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## 1 A Header

```
#define FastIO ios::sync_with_stdio(false);
cin.tie(0);cout.tie(0)
#include <ext/pb_ds/assoc_container.hpp> // Common file
using namespace __gnu_pbds;
/*
find_by_order(k) --> returns iterator to the kth largest
element counting from 0
order_of_key(val) --> returns the number of items in a set that
are strictly smaller than our item
*/
typedef tree<
int,
null_type,
less<int>,
rb_tree_tag,
tree_order_statistics_node_update>
ordered_set;
// #pragma GCC optimize("O3,unroll-loops")
// #pragma GCC target("avx2,bmi,bmi2,lzcnt")
// mt19937
rng(chrono::system_clock::now().time_since_epoch().count());
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
struct custom_hash {
static uint64_t splitmix64(uint64_t x) {
x += 0x9e3779b97f4a7c15; //Random
x=(x^(x>>30))*0xbf58476d1ce4e5b9; //Random
x=(x^(x>>27))*0x94d049bb133111eb; //Random
return x^(x>>31);
}
}
const uint64_t FIXED_RANDOM = chrono::
```

```
steady_clock::now().time_since_epoch().count();
size_t operator()(uint64_t x) const {
return splitmix64(x + FIXED_RANDOM);
}
size_t operator()(pair<int, int> x) const {
return splitmix64((uint64_t(x.first)<<32) +
x.second + FIXED_RANDOM);
}
};
gp_hash_table<pair<int, int>, int, custom_hash> ht;
namespace my_gcc_ints {
#pragma GCC diagnostic push
#pragma GCC diagnostic ignored "-Wpedantic"
using int128 = __int128;
#pragma GCC diagnostic pop
}
# stresstester GENERATOR SOL1 SOL2 ITERATIONS
for i in $(seq 1 "$4") ; do
echo -en "\rAttempt $i/$4"
$1 > in.txt
$2 < in.txt > out1.txt
$3 < in.txt > out2.txt
diff -y out1.txt out2.txt > diff.txt
if [ $? -ne 0 ] ; then
echo -e "\nTestcase Found:"; cat in.txt
echo -e "\nOutputs:"; cat diff.txt
exit
fi
done
```

## 2 DP

## 2.1 divide-and-conquer-optimization

```
int m, n;
vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr){
if (l > r)
return;
int mid = (l + r) >> 1;
pair<long long, int> best = {LLONG_MAX, -1};
for (int k = optl; k <= min(mid, optr); k++){
best = min(best, {(k ? dp_before[k - 1] : 0) + C(k,
mid), k});
}
dp_cur[mid] = best.first;
int opt = best.second;
compute(l, mid - 1, optl, opt);
compute(mid + 1, r, opt, optr);
}
int solve(){
for (int i = 0; i < n; i++){
dp_before[i] = C(0, i);
for (int i = 1; i < m; i++){
compute(0, n - 1, 0, n - 1);
dp_before = dp_cur;
}
return dp_before[n - 1];
}
```

## 2.2 knuth-optimization

```
int solve() {
int N;
... // read N and input
int dp[N][N], opt[N][N];
auto C = [&](int i, int j) {
... // Implement cost function C.
};
for (int i = 0; i < N; i++) {
opt[i][i] = i;
... // Initialize dp[i][i] according to the problem
}
for (int i = N-2; i >= 0; i--) {
for (int j = i+1; j < N; j++) {
int mn = INT_MAX;
int cost = C(i, j);
```

```

    for (int k = opt[i][j-1]; k <= min(j-1,
        opt[i+1][j]); k++) {
        if (mn >= dp[i][k] + dp[k+1][j] + cost) {
            opt[i][j] = k;
            mn = dp[i][k] + dp[k+1][j] + cost;
        }
    }
    dp[i][j] = mn;
}
}
cout << dp[0][N-1] << endl;
}

```

## 2.3 li-chao-tree

```

typedef long long ll;
class LiChaoTree{
    ll L,R;
    bool minimize;
    int lines;
    struct Node{
        complex<ll> line;
        Node *children[2];
        Node(complex<ll> ln= {0,10000000000000000000}){
            line=ln;
            children[0]=0;
            children[1]=0;
        }
    } *root;
    ll dot(complex<ll> a, complex<ll> b){
        return (conj(a) * b).real();
    }
    ll f(complex<ll> a, ll x){
        return dot(a, {x, 1});
    }
    void clear(Node* &node){
        if(node->children[0]){
            clear(node->children[0]);
        }
        if(node->children[1]){
            clear(node->children[1]);
        }
        delete node;
    }
    void add_line(complex<ll> nw, Node* &node, ll l, ll r){
        if(node==0){
            node=new Node(nw);
            return;
        }
        ll m = (l + r) / 2;
        bool lef = (f(nw, l) < f(node->line,
            l)&&minimize)||(!minimize)&&f(nw, l) >
            f(node->line, l));
        bool mid = (f(nw, m) < f(node->line,
            m)&&minimize)||(!minimize)&&f(nw, m) >
            f(node->line, m));
        if(mid){
            swap(node->line, nw);
        }
        if(r - l == 1){
            return;
        }
        else if(lef != mid){
            add_line(nw, node->children[0], l, m);
        }
        else{
            add_line(nw, node->children[1], m, r);
        }
    }
    ll get(ll x, Node* &node, ll l, ll r){
        ll m = (l + r) / 2;
        if(r - l == 1){
            return f(node->line, x);
        }
        else if(x < m){
            if(node->children[0]==0) return f(node->line, x);
            if(minimize) return min(f(node->line, x), get(x,
                node->children[0], l, m));
            else return max(f(node->line, x), get(x,
                node->children[0], l, m));
        }
    }
}

```

```

    }
    else{
        if(node->children[1]==0) return f(node->line, x);
        if(minimize) return min(f(node->line, x), get(x,
            node->children[1], m, r));
        else return max(f(node->line, x), get(x,
            node->children[1], m, r));
    }
}
public:
    LiChaoTree(ll l=-10000000001, ll r=10000000001, bool mn=false){
        L=l;
        R=r;
        root=0;
        minimize=mn;
        lines=0;
    }
    void AddLine(pair<ll,ll> ln){
        add_line({ln.first,ln.second},root,L,R);
        lines++;
    }
    int number_of_lines(){
        return lines;
    }
    ll getOptimumValue(ll x){
        return get(x,root,L,R);
    }
    ~LiChaoTree(){
        if(root!=0) clear(root);
    }
};

```

## 2.4 zero-matrix

```

int zero_matrix(vector<vector<int>> a) {
    int n = a.size();
    int m = a[0].size();
    int ans = 0;
    vector<int> d(m, -1), d1(m), d2(m);
    stack<int> st;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            if (a[i][j] == 1)
                d[j] = i;
        }
        for (int j = 0; j < m; ++j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d1[j] = st.empty() ? -1 : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = m - 1; j >= 0; --j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d2[j] = st.empty() ? m : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = 0; j < m; ++j)
            ans = max(ans, (i - d[j]) * (d2[j] - d1[j] - 1));
    }
    return ans;
}

```

## 3 DS

### 3.1 Heavy light decomposition

```

int value[N], Tree[N];
int parent[N], depth[N], heavy[N], head[N], pos[N];
int cur_pos, n;
vector<int> adj[N];
int dfs(int v) {
    int size = 1, max_c_size = 0;
    for (int c : adj[v]) {
        if (c != parent[v]) {
            parent[c] = v, depth[c] = depth[v] + 1;

```

```

            int c_size = dfs(c);
            size += c_size;
            if (c_size > max_c_size)
                max_c_size = c_size, heavy[v] = c;
        }
    }
    return size;
}
void update(int idx, int x, int nn){
    while(idx<nn){
        Tree[idx]+=x; idx+=(idx&idx-1);
    }
}
void decompose(int v, int h) {
    head[v] = h, pos[v] = cur_pos;
    update(cur_pos, value[v], n+1);
    cur_pos++;
    if (heavy[v] != -1) decompose(heavy[v], h);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c);
    }
}
int query_bit(int idx){
    int sum=0;
    while(idx>0){
        sum+=Tree[idx]; idx=(idx&idx-1);
    }
    return sum;
}
void init_hld(int root, int n){
    memset(Tree, 0, sizeof Tree);
    memset(heavy, -1, sizeof heavy);
    cur_pos = 1; parent[root]=-1;
    assert(dfs(root)==n);
    decompose(root, root);
}
int segment_tree_query(int x, int y){
    if(y<x) swap(x,y);
    return query_bit(y)-query_bit(x-1);
}
int query_hld(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max =
            segment_tree_query(pos[head[b]], pos[b]);
        res += cur_heavy_path_max;
    }
    if (depth[a] > depth[b])
        swap(a, b);
    /// now a is the lca or quert(a,b)
    int last_heavy_path_max = segment_tree_query(pos[a],
        pos[b]);
    res += last_heavy_path_max;
    return res;
}

```

## 3.2 MO with update

```

const int N = 1e5 + 5;
const int P = 2000; // block size = (2*n^2)^(1/3)
struct query{
    int t, l, r, k, i;
};
vector<query> q;
vector<array<int, 3>> upd;
vector<int> ans, a;
void add(int x); void rem(int x); int get_answer();
void mos_algorithm(){
    sort(q.begin(), q.end(), [](const query &a, const query &b){
        if (a.t / P != b.t / P) return a.t < b.t;
        if (a.l / P != b.l / P) return a.l < b.l;
        if ((a.l / P) & 1) return a.r < b.r;
        return a.r > b.r;
    });
    for(int i=upd.size()-1; i>=0; --i) a[upd[i][0]] = upd[i][1];
    int L = 0, R = -1, T = 0;
    auto apply = [&](int i, int fl){

```

```

    int p = upd[i][0], x = upd[i][f1 + 1];
    if (L <= p && p <= R){ rem(a[p]); add(x);}
    a[p] = x;
};
ans.clear(); ans.resize(q.size());
for (auto qr : q){
    int t = qr.t, l = qr.l, r = qr.r, k = qr.k;
    while (T < t) apply(T++, 1);
    while (T > t) apply(--T, 0);
    while (R < r) add(a[++R]);
    while (L > l) add(a[--L]);
    while (R > r) rem(a[R--]);
    while (L < l) rem(a[L++]);
    ans[qr.i] = get_answer();
}
}
void TEST_CASES(int cas){
    cin>>n>>m; a.resize(n); for(int i=0;i<n;i++) cin>>a[i];
    for(int i=0;i<m;i++){ int tp; scanf("%d", &tp);
        if (tp == 1){ int l, r, k; cin>>l>>r>>k;
            q.push_back({upd.size(), l - 1, r - 1, k, q.size()});}
        else{int p, x;cin>>p>>x;--p;
            upd.push_back({p, a[p], x}); a[p] = x;
        }
    }
    mos_algorithm();
}

```

### 3.3 bipartite-disjoint-set-union

```

void make_set(int v) {
    parent[v] = make_pair(v, 0); rank[v] = 0; bipartite[v] = true;}
pair<int, int> find_set(int v) { if (v != parent[v].first) {
    int parity = parent[v].second; parent[v] = find_set(
    parent[v].first); parent[v].second ^= parity;}
    return parent[v];
}
void add_edge(int a, int b) {
    pair<int, int> pa = find_set(a);
    a = pa.first; int x = pa.second;
    pair<int, int> pb = find_set(b); b = pb.first;
    int y = pb.second;
    if (a == b) {
        if (x == y) bipartite[a] = false;
    } else {
        if (rank[a] < rank[b]) swap (a, b);
        parent[b] = make_pair(a, x^y^1);
        bipartite[a] ^= bipartite[b];
        if (rank[a] == rank[b]) ++rank[a];
    }
}
bool is_bipartite(int v){ return bipartite[find_set(v).first];}

```

### 3.4 bitset

```

int LEN;// length of Bitset array t
struct Bitset{
    ull t[N/64+5]; Bitset(){memset(t,0,sizeof t);}
    void set(int p){ t[p>>6]|=1llu<<(p&63); }
    void shift(){ ull last=0llu;
        for(int i=0;i<LEN;i++){
            ull curr=t[i]>>63llu; (t[i]<<=1)|=last; last =curr;
        }
    }
    int count(){ int ret=0;
        for(int i=0;i<LEN;i++) ret+=__builtin_popcountll(t[i]);
        return ret;
    }
    Bitset &operator = (Bitset const&b){
        memcpy(t,b.t,sizeof (t)); return *this;
    }
    Bitset &operator |= (Bitset &b){
        for(int i=0;i<LEN;i++)t[i]|=b.t[i]; return *this;
    }
    Bitset &operator &= (Bitset &b){
        for(int i=0;i<LEN;i++) t[i]&=b.t[i]; return *this;
    }
    Bitset &operator ^= (Bitset &b){
        for(int i=0;i<LEN;i++) t[i]^=b.t[i]; return *this;
    }
}

```

```

}
};
Bitset operator-(const Bitset &a,const Bitset &b){
    Bitset tmp; ull last=0;
    for(int i=0;i<LEN;i++){ ull curr=(a.t[i]< b.t[i] + last);
        tmp.t[i]=a.t[i]-b.t[i]-last; last = curr;
    }
    return tmp;
}
// https://loj.ac/p/6564 , string s,t, m_old = 0
// lcs formula: let x = m_old / Occurance set of s [char or t]
// m_new = ((x - ((m_old<<1)+1)) ^x)&x; finally m_new.count()

```

### 3.5 centroid decomposition

```

set<int> g[N];
int par[N],sub[N],level[N],ans[N]; int DP[LOGN][N];
int n,m; int nn;
void dfs1(int u,int p){
    sub[u]=1; nn++;
    for(auto it=g[u].begin();it!=g[u].end();it++) if(*it!=p){
        dfs1(*it,u); sub[u]+=sub[*it];}
}
int dfs2(int u,int p){
    for(auto it=g[u].begin();it!=g[u].end();it++)
        if(*it!=p && sub[*it]>nn/2)
            return dfs2(*it,u);
    return u;
}
void decompose(int root,int p){
    nn=0; dfs1(root,root); int centroid = dfs2(root,root);
    if(p==-1)p=centroid; par[centroid]=p;
    for(auto it=g[centroid].begin();it!=g[centroid].end();it++){
        g[*it].erase(centroid); decompose(*it,centroid); }
    g[centroid].clear();
}

```

### 3.6 dsu-rollback

```

struct dsu_save {
    int v, rnk, u, rnk; dsu_save() {}
    dsu_save(int _v, int _rnk, int _u, int _rnk)
        : v(_v), rnk(_rnk), u(_u), rnk(_rnk) {}
};
struct dsu_with_rollback {
    vector<int> p, rnk; int comps; stack<dsu_save> op;
    dsu_with_rollback() {}
    dsu_with_rollback(int n) { p.resize(n); rnk.resize(n);
        for (int i = 0; i < n; i++) { p[i] = i; rnk[i] = 0; }
        comps = n;
    }
    int find_set(int v){return (v == p[v])?v:find_set(p[v]);}
    bool unite(int v, int u) { v = find_set(v); u = find_set(u);
        if (v == u) return false; comps--;
        if (rnk[v] > rnk[u]) swap(v, u);
        op.push(dsu_save(v, rnk[v], u, rnk[u])); p[v] = u;
        if (rnk[u] == rnk[v]) rnk[u]++; return true;
    }
    void rollback() { if (op.empty()) return;
        dsu_save x = op.top(); op.pop(); comps++; p[x.v] = x.v;
        rnk[x.v] = x.rnk; p[x.u] = x.u; rnk[x.u] = x.rnk;
    }
};
struct query {
    int v, u; bool united;
    query(int _v, int _u) : v(_v), u(_u) {}
};
struct QueryTree {
    vector<vector<query>> t; dsu_with_rollback dsu; int T;
    QueryTree() {}
    QueryTree(int _T, int n) : T(_T) {
        dsu = dsu_with_rollback(n); t.resize(4 * T + 4); }
    void add_to_tree(int v,int l,int r,int ul,int ur,query& q){
        if (ul > ur) return;
        if (l == ul && r == ur) { t[v].push_back(q); return; }
        int mid = (l + r) / 2;
        add_to_tree(2 * v, l, mid, ul, min(ur, mid), q);
        add_to_tree(2*v+1,mid+1,r,max(ul, mid + 1), ur, q);
    }
}

```

```

void add_query(query q, int l, int r) {
    add_to_tree(1, 0, T - 1, l, r, q); }
void dfs(int v, int l, int r, vector<int>& ans) {
    for (query& q : t[v]) q.united = dsu.unite(q.v, q.u);
    if (l == r) ans[l] = dsu.comps;
    else { int mid = (l + r) / 2;
        dfs(2 * v, l, mid, ans); dfs(2 * v + 1, mid + 1, r, ans); }
    for (query q : t[v]) { if (q.united) dsu.rollback(); }
}
vector<int> solve() {
    vector<int> ans(T); dfs(1, 0, T - 1, ans); return ans;
}
};

```

### 3.7 link cut tree

```

const int MOD = 998244353;
int sum(int a, int b) {
    return a+b >= MOD ? a+b-MOD : a+b;
}
int mul(int a, int b) {
    return (a*1LL*b)%MOD;
}
typedef pair< int , int >Linear;
Linear compose(const Linear &p, const Linear &q) {
    return Linear(mul(p.first, q.first), sum(mul(q.second,
    p.first), p.second));
}
struct SplayTree {
    struct Node {
        int ch[2] = {0, 0}, p = 0;
        long long self = 0, path = 0; // Path aggregates
        long long sub = 0, vir = 0; // Subtree aggregates
        bool flip = 0; // Lazy tags
        int size = 1;
        Linear _self{1, 0}, _path_shoja{1, 0}, _path_ulta{1, 0};
    };
    vector<Node> T;
    SplayTree(int n) : T(n + 1) {
        T[0].size = 0;
    }
    void push(int x) {
        if (!x || !T[x].flip) return;
        int l = T[x].ch[0], r = T[x].ch[1];
        T[l].flip ^= 1, T[r].flip ^= 1;
        swap(T[x].ch[0], T[x].ch[1]);
        T[x].flip = 0;
        swap(T[x]._path_shoja, T[x]._path_ulta);
    }
    void pull(int x) {
        int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);
        T[x].size = T[l].size + T[r].size + 1;
        T[x].path = T[l].path + T[x].self + T[r].path;
        T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
        T[x]._path_shoja = compose(T[r]._path_shoja,
        compose(T[x]._self, T[l]._path_shoja));
        T[x]._path_ulta = compose(T[l]._path_ulta,
        compose(T[x]._self, T[r]._path_ulta));
    }
    void set(int x, int d, int y) {
        T[x].ch[d] = y; T[y].p = x; pull(x);
    }
    void splay(int x) {
        auto dir = [&](int x) {
            int p = T[x].p; if (!p) return -1;
            return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 :
            -1;
        };
        auto rotate = [&](int x) {
            int y = T[x].p, z = T[y].p, dx = dir(x), dy =
            dir(y);
            set(y, dx, T[x].ch[!dx]);
            set(x, !dx, y);
            if (~dy) set(z, dy, x);
            T[x].p = z;
        };
        for (push(x); ~dir(x); ) {
            int y = T[x].p, z = T[y].p;

