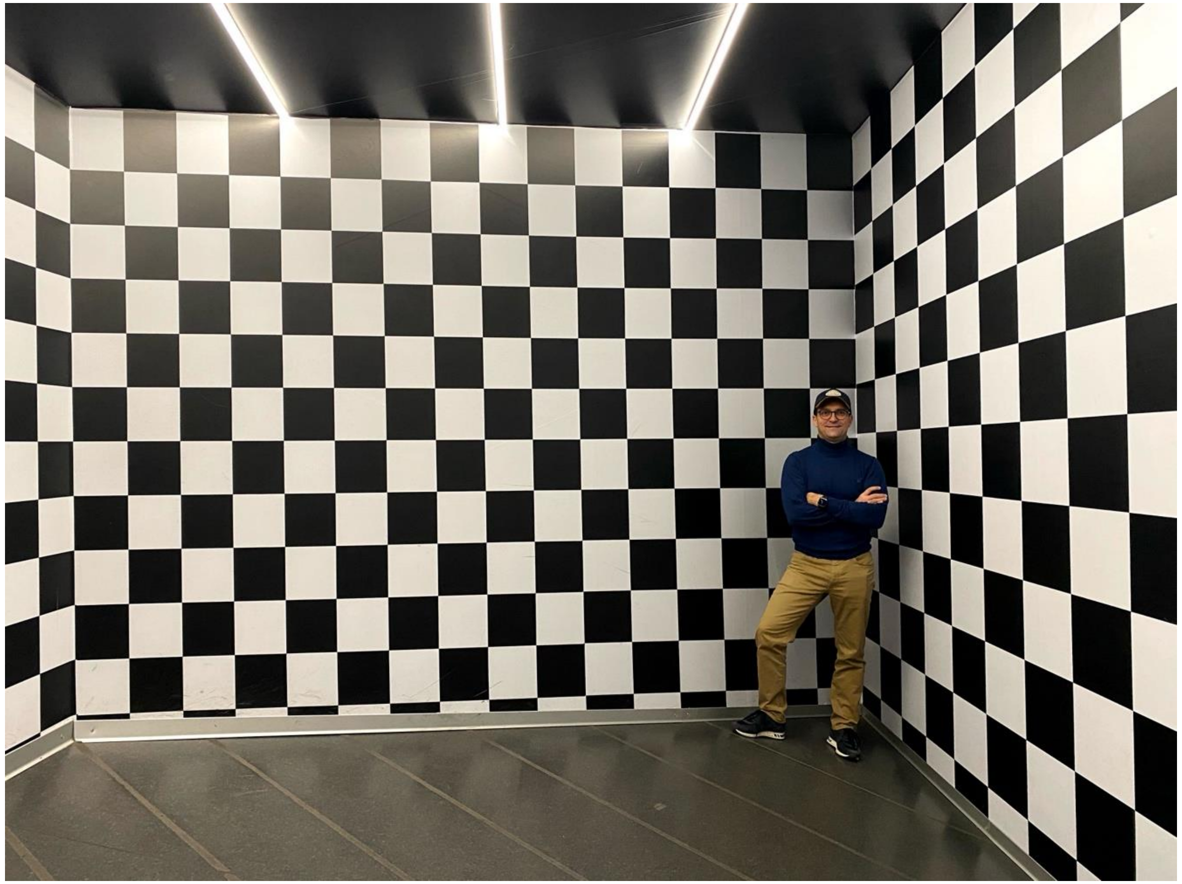
Kenzie Saunders | CC BY-ND 2.0

On Hands *2011*

Seongbin Im | CC BY-SA 2.0

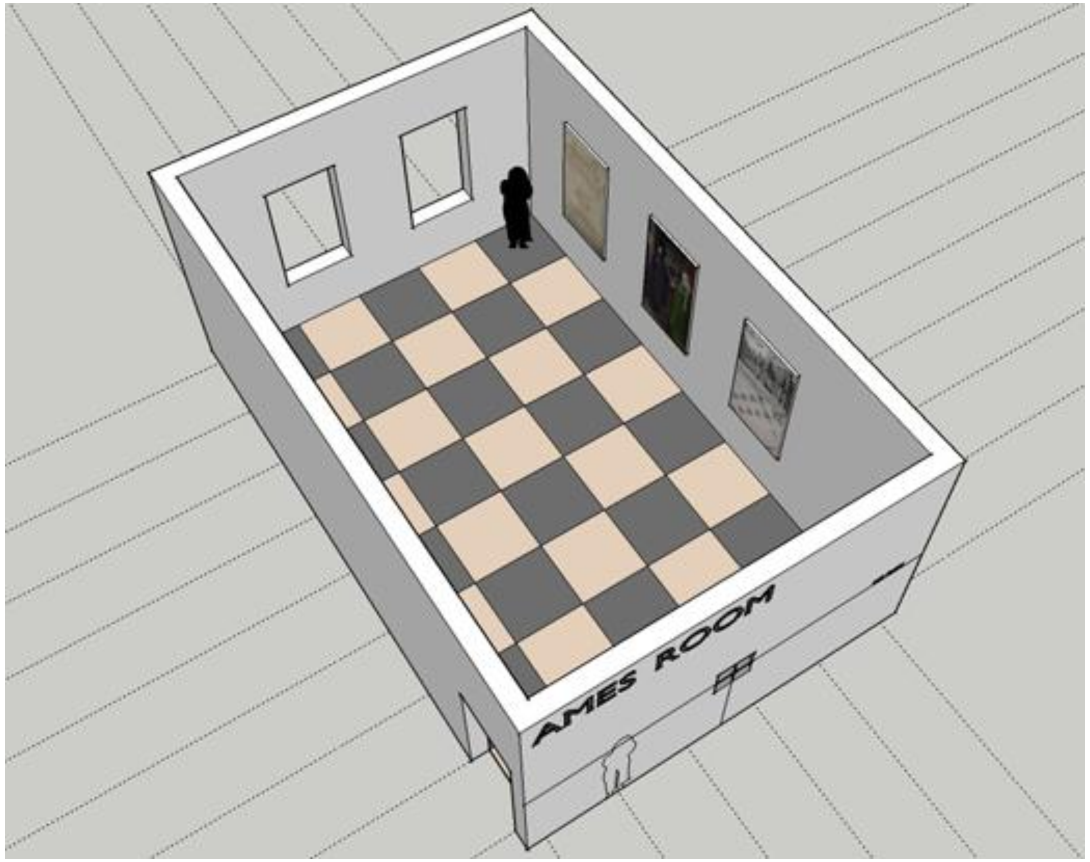
Ames Room

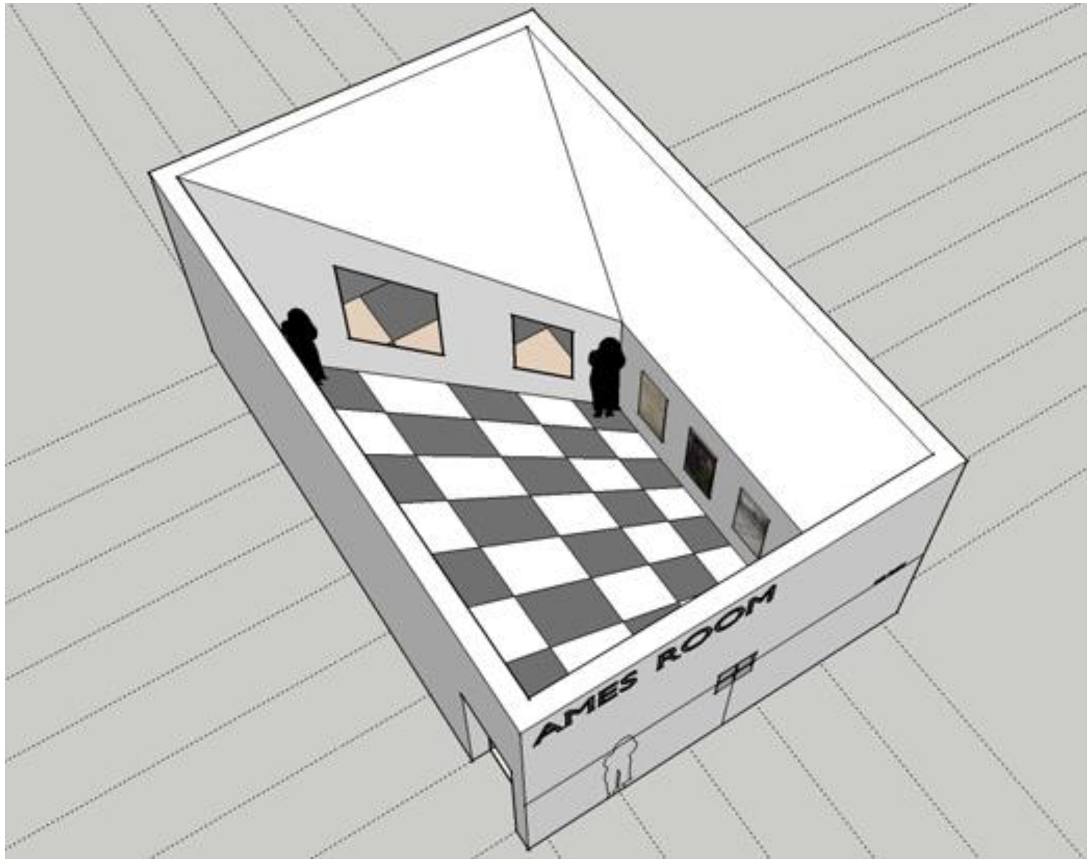





UNIVERSITY
OF VIRGINIA
THE NORTH FACE

Ames Room



