Q (2)

(1) $J = \sum_i w_i^{(i)} \left(y^{(i)} - \theta^T x^{(i)}\right)^2$

→ minimize this (aim)

$$w^{(i)} = \exp\left(-\frac{(x^{(i)} - x)^2}{2\tau^2}\right)$$

$x = 6.2532$
$\tau = 0.5$] → Given

$$w^{i} = \exp\left(-\frac{(x^{(i)} - 6.2532)^2}{2 \cdot (0.5)^2}\right)$$

$$\sum_i \exp\left(-\frac{(x^{(i)} - 6.2532)^2}{0.5}\right)\left(y^i - \theta^T x^{(i)}\right)^2$$

$$J(\theta) = \exp\left(-\frac{(x^i - 6.2532)^2}{0.5}\right)\left(y^{(i)} - \theta^T x^{(i)}\right)^2$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \exp\left(-\frac{(x^{(i)} - 6.2532)^2}{0.5}\right) 2\left(y^{(i)} - \theta^T x^{(i)}\right) \times \left(-x^{(i)}_j\right)$$

Take 4 data points from given data —

(1)	6.1814	—	1.444
(2)	5.3865	—	5.3014
(3)	6.6761	—	4.7875
(4)	6.0577	—	1.4451

Assume, intial $\theta = [0, 0]$

$$\frac{\partial J(\theta)}{\partial \theta_j} = -2\left(y^{(i)} - \theta^T x^{(i)}\right) x_j^{(i)} \exp\left(-\frac{(x^{(i)} - 6.2032)^2}{0.5}\right)$$

Iteration 1:

$$\theta^T x^{(1)} = \theta_0 + \theta_1 x^{(1)} = 0$$

$$\theta_0 = \theta_0 - \alpha\left[-2(1.444 - 0)(1)\right](1)\exp\left(-\frac{(6.1619 - 6.2032)^2}{0.5}\right)$$

Assume $\alpha = 0.001$

~~$\theta_0 = 0.028875$~~

$$\theta_0 = 0.028575$$

$$\theta_1 = \theta_1 - \alpha\left[-2(1.444 - 0)\right](6.1619)\exp\left[-\frac{(6.1619 - 6.453)^2}{0.5}\right]$$

$$\theta_1 = 0.0174$$

Iteration 2:

$$\theta^T x^{(2)} = 0.028575 + 0.0174 \cdot (5.3865)$$
$$= 0.122117$$

$$\theta_0 = 0.028575 - \alpha\left[-2(5.3014 - 0.122012)\right](1)$$
$$\times \exp\left(-\frac{(5.3865 - 6.4532)^2}{0.5}\right)$$

$$\theta_0 = 0.0305934$$

$$\Theta_1 = 0.0174 - \alpha \left[-2\left(5.3014 - 0.122017\right)\right](5.3866) \times \exp\left[-\frac{(5.3866 - 6.2832)^2}{0.5}\right]$$

$$\Theta_1 = 0.029821$$

Iteration 3

$$\theta^T x^{(3)} = 0.0308939(1) + 0.029821(6.6761)$$

$$= 0.238681$$

$$\Theta_0 = 0.0469678$$
$$\Theta_1 = 0.039465$$

Iteration 4

$$\theta^T x^{(4)} = 0.0469678 + 0.039465 \times (6.0577)$$

$$= 0.0287108$$

$$\Theta_0 = 0.038169$$
$$\Theta_1 = 0.053674$$