```

```

    push(z); push(y); push(x);
    int dx = dir(x), dy = dir(y);
    if (~dy) rotate(dx != dy ? x : y);
    rotate(x);
}
}
int KthNext(int x, int k) {
    assert(k > 0);
    splay(x);
    x = T[x].ch[1];
    if (T[x].size < k) return -1;
    while (true) {
        push(x);
        int l = T[x].ch[0], r = T[x].ch[1];
        if (T[l].size+1 == k) return x;
        if (k <= T[l].size) x = l;
        else k -= T[l].size+1, x = r;
    }
}
};
struct LinkCut : SplayTree {
    LinkCut(int n) : SplayTree(n) {}
    int access(int x) {
        int u = x, v = 0;
        for (; u; v = u, u = T[u].p) {
            splay(u);
            int& ov = T[u].ch[1];
            T[u].vir += T[ov].sub;
            T[u].vir -= T[v].sub;
            ov = v; pull(u);
        }
        splay(x);
        return v;
    }
    void reroot(int x) {
        access(x); T[x].flip ^= 1; push(x);
    }
    //makes v parent of u (optional: u must be a root)
    void Link(int u, int v) {
        reroot(u); access(v);
        T[v].vir += T[u].sub;
        T[u].p = v; pull(v);
    }
    //removes edge between u and v
    void Cut(int u, int v) {
        int _u = FindRoot(u);
        reroot(u); access(v);
        T[v].ch[0] = T[u].p = 0; pull(v);
        reroot(_u);
    }
    // Rooted tree LCA. Returns 0 if u and v arent connected.
    int LCA(int u, int v) {
        if (u == v) return u;
        access(u); int ret = access(v);
        return T[u].p ? ret : 0;
    }
    // Query subtree of u where v is outside the subtree.
    long long Subtree(int u, int v) {
        int _v = FindRoot(v);
        reroot(v); access(u);
        long long ans = T[u].vir + T[u].self;
        reroot(_v);
        return ans;
    }
    // Query path [u..v]
    long long Path(int u, int v) {
        int _u = FindRoot(u);
        reroot(u); access(v);
        long long ans = T[v].path;
        reroot(_u);
        return ans;
    }
    Linear_Path(int u, int v) {
        reroot(u); access(v); return T[v]._path_shoja;
    }
    // Update vertex u with value v
    void Update(int u, long long v) {
        access(u); T[u].self = v; pull(u);
    }
}

```

```

// Update vertex u with value v
void _Update(int u, Linear v) {
    access(u); T[u]._self = v; pull(u);
}
int FindRoot(int u) {
    access(u);
    while (T[u].ch[0]) {
        u = T[u].ch[0];
        push(u);
    }
    access(u);
    return u;
}
//k-th node (0-indexed) on the path from u to v
int KthOnPath(int u, int v, int k) {
    if (u == v) return k == 0 ? u : -1;
    int _u = FindRoot(u);
    reroot(u); access(v);
    int ans = KthNext(u, k);
    reroot(_u);
    return ans;
}
}
int main() {
    cin >> n >> q;
    LinkCut lct(n);
    for (int i = 1; i <= n; i++) {
        Linear l;
        cin >> l.first >> l.second;
        lct._Update(i, l);
    }
    for (int i = 1; i < n; i++) {
        int u, v;
        cin >> u >> v;
        lct.Link(u+1, v+1);
    }
    while (q--) {
        int op;
        cin >> op;
        if (op == 0) {
            int u, v, w, x;
            cin >> u >> v >> w >> x;
            lct.Cut(u+1, v+1);
            lct.Link(w+1, x+1);
        } else if (op == 1) {
            int p; Linear l;
            cin >> p >> l.first >> l.second;
            lct._Update(p+1, l);
        } else {
            int u, v, x;
            cin >> u >> v >> x;
            Linear l = lct.Path(u+1, v+1);
            cout << sum(mul(l.first, x), l.second) << "\n";
        }
    }
    return 0;
}

```

### 3.8 sparse table 2d

```

const int N=500; const int K = 8 ; // k >= ceil(lg22(n)) +1
int arr[N][N]; int st[K+1][K+1][N][N]; int lg2[N+1];
void ini(){ lg2[1] = 0;
    for (int i = 2; i <= N; i++) lg2[i] = lg2[i/2] + 1; }
int f(int i,int j){ return max(i,j); }
void pre( int n,int m){
    for(int x=0;x<=K;x++){
        for(int y=0;y<=K;y++){
            for(int i=0;i<n;i++){
                for(int j=0;j<m;j++){
                    if(i+(1<<x)>n || j+(1<<y) > m) continue;
                    if(x>0) st[x][y][i][j] = f(st[x-1][y][i][j] ,
                        st[x-1][y][i+(1<<(x-1))][j]);
                    else if(y>0) st[x][y][i][j] = f(st[x][y-1][i][j] ,
                        st[x][y-1][i][j+(1<<(y-1))]);
                    else st[x][y][i][j] = f(arr[i][j] , arr[i][j]);
                } } } }
    int getf( int R1 ,int C1 , int R2 , int C2){
        if(R1>R2) swap(R1,R2); if(C1>C2) swap(C1,C2);
        int x = lg2[R2 - R1 + 1]; int y = lg2[C2 - C1 + 1];
    }
}

```

```

    return f(f(f(st[x][y][R1][C1],st[x][y][R2-(1<<x)+1][C1]),
st[x][y][R1][C2-(1<<y)+1]),st[x][y][R2-(1<<x)+1][C2-(1<<y)+1]);
}

```

### 3.9 treap

```

template <class T>
class treap{
    struct item{
        int prior, cnt;
        T key;
        item *l,*r;
        item(T v)
        {
            key=v;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root,*node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){
        if (it) it->cnt = cnt(it->l) + cnt(it->r) + 1;
    }
    void split (item * t, T key, item * &l, item * &r){
        if (!t)
            l = r = NULL;
        else if (key < t->key)
            split (t->l, key, l, t->l), r = t;
        else
            split (t->r, key, t->r, r), l = t;
        upd_cnt(t);
    }
    void insert (item * &t, item * it){
        if (!t)
            t = it;
        else if (it->prior > t->prior)
            split (t, it->key, it->l, it->r), t = it;
        else
            insert (it->key < t->key ? t->l : t->r, it);
        upd_cnt(t);
    }
    // keys(l) < keys(r)
    void merge (item * &t, item * l, item * r){
        if (!l || !r)
            t = l ? l : r;
        else if (l->prior > r->prior)
            merge (l->r, l->r, r), t = l;
        else
            merge (r->l, l, r->l), t = r;
        upd_cnt(t);
    }
    void erase (item * &t, T key){
        if (t->key == key)
            merge (t, t->l, t->r);
        else
            erase (key < t->key ? t->l : t->r, key);
        upd_cnt(t);
    }
    T elementAt(item * &t,int key){
        T ans;
        if(cnt(t->l)==key) ans=t->key;
        else if(cnt(t->l)>key) ans=elementAt(t->l,key);
        else ans=elementAt(t->r,key-1-cnt(t->l));
        upd_cnt(t);
        return ans;
    }
    item * unite (item * l, item * r){
        if (!l || !r) return l ? l : r;
        if (l->prior < r->prior) swap (l, r);
        item * lt, * rt;
        split (r, l->key, lt, rt);
        l->l = unite (l->l, lt);
        l->r = unite (l->r, rt);
        upd_cnt(l);
        upd_cnt(r);
        return l;
    }
}

```

```

}
void heapify (item * t){
    if (!t) return;
    item * max = t;
    if (t->l != NULL && t->l->prior > max->prior)
        max = t->l;
    if (t->r != NULL && t->r->prior > max->prior)
        max = t->r;
    if (max != t)
    {
        swap (t->prior, max->prior);
        heapify (max);
    }
}
item * build (T * a, int n){
    if (n == 0) return NULL;
    int mid = n / 2;
    item * t = new item (a[mid], rand ());
    t->l = build (a, mid);
    t->r = build (a + mid + 1, n - mid - 1);
    heapify (t);
    return t;
}
void output (item * t,vector<T> &arr){
    if (!t) return;
    output (t->l,arr);
    arr.push_back(t->key);
    output (t->r,arr);
}
public:
    treap(){
        root=NULL;
    }
    treap(T *a,int n){
        build(a,n);
    }
    void insert(T value){
        node=new item(value);
        insert(root,node);
    }
    void erase(T value){
        erase(root,value);
    }
    T elementAt(int position){
        return elementAt(root,position);
    }
    int size(){
        return cnt(root);
    }
    void output(vector<T> &arr){
        output(root,arr);
    }
    int range_query(T l,T r){ //([l,r]
        item *previous,*next,*current;
        split(root,l,previous,current);
        split(current,r,current,next);
        int ans=cnt(current);
        merge(root,previous,current);
        merge(root,root,next);
        previous=NULL;
        current=NULL;
        next=NULL;
        return ans;
    }
};
template <class T>
class implicit_treap{
    struct item{
        int prior, cnt;
        T value;
        bool rev;
        item *l,*r;
        item(T v){
            value=v;
            rev=false;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    }

```

```

} *root,*node;
int cnt (item * it){
    return it ? it->cnt : 0;
}
void upd_cnt (item * it){
    if (it)
        it->cnt = cnt(it->l) + cnt(it->r) + 1;
}
void push (item * it){
    if (it && it->rev){
        it->rev = false;
        swap (it->l, it->r);
        if (it->l) it->l->rev ^= true;
        if (it->r) it->r->rev ^= true;
    }
}
void merge (item * &t, item * l, item * r){
    push (l);
    push (r);
    if (!l || !r)
        t = l ? l : r;
    else if (l->prior > r->prior)
        merge (l->r, l->r, r), t = l;
    else
        merge (r->l, l, r->l), t = r;
    upd_cnt (t);
}
void split (item * t, item * &l, item * &r, int key, int
    add = 0){
    if (!t)
        return void( l = r = 0 );
    push (t);
    int cur_key = add + cnt(t->l);
    if (key <= cur_key)
        split (t->l, l, t->l, key, add), r = t;
    else
        split (t->r, t->r, r, key, add + 1 + cnt(t->l)), l
            = t;
    upd_cnt (t);
}
void insert(item * &t,item * element,int key){
    item *l,*r;
    split(t,l,r,key);
    merge(l,l,element);
    merge(t,l,r);
    l=NULL;
    r=NULL;
}
T elementAt(item * &t,int key){
    push(t);
    T ans;
    if(cnt(t->l)==key) ans=t->value;
    else if(cnt(t->l)>key) ans=elementAt(t->l,key);
    else ans=elementAt(t->r,key-1-cnt(t->l));
    return ans;
}
void erase (item * &t, int key){
    push(t);
    if(!t) return;
    if (key == cnt(t->l))
        merge (t, t->l, t->r);
    else if(key<cnt(t->l))
        erase(t->l,key);
    else
        erase(t->r,key-cnt(t->l)-1);
    upd_cnt(t);
}
void reverse (item * &t, int l, int r){
    item *t1, *t2, *t3;
    split (t, t1, t2, l);
    split (t2, t2, t3, r-1+1);
    t2->rev ^= true;
    merge (t, t1, t2);
    merge (t, t, t3);
}
void cyclic_shift(item * &t,int L,int R){
    if(L==R) return;
    item *l,*r,*m;
    split(t,t,l,L);

```

```

        split(l,l,m,R-L+1);
        split(l,l,r,R-L);
        merge(t,t,r);
        merge(t,t,l);
        merge(t,t,m);
        l=NULL;
        r=NULL;
        m=NULL;
    }
    void output (item * t,vector<T> &arr){
        if (!t) return;
        push (t);
        output (t->l,arr);
        arr.push_back(t->value);
        output (t->r,arr);
    }
}
public:
    implicit_treap(){
        root=NULL;
    }
    void insert(T value,int position){
        node=new item(value);
        insert(root,node,position);
    }
    void erase(int position){
        erase(root,position);
    }
    void reverse(int l,int r){
        reverse(root,l,r);
    }
    T elementAt(int position){
        return elementAt(root,position);
    }
    void cyclic_shift(int L,int R){
        cyclic_shift(root,L,R);
    }
    int size(){
        return cnt(root);
    }
    void output(vector<T> &arr){
        output(root,arr);
    }
};

```

### 3.10 wavelet tree

```

#include <bits/stdc++.h>
using namespace std;
#define fo(i,n) for(i=0;i<n;i++)
#define ll long long
#define pb push_back
#define mp make_pair
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef vector<int> vi;
const int N = 3e5, M = N;
//=====
const int MAX = 1e6;
int a[N];
struct wavelet_tree{
    #define vi vector<int>
    #define pb push_back
    int lo, hi;
    wavelet_tree *l=0, *r=0;
    vi b;
    vi c; // c holds the prefix sum of elements
    //nos are in range [x,y]
    //array indices are [from, to)
    wavelet_tree(int *from, int *to, int x, int y){
        lo = x, hi = y;
        if (from >= to)
            return;
        if (hi == lo ){
            b.reserve(to-from+1);
            b.pb(0);
            c.reserve(to-from+1);
            c.pb(0);
            for(auto it = from; it != to; it++){
                b.pb(b.back() + 1);
                c.pb(c.back()+*it);
            }
        }
    }
};

```

```

    }
    return ;
}
int mid = (lo+hi)/2;
auto f = [mid](int x){
    return x <= mid;
};
b.reserve(to-from+1);
b.pb(0);
c.reserve(to-from+1);
c.pb(0);
for(auto it = from; it != to; it++){
    b.pb(b.back() + f(*it));
    c.pb(c.back() + *it);
}
//see how lambda function is used here
auto pivot = stable_partition(from, to, f);
l = new wavelet_tree(from, pivot, lo, mid);
r = new wavelet_tree(pivot, to, mid+1, hi);
}
// swap a[i] with a[i+1] , if a[i]!=a[i+1] call
swapadjacent(i)
void swapadjacent(int i){
    if(lo == hi)
        return ;
    b[i] = b[i-1] + b[i+1] - b[i];
    c[i] = c[i-1] + c[i+1] - c[i];
    if( b[i+1]-b[i] == b[i] - b[i-1]){
        if(b[i]-b[i-1])
            return this->l->swapadjacent(b[i]);
        else
            return this->r->swapadjacent(i-b[i]);
    }
    else
        return ;
}
//kth smallest element in [l, r]
int kth(int l, int r, int k){
    if(l > r)
        return 0;
    if(lo == hi)
        return lo;
    int inLeft = b[r] - b[l-1];
    int lb = b[l-1]; //amt of nos in first (l-1) nos that
                    //go in left
    int rb = b[r]; //amt of nos in first (r) nos that go in
                    //left
    if(k <= inLeft)
        return this->l->kth(lb+1, rb, k);
    return this->r->kth(l-lb, r-rb, k-inLeft);
}
//count of nos in [l, r] Less than or equal to k
int LTE(int l, int r, int k){
    if(l > r or k < lo)
        return 0;
    if(hi <= k)
        return r - l + 1;
    int lb = b[l-1], rb = b[r];
    return this->l->LTE(lb+1, rb, k) + this->r->LTE(l-lb,
        r-rb, k);
}
//count of nos in [l, r] equal to k
int count(int l, int r, int k){
    if(l > r or k < lo or k > hi)
        return 0;
    if(lo == hi)
        return r - l + 1;
    int lb = b[l-1], rb = b[r], mid = (lo+hi)/2;
    if(k <= mid)
        return this->l->count(lb+1, rb, k);
    return this->r->count(l-lb, r-rb, k);
}
//sum of nos in [l, r] less than or equal to k
int sumk(int l, int r, int k){
    if(l > r or k < lo)
        return 0;
    if(hi <= k)
        return c[r] - c[l-1];
    int lb = b[l-1], rb = b[r];

```

```

    return this->l->sumk(lb+1, rb, k) + this->r->sumk(l-lb,
        r-rb, k);
}
~wavelet_tree(){
    if(l)
        delete l;
    if(r)
        delete r;
}
};
int main(){
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    srand(time(NULL));
    int i,n,k,j,q,l,r;
    cin >> n;
    fo(i, n) cin >> a[i+1];
    wavelet_tree T(a+1, a+n+1, 1, MAX);
    cin >> q;
    while(q--){
        int x;
        cin >> x;
        cin >> l >> r >> k;
        if(x == 0){
            //kth smallest
            cout << "Kth smallest: ";
            cout << T.kth(l, r, k) << endl;
        }
        if(x == 1){
            //less than or equal to K
            cout << "LTE: ";
            cout << T.LTE(l, r, k) << endl;
        }
        if(x == 2){
            //count occurence of K in [l, r]
            cout << "Occurence of K: ";
            cout << T.count(l, r, k) << endl;
        }
        if(x == 3){
            //sum of elements less than or equal to K in [l, r]
            cout << "Sum: ";
            cout << T.sumk(l, r, k) << endl;
        }
    }
    return 0;
}

```

## 4 Game

### 4.1 HackenBush

```

/* tree case: g[u] = for all v : XOR[ g[v] + 1 ]
lose if no moves available
1. Colon Principle: Grundy number of a tree is the
xor of Grundy number of child subtrees.
2. Fusion Principle: Consider a pair of adjacent
vertices u, v that has another path (i.e.,
they are in a cycle). Then, we can contract u
and v without changing Grundy number.
We first decompose graph into two-edge connected
components. Then, by contracting each components by
using Fusion Principle, we obtain a tree (and many
self loops) that has the same Grundy number to the
original graph. By using Colon Principle, we can
compute the Grundy number. O(m + n). */
struct hackenbush {
    int n; vector<vector<int>>> adj;
    hackenbush(int n) : n(n), adj(n) {}
    void add_edge(int u, int v) {
        adj[u].push_back(v);
        if(u!=v) adj[v].push_back(u);
    }
    int Grundy(int r) {
        vector<int> num(n), low(n); int t = 0;
        function<int(int, int)> dfs=[&](int p, int u) {
            num[u] = low[u] = ++t; int ans = 0;
            for (int v : adj[u]) {
                if (v == p) { p += 2 * n; continue; }
                if (num[v] == 0) {
                    int res = dfs(u, v);
                    low[u] = min(low[u], low[v]);

```

```

                if (low[v] > num[u]) ans ^= (1+res)^1;
                else ans ^= res; // non bridge
            }
            else low[u] = min(low[u], num[v]);
        }
        if (p > n) p -= 2 * n;
        for (int v : adj[u])
            if (v != p && num[u] <= num[v]) ans ^= 1;
        return ans;
    };
    return dfs(-1, r);
}
};

```

## 5 Geo

### 5.1 3dGeo

```

int dcmp(double x) { return abs(x) < EPS ? 0 : (x<0 ? -1 : 1); }
double degreeToRadian(double rad) { return rad*PI/180; }
struct Point {
    double x, y, z; Point() : x(0), y(0), z(0) {}
    Point(double X, double Y, double Z) : x(X), y(Y), z(Z) {}
    Point operator + (const Point& u) const {
        return Point(x + u.x, y + u.y, z + u.z); }
    Point operator - (const Point& u) const {
        return Point(x - u.x, y - u.y, z - u.z); }
    Point operator * (const double u) const {
        return Point(x * u, y * u, z * u); }
    Point operator / (const double u) const {
        return Point(x / u, y / u, z / u); }
};
double dot(Point a, Point b){ return a.x*b.x+a.y*b.y+a.z*b.z; }
Point cross(Point a, Point b) { return
    Point(a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y - a.y*b.x);
}
double length(Point a) { return sqrt(dot(a, a)); }
double distance(Point a, Point b) { return length(a-b); }
Point unit(const Point &p) { return p/length(p); }
// Rotate p around axis x, with angle radians.
Point rotate(Point p, Point axis, double angle) {
    axis = unit(axis); Point comp1 = p * cos(angle);
    Point comp2 = axis * (1-cos(angle)) * dot(axis, p);
    Point comp3 = cross(axis, p) * sin(angle);
    return comp1 + comp2 + comp3;
}
struct Line {Point a, v;}; //a+tv
// returns the distance from point a to line l
double distancePointLine(Point p, Line l) {
    return length(cross(l.v, p - l.a)) / length(l.v); }
// distance from Line ab to Line cd
double distanceLineLine(Line a, Line b) {
    Point cr = cross(a.v, b.v); double crl = length(cr);
    if (dcmp(crl) == 0) return distancePointLine(a.a, b);
    return abs(dot(cr, a.a-b.a))/crl; }
struct Plane {
    Point normal; // Normal = (A, B, C)
    double d; // dot(Normal) = d <--> Ax + By + Cz = d
    Point P; // anyPoint on the plane, optional
    Plane(Point normal, double d) {
        double len = length(normal); assert(dcmp(len) > 0);
        normal = normal / len; d = d / len;
        if (dcmp(normal.x)) P = Point(d/normal.x, 0, 0);
        else if (dcmp(normal.y)) P = Point(0, d/normal.y, 0);
        else P = Point(0, 0, d/normal.z);
    }
    //Plane given by three Non-Collinear Points
    Plane(Point a, Point b, Point c) {
        normal = unit(cross(b-a, c-a)); d = dot(normal, a); P=a;
    }
    bool onPlane(Point a) { return dcmp(dot(normal, a)-d)== 0; }
    double distance(Point a) { return abs(dot(normal, a) - d); }
    double isParallel(Line l){return dcmp(dot(l.v, normal))==0;}
    //return t st l.a + t*l.v is a point on the plane, check
    //parallel first
    double intersectLine(Line l) {
        return dot(P-l.a, normal)/dot(l.v, normal); } };