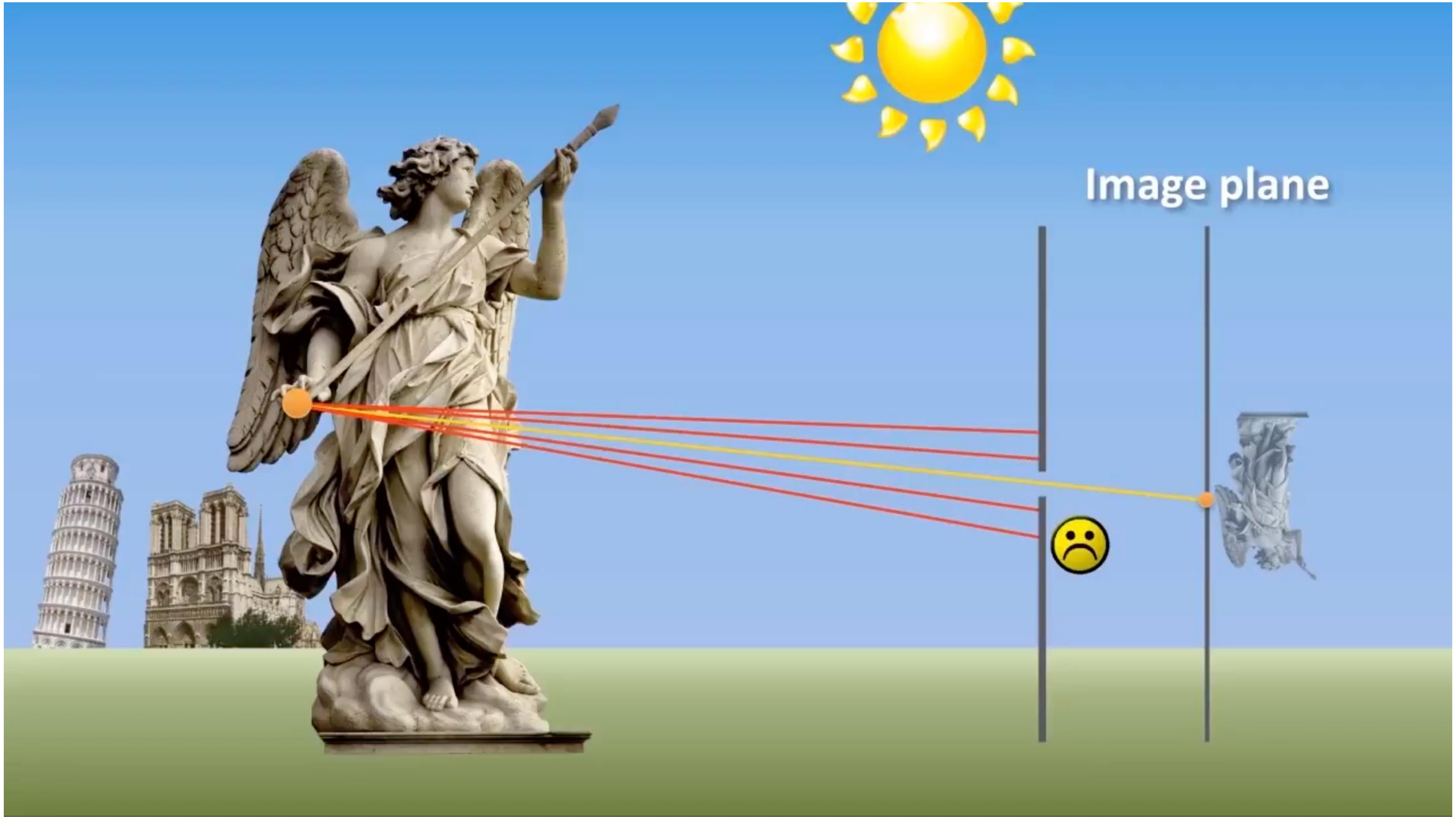


http://perspectiveresources.blogspot.com/2011/06/how-to-construct-ames-room.html

Pinhole Model

Slide by Peter Corke

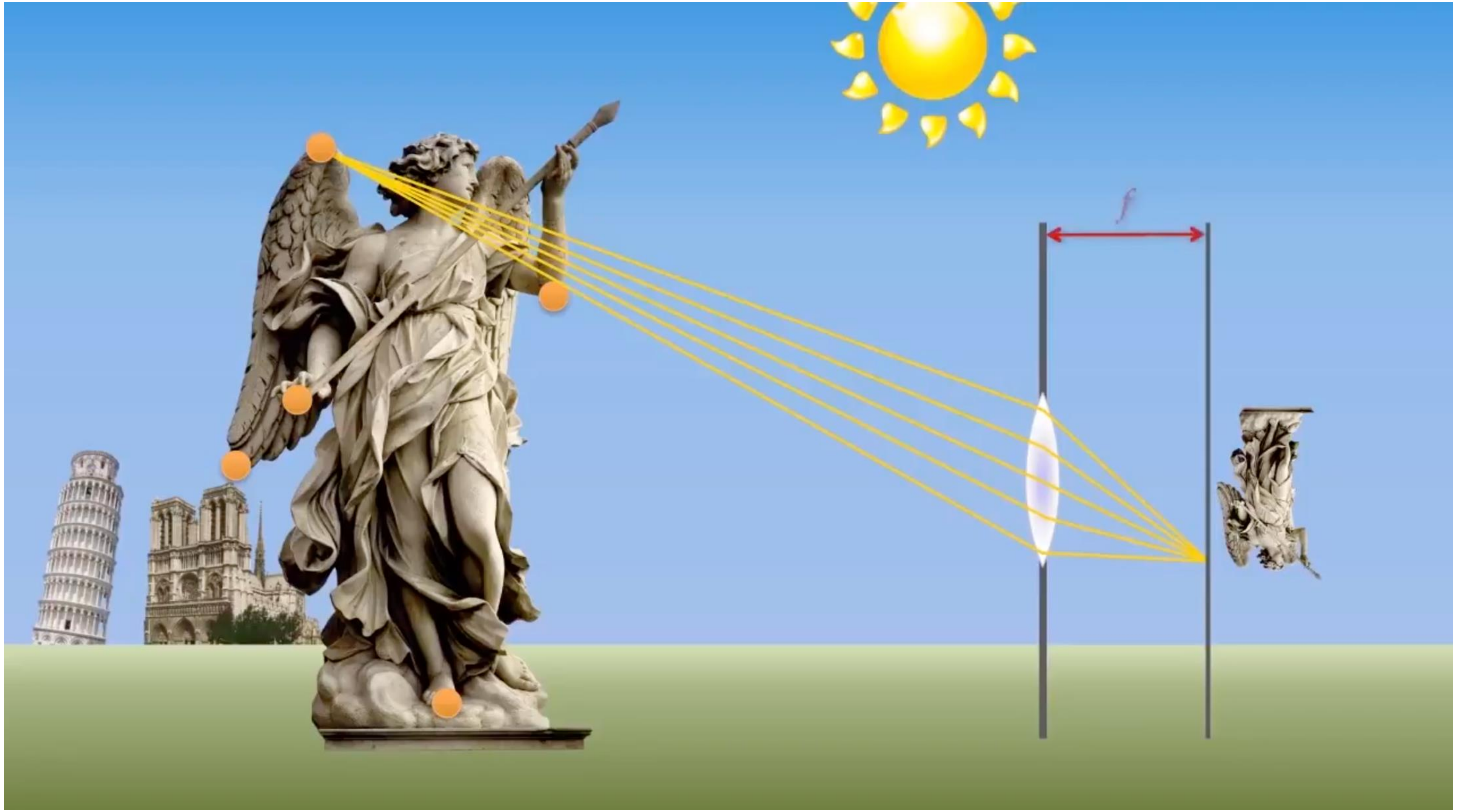
Pinhole Model



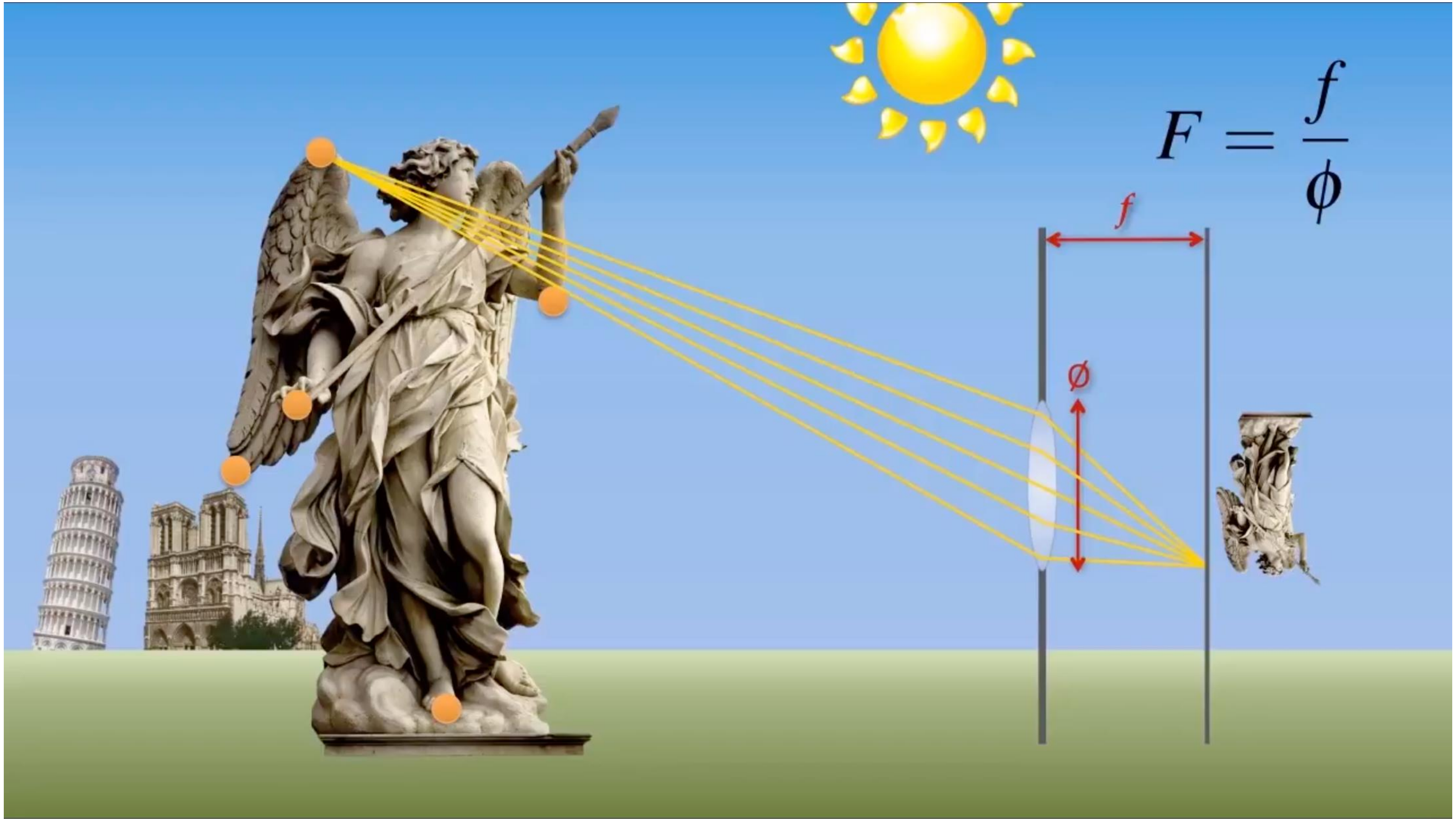
Slide by Peter Corke

Cameras use LENSES

