CLL788 MOJAHID HUSSAIN 2020CLL70182

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Y_1 :	2	3	5	7	9
Y_2 :	1	4	0	6	2

$$\bar{Y}_1 = \frac{2+3+5+7+9}{5} = 5.2$$

$$\bar{Y}_2 = \frac{1+4+0+6+2}{5} = 2.6$$

Standardising the data by centering it.

$(Y_{1i} - \bar{Y}_1)$:	-3.2	-2.2	-0.2	1.8	3.8
$(Y_{2i} - \bar{Y}_2)$:	-1.6	1.4	-2.6	3.4	-0.6

Covariance Matrix

$$C = \begin{bmatrix} Cov(Y_1, Y_1) & Cov(Y_1, Y_2) \\ Cov(Y_2, Y_1) & Cov(Y_2, Y_2) \end{bmatrix}$$

$$Cov(Y_1, Y_1) = \sum_{i=1}^{5} (Y_{1i} - \bar{Y}_1)(Y_{1i} - \bar{Y}_1)$$

$$= \frac{(-3.2)^2 + (-2.2)^2 + (-0.2)^2 + (1.8)^2 + (3.8)^2}{5}$$

$$= 6.56$$

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$$\text{Cov}(Y_2, Y_2) = \frac{\sum_{i=1}^{5} (Y_{2i} - \bar{Y}_2)^2}{5}$$

$$= \frac{(-1.6)^2 + (1.4)^2 + (-2.6)^2 + (3.4)^2 + (-0.6)^2}{5}$$

$$= 4.64$$

$$\text{Cov}(Y_1, Y_2) = \frac{\sum_{i=1}^{5} (Y_{1i} - \bar{Y}_1)(Y_{2i} - \bar{Y}_2)}{5}$$

$$= \frac{(-3.2)(-1.6) + (-2.2)(1.4) + (-0.2)(-2.6) + (1.8)(3.4) + (3.8)(-0.6)}{5}$$

$$= 1.28$$

Covariance Matrix:

$$C = \begin{bmatrix} 6.56 & 1.28 \\ 1.28 & 4.64 \end{bmatrix}$$

Eigen vectors corresponding to two principal directions

$$C - \lambda I = 0$$

$$\begin{bmatrix} 6.56 & 1.28 \\ 1.28 & 4.64 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

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$$\begin{bmatrix} 6.56-\lambda & 1.28 \\ 1.28 & 4.64-\lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 6.56-\lambda & 1.28 \\ 1.28 & 4.64-\lambda \end{bmatrix} = 0$$

$$(6.56-\lambda)(4.64-\lambda) - 1.28(1.28) = 0$$

$$30.4384 - 6.56\lambda - 4.64\lambda + \lambda^2 - 1.6384 = 0$$

$$\lambda^2 - 11.2\lambda + 28.8 = 0$$

$$\Rightarrow \boxed{\begin{aligned} \lambda_1 &= 7.2 \\ \lambda_2 &= 4 \end{aligned}}$$

Eigen vectors corresponding to eigen values.

$$CQ = \lambda V$$

for $\lambda = 7.2$

$$\begin{bmatrix} 6.56 & 1.28 \\ 1.28 & 4.64 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 7.2 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

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$6.56x_1 + 1.28y_1 = 7.2x_1$

$\Rightarrow 0.64x_1 = 1.28y_1$ — (i)

$1.28x_1 + 4.64y_1 = 7.2y_1$

$\Rightarrow 1.28x_1 = 2.56y_1$ — (ii)

from (i) & (ii),

$\Rightarrow x_1 = 2y_1$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix} = \begin{bmatrix} 0.8944 \\ 0.4472 \end{bmatrix}$

eigen vector corresponding to 7.2 eigen value

$= \begin{bmatrix} 0.8944 \\ 0.4472 \end{bmatrix}$

$\lambda_2 = 4$

$\begin{bmatrix} 6.56 & 1.28 \\ 1.28 & 4.64 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 4 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

$6.56x_1 + 1.28y_1 = 4x_1 \Rightarrow 2.56x_1 = -1.28y_1$

$1.28x_1 + 4.64y_1 = 4y \Rightarrow 1.28x_1 = -0.64y_1$

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$$\Rightarrow x_1 = -0.6y_1$$

$$\begin{bmatrix} -0.6 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{-0.6}{\sqrt{1.36}} \\ \frac{1}{\sqrt{1.36}} \end{bmatrix} = \begin{bmatrix} -0.4972 \\ 0.8944 \end{bmatrix}$$

eigen vector correspond to $\lambda_2 = 4$.

$$= \begin{bmatrix} -0.4972 \\ 0.8944 \end{bmatrix}$$

Transforming the data into the new coordinate plane.

$$Y' = \begin{bmatrix} \text{standardised} \\ \text{vectors} \end{bmatrix} \begin{bmatrix} \text{eigen} \\ \text{vector} \end{bmatrix}$$

$$= \begin{bmatrix} -3.2 & -1.6 \\ -2.2 & 1.4 \\ -0.2 & -2.6 \\ 1.8 & 3.4 \\ 3.8 & -0.6 \end{bmatrix} \begin{bmatrix} 0.8944 & -0.4972 \\ -0.4972 & 0.8944 \end{bmatrix}$$

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$$Y' = \begin{bmatrix} -2.14666 & 0 \\ -2.69376 & 2.236 \\ 0.9838 & -2.236 \\ 0.08944 & 2.236 \\ 3.66704 & -2.236 \end{bmatrix}$$

$$Y_1' = \{ -2.14666, -2.69376, 0.9838, 0.08944, 0.08944, 3.66704 \}$$

$$Y_2' = \{ 0, 2.236, -2.236, 2.236, -2.236 \}$$