```

## 5.2 Circle Cover

```

//Check if the all of the area of circ(0, R) in
//Circ(00, RR) is covered by some other circle
bool CoverCircle(PT O, double R, vector<PT> &cen,
    vector<double> &rad, PT OO, double RR) {
    int n = cen.size();
    vector<pair<double, double>> arcs;
    for (int i=0; i<n; i++) {
        PT P = cen[i]; double r = rad[i];
        if (i!=0 && R + sqrt(dist2(O, P))<r) return 1;
        if (i==0 && r + sqrt(dist2(O, P))<R) return 1;
        vector<PT> inter =
            CircleCircleIntersection(O, P, R, r);
        if (inter.size() <= 1) continue;
        PT X = inter[0], Y = inter[1];
        if (cross(O, X, Y) < 0) swap(X, Y);
        if (!(cross(O, X, P) >= 0 &&
            cross(O, Y, P) <= 0)) swap(X, Y);
        if (i==0) swap(X, Y);
        X = X-O; Y=Y-O;
        double ll = atan2(X.y, X.x);
        double rr = atan2(Y.y, Y.x);
        if (rr < ll) rr += 2*PI;
        arcs.emplace_back(ll, rr);
    }
    if (arcs.empty()) return false;
    sort(arcs.begin(), arcs.end());
    double st = arcs[0].ff, en = arcs[0].ss, ans = 0;
    for (int i=1; i<arcs.size(); i++) {
        if (arcs[i].first <= en + EPS)
            en = max(en, arcs[i].second);
        else st = arcs[i].first, en = arcs[i].second;
        ans = max(ans, en-st);
    }
    return ans >= 2*PI;
}

```

## 5.3 Circle Union Area

```

struct Point {
    LD x,y ;
    LD operator*(const Point &a)const {
        return x*a.y-y*a.x;}
    LD operator/(const Point &a)const {
        return sqrt((a.x-x)*(a.x-x)+(a.y-y)*(a.y-y));}
}po[N];
LD r[N];
int sgn(LD x) {return fabs(x)<EPS?0:(x>0.0?1:-1);}
pair<LD,bool> ARG[2*N];
LD cir_union(Point c[],LD r[],int n) {
    LD sum = 0.0 , sum1 = 0.0 ,d,p1,p2,p3 ;
    for(int i = 0 ; i < n ; i++) {
        bool f = 1 ;
        for(int j = 0 ; f&&j<n ; j++)
            if(i!=j && sgn(r[j]-r[i]-c[i]/c[j])!=-1)f=0;
        if(!f) swap(r[i],r[--n]),swap(c[i--],c[n]);
    }
    for(int i = 0; i < n; i++) {
        int k = 0, cnt = 0;
        for(int j = 0; j < n; j++) {
            if(i!=j&&sgn((d=c[i]/c[j])-r[i]-r[j])<=0){
                p3=acos((r[i]*r[i]+d*d-r[j]*r[j])/(
                    2.0*r[i]*d));
                p2=atan2(c[j].y-c[i].y,c[j].x-c[i].x);
                p1 = p2-p3; p2 = p2+p3;
                if(sgn(p1+PI)==-1) p1+=2*PI,cnt++;
                if(sgn(p2-PI)==1) p2-=2*PI,cnt++;
                ARG[k++] = make_pair(p1,0);
                ARG[k++] = make_pair(p2,1);
            }
        }
        if(k) {
            sort(ARG,ARG+k);
            p1 = ARG[k-1].first-2*PI;
            p3 = r[i]*r[i];
            for(int j = 0 ; j < k ; j++) {
                p2 = ARG[j].first;

```

```

        if(cnt==0) {
            sum+=(p2-p1-sin(p2-p1))*p3 ;
            sum1+=(c[i]+Point(cos(p1),sin(p1))*
                r[i])*c[i]+
                Point(cos(p2),sin(p2))*r[i]);
        }
        p1 = p2;
        ARG[j].second ? cnt--:cnt++;
    }
    }
    else sum += 2*PI*r[i]*r[i];
}
return (sum+fabs(sum1))*0.5 ;
}

```

## 5.4 basic-area-geometry

```

struct point2d {
    ftype x, y; point2d() {}
    point2d(ftype x, ftype y): x(x), y(y) {}
    point2d& operator+=(const point2d &t) {
        x += t.x; y += t.y; return *this;
    }
    point2d& operator-=(const point2d &t) {
        x -= t.x; y -= t.y; return *this;
    }
    point2d& operator*=(ftype t) {
        x *= t; y *= t; return *this;
    }
    point2d& operator/=(ftype t) {
        x /= t; y /= t; return *this;
    }
    point2d operator+(const point2d &t) const {
        return point2d(*this) += t;
    }
    point2d operator-(const point2d &t) const {
        return point2d(*this) -= t;
    }
    point2d operator*(ftype t) const {
        return point2d(*this) *= t;
    }
    point2d operator/(ftype t) const {
        return point2d(*this) /= t;
    }
};
point2d operator*(ftype a, point2d b) {
    return b * a;
}
struct point3d {
    ftype x, y, z; point3d() {}
    point3d(ftype x, ftype y, ftype z): x(x), y(y), z(z) {}
    point3d& operator+=(const point3d &t) {
        x += t.x; y += t.y; z += t.z; return *this;
    }
    point3d& operator-=(const point3d &t) {
        x -= t.x; y -= t.y; z -= t.z; return *this;
    }
    point3d& operator*=(ftype t) {
        x *= t; y *= t; z *= t; return *this;
    }
    point3d& operator/=(ftype t) {
        x /= t; y /= t; z /= t; return *this;
    }
    point3d operator+(const point3d &t) const {
        return point3d(*this) += t;
    }
    point3d operator-(const point3d &t) const {
        return point3d(*this) -= t;
    }
    point3d operator*(ftype t) const {
        return point3d(*this) *= t;
    }
    point3d operator/(ftype t) const {
        return point3d(*this) /= t;
    }
};
point3d operator*(ftype a, point3d b) {
    return b * a;
}

```

```

}
ftype dot(point2d a, point2d b) {
    return a.x * b.x + a.y * b.y;
}
ftype dot(point3d a, point3d b) {
    return a.x * b.x + a.y * b.y + a.z * b.z;
}
ftype norm(point2d a) {
    return dot(a, a);
}
double abs(point2d a) {
    return sqrt(norm(a));
}
double proj(point2d a, point2d b) {
    return dot(a, b) / abs(b);
}
double angle(point2d a, point2d b) {
    return acos(dot(a, b) / abs(a) / abs(b));
}
point3d cross(point3d a, point3d b) {
    return point3d(a.y * b.z - a.z * b.y,
        a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
}
ftype triple(point3d a, point3d b, point3d c) {
    return dot(a, cross(b, c));
}
ftype cross(point2d a, point2d b) {
    return a.x * b.y - a.y * b.x;
}
point2d intersect(point2d a1, point2d d1,
    point2d a2, point2d d2) {
    return a1 + cross(a2 - a1, d2) / cross(d1, d2) * d1;
}
point3d intersect(point3d a1, point3d n1, point3d a2,
    point3d n2, point3d a3, point3d n3) {
    point3d x(n1.x, n2.x, n3.x); point3d y(n1.y, n2.y, n3.y);
    point3d z(n1.z, n2.z, n3.z);
    point3d d(dot(a1, n1), dot(a2, n2), dot(a3, n3));
    return point3d(triple(d, y, z), triple(x, d, z),
        triple(x, y, d)) / triple(n1, n2, n3);
}
int signed_area_parallelogram(p2d p1, point2d p2, point2d p3) {
    return cross(p2 - p1, p3 - p2);
}
double triangle_area(point2d p1, point2d p2, point2d p3) {
    return abs(signed_area_parallelogram(p1, p2, p3)) / 2.0;
}
bool clockwise(point2d p1, point2d p2, point2d p3) {
    return signed_area_parallelogram(p1, p2, p3) < 0;
}
bool counter_clockwise(point2d p1, point2d p2, point2d p3) {
    return signed_area_parallelogram(p1, p2, p3) > 0;
}
double area(const vector<point>& fig) {
    double res = 0;
    for (unsigned i = 0; i < fig.size(); i++) {
        point p = i? fig[i - 1] : fig.back(); point q = fig[i];
        res += (p.x - q.x) * (p.y + q.y);
    }
    return fabs(res) / 2;
}
//Pick: S = I + B/2 - 1
int count_lattices(Fraction k, Fraction b, long long n) {
    auto fk = k.floor(); auto fb = b.floor(); auto cnt = 0LL;
    if (k >= 1 || b >= 1) {
        cnt+=(fk*(n - 1) + 2 * fb) * n / 2; k -= fk; b -= fb;
    }
    auto t = k * n + b; auto ft = t.floor();
    if (ft >= 1)
        cnt += count_lattices(1 / k, (t - t.floor()) / k, t.floor());
    return cnt;
}

```

## 5.5 delaunay-voronoi

```

typedef long long ll;
bool ge(const ll& a, const ll& b) { return a >= b; }
bool le(const ll& a, const ll& b) { return a <= b; }

```

```

bool eq(const ll& a, const ll& b) { return a == b; }
bool gt(const ll& a, const ll& b) { return a > b; }
bool lt(const ll& a, const ll& b) { return a < b; }
int sgn(const ll& a) { return a >= 0 ? a : 1 : 0 : -1; }
struct pt {
    ll x, y;
    pt() {}
    pt(ll _x, ll _y) : x(_x), y(_y) {}
    pt operator-(const pt& p) const {
        return pt(x - p.x, y - p.y);
    }
    ll cross(const pt& p) const {
        return x * p.y - y * p.x;
    }
    ll cross(const pt& a, const pt& b) const {
        return (a - *this).cross(b - *this);
    }
    ll dot(const pt& p) const {
        return x * p.x + y * p.y;
    }
    ll dot(const pt& a, const pt& b) const {
        return (a - *this).dot(b - *this);
    }
    ll sqrLength() const {
        return this->dot(*this);
    }
    bool operator==(const pt& p) const {
        return eq(x, p.x) && eq(y, p.y);
    }
};
const pt inf_pt = pt(1e18, 1e18);
struct QuadEdge {
    pt origin;
    QuadEdge* rot = nullptr;
    QuadEdge* onext = nullptr;
    bool used = false;
    QuadEdge* rev() const {
        return rot->rot;
    }
    QuadEdge* lnext() const {
        return rot->rev()->onext->rot;
    }
    QuadEdge* oprev() const {
        return rot->onext->rot;
    }
    pt dest() const {
        return rev()->origin;
    }
};
QuadEdge* make_edge(pt from, pt to) {
    QuadEdge* e1 = new QuadEdge;
    QuadEdge* e2 = new QuadEdge;
    QuadEdge* e3 = new QuadEdge;
    QuadEdge* e4 = new QuadEdge;
    e1->origin = from; e2->origin = to;
    e3->origin = e4->origin = inf_pt;
    e1->rot = e3; e2->rot = e4;
    e3->rot = e2; e4->rot = e1;
    e1->onext = e1; e2->onext = e2;
    e3->onext = e4; e4->onext = e3;
    return e1;
}
void splice(QuadEdge* a, QuadEdge* b) {
    swap(a->onext->rot->onext, b->onext->rot->onext);
    swap(a->onext, b->onext);
}
void delete_edge(QuadEdge* e) {
    splice(e, e->oprev());
    splice(e->rev(), e->rev()->oprev());
    delete e->rev()->rot; delete e->rev();
    delete e->rot; delete e;
}
QuadEdge* connect(QuadEdge* a, QuadEdge* b) {
    QuadEdge* e = make_edge(a->dest(), b->origin);
    splice(e, a->lnext()); splice(e->rev(), b);
    return e;
}
bool left_of(pt p, QuadEdge* e) {
    return gt(p.cross(e->origin, e->dest()), 0);
}

```

```

}
bool right_of(pt p, QuadEdge* e) {
    return lt(p.cross(e->origin, e->dest()), 0);
}
template <class T>
T det3(T a1, T a2, T a3, T b1, T b2, T b3, T c1, T c2, T c3) {
    return a1 * (b2 * c3 - c2 * b3) - a2 * (b1 * c3 - c1 * b3) +
        a3 * (b1 * c2 - c1 * b2);
}
bool in_circle(pt a, pt b, pt c, pt d) {
    // If there is __int128, calculate directly.
    // Otherwise, calculate angles.
    #if defined(__LP64__) || defined(_WIN64)
        __int128 det = -det3<__int128>(b.x, b.y, b.sqrLength(),
            c.x, c.y, c.sqrLength(), d.x, d.y, d.sqrLength());
        det += det3<__int128>(a.x, a.y, a.sqrLength(), c.x, c.y,
            c.sqrLength(), d.x, d.y, d.sqrLength());
        det -= det3<__int128>(a.x, a.y, a.sqrLength(), b.x, b.y,
            b.sqrLength(), d.x, d.y, d.sqrLength());
        det += det3<__int128>(a.x, a.y, a.sqrLength(), b.x, b.y,
            b.sqrLength(), c.x, c.y, c.sqrLength());
        return det > 0;
    #else
        auto ang = [](pt l, pt mid, pt r) {
            ll x = mid.dot(l, r); ll y = mid.cross(l, r);
            long double res = atan2((long double)x, (long double)y);
            return res;
        };
        long double kek = ang(a, b, c) + ang(c, d, a) - ang(b, c, d) - ang(d, a, b);
        if (kek > 1e-8) return true;
        else return false;
    #endif
}
pair<QuadEdge*, QuadEdge*> build_tr(int l, int r, vector<pt>& p) {
    if (r - l + 1 == 2) {
        QuadEdge* res = make_edge(p[l], p[r]);
        return make_pair(res, res->rev());
    }
    if (r - l + 1 == 3) {
        QuadEdge* a = make_edge(p[l], p[l+1]);
        *b = make_edge(p[l+1], p[r]); splice(a->rev(), b);
        int sg = sgn(p[l].cross(p[l+1], p[r]));
        if (sg == 0) return make_pair(a, b->rev());
        QuadEdge* c = connect(b, a);
        if (sg == 1) return make_pair(a, b->rev());
        else return make_pair(c->rev(), c);
    }
    int mid = (l + r) / 2; QuadEdge* ldo, *ldi, *rdo, *rdi;
    tie(ldo, ldi) = build_tr(l, mid, p);
    tie(rdi, rdo) = build_tr(mid + 1, r, p);
    while (true) {
        if (left_of(rdi->origin, ldi)) {
            ldi = ldi->lnext(); continue;
        }
        if (right_of(ldi->origin, rdi)) {
            rdi = rdi->rev()->onext; continue;
        }
        break;
    }
    QuadEdge* basel = connect(rdi->rev(), ldi);
    auto valid = [&basel](QuadEdge* e) {
        return right_of(e->dest(), basel);
    };
    if (ldi->origin == ldo->origin) ldo = basel->rev();
    if (rdi->origin == rdo->origin) rdo = basel;
    while (true) {
        QuadEdge* lcand = basel->rev()->onext;
        if (valid(lcand)) {
            while (in_circle(basel->dest(), basel->origin, lcand->dest(),
                lcand->onext->dest())) {
                QuadEdge* t = lcand->onext;
                delete_edge(lcand);
                lcand = t;
            }
        }
        QuadEdge* rcand = basel->oprev();
        if (valid(rcand)) {
            while (in_circle(basel->dest(), basel->origin, rcand->dest(),
                rcand->oprev()->dest())) {
                QuadEdge* t = rcand->oprev();
                delete_edge(rcand);
            }
        }
    }
}

```

```

    rcand = t;
}
    }
    if (!valid(lcand) && !valid(rcand)) break;
    if (!valid(lcand) || (valid(rcand) && in_circle(
        lcand->dest(), lcand->origin, rcand->origin, rcand->dest()))
        basel = connect(rcand, basel->rev());
        else basel = connect(basel->rev(), lcand->rev());
    } return make_pair(ldo, rdo);
}
vector<tuple<pt, pt, pt>> delaunay(vector<pt> p) {
    sort(p.begin(), p.end(), [](const pt& a, const pt& b) {
        return lt(a.x, b.x) || (eq(a.x, b.x) && lt(a.y, b.y));
    });
    auto res = build_tr(0, (int)p.size() - 1, p);
    QuadEdge* e = res.first; vector<QuadEdge*> edges = {e};
    while (lt(e->onext->dest().cross(e->dest(), e->origin), 0))
        e = e->onext;
    auto add = [&p, &e, &edges]() { QuadEdge* curr = e;
        do { curr->used = true; p.push_back(curr->origin);
            edges.push_back(curr->rev()); curr = curr->lnext();
        } while (curr != e);
    };
    add(); p.clear(); int kek = 0;
    while (kek < (int)edges.size()) {
        if (!e = edges[kek++]>->used) add();
    }
    vector<tuple<pt, pt, pt>> ans;
    for (int i = 0; i < (int)p.size(); i += 3) {
        ans.push_back(make_tuple(p[i], p[i+1], p[i+2]));
    }
    return ans;
}

```

## 5.6 half-plane-intersection

```

class HalfPlaneIntersection {
    static double eps, inf;
public:
    struct Point {
        double x, y;
        explicit Point(double x = 0, double y = 0) : x(x), y(y) {}
        // Addition, subtraction, multiply by constant, cross product.
        friend Point operator + (const Point& p, const Point& q) {
            return Point(p.x + q.x, p.y + q.y);
        }
        friend Point operator - (const Point& p, const Point& q) {
            return Point(p.x - q.x, p.y - q.y);
        }
        friend Point operator * (const Point& p, const double& k) {
            return Point(p.x * k, p.y * k);
        }
        friend double cross(const Point& p, const Point& q) {
            return p.x * q.y - p.y * q.x;
        }
    };
    // Basic half-plane struct.
    struct Halfplane {
        // 'p' is a passing point of the line and 'pq' is the direction vector of the line.
        Point p, pq;
        double angle;
        Halfplane() {}
        Halfplane(const Point& a, const Point& b) : p(a), pq(b - a) {
            angle = atan2(pq.y, pq.x);
        }
        // Check if point 'r' is outside this half-plane.
        // Every half-plane allows the region to the LEFT of its line.
        bool out(const Point& r) {
            return cross(pq, r - p) < -eps;
        }
    };
}