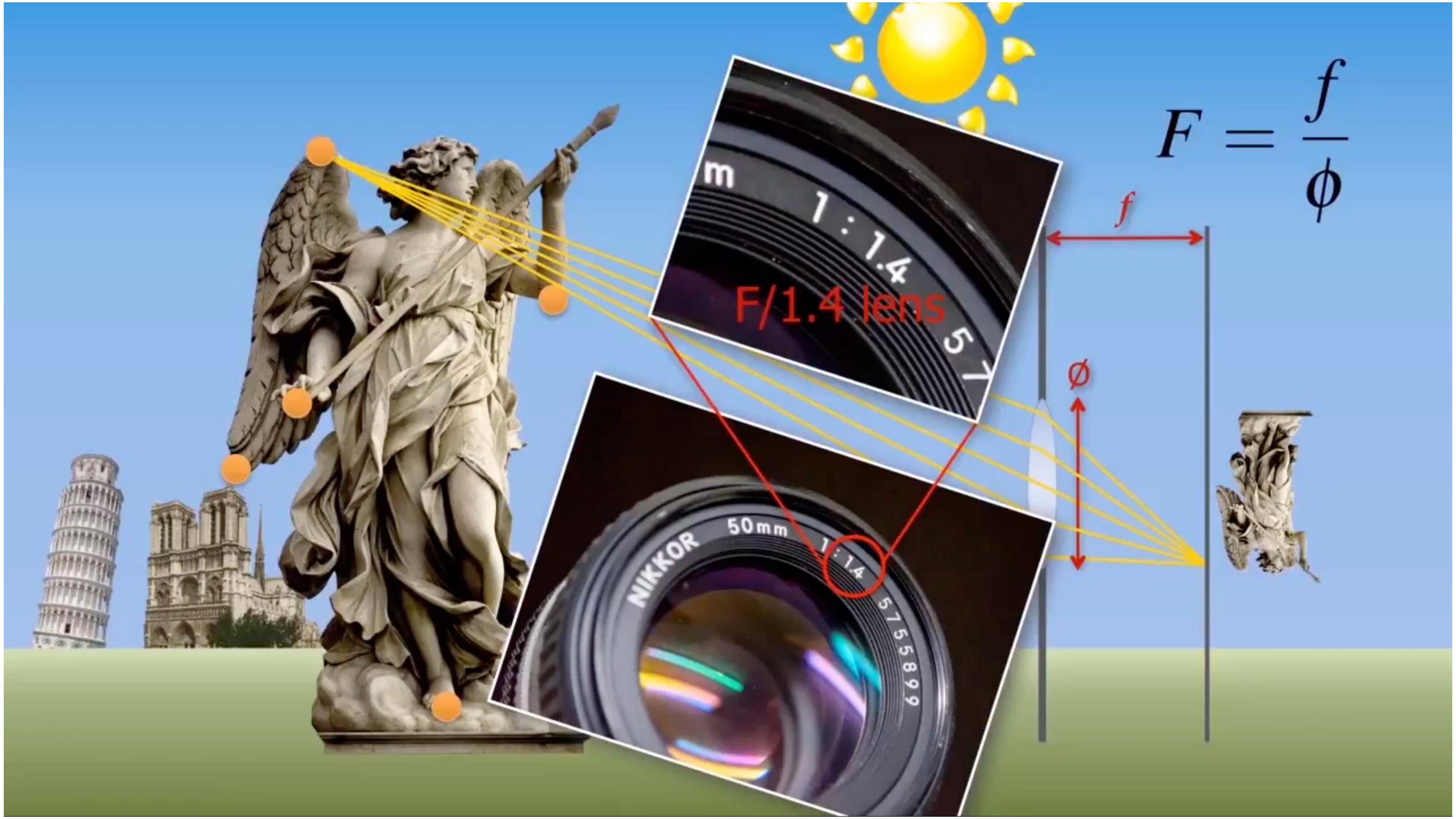
Slide by Peter Corke



Slide by Peter Corke

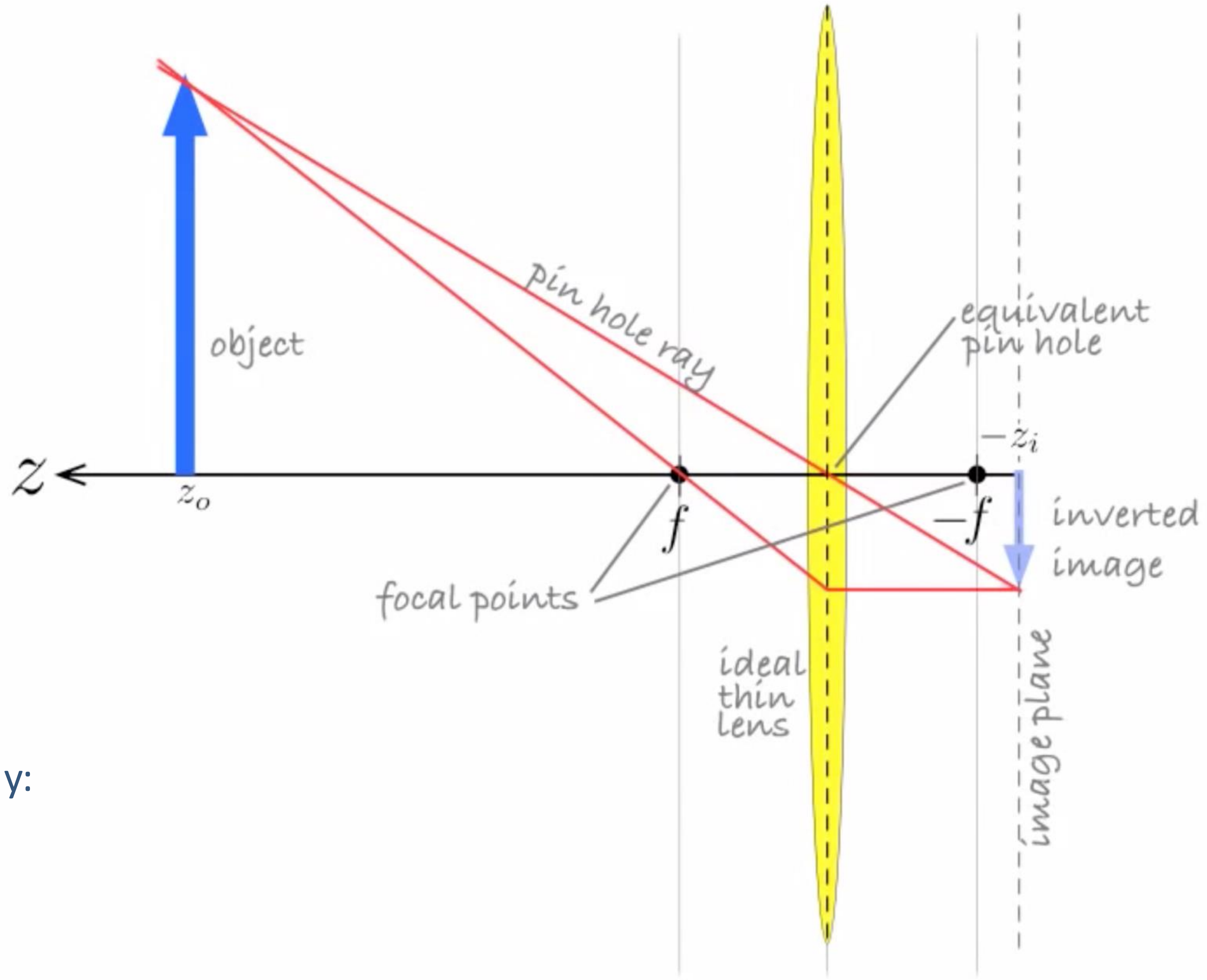


Slide by Peter Corke



Slide by Peter Corke

Thin Lens Model



$$\frac{1}{z_0} + \frac{1}{z_i} = \frac{1}{f}$$

Foscusing of objects far away:

