K means

①

	Var 1	Var 2
①	− 1.54	2.29
②	−0.44	2.34
③	0.03	0.41
④	1.2	1.87
⑤	0.65	2.89
⑥	− 4.67	− 4.8
⑦	− 3.37	− 5.41
⑧	− 3.93	− 4.64
⑨	− 4.78	− 4.96
⑩	− 4.12	− 5.36

initial.

$$\text{mean}_1 = (1 \quad 1)$$
$$\text{mean}_2 = (-4, -4)$$

← initial guess

cost / distortion

point ①

	mean 1	mean 2

~~①~~

① $(-1.54 - 1)^2 + (2.29 - 1)^2$
$= 8.1157$ $<$ $(-1.54 - (-4))^2 + (2.29 - (-4))^2$
$= 48.6157 \, (r_{11} = 1, r_{12}$

② $(-0.44 - 1)^2 + (2.34 - 1)^2 <$ $(-0.44 + 4)^2 + (2.34 + 4)^2$
$= 3.8692$ $<$ $52.8692 \, (r_{21} = 1, r_{22} = 0$

③ $\quad$ 1.289 $\quad$ < $\quad$ 35.689 $\qquad$ $r_{31} = 1 \quad r_{32} = 0$

④ $\quad$ 0.7969 $\quad$ < $\quad$ 61.4969 $\qquad$ $r_{41} = 1 \quad r_{42} = 0$

⑤ $\quad$ 2.0546 $\quad$ < $\quad$ 62.4546 $\qquad$ $r_{51} = 1 \quad r_{52} = 0$

⑥ $\quad$ 65.7889 $\quad$ > $\quad$ 1.0889 $\qquad$ $r_{61} = 0 \quad r_{62} = 1$

⑦ $\quad$ 60.185 $\quad$ > $\quad$ 2.385 $\qquad$ $r_{71} = 0 \quad r_{72} = 1$

⑧ $\quad$ 56.1145 $\quad$ > $\quad$ 0.4145 $\qquad$ $r_{81} = 0 \quad r_{32} = 1$

⑨ $\quad$ 68.93 $\quad$ > $\quad$ 1.53 $\qquad$ $r_{91} = 0 \quad r_{92} = 1$

⑩ $\quad$ 66. $\quad$ > $\quad$ 1.864 $\qquad$ $r_{10,1} = 0 \quad r_{10,2} = 0$

$$J = \sum_{n=1}^{N} \sum_{k=1}^{k} r_{nk} \left|\left| x_n - u_k \right|\right|^2$$
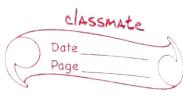
$$= 8.1157 + 3.8692 + 1.289 + 0.7979$$
$$+ 2.0546 + 1.0889 + 2.385 + 0.4145$$
$$+ 1.53 + 1.864 = 23.108$$

points 1,2'u in cluster 1 with mean L are
= ①, ②, ③, ④, ⑤

points lies in cluster 2 with mean2 are
6, 7, 8, 9, 10.

$$mean1 = \frac{① + ② + ③ + ④ + ⑤}{5}$$ ← There are points

$$mean1 = \frac{-1.54 - 0.44 + 0.03 + 1.2 + 0.65}{5} \quad , \quad \frac{2.29 + 2.34 + 0.414 + 1.87 + 2.39}{5}$$

$$mean1 = (-0.028 , 1.86)$$

$$mean2 = \frac{4.67 - 3.37 - 3.93 - 4.78 - 4.12}{5} \quad , \quad \frac{-4.8 - 5.41 - 4.64 - 4.96 - 5.38}{5}$$

$$= (-4.174 , -5.034)$$

Repeat the same process as before with new mean in order to update rank.

$\underline{mean1}$ $\underline{mean2}$

① $(-1.54 + 0.02)^2 + (2.29 - 1.86)^2$

$= 2.4953$ $<$ $(-1.54 + 4.174)^2 + (2.29 + 5.034)^2$

$= 60.57893 \left(\begin{array}{c} r_{11} = 1 \\ r_{12} = 0 \end{array}\right)$

②

$$(-0.44 + 0.02)^2 + (2.34 - 1.86)^2$$

$$= 0.4068 \qquad < \qquad (-0.44 + 4.17)^2$$
$$+ (2.34 + 5.034)^2$$
$$67.58373 \quad (r_{21} = 1, \ r_{22} = 0)$$

③ $\quad 2.106 \quad < \quad 71.31489 \qquad \left(r_{31} = 1, \quad r_{32} = 0 \right)$

④ $\quad 1.4885 < \quad 82.520 \qquad \left(r_{41} = 1, \quad r_{42} = 0 \right)$

⑤ $\quad 0.7298 \ < \ 76.91 \qquad \left(r_{51} = 1, \quad r_{52} = 0 \right)$

⑥ $\quad 65.978 > \quad 53.886 \qquad \left(r_{61} = 0, \quad r_{62} = 1 \right)$

⑦ $\quad 64.0764 > 54.287 \qquad \left(r_{71} = 0, \quad r_{72} = 1 \right)$

⑧ $\quad 57.5381 \ > \ 53.70051 \qquad \left(r_{81} = 0, \quad r_{82} = 1 \right)$

⑨ $\quad 69.17 \quad > \quad 54.00821 \qquad \left(r_{91} = 0, \quad r_{92} = 1 \right)$

⑩ $\quad 68.9384 \quad > \ 53.8438 \qquad \left(r_{10,1} = 0, \quad r_{10,2} = 1 \right)$

cluster 1 — 1, 2, 3, 4, 5
Cluster 2 — 6, 7, 8, 9, 10

$$J = 276.63 \qquad \left(\begin{array}{l} \text{Applied same formular} \\ \text{as before} \end{array} \right)$$

mean 1 $= (-0.02, \ 1.86)$

mean 2 $= (-4.17, \ -5.034)$

Since means 1 & mean 2 not changing.

(rank also won't change)

$\Rightarrow$ Converged

$\Rightarrow$ Cluster 1 — ①, ②, ③, ④, ⑤

$\Rightarrow$ Cluster 2 — ⑥, ⑦, ⑧, ⑨, ⑩