```



```
// Comparator for sorting.
// If the angle of both half-planes is equal, the
// leftmost one should go first.
bool operator < (const Halfplane& e) const{
    if (fabs(angle - e.angle) < eps) return cross(pq,
        e.p - p) < 0;
    return angle < e.angle;
}
// We use equal comparator for std::unique to easily
// remove parallel half-planes.
bool operator == (const Halfplane& e) const{
    return fabs(angle - e.angle) < eps;
}
// Intersection point of the lines of two half-planes.
// It is assumed they're never parallel.
friend Point inter(const Halfplane& s, const Halfplane&
    t){
    double alpha = cross((t.p - s.p), t.pq) /
        cross(s.pq, t.pq);
    return s.p + (s.pq * alpha);
}
};
static vector<Point> hp_intersect(vector<Halfplane>& H){
    Point box[4] = // Bounding box in CCW order{
        Point(-inf, -inf),
        Point(inf, inf),
        Point(inf, -inf),
        Point(-inf, inf)
    };
    for(int i = 0; i < 4; i++) // Add bounding box
        half-planes.{
            Halfplane aux(box[i], box[(i+1) % 4]);
            H.push_back(aux);
        }
    // Sort and remove duplicates
    sort(H.begin(), H.end());
    H.erase(unique(H.begin(), H.end()), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); i++){
        // Remove from the back of the deque while last
        // half-plane is redundant
        while (len > 1 && H[i].out(inter(dq[len-1],
            dq[len-2]))){
            dq.pop_back();
            --len;
        }
        // Remove from the front of the deque while first
        // half-plane is redundant
        while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
            dq.pop_front();
            --len;
        }
        // Add new half-plane
        dq.push_back(H[i]);
        ++len;
    }
    // Final cleanup: Check half-planes at the front
    // against the back and vice-versa
    while (len > 2 && dq[0].out(inter(dq[len-1],
        dq[len-2]))){
        dq.pop_back();
        --len;
    }
    while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
        dq.pop_front();
        --len;
    }
    // Report empty intersection if necessary
    if (len < 3) return vector<Point>();
    // Reconstruct the convex polygon from the remaining
    // half-planes.
    vector<Point> ret(len);
    for(int i = 0; i < len; i++){
        ret[i] = inter(dq[i], dq[i+1]);
    }
    ret.back() = inter(dq[len-1], dq[0]);
    return ret;
}
```

```
}
};
double HalfPlaneIntersection::eps=1e-9;
double HalfPlaneIntersection::inf=1e9;

5.7 heart-of-geometry-2d

typedef double ftype;
const double EPS = 1E-9;
struct pt{
    ftype x, y;
    int id;
    pt() {}
    pt(ftype _x, ftype _y):x(_x), y(_y) {}
    pt operator+(const pt & p) const{
        return pt(x + p.x, y + p.y);
    }
    pt operator-(const pt & p) const{
        return pt(x - p.x, y - p.y);
    }
    ftype cross(const pt & p) const{
        return x * p.y - y * p.x;
    }
    ftype dot(const pt & p) const{
        return x * p.x + y * p.y;
    }
    ftype cross(const pt & a, const pt & b) const{
        return (a - *this).cross(b - *this);
    }
    ftype dot(const pt & a, const pt & b) const{
        return (a - *this).dot(b - *this);
    }
    ftype sqrLen() const{
        return this->dot(*this);
    }
    bool operator<(const pt& p) const{
        return x < p.x - EPS || (abs(x - p.x) < EPS && y < p.y
            - EPS);
    }
    bool operator==(const pt& p) const{
        return abs(x-p.x)<EPS && abs(y-p.y)<EPS;
    }
};
int sign(double x) { return (x > EPS) - (x < -EPS); }
inline int orientation(pt a, pt b, pt c) { return
    sign(a.cross(b,c)); }
bool is_point_on_seg(pt a, pt b, pt p) {
    if (fabs(b.cross(p,a)) < EPS) {
        if (p.x < min(a.x, b.x) - EPS || p.x > max(a.x, b.x) +
            EPS) return false;
        if (p.y < min(a.y, b.y) - EPS || p.y > max(a.y, b.y) +
            EPS) return false;
        return true;
    }
    return false;
}
bool is_point_on_polygon(vector<pt> &p, const pt& z) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        if (is_point_on_seg(p[i], p[(i + 1) % n], z)) return 1;
    }
    return 0;
}
int winding_number(vector<pt> &p, const pt& z) { // O(n)
    if (is_point_on_polygon(p, z)) return 1e9;
    int n = p.size(), ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) % n;
        bool below = p[i].y < z.y;
        if (below != (p[j].y < z.y)) {
            auto orient = orientation(z, p[j], p[i]);
            if (orient == 0) return 0;
            if (below == (orient > 0)) ans += below ? -1 : 1;
        }
    }
    return ans;
}
double dist_sqr(pt a, pt b){
    return ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
```

```
}
double dist(pt a, pt b){
    return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
double angle(pt a, pt b, pt c){
    if(b==a || b==c) return 0;
    double A2 = dist_sqr(b,c);
    double C2 = dist_sqr(a,b);
    double B2 = dist_sqr(c,a);
    double A = sqrt(A2), C = sqrt(C2);
    double ans = (A2 + C2 - B2)/(A*C*2);
    if(ans<-1) ans=acos(-1);
    else if(ans>1) ans=acos(1);
    else ans = acos(ans);
    return ans;
}
bool cmp(pt a, pt b){
    return a.x < b.x || (a.x == b.x && a.y < b.y);
}
bool ccw(pt a, pt b, pt c, bool include_collinear=false) {
    int o = orientation(a, b, c);
    return o > 0 || (include_collinear && o == 0);
}
bool cw(pt a, pt b, pt c, bool include_collinear=false) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c)
    == 0; }
double area(pt a, pt b, pt c){
    return (a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y))/2;
}
struct cmp_x{
    bool operator()(const pt & a, const pt & b) const{
        return a.x < b.x || (a.x == b.x && a.y < b.y);
    }
};
struct cmp_y{
    bool operator()(const pt & a, const pt & b) const{
        return a.y < b.y || (a.y == b.y && a.x < b.x);
    }
};
struct circle : pt {
    ftype r;
};
bool insideCircle(circle c, pt p){
    return dist_sqr(c,p) <= c.r*c.r + EPS;
}
struct line {
    ftype a, b, c;
    line() {}
    line(pt p, pt q){
        a = p.y - q.y;
        b = q.x - p.x;
        c = -a * p.x - b * p.y;
        norm();
    }
    void norm(){
        double z = sqrt(a * a + b * b);
        if (abs(z) > EPS)
            a /= z, b /= z, c /= z;
    }
    line getParallel(pt p){
        line ans = *this;
        ans.c = -(ans.a*p.x+ans.b*p.y);
        return ans;
    }
    ftype getValue(pt p){
        return a*p.x+b*p.y+c;
    }
    line getPerpend(pt p){
        line ans;
        ans.a=this->b;
        ans.b=-(this->a);
        ans.c = -(ans.a*p.x+ans.b*p.y);
        return ans;
    }
};
//dist formula is wrong but don't change
double dist(pt p) const { return a * p.x + b * p.y + c; }
```

```

};
double sqr (double a) {
    return a * a;
}
double det(double a, double b, double c, double d) {
    return a*d - b*c;
}
bool intersect(line m, line n, pt & res) {
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS)
        return false;
    res.x = -det(m.c, m.b, n.c, n.b) / zn;
    res.y = -det(m.a, m.c, n.a, n.c) / zn;
    return true;
}
bool parallel(line m, line n) {
    return abs(det(m.a, m.b, n.a, n.b)) < EPS;
}
bool equivalent(line m, line n) {
    return abs(det(m.a, m.b, n.a, n.b)) < EPS
        && abs(det(m.a, m.c, n.a, n.c)) < EPS
        && abs(det(m.b, m.c, n.b, n.c)) < EPS;
}
double det(double a, double b, double c, double d){
    return a * d - b * c;
}
inline bool betw(double l, double r, double x){
    return min(l, r) <= x + EPS && x <= max(l, r) + EPS;
}
inline bool intersect_1d(double a, double b, double c, double d){
    if (a > b)
        swap(a, b);
    if (c > d)
        swap(c, d);
    return max(a, c) <= min(b, d) + EPS;
}
bool intersect_segment(pt a, pt b, pt c, pt d, pt& left, pt& right){
    if (!intersect_1d(a.x, b.x, c.x, d.x) ||
        !intersect_1d(a.y, b.y, c.y, d.y))
        return false;
    line m(a, b);
    line n(c, d);
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS) {
        if (abs(m.dist(c)) > EPS || abs(n.dist(a)) > EPS)
            return false;
        if (b < a)
            swap(a, b);
        if (d < c)
            swap(c, d);
        left = max(a, c);
        right = min(b, d);
        return true;
    } else {
        left.x = right.x = -det(m.c, m.b, n.c, n.b) / zn;
        left.y = right.y = -det(m.a, m.c, n.a, n.c) / zn;
        return betw(a.x, b.x, left.x) && betw(a.y, b.y, left.y)
            && betw(c.x, d.x, left.x) && betw(c.y, d.y, left.y);
    }
}
void tangents (pt c, double r1, double r2, vector<line> & ans)
{
    double r = r2 - r1;
    double z = sqr(c.x) + sqr(c.y);
    double d = z - sqr(r);
    if (d < -EPS) return;
    d = sqrt (abs (d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back (l);
}
vector<line> tangents (circle a, circle b) {
    vector<line> ans;
    for (int i=-1; i<=1; i+=2)
        for (int j=-1; j<=1; j+=2)

```

```

        tangents (b-a, a.r*i, b.r*j, ans);
    for (size_t i=0; i<ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}
class pointLocationInPolygon{
    bool lexComp(const pt & l, const pt & r){
        return l.x < r.x || (l.x == r.x && l.y < r.y);
    }
    int sgn(ftype val){
        return val > 0 ? 1 : (val == 0 ? 0 : -1);
    }
    vector<pt> seq;
    int n;
    pt translate;
    bool pointInTriangle(pt a, pt b, pt c, pt point){
        ftype s1 = abs(a.cross(b, c));
        ftype s2 = abs(point.cross(a, b)) + abs(point.cross(b, c)) + abs(point.cross(c, a));
        return s1 == s2;
    }
public:
    pointLocationInPolygon(){
    }
    pointLocationInPolygon(vector<pt> & points){
        prepare(points);
    }
    void prepare(vector<pt> & points){
        seq.clear();
        n = points.size();
        int pos = 0;
        for(int i = 1; i < n; i++){
            if(lexComp(points[i], points[pos]))
                pos = i;
        }
        translate.x=points[pos].x;
        translate.y=points[pos].y;
        rotate(points.begin(), points.begin() + pos, points.end());
        n--;
        seq.resize(n);
        for(int i = 0; i < n; i++){
            seq[i] = points[i + 1] - points[0];
        }
    }
    bool pointInConvexPolygon(pt point){
        point.x-=translate.x;
        point.y-=translate.y;
        if(seq[0].cross(point) != 0 && sgn(seq[0].cross(point))
            != sgn(seq[0].cross(seq[n - 1])))
            return false;
        if(seq[n - 1].cross(point) != 0 && sgn(seq[n - 1].cross(seq[0]))
            != sgn(seq[n - 1].cross(seq[0])))
            return false;
        if(seq[0].cross(point) == 0)
            return seq[0].sqrLen() >= point.sqrLen();
        int l = 0, r = n - 1;
        while(r - l > 1){
            int mid = (l + r)/2;
            int pos = mid;
            if(seq[pos].cross(point) >= 0) l = mid;
            else r = mid;
        }
        int pos = l;
        return pointInTriangle(seq[pos], seq[pos + 1], pt(0, 0), point);
    }
    ~pointLocationInPolygon(){
        seq.clear();
    }
}
class Minkowski{
    static void reorder_polygon(vector<pt> & P){
        size_t pos = 0;
        for(size_t i = 1; i < P.size(); i++){
            if(P[i].y < P[pos].y || (P[i].y == P[pos].y &&
                P[i].x < P[pos].x))
                pos = i;
        }
        rotate(P.begin(), P.begin() + pos, P.end());
    }
}

```

```

}
public:
    static vector<pt> minkowski(vector<pt> P, vector<pt> Q){
        // the first vertex must be the lowest
        reorder_polygon(P);
        reorder_polygon(Q);
        // we must ensure cyclic indexing
        P.push_back(P[0]);
        P.push_back(P[1]);
        Q.push_back(Q[0]);
        Q.push_back(Q[1]);
        // main part
        vector<pt> result;
        size_t i = 0, j = 0;
        while(i < P.size() - 2 || j < Q.size() - 2){
            result.push_back(P[i] + Q[j]);
            auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j]);
            if(cross >= 0)
                ++i;
            if(cross <= 0)
                ++j;
        }
        return result;
    }
};
vector<pt> circle_line_intersections(circle cir,line l){
    double r = cir.r, a = l.a, b = l.b, c = l.c + l.a*cir.x + l.b*cir.y;
    vector<pt> ans;
    double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
    if (c*c > r*r*(a*a+b*b)+EPS);
    else if (abs (c*c - r*r*(a*a+b*b)) < EPS){
        pt p;
        p.x=x0;
        p.y=y0;
        ans.push_back(p);
    }
    else{
        double d = r*r - c*c/(a*a+b*b);
        double mult = sqrt (d / (a*a+b*b));
        double ax, ay, bx, by;
        ax = x0 + b * mult;
        bx = x0 - b * mult;
        ay = y0 - a * mult;
        by = y0 + a * mult;
        pt p;
        p.x = ax;
        p.y = ay;
        ans.push_back(p);
        p.x = bx;
        p.y = by;
        ans.push_back(p);
    }
    for(int i=0;i<ans.size();i++){
        ans[i] = ans[i] + cir;
    }
    return ans;
}
double circle_polygon_intersection(circle c,vector<pt> &V){
    int n = V.size();
    double ans = 0;
    for(int i=0; i<n; i++){
        line l(V[i],V[(i+1)%n]);
        vector<pt> lpts = circle_line_intersections(c,l);
        int sz=lpts.size();
        for(int j=sz-1; j>=0; j--){
            if(!is_point_on_seg(V[i],V[(i+1)%n],lpts[j])){
                swap(lpts.back(),lpts[j]);
                lpts.pop_back();
            }
        }
        lpts.push_back(V[i]);
        lpts.push_back(V[(i+1)%n]);
        sort(lpts.begin(),lpts.end());
        sz=lpts.size();
        if(V[(i+1)%n]<V[i])
            reverse(lpts.begin(),lpts.end());
        for(int j=1; j<sz; j++){
            if(insideCircle(c,lpts[j-1])

```

```

    &&insideCircle(c,lpts[j]))
    ans = ans + area(lpts[j-1],lpts[j],c);
else{
    double ang = angle(lpts[j-1],c,lpts[j]);
    double aa = c.r*c.r*ang/2;
    if(ccw(lpts[j-1],lpts[j],c))
        ans = ans+aa;
    else
        ans = ans-aa;
}
}
ans = abs(ans);
return ans;
}

void convex_hull(vector<pt>& a, bool include_collinear = false) {
    pt p0 = *min_element(a.begin(), a.end(), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x-a.x)*(p0.x-a.x) + (p0.y-a.y)*(p0.y-a.y) < (p0.x-b.x)*(p0.x-b.x) + (p0.y-b.y)*(p0.y-b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = (int)a.size()-1;
        while (i >= 0 && collinear(p0, a[i], a.back())) i--;
        reverse(a.begin()+i+1, a.end());
    }
    vector<pt> st;
    for (int i = 0; i < (int)a.size(); i++) {
        while (st.size() > 1 && !cw(st[st.size()-2], st.back(), a[i], include_collinear))
            st.pop_back();
        st.push_back(a[i]);
    }
    a = st;
    int m = a.size();
    for(int i = 0; i < m-1; i++){
        swap(a[i], a[m-1-i]);
    }
}

double mindist;
pair<int, pair<int, int> > best_pair;
void upd_ans(const pt & a, const pt & b, const pt & c){
    double distC = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.y));
    double distA = sqrt((c.x - b.x)*(c.x - b.x) + (c.y - b.y)*(c.y - b.y));
    double distB = sqrt((a.x - c.x)*(a.x - c.x) + (a.y - c.y)*(a.y - c.y));
    if (distA + distB + distC < mindist){
        mindist = distA + distB + distC;
        best_pair = make_pair(a.id, make_pair(b.id, c.id));
    }
}

vector<pt> t;
//Min possible triplet distance
void rec(int l, int r){
    if (r - l <= 3 && r - l >= 2){
        for (int i = l; i < r; ++i){
            for (int j = i + 1; j < r; ++j){
                for(int k=j+1; k<r; k++){
                    upd_ans(a[i], a[j], a[k]);
                }
            }
        }
    }
    sort(a.begin() + l, a.begin() + r, cmp_y());
    return;
}

int m = (l + r) >> 1;
int midx = a[m-1].x;
/*
* Got WA in a team contest
* for putting midx = a[m].x;
* Don't know why. Maybe due to
* floating point numbers.
*/

```

```

*/
rec(l, m);
rec(m, r);
merge(a.begin() + l, a.begin() + m, a.begin() + m,
        a.begin() + r, t.begin(), cmp_y());
copy(t.begin(), t.begin() + r - l, a.begin() + l);
int tsz = 0;
for (int i = l; i < r; ++i){
    if (abs(a[i].x - midx) < mindist/2){
        for (int j = tsz - 1; j >= 0 && a[i].y - t[j].y < mindist/2; --j){
            if(i+1<r) upd_ans(a[i], a[i+1], t[j]);
            if(j>0) upd_ans(a[i], t[j-1], t[j]);
        }
        t[tsz++] = a[i];
    }
}
}
}

```

## 5.8 intersecting-segments-pair

```

const double EPS = 1E-9;
struct pt {
    double x, y;
};
struct seg {
    pt p, q;
    int id;
    double get_y(double x) const {
        if (abs(p.x - q.x) < EPS)
            return p.y;
        return p.y + (q.y - p.y) * (x - p.x) / (q.x - p.x);
    }
};
bool intersectId(double l1, double r1, double l2, double r2) {
    if (l1 > r1)
        swap(l1, r1);
    if (l2 > r2)
        swap(l2, r2);
    return max(l1, l2) <= min(r1, r2) + EPS;
}
int vec(const pt& a, const pt& b, const pt& c) {
    double s = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);
    return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
}
bool intersect(const seg& a, const seg& b){
    return intersectId(a.p.x, a.q.x, b.p.x, b.q.x) &&
        intersectId(a.p.y, a.q.y, b.p.y, b.q.y) &&
        vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
        vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
}
bool operator<(const seg& a, const seg& b){
    double x = max(min(a.p.x, a.q.x), min(b.p.x, b.q.x));
    return a.get_y(x) < b.get_y(x) - EPS;
}
struct event {
    double x;
    int tp, id;
    event() {}
    event(double x, int tp, int id) : x(x), tp(tp), id(id) {}
    bool operator<(const event& e) const {
        if (abs(x - e.x) > EPS)
            return x < e.x;
        return tp > e.tp;
    }
};
set<seg> s;
vector<set<seg>::iterator> where;
set<seg>::iterator prev(set<seg>::iterator it) {
    return it == s.begin() ? s.end() : --it;
}
set<seg>::iterator next(set<seg>::iterator it) {
    return ++it;
}
}
pair<int, int> solve(const vector<seg>& a) {
    int n = (int)a.size();
    vector<event> e;
    for (int i = 0; i < n; ++i) {

```

```

        e.push_back(event(min(a[i].p.x, a[i].q.x), +1, i));
        e.push_back(event(max(a[i].p.x, a[i].q.x), -1, i));
    }
    sort(e.begin(), e.end());
    s.clear();
    where.resize(a.size());
    for (size_t i = 0; i < e.size(); ++i) {
        int id = e[i].id;
        if (e[i].tp == +1) {
            set<seg>::iterator nxt = s.lower_bound(a[id]), prv = prev(nxt);
            if (nxt != s.end() && intersect(*nxt, a[id]))
                return make_pair(nxt->id, id);
            if (prv != s.end() && intersect(*prv, a[id]))
                return make_pair(prv->id, id);
            where[id] = s.insert(nxt, a[id]);
        } else {
            set<seg>::iterator nxt = next(where[id]), prv = prev(where[id]);
            if (nxt != s.end() && prv != s.end() && intersect(*nxt, *prv))
                return make_pair(prv->id, nxt->id);
            s.erase(where[id]);
        }
    }
    return make_pair(-1, -1);
}