$z_0 \rightarrow \infty$

$z_i \rightarrow f$

Slide by Peter Corke


OLYMPUS

Price to pay when projecting the world in 2D

$$(X, Y, Z) \mapsto (x, y)$$

$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

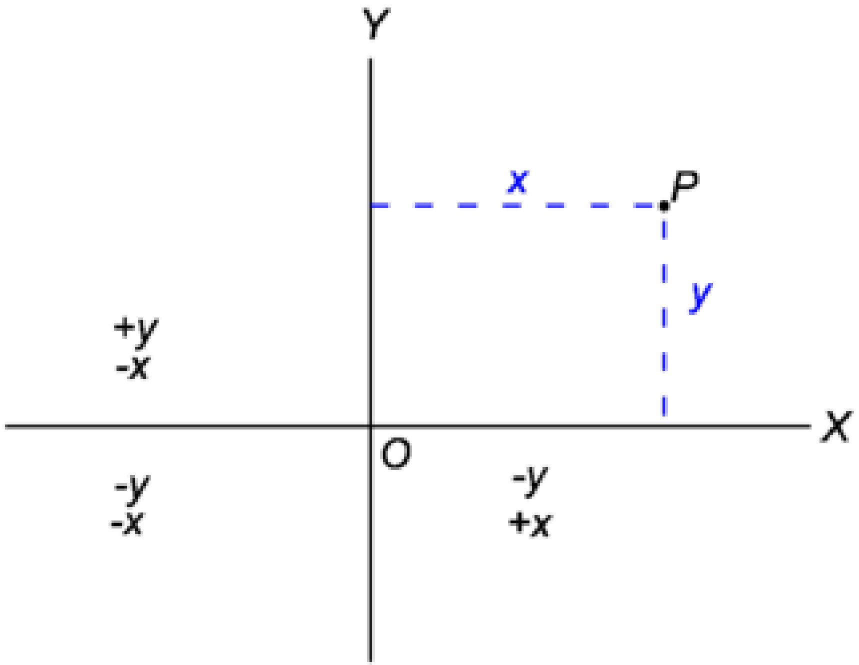
Homogeneous Coordinates

Universitat de Girona

Cartesian Coord. → Homogeneous Coord.

$$\boldsymbol{p} = (x, y)^T \qquad \tilde{\boldsymbol{p}} = (x, y, 1)^T$$

$$\boldsymbol{p} \in \mathbb{R}^2 \qquad \boldsymbol{p} \in \mathbb{P}^2$$



Homogeneous Coord. → Cartesian C.

$$\tilde{\boldsymbol{p}} = (\tilde{x}, \tilde{y}, \tilde{z})^T$$

$$x = \frac{\tilde{x}}{\tilde{z}}$$

$$y = \frac{\tilde{y}}{\tilde{z}}$$

$$\boldsymbol{p} = (x, y)^T$$

Eq. of a Line in homogeneous Coordinates

$\tilde{\ell} = (l_1, l_2, l_3)^T$

"Point equation of a line"

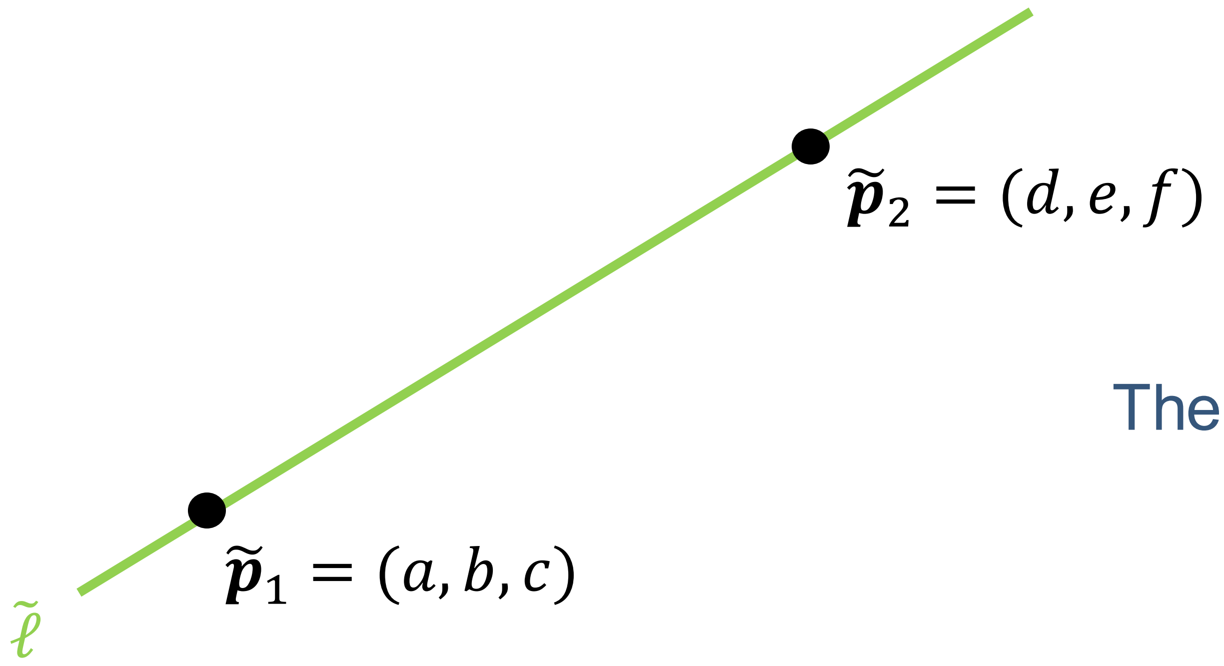
Then $\rightarrow$ $\tilde{\ell}^T \tilde{\boldsymbol{p}} = 0$

$$l_1\tilde{x} + l_2\tilde{y} + l_3\tilde{z} = 0$$

Instead of $\rightarrow$ $y = mx + c$

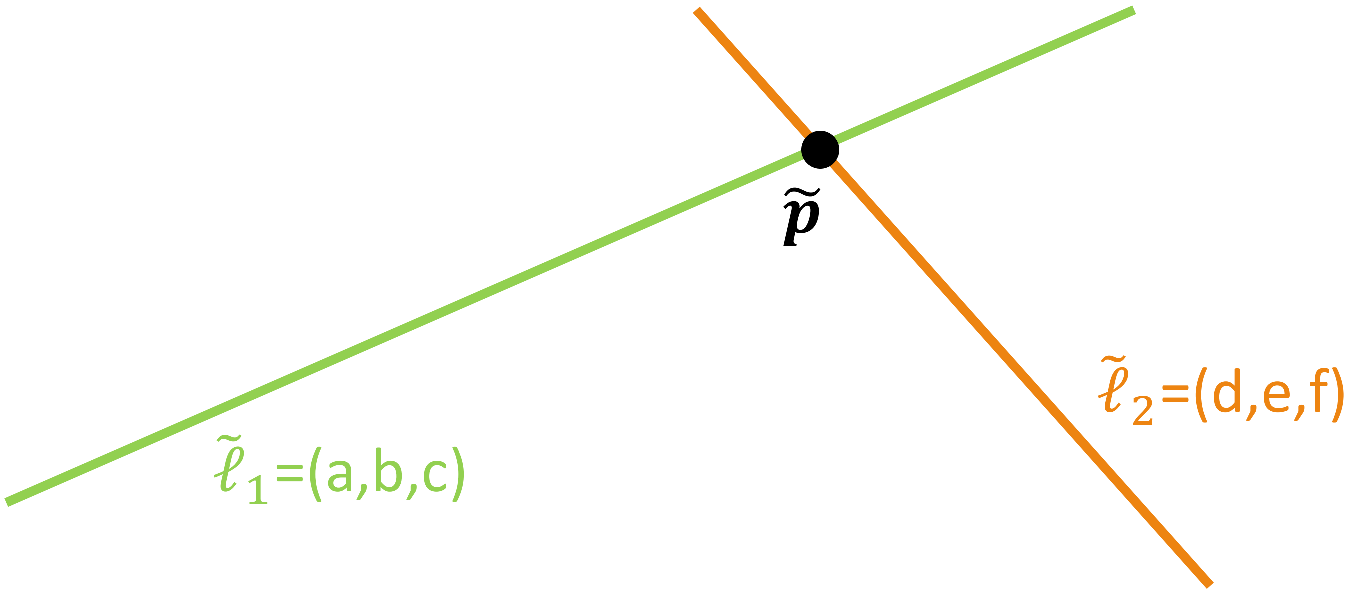
Equació d'una línia que passa per dos punts