```

## 5.9 point-location

```

typedef long long ll;
bool ge(const ll& a, const ll& b) { return a >= b; }
bool le(const ll& a, const ll& b) { return a <= b; }
bool eq(const ll& a, const ll& b) { return a == b; }
bool gt(const ll& a, const ll& b) { return a > b; }
bool lt(const ll& a, const ll& b) { return a < b; }
int sgn(const ll& x) { return le(x, 0) ? eq(x, 0) ? 0 : -1 : 1; }
struct pt {
    ll x, y;
    pt() {}
    pt(ll _x, ll _y) : x(_x), y(_y) {}
    pt operator-(const pt& a) const { return pt(x - a.x, y - a.y); }
    ll dot(const pt& a) const { return x * a.x + y * a.y; }
    ll dot(const pt& a, const pt& b) const { return (a - *this).dot(b - *this); }
    ll cross(const pt& a) const { return x * a.y - y * a.x; }
    ll cross(const pt& a, const pt& b) const { return (a - *this).cross(b - *this); }
    bool operator==(const pt& a) const { return a.x == x && a.y == y; }
};
struct Edge {
    pt l, r;
};
bool edge_cmp(Edge* edge1, Edge* edge2){
    const pt a = edge1->l, b = edge1->r;
    const pt c = edge2->l, d = edge2->r;
    int val = sgn(a.cross(b, c)) + sgn(a.cross(b, d));
    if (val != 0)
        return val > 0;
    val = sgn(c.cross(d, a)) + sgn(c.cross(d, b));
    return val < 0;
}
enum EventType { DEL = 2, ADD = 3, GET = 1, VERT = 0 };
struct Event {
    EventType type;
    int pos;
    bool operator<(const Event& event) const { return type < event.type; }
};
vector<Edge*> sweepline(vector<Edge*> planar, vector<pt> queries){
    using pt_type = decltype(pt::x);
    // collect all x-coordinates
    auto s =

```

```

    set<pt_type, std::function<bool(const pt_type&, const
        pt_type&)>>(lt);
    for (pt p : queries)
        s.insert(p.x);
    for (Edge* e : planar) {
        s.insert(e->l.x);
        s.insert(e->r.x);
    }
    // map all x-coordinates to ids
    int cid = 0;
    auto id =
        map<pt_type, int, std::function<bool(const pt_type&,
            const pt_type&)>>(lt);
    for (auto x : s)
        id[x] = cid++;
    // create events
    auto t = set<Edge*, decltype(*edge_cmp)>(edge_cmp);
    auto vert_cmp = [] (const pair<pt_type, int>& l,
        const pair<pt_type, int>& r) {
        if (!eq(l.first, r.first))
            return lt(l.first, r.first);
        return l.second < r.second;
    };
    auto vert = set<pair<pt_type, int>,
        decltype(vert_cmp)>(vert_cmp);
    vector<vector<Event>> events(cid);
    for (int i = 0; i < (int)queries.size(); i++) {
        int x = id[queries[i].x];
        events[x].push_back(Event{GET, i});
    }
    for (int i = 0; i < (int)planar.size(); i++) {
        int lx = id[planar[i]->l.x], rx = id[planar[i]->r.x];
        if (lx > rx) {
            swap(lx, rx);
            swap(planar[i]->l, planar[i]->r);
        }
        if (lx == rx) {
            events[lx].push_back(Event{VERT, i});
        } else {
            events[lx].push_back(Event{ADD, i});
            events[rx].push_back(Event{DEL, i});
        }
    }
    // perform sweep line algorithm
    vector<Edge*> ans(queries.size(), nullptr);
    for (int x = 0; x < cid; x++) {
        sort(events[x].begin(), events[x].end());
        vert.clear();
        for (Event event : events[x]) {
            if (event.type == DEL) {
                t.erase(planar[event.pos]);
            }
            if (event.type == VERT) {
                vert.insert(make_pair(
                    min(planar[event.pos]->l.y,
                        planar[event.pos]->r.y),
                    event.pos));
            }
            if (event.type == ADD) {
                t.insert(planar[event.pos]);
            }
            if (event.type == GET) {
                auto jt = vert.upper_bound(
                    make_pair(queries[event.pos].y,
                        planar.size()));
                if (jt != vert.begin()) {
                    --jt;
                    int i = jt->second;
                    if (ge(max(planar[i]->l.y, planar[i]->r.y),
                        queries[event.pos].y)) {
                        ans[event.pos] = planar[i];
                        continue;
                    }
                }
                Edge* e = new Edge;
                e->l = e->r = queries[event.pos];
                auto it = t.upper_bound(e);

```

```

                if (it != t.begin())
                    ans[event.pos] = *(--it);
                delete e;
            }
        }
        for (Event event : events[x]) {
            if (event.type != GET)
                continue;
            if (ans[event.pos] != nullptr &&
                eq(ans[event.pos]->l.x, ans[event.pos]->r.x))
                continue;
            Edge* e = new Edge;
            e->l = e->r = queries[event.pos];
            auto it = t.upper_bound(e);
            delete e;
            if (it == t.begin())
                e = nullptr;
            else
                e = *(--it);
            if (ans[event.pos] == nullptr) {
                ans[event.pos] = e;
                continue;
            }
            if (e == nullptr)
                continue;
            if (e == ans[event.pos])
                continue;
            if (id[ans[event.pos]->r.x] == x) {
                if (id[e->l.x] == x) {
                    if (gt(e->l.y, ans[event.pos]->r.y))
                        ans[event.pos] = e;
                }
            } else {
                ans[event.pos] = e;
            }
        }
        return ans;
    }
    struct DCEL {
        struct Edge {
            pt origin;
            Edge* nxt = nullptr;
            Edge* twin = nullptr;
            int face;
        };
        vector<Edge*> body;
    };
    vector<pair<int, int>> point_location(DCEL planar, vector<pt>
        queries){
        vector<pair<int, int>> ans(queries.size());
        vector<Edge*> planar2;
        map<intptr_t, int> pos;
        map<intptr_t, int> added_on;
        int n = planar.body.size();
        for (int i = 0; i < n; i++) {
            if (planar.body[i]->face > planar.body[i]->twin->face)
                continue;
            Edge* e = new Edge;
            e->l = planar.body[i]->origin;
            e->r = planar.body[i]->twin->origin;
            added_on[(intptr_t)e] = i;
            pos[(intptr_t)e] =
                lt(planar.body[i]->origin.x,
                    planar.body[i]->twin->origin.x)
                    ? planar.body[i]->face
                    : planar.body[i]->twin->face;
            planar2.push_back(e);
        }
        auto res = sweepline(planar2, queries);
        for (int i = 0; i < (int)queries.size(); i++) {
            if (res[i] == nullptr) {
                ans[i] = make_pair(1, -1);
                continue;
            }
            pt p = queries[i];
            pt l = res[i]->l, r = res[i]->r;
            if (eq(p.cross(l, r), 0) && le(p.dot(l, r), 0)) {
                ans[i] = make_pair(0, added_on[(intptr_t)res[i]]);

```

```

                continue;
            }
            ans[i] = make_pair(1, pos[(intptr_t)res[i]]);
        }
        for (auto e : planar2)
            delete e;
        return ans;
    }

```

## 5.10 vertical-decomposition

```

typedef double dbl;
const dbl eps = 1e-9;
inline bool eq(dbl x, dbl y){
    return fabs(x - y) < eps;
}
inline bool lt(dbl x, dbl y){
    return x < y - eps;
}
inline bool gt(dbl x, dbl y){
    return x > y + eps;
}
inline bool le(dbl x, dbl y){
    return x < y + eps;
}
inline bool ge(dbl x, dbl y){
    return x > y - eps;
}
struct pt{
    dbl x, y;
    inline pt operator - (const pt & p)const{
        return pt{x - p.x, y - p.y};
    }
    inline pt operator + (const pt & p)const{
        return pt{x + p.x, y + p.y};
    }
    inline pt operator * (dbl a)const{
        return pt{x * a, y * a};
    }
    inline dbl cross(const pt & p)const{
        return x * p.y - y * p.x;
    }
    inline dbl dot(const pt & p)const{
        return x * p.x + y * p.y;
    }
    inline bool operator == (const pt & p)const{
        return eq(x, p.x) && eq(y, p.y);
    }
};
struct Line{
    pt p[2];
    Line(){
        Line(pt a, pt b):p{a, b}{}
    }
    pt vec()const{
        return p[1] - p[0];
    }
    pt& operator [] (size_t i){
        return p[i];
    }
};
inline bool lexComp(const pt & l, const pt & r){
    if(fabs(l.x - r.x) > eps){
        return l.x < r.x;
    }
    else return l.y < r.y;
}
vector<pt> interSegSeg(Line l1, Line l2){
    if(eq(l1.vec().cross(l2.vec(), 0))){
        if(!eq(l1.vec().cross(l2[0] - l1[0], 0))
            return {};
        if(!lexComp(l1[0], l1[1]))
            swap(l1[0], l1[1]);
        if(!lexComp(l2[0], l2[1]))
            swap(l2[0], l2[1]);
        pt l = lexComp(l1[0], l2[0]) ? l2[0] : l1[0];
        pt r = lexComp(l1[1], l2[1]) ? l1[1] : l2[1];
        if(l == r)
            return {l};
    }

```

```

    else return lexComp(l, r) ? vector<pt>{l, r} :
        vector<pt>();
}
else{
    dbl s = (l2[0] - l1[0]).cross(l2.vec()) /
        l1.vec().cross(l2.vec());
    pt inter = l1[0] + l1.vec() * s;
    if(ge(s, 0) && le(s, 1) && le((l2[0] - inter).dot(l2[1]
        - inter), 0))
        return {inter};
    else
        return {};
}
}
inline char get_segtype(Line segment, pt other_point){
    if(eq(segment[0].x, segment[1].x))
        return 0;
    if(!lexComp(segment[0], segment[1]))
        swap(segment[0], segment[1]);
    return (segment[1] - segment[0]).cross(other_point -
        segment[0]) > 0 ? 1 : -1;
}
dbl union_area(vector<tuple<pt, pt, pt> > triangles){
    vector<Line> segments(3 * triangles.size());
    vector<char> segtype(segments.size());
    for(size_t i = 0; i < triangles.size(); i++){
        pt a, b, c;
        tie(a, b, c) = triangles[i];
        segments[3 * i] = lexComp(a, b) ? Line(a, b) : Line(b, a);
        segtype[3 * i] = get_segtype(segments[3 * i], c);
        segments[3 * i + 1] = lexComp(b, c) ? Line(b, c) : Line(c, b);
        segtype[3 * i + 1] = get_segtype(segments[3 * i + 1], a);
        segments[3 * i + 2] = lexComp(c, a) ? Line(c, a) : Line(a, c);
        segtype[3 * i + 2] = get_segtype(segments[3 * i + 2], b);
    }
    vector<dbl> k(segments.size()), b(segments.size());
    for(size_t i = 0; i < segments.size(); i++){
        if(segtype[i]){
            k[i] = (segments[i][1].y - segments[i][0].y) /
                (segments[i][1].x - segments[i][0].x);
            b[i] = segments[i][0].y - k[i] * segments[i][0].x;
        }
    }
    dbl ans = 0;
    for(size_t i = 0; i < segments.size(); i++){
        if(!segtype[i])
            continue;
        dbl l = segments[i][0].x, r = segments[i][1].x;
        vector<pair<dbl, int> > evts;
        for(size_t j = 0; j < segments.size(); j++){
            if(!segtype[j] || i == j)
                continue;
            dbl l1 = segments[j][0].x, r1 = segments[j][1].x;
            if(ge(l1, r) || ge(l, r1))
                continue;
            dbl common_l = max(l, l1), common_r = min(r, r1);
            auto pts = interSegSeg(segments[i], segments[j]);
            if(pts.empty()){
                dbl y11 = k[j] * common_l + b[j];
                dbl y1 = k[i] * common_l + b[i];
                if(lt(y11, y1) == (segtype[i] == 1)){
                    int evt_type = -segtype[i] * segtype[j];
                    evts.emplace_back(common_l, evt_type);
                    evts.emplace_back(common_r, -evt_type);
                }
            }
            else if(pts.size() == 1u){
                dbl y1 = k[i] * common_l + b[i], y11 = k[j] * common_l + b[j];
                int evt_type = -segtype[i] * segtype[j];
                if(lt(y11, y1) == (segtype[i] == 1)){
                    evts.emplace_back(common_l, evt_type);
                    evts.emplace_back(pts[0].x, -evt_type);
                }
            }
            y1 = k[i] * common_r + b[i], y11 = k[j] * common_r + b[j];
            if(lt(y11, y1) == (segtype[i] == 1)){
                evts.emplace_back(pts[0].x, evt_type);
                evts.emplace_back(common_r, -evt_type);
            }
        }
    }
}

```

```

    }
}
else{
    if(segtype[j] != segtype[i] || j > i){
        evts.emplace_back(common_l, -2);
        evts.emplace_back(common_r, 2);
    }
}
}
evts.emplace_back(l, 0); sort(evts.begin(), evts.end());
size_t j = 0; int balance = 0;
while(j < evts.size()){
    size_t ptr = j;
    while(ptr < evts.size() && eq(evts[j].first,
        evts[ptr].first)){
        balance += evts[ptr].second;
        ++ptr;
    }
    if(!balance && !eq(evts[j].first, r)){
        dbl next_x = ptr == evts.size() ? r :
            evts[ptr].first;
        ans -= segtype[i] * k[i] * (next_x +
            evts[j].first) + 2 * b[i] * (next_x -
            evts[j].first);
        j = ptr;
    }
}
return ans/2;
}
pair<dbl,dbl> union_perimeter(vector<tuple<pt, pt, pt> >
    triangles){
    //Same as before
    pair<dbl,dbl> ans = make_pair(0,0);
    for(size_t i = 0; i < segments.size(); i++){
        //Same as before
        double dist=(segments[i][1].x-segments[i][0].x)
            *(segments[i][1].x-segments[i][0].x)
            +(segments[i][1].y-segments[i][0].y)
            *(segments[i][1].y-segments[i][0].y);
        dist=sqrt(dist);
        while(j < evts.size()){
            size_t ptr = j;
            while(ptr < evts.size() && eq(evts[j].first,
                evts[ptr].first)){
                balance += evts[ptr].second; ++ptr;
            }
            if(!balance && !eq(evts[j].first, r)){
                dbl next_x = ptr == evts.size() ? r :
                    evts[ptr].first;
                ans.first += dist * (next_x - evts[j].first) /
                    (r-1);
                if(eq(segments[i][1].y, segments[i][0].y))
                    ans.second += (next_x - evts[j].first);
                j = ptr;
            }
        }
    }
    return ans;
}

```

## 6 Graph

### 6.1 DMST with solution

```

// not tested yet
const int INF = 1029384756;
#define MAXN 1000
#define FOR(i,x) for(auto i : x)
struct edge_t {
    int u,v,w;
    set< pair<int,int> > add, sub;
    edge_t() : u(-1), v(-1), w(0) {}
    edge_t(int _u, int _v, int _w) {
        u = _u;
        v = _v;
        w = _w;
        add.insert({u, v});
    }
    edge_t& operator += (const edge_t& obj) {

```

```

        w += obj.w;
        for (auto it : obj.add) {
            if (!sub.count(it)) add.insert(it);
            else sub.erase(it);
        }
        for (auto it : obj.sub) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
    edge_t& operator -= (const edge_t& obj) {
        w -= obj.w;
        for (auto it : obj.sub) {
            if (!sub.count(it)) add.insert(it);
            else sub.erase(it);
        }
        for (auto it : obj.add) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
}
eg[MAXN*MAXN], prv[MAXN], EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MAXN],contracted[MAXN];
vector<int> E[MAXN];
edge_t dmst(int rt) {
    edge_t cost;
    for (int i=0; i<N; i++) {
        contracted[i] = incyc[i] = 0;
        prv[i] = EDGE_INF;
    }
    cid = 0;
    int u,v;
    while (true) {
        for (v=0; v<N; v++) {
            if (v != rt && !contracted[v] && prv[v].w == INF)
                break;
        }
        if (v >= N) break; // end
        for (int i=0; i<M; i++) {
            if (eg[i].v == v && eg[i].w < prv[v].w)
                prv[v] = eg[i];
        }
        if (prv[v].w == INF) // not connected
            return EDGE_INF;
        cost += prv[v];
        for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
        if (u == -1) continue;
        incyc[v] = ++cid;
        for (u=prv[v].u; u!=v; u=prv[u].u) {
            contracted[u] = 1;
            incyc[u] = cid;
        }
        for (int i=0; i<M; i++) {
            if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
                cid) {
                eg[i] -= prv[eg[i].v];
            }
        }
        for (int i=0; i<M; i++) {
            if (incyc[eg[i].u] == cid) eg[i].u = v;
            if (incyc[eg[i].v] == cid) eg[i].v = v;
            if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
        }
        for (int i=0; i<N; i++) {
            if (contracted[i]) continue;
            if (prv[i].u>=0 && incyc[prv[i].u] == cid)
                prv[i].u = v;
        }
        prv[v] = EDGE_INF;
    }
    return cost;
}
#define F first
#define S second
void solve() {
    edge_t cost = dmst(0);
}

```

```

for (auto it : cost.add) { // find a solution
    E[it.F].push_back(it.S);
    prv[it.S] = edge_t(it.F,it.S,0);
}
}

```

## 6.2 DMST

```

// tested on https://lightoj.com/problem/teleport
const int inf = 1e9;
struct edge {
    int u, v, w;
    edge() {}
    edge(int a,int b,int c) : u(a), v(b), w(c) {}
    bool operator < (const edge& o) const {
        if (u == o.u)
            if (v == o.v) return w < o.w;
            else return v < o.v;
        return u < o.u;
    }
};
int dmst(vector<edge> &edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }
        lo[root] = 0;
        for (int i = 0; i < lo.size(); ++i) {
            if (i == root) continue;
            if (lo[i] == inf) return -1;
        }
        int cur_id = 0;
        vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
        for (int i = 0; i < cur_nodes; ++i) {
            ans += lo[i];
            int u = i;
            while (u != root and id[u] < 0 and mark[u] != i) {
                mark[u] = i;
                u = pi[u];
            }
            if (u != root and id[u] < 0) { // Cycle
                for (int v = pi[u]; v != u; v = pi[v]) id[v] = cur_id;
                id[u] = cur_id++;
            }
        }
        if (cur_id == 0) break;
        for (int i = 0; i < cur_nodes; ++i)
            if (id[i] < 0) id[i] = cur_id++;
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            edges[i].u = id[u];
            edges[i].v = id[v];
            if (id[u] != id[v]) edges[i].w -= lo[v];
        }
        cur_nodes = cur_id;
        root = id[root];
    }
    return ans;
}

```

## 6.3 Flow With Demands

**Finding an arbitrary flow** Consider flow networks, where we additionally require the flow of each edge to have a certain amount, i.e. we bound the flow from below by a **demand** function  $d(e)$ :

$$d(e) \leq f(e) \leq c(e)$$

So next each edge has a minimal flow value, that we have to pass along the edge.

We make the following changes in the network. We add a new source  $s'$  and a new sink  $t'$ , a new edge from the source  $s'$  to every other vertex, a new edge for every vertex to the sink  $t'$ , and one edge from  $t$  to  $s$ . Additionally we define the new capacity function  $c'$  as:

- $c'((s', v)) = \sum_{u \in V} d((u, v))$  for each edge  $(s', v)$ .
- $c'((v, t')) = \sum_{w \in V} d((v, w))$  for each edge  $(v, t')$ .
- $c'((u, v)) = c((u, v)) - d((u, v))$  for each edge  $(u, v)$  in the old network.
- $c'((t, s)) = \infty$

If the new network has a saturating flow (a flow where each edge outgoing from  $s'$  is completely filled, which is equivalent to every edge incoming to  $t'$  is completely filled), then the network with demands has a valid flow, and the actual flow can be easily reconstructed from the new network. Otherwise there doesn't exist a flow that satisfies all conditions. Since a saturating flow has to be a maximum flow, it can be found by any maximum flow algorithm.

**Minimal flow** Note that along the edge  $(t, s)$  (from the old sink to the old source) with the capacity  $\infty$  flows the entire flow of the corresponding old network. I.e. the capacity of this edge effects the flow value of the old network. By giving this edge a sufficient large capacity (i.e.  $\infty$ ), the flow of the old network is unlimited. By limiting this edge by smaller capacities, the flow value will decrease. However if we limit this edge by a too small value, than the network will not have a saturated solution, e.g. the corresponding solution for the original network will not satisfy the demand of the edges. Obviously here can use a binary search to find the lowest value with which all constraints are still satisfied. This gives the minimal flow of the original network.

## 6.4 articulation-vertex

```

int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] >= tin[v] && p != -1)
                IS_CUTPOINT(v);
            ++children;
        }
    }
    if (p == -1 && children > 1)
        IS_CUTPOINT(v);
}
void find_cutpoints() {

```

```

timer = 0;
visited.assign(n, false);
tin.assign(n, -1);
low.assign(n, -1);
for (int i = 0; i < n; ++i) {
    if (!visited[i])
        dfs(i);
}
}

```

## 6.5 bellman-ford

```

struct Edge {
    int a, b, cost;
};
int n, m;
vector<Edge> edges;
const int INF = 1000000000;
void solve() {
    vector<int> d(n);
    vector<int> p(n, -1);
    int x;
    for (int i = 0; i < n; ++i) {
        x = -1;
        for (Edge e : edges) {
            if (d[e.a] + e.cost < d[e.b]) {
                d[e.b] = d[e.a] + e.cost;
                p[e.b] = e.a;
                x = e.b;
            }
        }
    }
    if (x == -1) {
        cout << "No negative cycle found.";
    } else {
        for (int i = 0; i < n; ++i)
            x = p[x];
        vector<int> cycle;
        for (int v = x; v = p[v]) {
            cycle.push_back(v);
            if (v == x && cycle.size() > 1)
                break;
        }
        reverse(cycle.begin(), cycle.end());
        cout << "Negative cycle: ";
        for (int v : cycle)
            cout << v << ' ';
        cout << endl;
    }
}

```

## 6.6 bridge

```

const int vmax = 2e5+10, emax = 2e5+10;
namespace Bridge { //edge, nodes, comps 1 indexed
    vector<int> adj[vmax]; // edge-id
    pair<int, int> edges[emax]; // (u, v)
    bool isBridge[emax];
    int visited[vmax]; // 0-unvis, 1-vising, 2-vis
    int st[vmax], low[vmax], clk = 0, edgeId = 0;
    // For bridge tree components
    int who[vmax], compId = 0;
    vector<int> stk;
    // For extra end time calc
    int en[vmax];
    void dfs(int u, int parEdge) {
        visited[u] = 1; low[u] = st[u] = ++clk;
        stk.push_back(u);
        for (auto e : adj[u]) {
            if (e == parEdge) continue;
            int v = edges[e].first^edges[e].second^u;
            if (visited[v] == 1) {
                low[u] = min(low[u], st[v]);
            } else if (visited[v] == 0) {
                dfs(v, e); low[u] = min(low[u], low[v]);
            }
        }
        visited[u] = 2;
        if (st[u] == low[u]) { // found
            ++compId; int cur;

```

```

do{
    cur = stk.back(); stk.pop_back();
    who[cur] = compId;
}while(cur != u);
if(parEdge != -1){isBridge[parEdge] = true;}
}
en[u] = clk;
}
void clearAll(int n){
    for(int i = 0; i<n; i++) {
        adj[i].clear(); visited[i] = st[i] = 0; }
    for(int i = 0; i<edgeId; i++) isBridge[i]=0;
    clk = compId = edgeId = 0;
}
void findBridges(int n){
    for(int i = 1; i<n; i++){
        if(visited[i] == 0) dfs(i, -1); }
}
bool isReplacable(int eid, int u, int v){
    if(!isBridge[eid]) return true;
    int a=edges[eid].first,b=edges[eid].second;
    if(st[a] > st[b]) swap(a, b);
    return (st[b] <= st[u] && st[u] <= en[b])
    != (st[b] <= st[v] && st[v] <= en[b]);
}
void addEdge(int u, int v){
    edgeId++; edges[edgeId] = {u, v};
    adj[u].emplace_back(edgeId);
    adj[v].emplace_back(edgeId);
}
}

```

## 6.7 edmond-blossom

```

/**Copied from https://codeforces.com/blog/entry/49402**/
/*
GETS:
V->number of vertices
E->number of edges
pair of vertices as edges (vertices are 1..V)
GIVES:
output of edmonds() is the maximum matching
match[i] is matched pair of i (-1 if there isn't a matched pair)
*/
const int M=500;
struct struct_edge
{
    int v;
    struct_edge* n;
};
typedef struct_edge* edge;
struct_edge pool[M*M*2];
edge top=pool,adj[M];
int V,E,match[M],qh,qt,q[M],father[M],base[M];
bool inq[M],inb[M],ed[M][M];
void add_edge(int u,int v)
{
    top->v=v,top->n=adj[u],adj[u]=top++;
    top->v=u,top->n=adj[v],adj[v]=top++;
}
int LCA(int root,int u,int v)
{
    static bool inp[M];
    memset(inp,0,sizeof(inp));
    while(1)
    {
        inp[u=base[u]]=true;
        if (u==root) break;
        u=father[match[u]];
    }
    while(1)
    {
        if (inp[v=base[v]]) return v;
        else v=father[match[v]];
    }
}
void mark_blossom(int lca,int u)
{
    while (base[u]!=lca)

```

```

{
    int v=match[u];
    inb[base[u]]=inb[base[v]]=true;
    u=father[v];
    if (base[u]!=lca) father[u]=v;
}
}
void blossom_contraction(int s,int u,int v)
{
    int lca=LCA(s,u,v);
    memset(inb,0,sizeof(inb));
    mark_blossom(lca,u);
    mark_blossom(lca,v);
    if (base[u]!=lca)
        father[u]=v;
    if (base[v]!=lca)
        father[v]=u;
    for (int u=0; u<V; u++)
        if (inb[base[u]])
        {
            base[u]=lca;
            if (!inq[u])
                inq[q[++qt]=u]=true;
        }
}
int find_augmenting_path(int s)
{
    memset(inq,0,sizeof(inq));
    memset(father,-1,sizeof(father));
    for (int i=0; i<V; i++) base[i]=i;
    inq[q[qh=qt=0]=s]=true;
    while (qh<=qt)
    {
        int u=q[qh++];
        for (edge e=adj[u]; e; e=e->n)
        {
            int v=e->v;
            if (base[u]!=base[v]&&match[u]!=v)
            {
                if ((v==s)|| (match[v]==-1 &&
                    father[match[v]]!=-1))
                    blossom_contraction(s,u,v);
                else if (father[v]==-1)
                {
                    father[v]=u;
                    if (match[v]==-1)
                        return v;
                    else if (!inq[match[v]])
                        inq[q[++qt]=match[v]]=true;
                }
            }
        }
        return -1;
    }
}
int augment_path(int s,int t)
{
    int u=t,v,w;
    while (u!=-1)
    {
        v=father[u];
        w=match[v];
        match[v]=u;
        match[u]=v;
        u=w;
    }
    return t!=-1;
}
int edmonds()//Gives number of matchings
{
    int matchc=0;
    memset(match,-1,sizeof(match));
    for (int u=0; u<V; u++)
        if (match[u]==-1)
            matchc+=augment_path(u,find_augmenting_path(u));
    return matchc;
}
//To add edge add_edge(u-1,v-1); ed[u-1][v-1]=ed[v-1][u-1]=true;

```

## 6.8 euler-path

```
int main() {
```

```

int n;
vector<vector<int>> g(n, vector<int>(n));
// reading the graph in the adjacency matrix
vector<int> deg(n);
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j)
        deg[i] += g[i][j];
}
int first = 0;
while (first < n && !deg[first])
    ++first;
if (first == n) {
    cout << -1;
    return 0;
}
int v1 = -1, v2 = -1;
bool bad = false;
for (int i = 0; i < n; ++i) {
    if (deg[i] & 1) {
        if (v1 == -1)
            v1 = i;
        else if (v2 == -1)
            v2 = i;
        else
            bad = true;
    }
}
if (v1 != -1)
    ++g[v1][v2], ++g[v2][v1];
stack<int> st;
st.push(first);
vector<int> res;
while (!st.empty()) {
    int v = st.top();
    int i;
    for (i = 0; i < n; ++i)
        if (g[v][i])
            break;
    if (i == n) {
        res.push_back(v);
        st.pop();
    } else {
        --g[v][i];
        --g[i][v];
        st.push(i);
    }
}
if (v1 != -1) {
    for (size_t i = 0; i + 1 < res.size(); ++i) {
        if ((res[i] == v1 && res[i + 1] == v2) ||
            (res[i] == v2 && res[i + 1] == v1)) {
            vector<int> res2;
            for (size_t j = i + 1; j < res.size(); ++j)
                res2.push_back(res[j]);
            for (size_t j = 1; j <= i; ++j)
                res2.push_back(res[j]);
            res = res2;
            break;
        }
    }
}
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
        if (g[i][j])
            bad = true;
    }
}
if (bad) {
    cout << -1;
} else {
    for (int x : res)
        cout << x << " ";
}
}

```

## 6.9 hopcraft-karp

```

/** Source: https://iq.opengenus.org/hopcroft-karp-algorithm/
**/
// A class to represent Bipartite graph for

```

```

// If this node is not NIL and can provide a shorter
    path to NIL then
    if (dist[u] < dist[NIL]){
        // Get all the adjacent vertices of the dequeued
        vertex u
        std::list<int>::iterator it;
        for (it = adj[u].begin(); it != adj[u].end(); ++it){
            int v = *it;
            // If pair of v is not considered so far
            // i.e. (v, pair_v[v]) is not yet explored edge.
            if (dist[pair_v[v]] == INF){
                // Consider the pair and push it to queue
                dist[pair_v[v]] = dist[u] + 1;
                q.push(pair_v[v]);
            }
        }
    }
// If we could come back to NIL using alternating path of
    distinct
    // vertices then there is an augmenting path available
    return (dist[NIL] != INF);
}

// Returns true if there is an augmenting path beginning with
    free vertex u
bool BGraph::dfs(int u){
    if (u != NIL){
        std::list<int>::iterator it;
        for (it = adj[u].begin(); it != adj[u].end(); ++it){
            // Adjacent vertex of u
            int v = *it;
            // Follow the distances set by BFS search
            if (dist[pair_v[v]] == dist[u] + 1){
                // If dfs for pair of v also return true then
                if (dfs(pair_v[v]) == true){ // new matching
                    // possible, store the matching
                    pair_v[v] = u;
                    pair_u[u] = v;
                    return true;
                }
            }
        }
        // If there is no augmenting path beginning with u then.
        dist[u] = INF;
        return false;
    }
    return true;
}

// Constructor for initialization
BGraph::BGraph(int m, int n){
    this->m = m;
    this->n = n;
    adj = new std::list<int>[m + 1];
}

// function to add edge from u to v
void BGraph::addEdge(int u, int v){
    adj[u].push_back(v); // Add v to us list.
}

```

```
class HungarianAlgorithm{
    int N, inf, n, max_match;
    int *lx, *ly, *xy, *yx, *slack, *slackx, *prev;
    int **cost;
    bool *S, *T;
    void init_labels(){
        for(int x=0; x<n; x++) lx[x]=0;
        for(int y=0; y<n; y++) ly[y]=0;
        for (int x = 0; x < n; x++)
            for (int y = 0; y < n; y++)
                lx[x] = max(lx[x], cost[x][y]);
    }
    void update_labels(){
        int x, y, delta = inf; //init delta as infinity
        for (y = 0; y < n; y++) //calculate delta using slack
            if (!T[y])
                delta = min(delta, slack[y]);
        for (x = 0; x < n; x++) //update X labels
```

```

    if (S[x]) lx[x] -= delta;
    for (y = 0; y < n; y++) //update Y labels
        if (T[y]) ly[y] += delta;
    for (y = 0; y < n; y++) //update slack array
        if (!T[y])
            slack[y] -= delta;
}
void add_to_tree(int x, int prevx)
//x - current vertex, prevx - vertex from X before x in the
//alternating path,
//so we add edges (prevx, xy[x]), (xy[x], x){
    S[x] = true; //add x to S
    prev[x] = prevx; //we need this when augmenting
    for (int y = 0; y < n; y++) //update slacks, because we
        add new vertex to S
        if (lx[x] + ly[y] - cost[x][y] < slack[y]){
            slack[y] = lx[x] + ly[y] - cost[x][y];
            slackx[y] = x;
        }
}
void augment() //main function of the algorithm{
    if (max_match == n) return; //check whether matching is
        already perfect
    int x, y, root; //just counters and root vertex
    int q[N], wr = 0, rd = 0; //q - queue for bfs, wr, rd -
        write and read
//pos in queue
    //memset(S, false, sizeof(S)); //init set S
    for (int i=0; i<n; i++) S[i]=false;
    //memset(T, false, sizeof(T)); //init set T
    for (int i=0; i<n; i++) T[i]=false;
    //memset(prev, -1, sizeof(prev)); //init set prev - for
        the alternating tree
    for (int i=0; i<n; i++) prev[i]=-1;
    for (x = 0; x < n; x++) //finding root of the tree{
        if (xy[x] == -1){
            q[wr++] = root = x;
            prev[x] = -2;
            S[x] = true;
            break;
        }
    }
    for (y = 0; y < n; y++) //initializing slack array{
        slack[y] = lx[root] + ly[y] - cost[root][y];
        slackx[y] = root;
    }
    while (true) //main cycle{
        while (rd < wr) //building tree with bfs cycle{
            x = q[rd++]; //current vertex from X part
            for (y = 0; y < n; y++) //iterate through all
                edges in equality graph{
                    if (cost[x][y] == lx[x] + ly[y] && !T[y]){
                        if (yx[y] == -1) break; //an exposed
                            vertex in Y found, so
//augmenting path exists!
                        T[y] = true; //else just add y to T,
                        q[wr++] = yx[y]; //add vertex yx[y],
                            which is matched
//with y, to the queue
                        add_to_tree(yx[y], x); //add edges (x,y)
                            and (y,yx[y]) to the tree
                    }
                }
            if (y < n) break; //augmenting path found!
        }
        if (y < n) break; //augmenting path found!
        update_labels(); //augmenting path not found, so
            improve labeling
        wr = rd = 0;
        for (y = 0; y < n; y++){
            //in this cycle we add edges that were added to
                the equality graph as a
//result of improving the labeling, we add edge (slackx[y], y)
            to the tree if
//and only if !T[y] && slack[y] == 0, also with this edge we
            add another one
// (y, yx[y]) or augment the matching, if y was exposed
            if (!T[y] && slack[y] == 0){

```



```

        if (yx[y] == -1) //exposed vertex in Y found
            - augmenting path exists!{
                x = slackx[y];
                break;
            }
        else{
            T[y] = true; //else just add y to T,
            if (!S[yx[y]]){
                q[wr++] = yx[y]; //add vertex yx[y],
                which is matched with
            }
            add_to_tree(yx[y], slackx[y]); //and
            add edges (x,y) and (y,
        }
    }
}
//y, to the queue
//yx[y] to the tree
}
}
}
if (y < n) break; //augmenting path found!
}
if (y < n) //we found augmenting path!{
    max_match++; //increment matching
    //in this cycle we inverse edges along augmenting path
    for (int cx = x, cy = y, ty; cx != -2; cx =
        prev[cx], cy = ty){
        ty = xy[cx];
        yx[cy] = cx;
        xy[cx] = cy;
    }
    augment(); //recall function, go to step 1 of the
    algorithm
}
} //end of augment() function
public:
    HungarianAlgorithm(int vv,int inf=1000000000){
        N=vv;
        n=N;
        max_match=0;
        this->inf=inf;
        lx=new int[N];
        ly=new int[N]; //labels of X and Y parts
        xy=new int[N]; //xy[x] - vertex that is matched with x,
        yx=new int[N]; //yx[y] - vertex that is matched with y
        slack=new int[N]; //as in the algorithm description
        slackx=new int[N]; //slackx[y] such a vertex, that
            l(slackx[y]) + l(y) - w(slackx[y],y) = slack[y]
        prev=new int[N]; //array for memorizing alternating paths
        S=new bool[N];
        T=new bool[N]; //sets S and T in algorithm
        cost=new int*[N]; //cost matrix
        for(int i=0; i<N; i++){
            cost[i]=new int[N];
        }
    }
    HungarianAlgorithm(){
        delete []lx;
        delete []ly;
        delete []xy;
        delete []yx;
        delete []slack;
        delete []slackx;
        delete []prev;
        delete []S;
        delete []T;
        int i;
        for(i=0; i<N; i++){
            delete []cost[i];
        }
        delete []cost;
    }
    void setCost(int i,int j,int c){
        cost[i][j]=c;
    }
    int* matching(bool first=true){
        int *ans;
        ans=new int[N];
        for(int i=0;i<N;i++){
            if(first) ans[i]=xy[i];

```

```

        else ans[i]=yx[i];
    }
    return ans;
}
int hungarian(){
    int ret = 0; //weight of the optimal matching
    max_match = 0; //number of vertices in current matching
    for(int x=0;x<n;x++) xy[x]=-1;
    for(int y=0;y<n;y++) yx[y]=-1;
    init_labels(); //step 0
    augment(); //steps 1-3
    for (int x = 0; x < n; x++) //forming answer there
        ret += cost[x][xy[x]];
    return ret;
}
};

```

## 6.11 max-flow-dinic

```

#include<bits/stdc++.h>
#include<vector>
using namespace std;
#define MAX 100
#define HUGE_FLOW 1000000000
#define BEGIN 1
#define DEFAULT_LEVEL 0
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u),
        cap(cap) {}
};
struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;
    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }
    void add_edge(int v, int u, long long cap) {
        edges.emplace_back(v, u, cap);
        edges.emplace_back(u, v, 0);
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }
    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }
    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size();
            cid++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap -
                edges[id].flow < 1)
                continue;

```

```

            long long tr = dfs(u, min(pushed, edges[id].cap -
                edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }
    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
};
int main(){
    return 0;
}

```

## 6.12 min-cost-max-flow

```

struct Edge{ int from, to, capacity, cost; };
vector<vector<int>> adj, cost, capacity;
const int INF = 1e9;
void shortest_paths(int n,int v0,vector<int>&d,vector<int>&p){
    d.assign(n, INF); d[v0] = 0; vector<bool> inq(n, false);
    queue<int> q; q.push(v0); p.assign(n, -1);
    while (!q.empty()) {
        int u = q.front(); q.pop(); inq[u] = false;
        for (int v : adj[u]) {
            if (capacity[u][v] > 0 && d[v] > d[u]+cost[u][v]){
                d[v] = d[u] + cost[u][v]; p[v] = u;
                if (!inq[v]) { inq[v] = true; q.push(v); }
            }
        }
    }
}
int min_cost_flow(int N,vector<Edge> edges,int K,int s,int t) {
    adj.assign(N,vector<int>());cost.assign(N,vector<int>(N,0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to); adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost; cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity; }
    int flow = 0; int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p); if (d[t] == INF) break;
        // find max flow on that path
        int f = K - flow; int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]); cur = p[cur];
        }
        // apply flow
        flow += f; cost += f * d[t]; cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f; capacity[cur][p[cur]]+= f;
            cur = p[cur]; }
    }
    if (flow < K) return -1;
    else return cost;
}

```

## 6.13 online-bridge

```

vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
int bridges; int lca_iteration;
vector<int> last_visit;