Universitat de Girona



Then → $\tilde{\ell} = \tilde{\boldsymbol{p}}_1 \times \tilde{\boldsymbol{p}}_1$

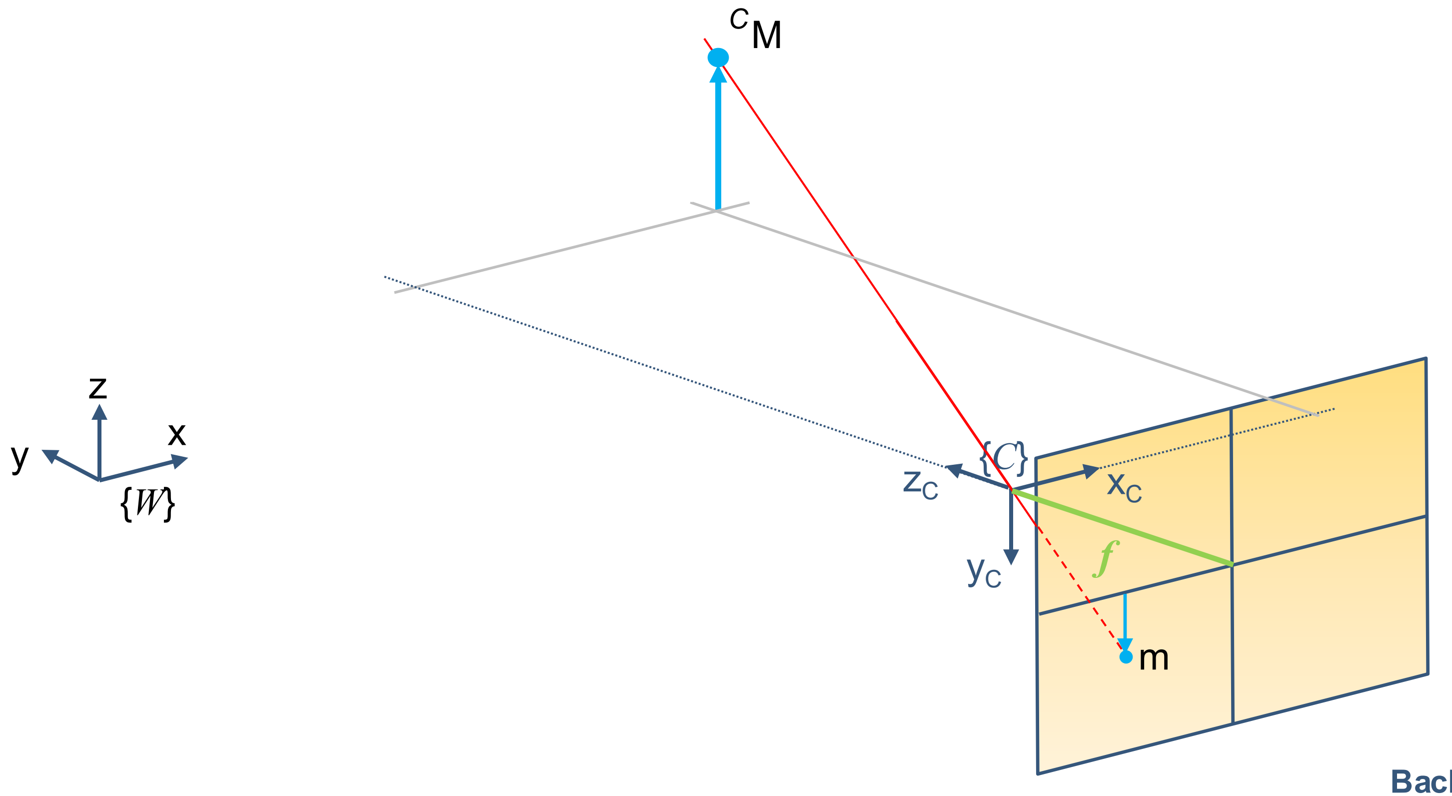
Intersection of two Lines



$$\tilde{\boldsymbol{p}} = \tilde{\ell}_1 \times \tilde{\ell}_2$$

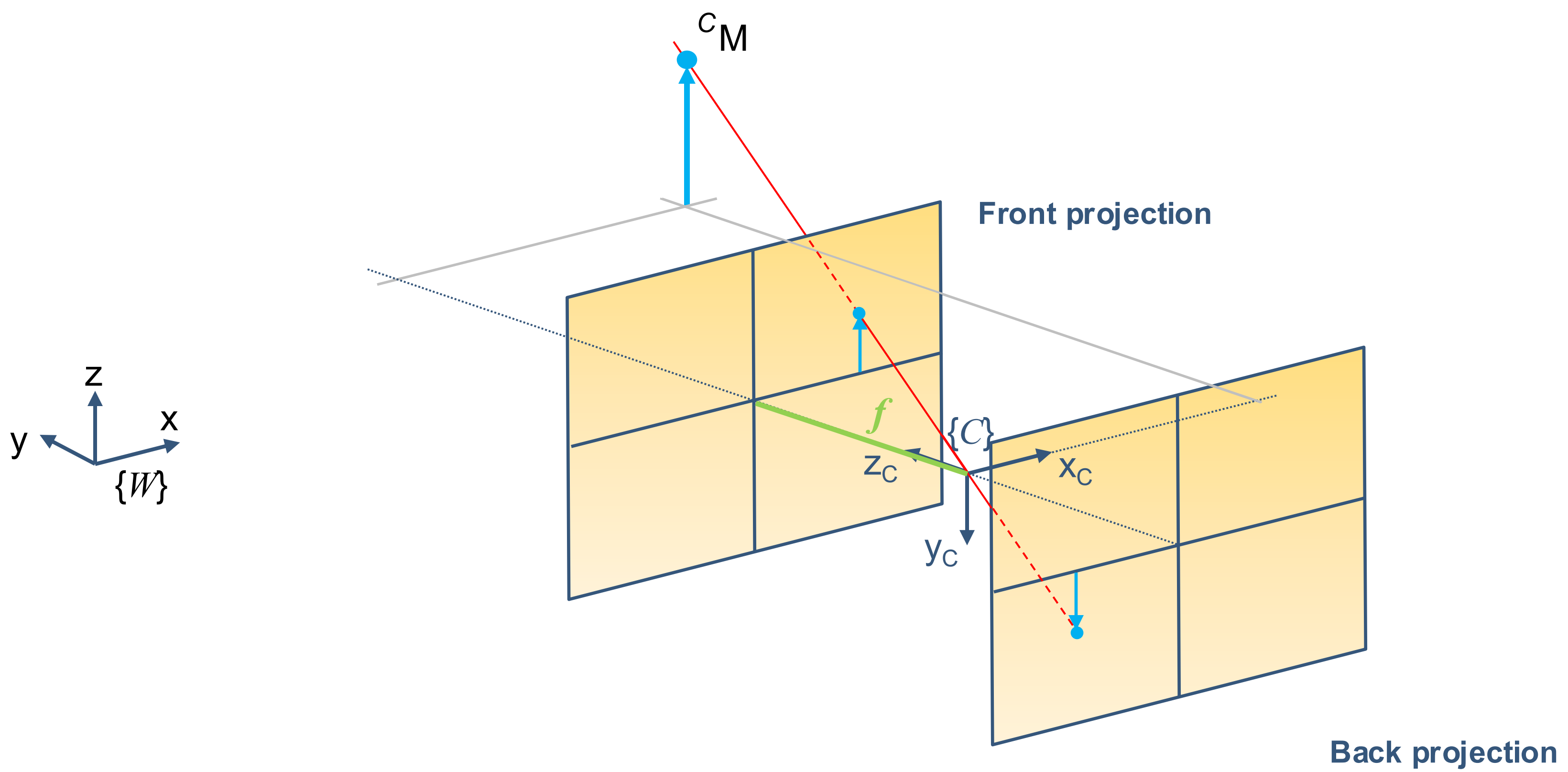
"Line equation of a point"