```

```

void init(int n) {
    par.resize(n); dsu_2ecc.resize(n); dsu_cc.resize(n);
    dsu_cc_size.resize(n); lca_iteration=0; last_visit.assign(n,0);
    for (int i=0; i<n; ++i) {
        dsu_2ecc[i] = i; dsu_cc[i] = i; dsu_cc_size[i] = 1;
        par[i] = -1;
    }
    bridges = 0;
}

int find_2ecc(int v) {
    if (v == -1) return -1;
    return dsu_2ecc[v]==v?v:dsu_2ecc[v]=find_2ecc(dsu_2ecc[v]);
}

int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
}

void make_root(int v) {
    v = find_2ecc(v); int root = v; int child = -1;
    while (v != -1) {
        int p = find_2ecc(par[v]); par[v] = child;
        dsu_cc[v] = root; child = v; v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}

void merge_path (int a, int b) {
    ++lca_iteration; vector<int> path_a, path_b; int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a); path_a.push_back(a);
            if (last_visit[a] == lca_iteration){
                lca = a; break; }
            last_visit[a] = lca_iteration; a = par[a];
        }
        if (b != -1) {
            b = find_2ecc(b); path_b.push_back(b);
            if (last_visit[b] == lca_iteration){
                lca = b; break; }
            last_visit[b] = lca_iteration; b = par[b];
        }
    }
    for (int v : path_a) {
        dsu_2ecc[v] = lca; if (v == lca) break; --bridges;
    }
    for (int v : path_b) {
        dsu_2ecc[v] = lca; if (v == lca) break; --bridges;
    }
}

void add_edge(int a, int b) {
    a = find_2ecc(a); b = find_2ecc(b);
    if (a == b) return;
    int ca = find_cc(a); int cb = find_cc(b);
    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b); swap(ca, cb); }
        make_root(a); par[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}

```

## 6.14 scc + 2 Sat

```

namespace SCC { //Everything 0-indexed.
const int N = 2e6+7; int which[N], vis[N], cc;
vector<int> adj[N], adjr[N]; vector<int> order;
void addEdge(int u, int v) {
    adj[u].push_back(v); adjr[v].push_back(u);
}

void dfs1(int u){
    if (vis[u]) return; vis[u] = true;
    for(int v: adj[u]) dfs1(v); order.push_back(u);
}

void dfs2(int u, int id) {
    if(vis[u]) return; vis[u] = true;
    for(int v: adjr[u]) dfs2(v, id); which[u] = id;
}
}

```

```

int last = 0;
void findSCC(int n) {
    cc=0,last=n; order.clear(); fill(vis, vis+n, 0);
    for(int i=0; i<n; i++) if(!vis[i]) dfs1(i);
    reverse(order.begin(), order.end());
    fill(vis, vis+n, 0);
    for (int u: order) {
        if (vis[u]) continue; dfs2(u, cc); ++cc;
    }
}

void clear() {
    for (int i=0; i<last; i++)
        adj[i].clear(), adjr[i].clear();
}

struct TwoSat {
    int n; int vars = 0; vector<bool> ans;
    TwoSat(int n) : n(n), ans(n) {
        SCC::clear(); vars = 2*n;
    }
    void implies(int x, int y) {
        SCC::addEdge(x, y); SCC::addEdge(y^1, x^1);
    }
    void OR(int x, int y) {
        SCC::addEdge(x^1, y); SCC::addEdge(y^1, x);
    }
    void XOR(int x, int y) {
        implies(x, y^1); implies(x^1, y);
    }
    void atmostOne(vector<int> v) {
        int k = v.size();
        for (int i=0; i<k; i++) {
            if (i+1<k) implies(vars+2*i, vars+2*i+2);
            implies(v[i], vars+2*i);
            if (i>0) implies(v[i], vars+2*i-1);
        }
        vars += 2*k;
    }
    bool solve() {
        SCC::findSCC(vars); ans.resize(vars/2);
        for (int i=0; i<vars; i+=2) {
            if (SCC::which[i]==SCC::which[i+1])return 0;
            if (i<2*n)
                ans[i/2] = SCC::which[i]>SCC::which[i+1];
        }
        return true;
    }
}

```

## 7 Math

### 7.1 BerleKampMassey

```

const int MOD = 998244353;
vector<LL> berlekampMassey(vector<LL> s) {
    if (s.empty()) return {};
    int n = s.size(), L = 0, m = 0;
    vector<LL> C(n), B(n), T;
    C[0] = B[0] = 1; LL b = 1;
    for (int i = 0; i < n; ++i) {
        ++m; LL d = s[i] % MOD;
        for (int j = 1; j <= L; ++j) d = (d + C[j] * s[i - j]) % MOD;
        if (!d) continue;
        T = C; LL coeff = d * bigMod(b, -1) % MOD;
        for (int j = m; j < n; ++j) C[j] = (C[j] - coeff * B[j - m]) % MOD;
        if (2*L > i) continue;
        L = i+1-L, B = T, b = d, m = 0;
    }
    C.resize(L + 1), C.erase(C.begin());
    for (LL &x : C) x = (MOD - x) % MOD;
    return C;
}

```

### 7.2 FloorSum

```

LL mod(LL a, LL m) {
    LL ans = a%m;
    return ans < 0 ? ans+m : ans;
}

```

```

//Sum(floor((a*x+b)/m)) for i=0 to n-1, (n,m >= 0)
LL floorSum(LL n, LL m, LL a, LL b) {
    LL ra = mod(a, m), rb = mod(b, m), k = (ra*n+rb);
    LL ans = ((a-ra)/m) * n*(n-1)/2 + ((b-rb)/m) * n;
    if (k < m) return ans;
    return ans + floorSum(k/m, ra, m, k%m);
}

```

### 7.3 Stern Brocot Tree

```

//finds x/y with min y st: L <= (x/y) < R
pair<LL,LL> solve(LD L, LD R){
    pair<LL, LL> l(0, 1), r(1, 1);
    if(L==0.0) return l; // corner case
    while(true) {
        pair<int, int> m(l.x+r.x, l.y+r.y);
        if(m.x<L*m.y){ // move to the right
            LL kl=1, kr=1;
            while(l.x+kr*r.x <= L*(l.y+kr*r.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(l.x+km*r.x < L*(l.y+km*r.y)) kl=km+1;
                else kr=km;
            }
            l={l.x+(kl-1)*r.x,l.y+(kl-1)*r.y};
        }
        else if(m.x>=R*m.y){ //move to left
            LL kl=1, kr=1;
            while(r.x+kr*l.x>=R*(r.y+kr*l.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(r.x+km*l.x>=R*(r.y+km*l.y)) kl = km+1;
                else kr = km;
            }
            r={r.x+(kl-1)*l.x,r.y+(kl-1)*l.y};
        }
        else return m;
    }
}

```

### 7.4 Sum Of Kth Power

```

LL mod; LL S[105][105];
// Find 1^k+2^k+...+n^k % mod
void solve() {
    LL n, k;
    /* x^k = sum (i=1 to k) Stirling2(k, i) * i! * ncr(x, i)
    sum (x = 0 to n) x^k
    = sum (i=0 to k) Stirling2(k, i) * i! * sum (x=0 to n)ncr(x, i)
    = sum (i=0 to k) Stirling2(k, i) * i! * ncr(n+1, i+1)
    = sum (i=0 to k) Stirling2(k, i) * i! * (n+1)! / (i+1)! / (n-i)!
    = sum (i=0 to k) Stirling2(k, i) * (n-i+1) *
      (n-i+2) * ... (n+1) / (i+1) */
    S[0][0] = 1 % mod;
    for (int i = 1; i <= k; i++) {
        for (int j = 1; j <= i; j++) {
            if (i == j) S[i][j] = 1 % mod;
            else S[i][j] = (j * S[i-1][j] + S[i-1][j-1]) % mod;
        }
    }
    LL ans = 0;
    for (int i = 0; i <= k; i++) {
        LL fact = 1, z = i + 1;
        for (LL j = n - i + 1; j <= n + 1; j++) {
            LL mul = j;
            if (mul % z == 0) {
                mul /= z; z /= z;
            }
            fact = (fact * mul) % mod;
        }
        ans = (ans + S[k][i] * fact) % mod;
    }
}

```

### 7.5 combination-generator

```

bool next_combination(vector<int>& a, int n) {
    int k = (int)a.size();
    for (int i = k - 1; i >= 0; i--) {

```

```

    if (a[i] < n - k + i + 1) {
        a[i]++;
        for (int j = i + 1; j < k; j++)
            a[j] = a[j - 1] + 1;
        return true;
    }
    return false;
}
vector<int> ans;
void gen(int n, int k, int idx, bool rev) {
    if (k > n || k < 0)
        return;
    if (!n) {
        for (int i = 0; i < idx; ++i) {
            if (ans[i])
                cout << i + 1;
        }
        cout << "\n";
        return;
    }
    ans[idx] = rev;
    gen(n - 1, k - rev, idx + 1, false);
    ans[idx] = !rev;
    gen(n - 1, k - !rev, idx + 1, true);
}
void all_combinations(int n, int k) {
    ans.resize(n);
    gen(n, k, 0, false);
}

```

## 7.6 continued-fractions

```

auto fraction(int p, int q) {
    vector<int> a;
    while(q) {
        a.push_back(p / q);
        tie(p, q) = make_pair(q, p % q);
    }
    return a;
}
auto convergents(vector<int> a) {
    vector<int> p = {0, 1};
    vector<int> q = {1, 0};
    for(auto it: a) {
        p.push_back(p[p.size() - 1] * it + p[p.size() - 2]);
        q.push_back(q[q.size() - 1] * it + q[q.size() - 2]);
    }
    return make_pair(p, q);
}

```

## 7.7 crt anachor

```

/// Chinese remainder theorem (special case): find z st z%m1 =
    r1, z%m2 = r2.
/// z is unique modulo M = lcm(m1, m2). Returns (z, M). On
    failure, M = -1.
PLL CRT(LL m1, LL r1, LL m2, LL r2) {
    LL s, t;
    LL g = egcd(m1, m2, s, t);
    if (r1%g != r2%g) return PLL(0, -1);
    LL M = m1*m2;
    LL ss = ((s*r2)%m2)*m1;
    LL tt = ((t*r1)%m1)*m2;
    LL ans = ((ss+tt)%M+M)%M;
    return PLL(ans/g, M/g);
}
// expected: 23 105
//          11 12
PLL ans = CRT({3,5,7}, {2,3,2});
cout << ans.first << " " << ans.second << endl;
ans = CRT({4,6}, {3,5});
cout << ans.first << " " << ans.second << endl;

```

## 7.8 discrete-root

```

#define MAX 100000
int prime[MAX+1], Phi[MAX+1];
vector<int> pr;
void sieve(){

```

```

    for (int i=2; i <= N; ++i) {
        if (prime[i] == 0) {
            prime[i] = i;
            pr.push_back(i);
        }
        for (int j=0; j < (int)pr.size() && pr[j] <= prime[i]
            && i*pr[j] <= N; ++j) {
            prime[i * pr[j]] = pr[j];
        }
    }
}
void PhiWithSieve(){
    int i;
    for(i=2; i<=MAX; i++){
        if(prime[i]==i){
            Phi[i]=i-1;
        }
        else if((i/prime[i])%prime[i]==0){
            Phi[i]=Phi[i/prime[i]]*prime[i];
        }
        else{
            Phi[i]=Phi[i/prime[i]]*(prime[i]-1);
        }
    }
}
int powmod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 1ll * a % p), --b;
        else
            a = int (a * 1ll * a % p), b >>= 1;
    return res;
}
int PrimitiveRoot(int p){
    vector<int> fact;
    int phi=Phi[p];
    int n=phi;
    while(n>1){
        if(prime[n]==n){
            fact.push_back(n);
            n=1;
        }
        else{
            int f=prime[n];
            while(n%f==0){
                n=n/f;
            }
            fact.push_back(f);
        }
    }
    int res;
    for(res=p-1; res>1; --res){
        for(n=0; n<fact.size(); n++){
            if(powmod(res,phi/fact[n],p)==1){
                break;
            }
        }
        if(n>=fact.size()) return res;
    }
    return -1;
}
int DiscreteLog(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    map<int, int> vals;
    for (int p = 1; p <= n; ++p)
        vals[powmod(a, (int) (1ll * p * n) % m, m)] = p;
    for (int q = 0; q <= n; ++q) {
        int cur = (powmod(a, q, m) * 1ll * b) % m;
        if (vals.count(cur)) {
            int ans = vals[cur] * n - q;
            return ans;
        }
    }
    return -1;
}
vector<int> DiscreteRoot(int n,int a,int k){
    int g = PrimitiveRoot(n);

```

```

    vector<int> ans;
    int any_ans = DiscreteLog(powmod(g,k,n),a,n);
    if (any_ans == -1){
        return ans;
    }
    int delta = (n-1) / gcd(k, n-1);
    for (int cur = any_ans % delta; cur < n-1; cur += delta)
        ans.push_back(powmod(g, cur, n));
    sort(ans.begin(), ans.end());
    return ans;
}

```

## 7.9 fast-fourier-transform

```

using cd = complex<double>;
const double PI = acos(-1);
typedef long long ll;
void fft(vector<cd>& a, bool invert){
    int n = a.size();
    for(int i = 1, j = 0; i < n; i++){
        int bit = n>>1;
        for(; j&bit; bit>>=1){
            j^=bit;
        }
        j ^= bit;
        if(i < j)
            swap(a[i], a[j]);
    }
    for(int len = 2; len <= n; len <= 1){
        double ang = 2*PI/len*(invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += len){
            cd w(1);
            for(int j = 0; j < len/2; j++){
                cd u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
        if(invert){
            for(cd &x: a)
                x /= n;
        }
    }
}
vector<int> multiply(vector<int> const& a, vector<int>
    const&b){
    vector<cd> fa(a.begin(), a.end());
    vector<cd> fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size()+b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for(int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
    vector<int> result(n);
    for(int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    return result;
}
/*Number Theoretic Transformation
ll int gcd(ll int a,ll int b){
    if(b==0) return a;
    else return gcd(b,a%b);
}
ll int egcd(ll int a, ll int b, ll int &x, ll int &y) {
    if (a == 0) {
        x = 0;
        y = 1;
        return b;
    }
    ll int x1, y1;
    ll int d = egcd(b % a, a, x1, y1);
    x = y1 - (b / a) * x1;
    y = x1;

```

```

    return d;
}
ll int ModuloInverse(ll int a, ll int n){
    ll int x, y;
    x = gcd(a, n);
    a = a/x;
    n = n/x;
    ll int res = egcd(a, n, x, y);
    x = (x%n+n)%n;
    return x;
}
const int mod = 998244353;
const int root = 15311432;
const int root_1 = 469870224;
const int root_pw = 1 << 23;
998244353 = 119 * 2^23 + 1, primitive root = 3
985661441 = 235 * 2^22 + 1, primitive root = 3
1012924417 = 483 * 2^21 + 1, primitive root = 5
void fft(vector<int> &a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <= 1)
            wlen = (int)(1LL * wlen * wlen % mod);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
                a[i+j] = u + v < mod ? u + v : u + v - mod;
                a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
                w = (int)(1LL * w * wlen % mod);
            }
        }
    }
    if (invert) {
        int n_1 = (int) ModuloInverse(n, mod);
        for (int &x : a)
            x = (int)(1LL * x * n_1 % mod);
    }
}
vector<int> multiply(vector<int> const& a, vector<int> const& b){
    vector<int> fa(a.begin(), a.end());
    vector<int> fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size()+b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for(int i = 0; i < n; i++)
        fa[i] = (int)(1LL*fa[i]*fb[i]%mod);
    fft(fa, true);
    vector<int> result(n);
    for(int i = 0; i < n; i++)
        result[i] = fa[i];
    return result;
}
*/

```

## 7.10 fast-walsh-hadamard

```

void FWHT(vector<LL> &p, bool inv) {
    int n = p.size(); assert((n&(n-1))==0);
    for (int len=1; 2*len<=n; len<= 1) {
        for (int i = 0; i < n; i += len+len){
            for (int j = 0; j < len; j++) {
                LL u = p[i+j], v = p[i+len+j];
                ///XOR p[i+j]=u+v; p[i+len+j]=u-v;
                ///OR if(!inv) p[i+j]=u, p[i+len+j]=u+v;
            }
        }
    }
}

```

```

///OR else p[i+j]=u+v, p[i+len+j]=u;
///AND if(!inv) p[i+j]=u+v, p[i+len+j]=u;
///AND else p[i+j]=u, p[i+len+j]=u-v;
    }
}
}
///XOR if(inv) for(int i=0; i<n; i++) p[i]/=n;
}
vector<LL> convo(vector<LL> a, vector<LL> b) {
    int n = 1, sz = max(a.size(), b.size());
    while(n<sz) n*=2;
    a.resize(n); b.resize(n); vector<LL> res(n, 0);
    FWHT(a, 0); FWHT(b, 0);
    for(int i=0; i<n; i++) res[i] = a[i] * b[i];
    FWHT(res, 1);
    return res;
}

```

## 7.11 find-root

```

double sqrt_newton(double n) {
    const double eps = 1E-15;
    double x = 1;
    for (;;) {
        double nx = (x + n / x) / 2;
        if (abs(x - nx) < eps)
            break;
        x = nx;
    }
    return x;
}
int isqrt_newton(int n) {
    int x = 1;
    bool decreased = false;
    for (;;) {
        int nx = (x + n / x) >> 1;
        if (x == nx || nx > x && decreased)
            break;
        decreased = nx < x;
        x = nx;
    }
    return x;
}

```

## 7.12 integer-factorization

```

long long pollards_p_minus_1(long long n) {
    int B = 10; long long g = 1;
    while (B <= 1000000 && g < n) {
        long long a = 2 + rand() % (n - 3); g = gcd(a, n);
        if (g > 1) return g;
        // compute a^M
        for (int p : primes) {
            if (p >= B) continue; long long p_power = 1;
            while (p_power * p <= B) p_power *= p;
            a = power(a, p_power, n); g = gcd(a - 1, n);
            if (g > 1 && g < n) return g;
        }
        B *= 2;
    }
    return 1;
}
long long mult(long long a, long long b, long long mod) {
    long long result = 0;
    while (b) {
        if (b & 1) result = (result + a) % mod;
        a = (a + a) % mod; b >>= 1;
    }
    return result;
}
long long f(long long x, long long c, long long mod) {
    return (mult(x, x, mod) + c) % mod;
}
long long rho(long long n, long long x0=2, long long c=1) {
    long long x = x0; long long y = x0; long long g = 1;
    while (g == 1) {
        x = f(x, c, n); y = f(y, c, n); y = f(y, c, n);
        g = gcd(abs(x - y), n);
    }
    return g;
}

```

```

long long brent(long long n, long long x0=2, long long c=1) {
    long long x = x0; long long g = 1; long long q = 1;
    long long xs, y; int m = 128; int l = 1;
    while (g == 1) {
        y = x; int k = 0;
        for (int i = 1; i < l; i++) x = f(x, c, n);
        while (k < l && g == 1) {
            xs = x;
            for (int i = 0; i < m && i < l - k; i++) {
                x = f(x, c, n); q = mult(q, abs(y - x), n);
            }
            g = gcd(q, n); k += m;
        }
        l *= 2;
    }
    if (g == n) {
        do {
            xs = f(xs, c, n); g = gcd(abs(xs - y), n);
        } while (g == 1);
    }
    return g;
}

```

## 7.13 integration-simpson

```

const int N = 1000 * 1000; // number of steps (already
    multiplied by 2)
double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final
        Simpson's formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}

```

## 7.14 linear-diophantine-equation-gray-code

```

int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
bool find_any_solution(int a, int b, int c, int &x0, int &y0,
    int &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
void shift_solution(int &x, int &y, int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}
int find_all_solutions(int a, int b, int c, int minx, int
    maxx, int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;
    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx)

```

```

    shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    int lx1 = x;
    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    int rx1 = x;
    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    int lx2 = x;
    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy)
        shift_solution(x, y, a, b, sign_a);
    int rx2 = x;
    if (lx2 > rx2)
        swap(lx2, rx2);
    int lx = max(lx1, lx2);
    int rx = min(rx1, rx2);
    if (lx > rx)
        return 0;
    return (rx - lx) / abs(b) + 1;
}

int g(int n) {
    return n ^ (n >> 1);
}

int rev_g(int g) {
    int n = 0;
    for (; g; g >>= 1)
        n ^= g;
    return n;
}