Projecting a point onto the image plane

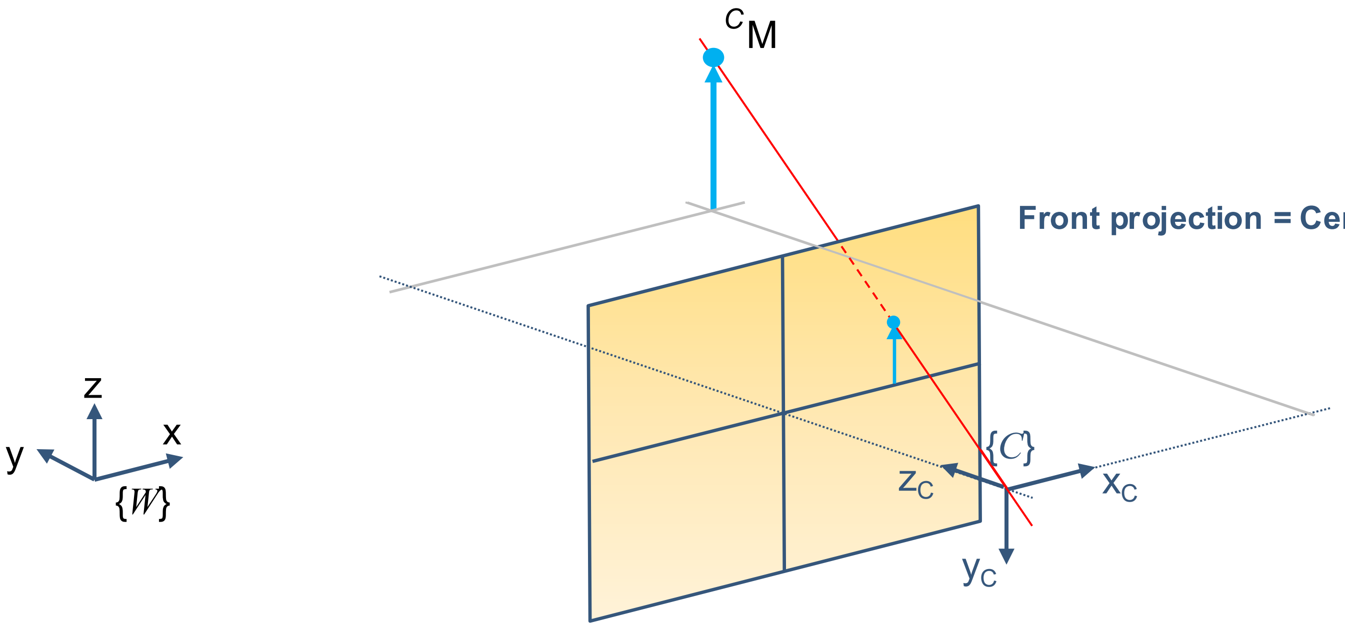


Back projection

Projecting a point onto the image plane

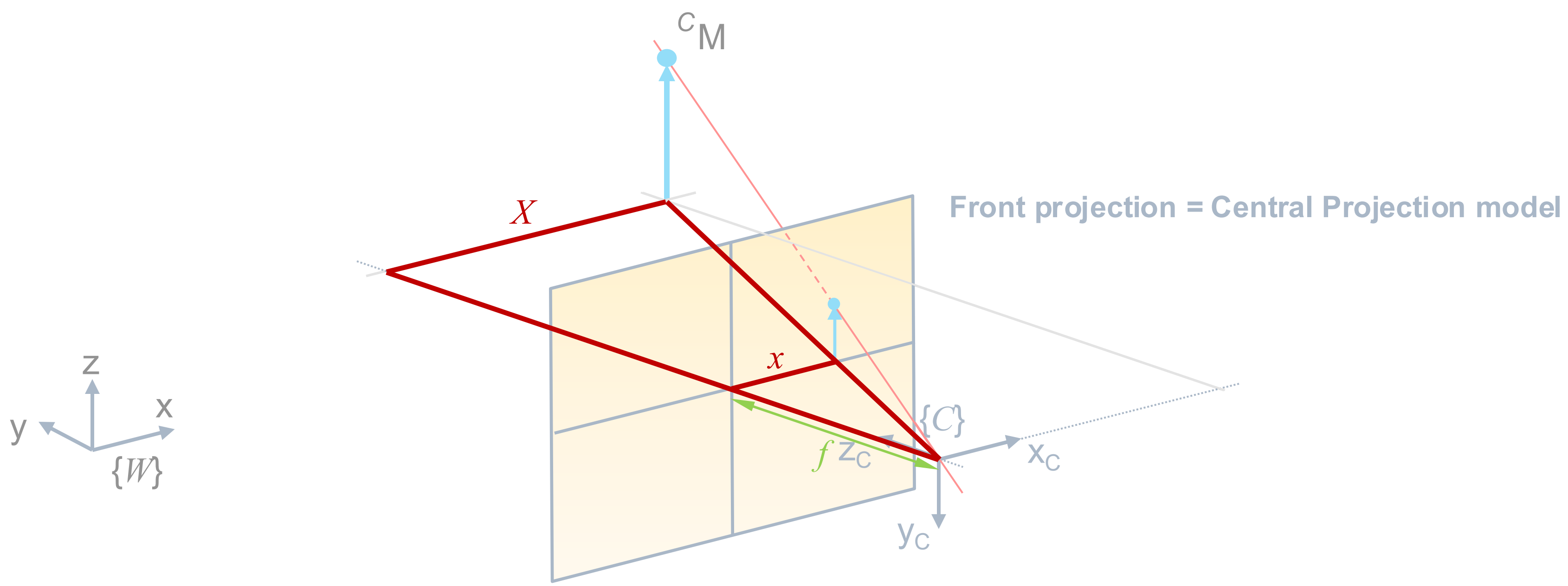


Projecting a point onto the image plane

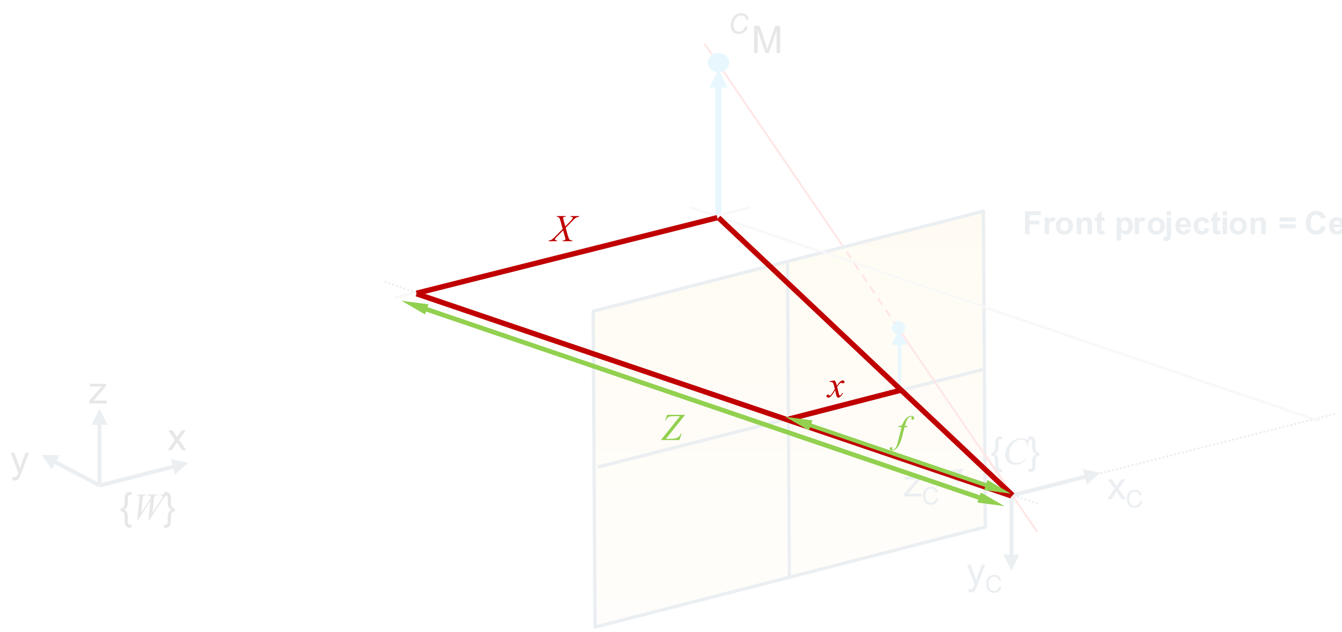


Front projection = Central Projection model

Projecting a point onto the image plane

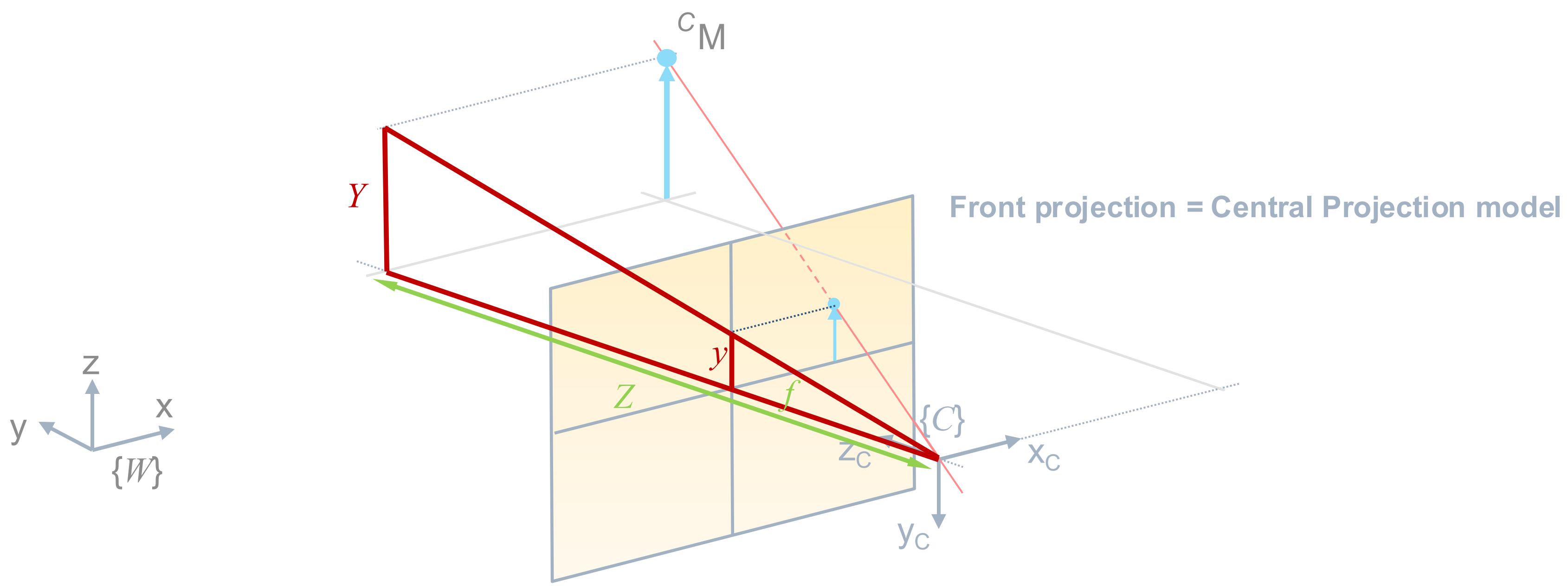


Projecting a point onto the image plane



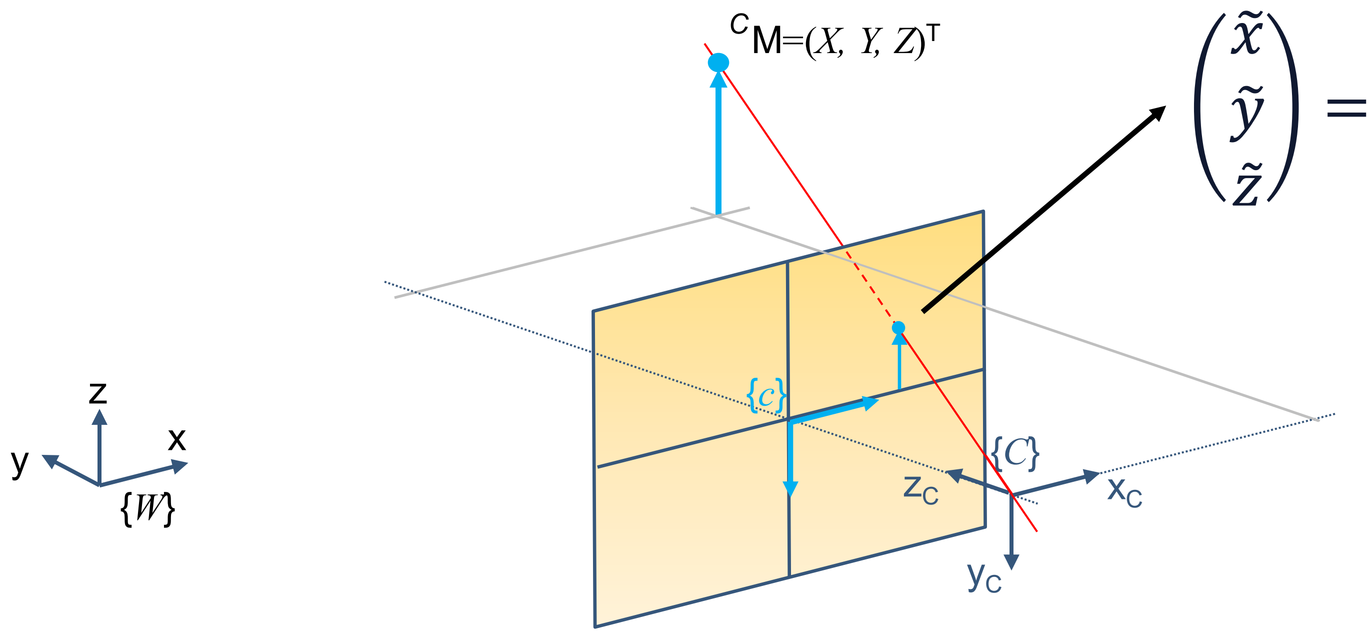
$$\frac{x}{f} = \frac{X}{Z} \quad \rightarrow \quad x = \frac{f \cdot X}{Z}$$

Projecting a point onto the image plane



$$\frac{y}{f} = \frac{Y}{Z} \quad \rightarrow \quad y = \frac{f \cdot Y}{Z}$$

Projecting a point onto the image plane



$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Perspective Projection

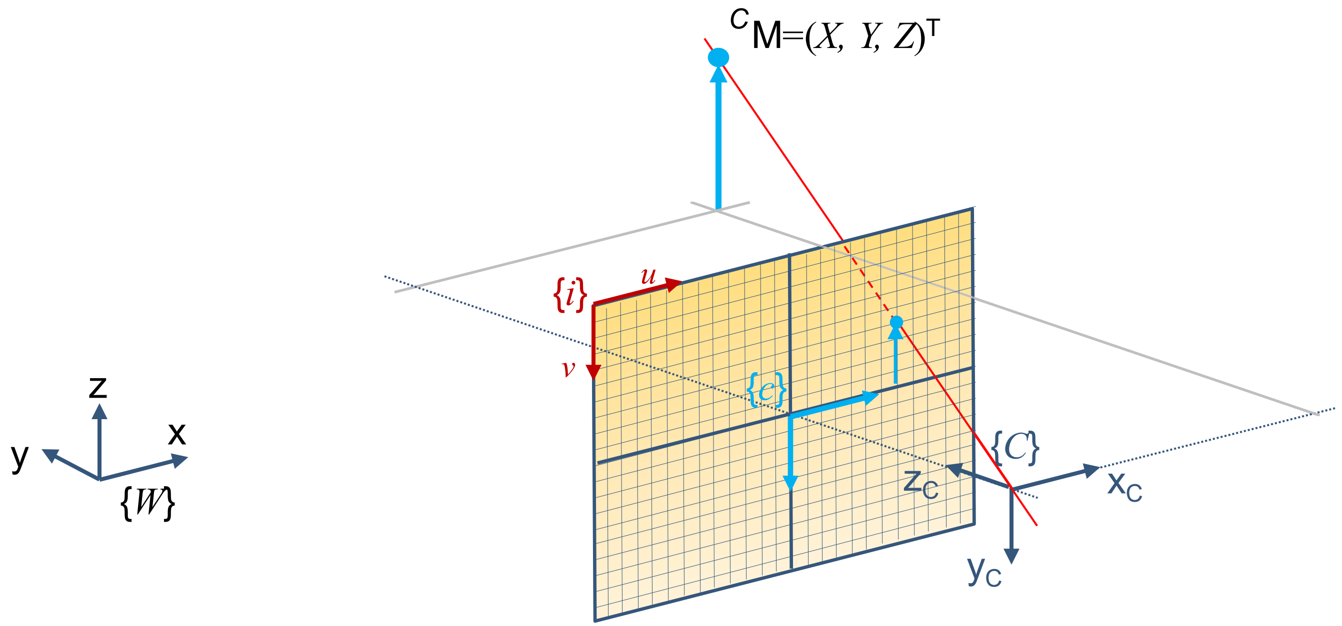
EXERCISE

Given a point M at position (20, 30, 4)
and a camera with focal distance $f = 2$

Where does M project onto the image?

(all the values are in mm)
Provide the results in cartesian coordinates

Projecting a point onto the image plane



$${}^{i}u = \frac{{}^{c}x}{\rho_u} + u_0$$

$${}^{i}v = \frac{{}^{c}y}{\rho_v} + v_0$$

- We should go from $\{c\}$ to $\{i\}$
- $\{c\}$ is in meters and $\{i\}$ in pixels
- Every pixel has size (ρ_u, ρ_v)

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix}$$

Iphone 12 Pro vs 13 Pro

Camera	**Apple iPhone 13 Pro Max**	**Apple iPhone 12 Pro Max**
Main camera	Sony IMX703, 1.9 µm, 26 mm	Sony IMX603, 1.7 µm, 26 mm
Ultra-wide-angle camera	Sony IMX772, 1µm, 13 mm	Sony IMX372, 1 µm, 13 mm
Telephoto camera	Sony IMX713, 1µm, 77 mm	Sony IMX613, 1 µm, 65 mm
ToF	Sony IMX590	
Selfie camera	Sony IMX514, 1µm	

Projection Matrix

- Complete Model :

$$\begin{pmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{pmatrix} = \underbrace{\boxed{\begin{pmatrix}\frac{1}{\rho_u} & 0 & u_0\\ 0 & \frac{1}{\rho_v} & v_0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}f & 0 & 0\\ 0 & f & 0\\ 0 & 0 & 1\end{pmatrix}}}_{\text{Intrinsic Parameters: K}}\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}\mathbf{R} & \mathbf{t}\\ \mathbf{0}_{1\times 3} & 1\end{pmatrix}^{-1}\begin{pmatrix}X\\ Y\\ Z\\ 1\end{pmatrix}$$

$$K = \begin{bmatrix}\alpha_x & \gamma & u_0 & 0\\ 0 & \alpha_y & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

- Projection Matrix:

$$\begin{pmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{pmatrix} = \begin{pmatrix}P_{11} & P_{12} & P_{13} & P_{14}\\ P_{21} & P_{22} & P_{23} & P_{24}\\ P_{31} & P_{32} & P_{33} & P_{34}\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\\ 1\end{pmatrix}$$

EXERCISE 1

Given a point M at 3D position (-2 1.3 5) (in meters)

Camera values:

f=0.012 m; principal point (512,512); $\rho_u = \rho_v = 10\mu m$

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

The camera is placed at $\{W\}$.

Where does M project?

EXERCISE 2

Given a point M at 3D position (5150 -150 6600) (in mm)

Camera values: main camera iphone 13.

f= 26 m*m*; principal point (2000, 1500); $\rho_u = \rho_v = 1.9\mu m$

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

The camera is placed at $\{W\}+$offset 5.5 meters in the X direction.

Where does M project in the image in pixels?

Projection Matrix (P)

- If we multiply to the right by an arbitrary value, the equality is maintained (homogeneous coordinates!)

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \lambda \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