```

### 7.15 linear-equation-system

```

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity
// or a big number
int gauss(vector<vector<double>> a, vector<double> &ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where(m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs(a[i][col]) > abs(a[sel][col]))
                sel = i;
        if (abs(a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap(a[sel][i], a[row][i]);
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign(m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs(sum - a[i][m]) > EPS)
            return 0;
    }
    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

### 7.16 matrix-determinant

```

const double EPS = 1E-9;
int n;
vector<vector<double>> a(n, vector<double>(n));
double det = 1;
for (int i=0; i<n; ++i) {
    int k = i;
    for (int j=i+1; j<n; ++j)
        if (abs(a[j][i]) > abs(a[k][i]))
            k = j;
    if (abs(a[k][i]) < EPS) {
        det = 0;
        break;
    }
    swap(a[i], a[k]);
    if (i != k)
        det = -det;
    det *= a[i][i];
    for (int j=i+1; j<n; ++j)
        a[i][j] /= a[i][i];
    for (int j=0; j<n; ++j)
        if (j != i && abs(a[j][i]) > EPS)
            for (int k=i+1; k<n; ++k)
                a[j][k] -= a[i][k] * a[j][i];
}
cout << det;

```

### 7.17 matrix-rank

```

const double EPS = 1E-9;
int compute_rank(vector<vector<double>> A) {
    int n = A.size(); int m = A[0].size(); int rank = 0;
    vector<bool> row_selected(n, false);
    for (int i = 0; i < m; ++i) {
        int j;
        for (j = 0; j < n; ++j) {
            if (!row_selected[j] && abs(A[j][i]) > EPS) break;
        }
        if (j != n) {
            ++rank; row_selected[j] = true;
            for (int p = i + 1; p < m; ++p) A[j][p] /= A[j][i];
            for (int k = 0; k < n; ++k) {
                if (k != j && abs(A[k][i]) > EPS) {
                    for (int p = i + 1; p < m; ++p)
                        A[k][p] -= A[j][p] * A[k][i];
                }
            }
        }
    }
    return rank;
}

```

### 7.18 $nCr \bmod p^a$

```

LL F[1000009];
void pre(LL mod, LL pp) { // mod is pp^a, pp is prime
    F[0] = 1;
    REPL(i, 1, mod) { // we keep in F factorial with
        // the terms which are coprime with pp
        if (i % pp != 0) F[i] = (F[i-1] * i) % mod;
        else F[i] = F[i-1];
    }
}

LL fact2(LL nn, LL mod) {
    LL cycle = nn / mod;
    LL n2 = nn % mod;
    return (bigmod(F[mod], cycle, mod) * F[n2]) % mod;
}

// returns highest power of pp that divides N and the coprime
// with pp part of N! % mod
PLL fact(LL N, LL pp, LL mod) {
    LL nn = N; LL ord = 0;
    while (nn > 0) { nn /= pp; ord += nn; }
    LL ans = 1; nn = N;
    while (nn > 0) { ans = (ans * fact2(nn, mod)) % mod;
        nn /= pp; }
    return MP(ord, ans);
}

LL ncrp(ULL n, ULL r, LL prm, LL pr) { // ncr mod prm^pr
    LL mod = bigmod(prm, pr, INF), temp;
    pre(mod, prm);
}

```

```

PLL x=fact(n,prm,mod),y=fact(r,prm,mod),z=fact(n-r,prm,mod);
LL guun=x.second*modInverse(y.second,mod,prm);
guun%=mod;guun*=modInverse(z.second,mod,prm);
guun%=mod;
LL guun2=x.first-y.first-z.first;
guun*=bigmod(prm,guun2,mod);
guun%=mod;
return guun;
}

```

### 7.19 primality-test

```

using u64 = uint64_t;
using u128 = __uint128_t;
u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1; base %= mod;
    while (e) {
        if (e & 1) result = (u128)result * base % mod;
        base = (u128)base * base % mod; e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1) return false;
    }
    return true;
}

// returns true if n is prime, else returns false.
bool MillerRabin(u64 n) {
    if (n < 2) return false;
    int r = 0; u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1; r++;
    }
    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37})
        if (n == a) return true;
        if (check_composite(n, a, d, r)) return false;
    return true;
}

bool probablyPrimeFermat(int n, int iter=5) {
    if (n < 4) return n == 2 || n == 3;
    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (binpower(a, n - 1, n) != 1)
            return false;
    }
    return true;
}

```

### 7.20 prime counting function

```

#define MAXN 500
#define MAXM 100010
#define MAXP 666666
#define MAX 10000010
#define ll long long int
#define chkbit(ar,i) (((ar[(i)>>6])&(1<<(((i)>>1)&31))))
#define setbit(ar,i) (((ar[(i)>>6])|= (1<<(((i)>>1)&31))))
#define isprime(x) (((x)&&((x)&1)&&!chkbit(ar,(x))))||((x)==2))
namespace pc{
    long long dp[MAXN][MAXM];
    unsigned int ar[(MAX>>6)+5] = {0};
    int len=0, primes[MAXP], counter[MAX];
    void Sieve(){ setbit(ar,0), setbit(ar,1);
        for (int i=3;(i*i)<MAX;i++){
            if(!chkbit(ar,i)){
                for(int j=(i*i);j<MAX;j+=i) setbit(ar,j);
            }
        }
        for(int i=1;i<MAX;i++){ counter[i]=counter[i-1];
            if(isprime(i)) primes[len++]=i, counter[i]++;
        }
    }
    void init(){
}

```

```

Sieve();
for(int n=0;n<MAXN;n++){
    for(int m=0;m<MAXM;m++){
        if(!n) dp[n][m]=m;
        else dp[n][m]=dp[n-1][m]-dp[n-1][m/primes[n-1]];
    }
}
11 phi(11 m,int n){
    if(n==0) return m;    if(primes[n-1]>m) return 1;
    if(m<MAXM && n<MAXN) return dp[n][m];
    return phi(m,n-1) - phi(m/primes[n-1],n-1);
}
11 Lehmer(long long m){
    if(m<MAX) return counter[m];
    11 w,res=0; int i,a,s,c,x,y;
    s=sqrt(0.9+m), y=cbrt(0.9+m);
    a=counter[y], res=phi(m,a)+a-1;
    for(i=a;primes[i]<=s;i++)
        res=res-Lehmer(m/primes[i])+Lehmer(primes[i])-1;
    return res;
}
}

```

## 8 String

### 8.1 Hashing

```

11 fmod(11 a, 11 b, int md=mods[0]) {
    unsigned long long x = (long long) a * b;
    unsigned xh = (unsigned) (x >> 32), xl = (unsigned) x, d, m;
    asm(
        "div %4; \n\t"
        : "=a" (d), "=d" (m)
        : "d" (xh), "a" (xl), "r" (md)
    );
    return m;
}
void Build1(const string &str) {
    for(11 i = str.size() - 1; i >= 0; i--){
        hsh[i] = fmod(hsh[i + 1], bases[j], mods[j]) + str[i];
        if (hsh[i] > mods[j]) hsh[i] -= mods[j];
    }
}
11 getSingleHash(11 i, 11 j) {
    assert(i <= j);
    11 tmp1 = (hsh[i] - fmod(hsh[j+1], pwbase[0][j-i+1]));
    if(tmp1 < 0) tmp1 += mods[0]; return tmp1;
}

```

### 8.2 aho-corasick

```

const int K = 26;
struct Vertex {
    int next[K]; bool leaf = false; int p = -1; char pch;
    int link = -1; int go[K];
    Vertex(int p=-1, char ch='$',) : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> t(1);
void add_string(string const& s) { int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size(); t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf = true;
}
int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0) t[v].link = 0;
        else t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}

```

```

}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1) t[v].go[c] = t[v].next[c];
        else t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}

```

### 8.3 manacher

```

char s[MAX]; vector<int> d1(n); vector<int> d2(n);
for (int i = 0, l = 0, r = -1; i < n; i++){
    int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
    while (0 <= i - k && i + k < n && s[i - k] == s[i + k])
        { k++; }
    d1[i] = k--;
    if (i + k > r) { l = i - k; r = i + k; }
}
for (int i = 0, l = 0, r = -1; i < n; i++){
    int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
    while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i + k])
        { k++; }
    d2[i] = k--;
    if (i + k > r) { l = i - k - 1; r = i + k; }
}

```

### 8.4 palindromic tree

```

struct node {
    int next[26]; int len; int sufflink; int num; };
int len; char s[MAXN]; node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root with len 0
int suff; // max suffix palindrome
bool addLetter(int pos) {
    int cur = suff, curlen = 0; int let = s[pos] - 'a';
    while (true) {
        curlen = tree[cur].len;
        if (pos-1-curlen >= 0 && s[pos-1-curlen] == s[pos])
            break;
        cur = tree[cur].sufflink;
    }
    if (tree[cur].next[let]) {
        suff = tree[cur].next[let]; return false;
    }
    num++; suff = num; tree[num].len = tree[cur].len + 2;
    tree[cur].next[let] = num;
    if (tree[num].len == 1) { tree[num].sufflink = 2;
        tree[num].num = 1; return true;
    }
    while (true) {
        cur = tree[cur].sufflink; curlen = tree[cur].len;
        if (pos-1-curlen >= 0 && s[pos-1-curlen] == s[pos]) {
            tree[num].sufflink = tree[cur].next[let]; break;
        }
    }
    tree[num].num = 1 + tree[tree[num].sufflink].num; return true;
}
void initTree() {
    num = 2; suff = 2; // memset tree must
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}
int main() { gets(s); len = strlen(s); initTree();
    for (int i = 0; i < len; i++) { addLetter(i);
        ans += tree[suff].num; }
    cout << ans << endl; return 0;
}

```

### 8.5 suffix array da

```

/* sa => ith smallest suffix of the string
rak => rak[i] indicates the position of suffix(i) in the suffix
array; height => height[i] indicates the LCP of i-1 and i th
suffix; LCP of suffix(i) & suffix(j) = { L = rak[i], R = rak[j]
        , min(height[L+1, R]); } */
const int maxn = 5e5+5;
int wa[maxn],wb[maxn],wv[maxn],wc[maxn];
int r[maxn],sa[maxn],rak[maxn], height[maxn],dp[maxn][22],

```

```

jump[maxn], SIGMA = 0 ;
int cmp(int *r,int a,int b,int l)
    {return r[a]==r[b]&&r[a+1]==r[b+1];}
void da(int *r,int *sa,int n,int m){
    int i,j,p,*x=wa,*y=wb,*t;
    for(i=0;i<n;i++) wc[i]=0;
    for(i=0;i<n;i++) wc[x[i]=r[i]] ++;
    for(i=1;i<n;i++) wc[i] += wc[i-1];
    for(i=n-1;i>=0;i--) sa[--wc[x[i]]] = i;
    for(j=1,p=1;p<n;j*=2,m=p){
        for(p=0,i=n-j;i<n;i++)y[p++] = i;
        for(i=0;i<n;i++)if(sa[i] >= j) y[p++] = sa[i] - j;
        for(i=0;i<n;i++)wv[i] = x[y[i]];
        for(i=0;i<n;i++) wc[i] = 0;
        for(i=0;i<n;i++) wc[wv[i]] ++;
        for(i=1;i<n;i++) wc[i] += wc[i-1];
        for(i=n-1;i>=0;i--) sa[--wc[wv[i]]] = y[i];
        for(t=x,x=y,y=t,p=1,x[sa[0]] = 0,i=1;i<n;i++)
            x[sa[i]] = cmp(y,sa[i-1],sa[i],j) ? p-1:p++;
    }
}
void calheight(int *r,int *sa,int n){
    int i,j,k=0;
    for(i=1;i<n;i++) rak[sa[i]] = i;
    for(i=0;i<n;height[rak[i++]] = k) {
        for(k?k--:0, j=sa[rak[i]-1]; r[i+k] == r[j+k]; k ++);
    }
}
void initRMQ(int n){
    for(int i = 0; i < n; i++) dp[i][0] = height[i];
    for(int j = 1; (1<<j) <= n; j ++ ){
        for(int i = 0; i + (1<<j) - 1 <= n; i ++ ) {
            dp[i][j] = min(dp[i][j-1], dp[i+(1<<(j-1))][j-1]);
        }
        for(int i = 1; i <= n; i ++ ) {
            int k = 0; while((1 << (k+1)) <= i) k++; jump[i] = k;
        }
    }
}
int askRMQ(int L,int R){
    int k = jump[R-L+1];
    return min(dp[L][k], dp[R - (1<<k) + 1][k]);
}
int main(){
    scanf("%s",s); int n = strlen(s);
    for(int i = 0; i < n; i ++ ) {
        r[i] = s[i] - 'a' + 1; SIGMA = max(SIGMA, r[i]);
    }
    r[n] = 0; da(r,sa,n+1,SIGMA + 1);
    calheight(r,sa,n);
    /* don't forget SIGMA + 1. It will ruin you. */
}

```

### 8.6 suffix-automaton

```

class SuffixAutomaton{
    bool complete; int last;
    set<char> alphabet;
    struct state{
        int len, link, endpos, first_pos,snas,height;
        long long substrings,sublen;
        bool is_clone;
        map<char, int> next;
        vector<int> inv_link;
        state(int leng=0,int li=0){
            len=leng; link=li;
            first_pos=-1; substrings=0;
            sublen=0; // length of all substrings
            snas=0; // shortest_non_appearing_string
            endpos=1; is_clone=false; height=0;
        }
    };
    vector<state> st;
    void process(int node){
        map<char, int> ::iterator mit;
        st[node].substrings=1;
        st[node].snas=st.size();
        if((int) st[node].next.size()<(int) alphabet.size())
            st[node].snas=1;
        for(mit=st[node].next.begin(); mit!=st[node].next.end();++mit){

```

```

    if(st[mit->second].substrings==0) process(mit->second);
    st[node].height=max(st[node].height,1+st[mit->second].height);
    st[node].substrings=
        st[node].substrings+st[mit->second].substrings;
    st[node].sublen=st[node].sublen
        +st[mit->second].sublen+st[mit->second].substrings;
    st[node].snas=min(st[node].snas,
        1+st[mit->second].snas);
}
if(st[node].link!=-1)
    st[st[node].link].inv_link.push_back(node);
}
void set_suffix_links(int node){
    int i;
    for(i=0; i<st[node].inv_link.size(); i++){
        set_suffix_links(st[node].inv_link[i]);
        st[node].endpos=
            st[node].endpos+st[st[node].inv_link[i]].endpos; }
}
void output_all_occurrences(int v,int P_length,vector<int>&pos){
    if (st[v].is_clone)
        pos.push_back(st[v].first_pos - P_length + 1);
    for (int u : st[v].inv_link)
        output_all_occurrences(u, P_length, pos);
}
void kth_smallest(int node,int k,vector<char> &str){
    if(k==0) return;
    map<char, int> ::iterator mit;
    for(mit=st[node].next.begin(); mit!=st[node].next.end();++mit){
        if(st[mit->second].substrings<k)k=k-st[mit->second].substrings;
        else{
            str.push_back(mit->first);
            kth_smallest(mit->second,k-1,str);
            return;
        }
    }
}
int find_occurrence_index(int node,int index,vector<char>&str){
    if(index==str.size()) return node;
    if(!st[node].next.count(str[index])) return -1;
    else return find_occurrence_index(st[node].next[str[index]]
        ,index+1,str);
}
void klen_smallest(int node,int k,vector<char> &str){
    if(k==0) return;
    map<char, int> ::iterator mit;
    for(mit=st[node].next.begin(); mit!=st[node].next.end();
    ++mit){ if(st[mit->second].height>=k-1){
        str.push_back(mit->first);
        klen_smallest(mit->second,k-1,str);
        return;
    }
    }
}
void minimum_non_existing_string(int node,vector<char> &str){
    map<char, int> ::iterator mit;
    set<char>::iterator sit;
    for(mit=st[node].next.begin(),sit=alphabet.begin();
        sit!=alphabet.end(); ++sit,++mit){
        if(mit==st[node].next.end()||mit->first!=(*sit)){
            str.push_back(*sit);
            return;
        }
        else if(st[node].snas==1+st[mit->second].snas){
            str.push_back(*sit);
            minimum_non_existing_string(mit->second,str);
            return;
        }
    }
}
void find_substrings(int node,int index,vector<char> &str,
    vector<pair<long long,long long> > &sub_info){
    sub_info.push_back(make_pair(st[node].substrings,
        st[node].sublen+st[node].substrings*index));
    if(index==str.size()) return;
    if(st[node].next.count(str[index])){ find_substrings(

```

```

        st[node].next[str[index]],index+1,str,sub_info);return;
    }
    else{
        sub_info.push_back(make_pair(0,0));
    }
}
void check(){
    if(!complete){
        process(0);
        set_suffix_links(0);
        int i;
        complete=true;
    }
}
public:
    SuffixAutomaton(set<char> &alpha){
        st.push_back(state(0,-1));
        last=0;
        complete=false;
        set<char>::iterator sit;
        for(sit=alpha.begin(); sit!=alpha.end(); sit++)
            alphabet.insert(*sit);
        st[0].endpos=0;
    }
    void sa_extend(char c){
        int cur = st.size();
        st.push_back(state(st[last].len + 1));
        st[cur].first_pos=st[cur].len-1;
        int p = last;
        while (p != -1 && !st[p].next.count(c)){
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if (p == -1){
            st[cur].link = 0;
        }
        else{
            int q = st[p].next[c];
            if (st[p].len + 1 == st[q].len){
                st[cur].link = q;
            }
            else{
                int clone = st.size();
                st.push_back(state(st[p].len + 1,st[q].link));
                st[clone].next = st[q].next;
                st[clone].is_clone=true;
                st[clone].endpos=0;
                st[clone].first_pos=st[q].first_pos;
                while (p != -1 && st[p].next[c] == q){
                    st[p].next[c] = clone; p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
        complete=false;
    }
    ~SuffixAutomaton(){
        int i;
        for(i=0; i<st.size(); i++){
            st[i].next.clear();
            st[i].inv_link.clear();
        }
        st.clear();
        alphabet.clear();
    }
    void kth_smallest(int k,vector<char> &str){
        check();
        kth_smallest(0,k,str);
    }
    int FindFirstOccurrenceIndex(vector<char> &str){
        check();
        int ind=find_occurrence_index(0,0,str);
        if(ind==0) return -1;
        else if(ind!=-1) return st.size();
        else return st[ind].first_pos+1-(int) str.size();
    }
}

```

```

}
void FindAllOccurrenceIndex(vector<char> &str,vector<int>&pos){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind!=-1) output_all_occurrences(ind,str.size(),pos);
}
int Occurrences(vector<char> &str){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind==0) return 1;
    else if(ind!=-1) return 0;
    else return st[ind].endpos;
}
void klen_smallest(int k,vector<char> &str){
    check();
    if(st[0].height>=k) klen_smallest(0,k,str);
}
void minimum_non_existing_string(vector<char> &str){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind!=-1) minimum_non_existing_string(ind,str);
}
long long cyclic_occurrence(vector<char> &str){
    check();
    int i,j,len;
    long long ans=0;
    int n=str.size();
    set<int> S;
    set<int>::iterator it;
    for(i=0,j=0,len=0; i<n*2-1; i++){
        if(st[j].next.count(str[i%n])){
            len++;
            j=st[j].next[str[i%n]];
        }
        else{
            while(j!=-1&&(!st[j].next.count(str[i%n]))){
                j=st[j].link;
            }
            if(j!=-1){
                len=st[j].len+1;
                j=st[j].next[str[i%n]];
            }
            else{
                len=0;
                j=0;
            }
        }
        while(st[j].link!=-1&&st[st[j].link].len>=n){
            j=st[j].link;
            len=st[j].len;
        }
        if(len>=n) S.insert(j);
    }
    for(it=S.begin();it!=S.end();++it){
        ans=ans+st[*it].endpos;
    }
    return ans;
}
}

```

## 8.7 z-algorithm

```

vector<int> z_function(string s) {
    int n = (int) s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min (r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

```