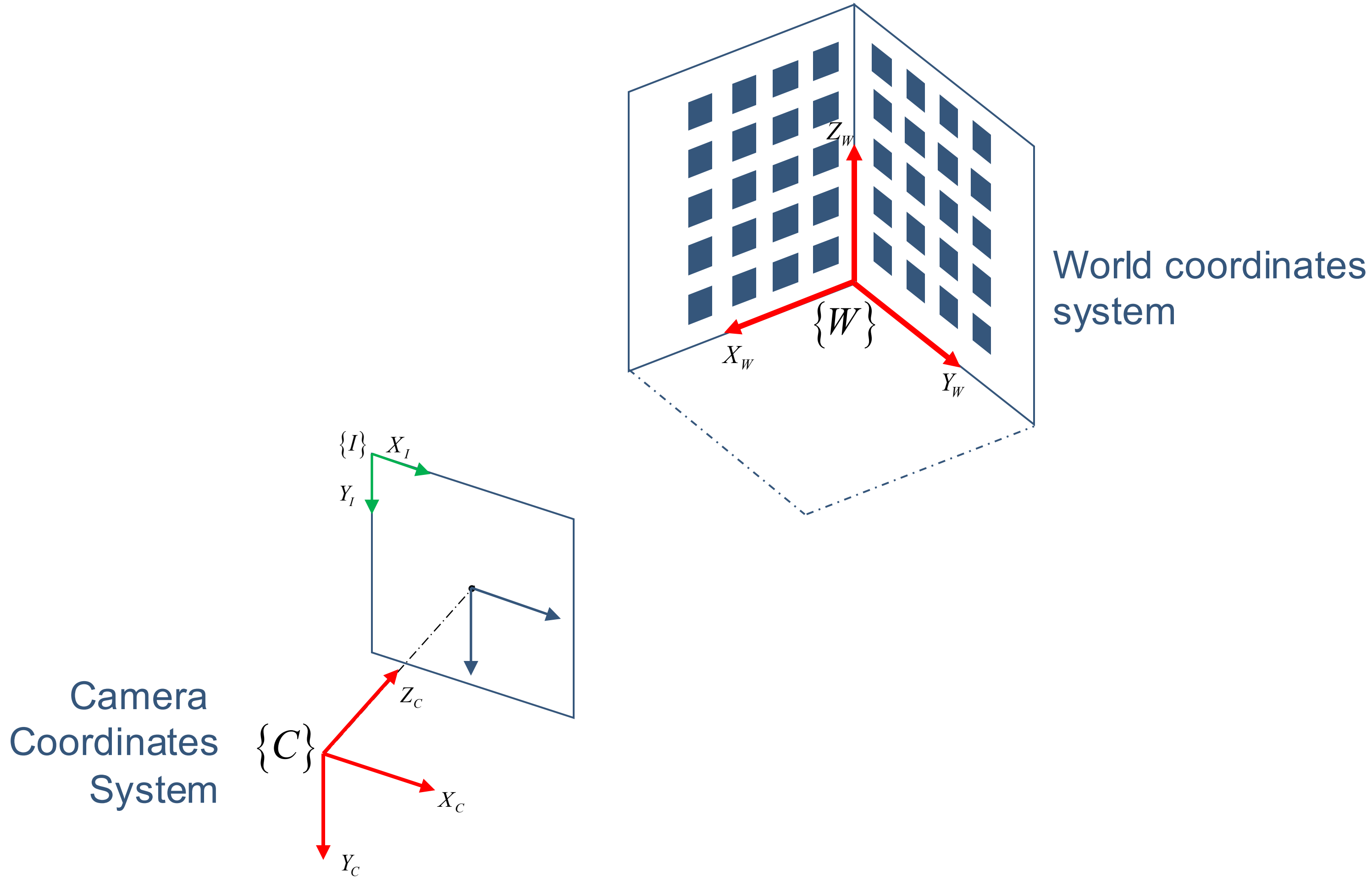
- Therefore, we can scale the matrix so that $P_{34} = 1$, and only 11 parameters need to be estimated

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Camera Calibration

- What is Camera Calibration?
  - Finding the parameters, internal to the camera, that affect the imaging process OR finding the projection matrix that encodes those parameters
  - Intrinsic camera parameters:
    - Focal length
    - Principal point
    - Scaling factors for row pixels
    - Lens distortion
  - Extrinsic camera parameters:
    - 3D position
    - 3D orientation

Hall Method to compute P



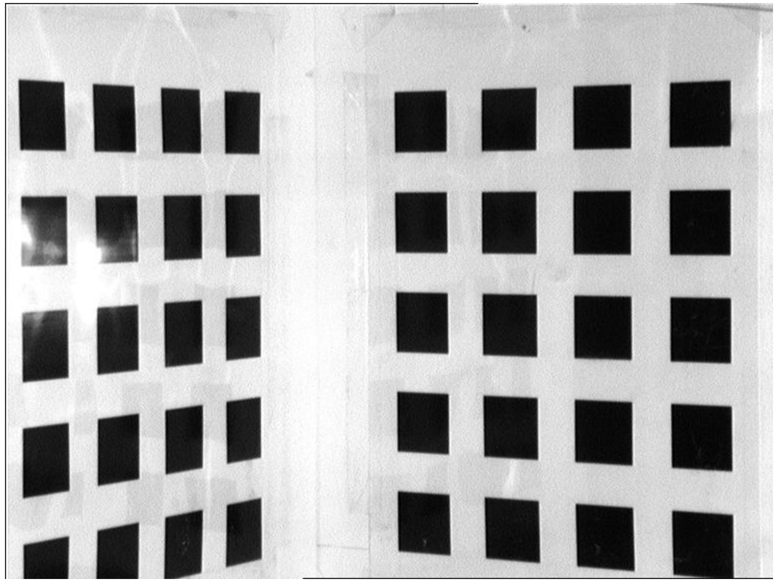


Image of the calibration pattern

Computing the projection Matrix

- Start with P

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- Change of nomenclature:

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

Computing the projection Matrix

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

$${}^{I}X_u = \frac{A_{11}\,{}^{W}X_w + A_{12}\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w + A_{14}}{A_{31}\,{}^{W}X_w + A_{32}\,{}^{W}Y_w + A_{33}\,{}^{W}Z_w + 1}$$

$${}^{I}Y_u = \frac{A_{21}\,{}^{W}X_w + A_{22}\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w + A_{24}}{A_{31}\,{}^{W}X_w + A_{32}\,{}^{W}Y_w + A_{33}\,{}^{W}Z_w + 1}$$

$$A_{11}\,{}^{W}X_w - A_{31}\,{}^{I}X_u\,{}^{W}X_w + A_{12}\,{}^{W}Y_w - A_{32}\,{}^{I}X_u\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w - A_{33}\,{}^{I}X_u\,{}^{W}Z_w + A_{14} = {}^{I}X_u$$

$$A_{21}\,{}^{W}X_w - A_{31}\,{}^{I}Y_u\,{}^{W}X_w + A_{22}\,{}^{W}Y_w - A_{32}\,{}^{I}Y_u\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w - A_{33}\,{}^{I}Y_u\,{}^{W}Z_w + A_{24} = {}^{I}Y_u$$

Computing the projection Matrix

- For every 3D-2D point, we obtain 2 equations:

$$A_{11}\,{}^{W}X_w - A_{31}\,{}^{I}X_u\,{}^{W}X_w + A_{12}\,{}^{W}Y_w - A_{32}\,{}^{I}X_u\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w - A_{33}\,{}^{I}X_u\,{}^{W}Z_w + A_{14} = {}^{I}X_u$$
$$A_{21}\,{}^{W}X_w - A_{31}\,{}^{I}Y_u\,{}^{W}X_w + A_{22}\,{}^{W}Y_w - A_{32}\,{}^{I}Y_u\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w - A_{33}\,{}^{I}Y_u\,{}^{W}Z_w + A_{24} = {}^{I}Y_u$$

- Place all the unknowns in a column vector:

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \end{pmatrix}^{\mathrm{T}}$$

- And we obtain a linear system of equations $QA = B$

- REMEMBER: we have 11 unknowns, therefore we need at least six 3D-2D pairs, which will generate two rows on matrix Q:

$$Q_{2i-1} = \begin{pmatrix} {}^{W}X_{wi} & {}^{W}Y_{wi} & {}^{W}Z_{wi} & 1 & 0 & 0 & 0 & 0 & -{}^{I}X_{ui}\,{}^{W}X_{wi} & -{}^{I}X_{ui}\,{}^{W}Y_{wi} & -{}^{I}X_{ui}\,{}^{W}Z_{wi} \end{pmatrix}$$
$$Q_{2i} = \begin{pmatrix} 0 & 0 & 0 & 0 & {}^{W}X_{wi} & {}^{W}Y_{wi} & {}^{W}Z_{wi} & 1 & -{}^{I}Y_{ui}\,{}^{W}X_{wi} & -{}^{I}Y_{ui}\,{}^{W}Y_{wi} & -{}^{I}Y_{ui}\,{}^{W}Z_{wi} \end{pmatrix}$$

Computing the projection Matrix

- REMEMBER: we have 11 unknowns, therefore we need at least six 3D-2D pairs, which will generate two rows on matrix Q:

$$Q_{2i-1} = \begin{pmatrix} {}^W X_{wi} & {}^W Y_{wi} & {}^W Z_{wi} & 1 & 0 & 0 & 0 & 0 & -{}^I X_{ui}\,{}^W X_{wi} & -{}^I X_{ui}\,{}^W Y_{wi} & -{}^I X_{ui}\,{}^W Z_{wi} \end{pmatrix}$$
$$Q_{2i} = \begin{pmatrix} 0 & 0 & 0 & 0 & {}^W X_{wi} & {}^W Y_{wi} & {}^W Z_{wi} & 1 & -{}^I Y_{ui}\,{}^W X_{wi} & -{}^I Y_{ui}\,{}^W Y_{wi} & -{}^I Y_{ui}\,{}^W Z_{wi} \end{pmatrix}$$

$$B_{2i-1} = \left({}^I X_{ui} \right)$$
$$B_{2i} = \left({}^I Y_{ui} \right)$$

- If we have more than 6 pairs, we can do least squares:

$$A = Q^{-1} B \quad \Rightarrow \quad \boxed{A = \left(Q^t Q \right)^{-1} Q^t B}$$

Extracting K and R from Projection Matrix P

We have the projection matrix P and need to extract:

- **K** - Intrinsic matrix (upper triangular)
- **R** - Rotation matrix (orthogonal)

$$\begin{vmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{vmatrix} = \begin{vmatrix} K_{11} & K_{12} & K_{13} \\ 0 & K_{22} & K_{23} \\ 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

Extracting K and R from Projection Matrix P

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ 0 & K_{22} & K_{23} \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

K - Intrinsic Matrix

- Upper triangular
- Positive diagonal elements
- K(3,3) = 1 (normalized)
- Contains focal lengths, principal point, skew

R - Rotation Matrix

- Orthogonal: $R \cdot R^T = I$
- det(R) = +1
- Columns are unit vectors
- Represents camera orientation

Summary

- Why modelling a camera is important. Applications.
- Intuition about the *pinhole* model
- Description of the projection equations 3D-2D
- Camera calibration procedure
- Point Projection equation using 4x3 matrix P