

## Contents

<b>1 DP</b>	<b>1</b>
1.1 divide-and-conquer-optimization	1
1.2 knuth-optimization	1
1.3 li-chao-tree	1
1.4 zero-matrix	1
<b>2 DS</b>	<b>1</b>
2.1 BIT	1
2.2 Heavy-Light-Decomposition	1
2.3 Lazy Propagation	2
2.4 MO with update	2
2.5 bipartite-disjoint-set-union	2
2.6 centroid decomposition	2
2.7 dsu-rollback	2
2.8 fenwick-tree-2d	3
2.9 link cut tree	3
2.10 sparse table 2d	4
2.11 treap	4
2.12 wavelet tree	5
<b>3 Extra</b>	<b>6</b>
3.1 Header	6
<b>4 Game</b>	<b>6</b>
4.1 HackenBush	6
<b>5 Geo</b>	<b>6</b>
5.1 3dGeo	6
5.2 Circle Cover	7
5.3 Circle Union Area	7
5.4 basic-area-geometry	7
5.5 geo-formulae	7
5.6 half-plane-intersection	8
5.7 heart-of-geometry-2d	8
5.8 intersecting-segments-pair	10
5.9 radiant-geo	11
5.10 trianlge-ear-clipping	13
5.11 vertical-decomposition	13
<b>6 Graph</b>	<b>14</b>
6.1 DMST with solution	14
6.2 DMST	15
6.3 Flow With Demands	15
6.4 LCA	15
6.5 articulation-vertex	15
6.6 bellman-ford	15
6.7 bridge	16
6.8 edmond-blossom	16
6.9 euler-path	16
6.10 hopcraft-karp	17
6.11 hungerian-algorithm	17
<b>7 Math</b>	<b>19</b>
7.1 BerleKampMassey	19
7.2 FloorSum	19
7.3 On Bit Count	19
7.4 Stern Brocot Tree	20
7.5 Sum Of Kth Power	20
7.6 combination-generator	20
7.7 continued-fractions	20
7.8 convolution	20
7.9 crt anchor	20
7.10 discrete-root	20
7.11 fast-walsh-hadamard	21
7.12 fft	21
7.13 formulas	21
7.14 integer-factorization	22
7.15 integration-simpson	22
7.16 linear-diophantine-equation-gray-code	22
7.17 linear-equation-system	22
7.18 math	22
7.19 nCr mod $p^a$	23
7.20 ntt	23
7.21 primality-test	23
7.22 prime counting function	23
<b>8 String</b>	<b>23</b>
8.1 Hashing	23
8.2 aho-corasick	23
8.3 manacher	24
8.4 palindromic tree	24
8.5 suffix array da	24
8.6 suffix-automaton	24
8.7 z-algorithm	25
<b>1 DP</b>	<b>13</b>
<b>1.1 divide-and-conquer-optimization</b>	<b>13</b>

```

int m, n;
vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr){
    if (l > r)
        return;
    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid, optr); k++){
        best = min(best, {k ? dp_before[k - 1] : 0 + C(k,
            mid), k});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}
int solve(){

```

```

for (int i = 0; i < n; i++)
    dp_before[i] = C(0, i);
for (int i = 1; i < m; i++){
    compute(0, n - 1, 0, n - 1);
    dp_before = dp_cur;
}
return dp_before[n - 1];
}

```

**1.2 knuth-optimization**

```

int solve() {
    int N;
    ... // read N and input
    int dp[N][N], opt[N][N];
    auto C = [&](int i, int j) {
        ... // Implement cost function C.
    };
    for (int i = 0; i < N; i++) {
        opt[i][i] = i;
        ... // Initialize dp[i][i] according to the problem
    }
    for (int i = N-2; i >= 0; i--) {
        for (int j = i+1; j < N; j++) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k <= min(j-1,
                opt[i+1][j]); k++) {
                if (mn >= dp[i][k] + dp[k+1][j] + cost) {
                    opt[i][j] = k;
                    mn = dp[i][k] + dp[k+1][j] + cost;
                }
            }
            dp[i][j] = mn;
        }
    }
    cout << dp[0][N-1] << endl;
}

```

**1.3 li-chao-tree**

```

typedef long long ll;
class LiChaoTree{
    ll L,R;
    bool minimize;
    int lines;
    struct Node{
        pair<ll,ll> line;
        Node *children[2];
        Node(pair<ll,ll> ln= {0,1000000000000000000}){
            line=ln;
            children[0]=0;
            children[1]=0;
        }
    } *root;
    ll f(pair<ll,ll> a, ll x){
        return a.first*x+a.second;
    }
    void clear(Node* &node){
        if(node->children[0]){
            clear(node->children[0]);
        }
        if(node->children[1]){
            clear(node->children[1]);
        }
        delete node;
    }
    void add_line(pair<ll,ll> nw, Node* &node, ll l, ll r){
        if(node==0){
            node=new Node(nw);
            return;
        }
        ll m = (l + r) / 2;
        bool lef = (f(nw, l) < f(node->line,
            l))&&minimize||(!minimize)&&f(nw, l) >
            f(node->line, l));
        bool mid = (f(nw, m) < f(node->line,
            m))&&minimize||(!minimize)&&f(nw, m) >
            f(node->line, m));
        if(mid){

```

```

        swap(node->line, nw);
    }
    if(r - 1 == 1){
        return;
    }
    else if(lef != mid){
        add_line(nw, node->children[0], 1, m);
    }
    else{
        add_line(nw, node->children[1], m, r);
    }
}
ll get(ll x, Node* &node, ll l, ll r){
    ll m = (l + r) / 2;
    if(r - l == 1){
        return f(node->line, x);
    }
    else if(x < m){
        if(node->children[0]==0) return f(node->line, x);
        if(minimize) return min(f(node->line, x), get(x,
            node->children[0], l, m));
        else return max(f(node->line, x), get(x,
            node->children[0], l, m));
    }
    else{
        if(node->children[1]==0) return f(node->line, x);
        if(minimize) return min(f(node->line, x), get(x,
            node->children[1], m, r));
        else return max(f(node->line, x), get(x,
            node->children[1], m, r));
    }
}
public:
    LiChaoTree(ll l=-1000000001, ll r=1000000001, bool mn=false){
        L=l;
        R=r;
        root=0;
        minimize=mn;
        lines=0;
    }
    void AddLine(pair<ll,ll> ln){
        add_line({ln.first,ln.second},root,L,R);
        lines++;
    }
    int number_of_lines(){
        return lines;
    }
    ll getOptimumValue(ll x){
        return get(x,root,L,R);
    }
    ~LiChaoTree(){
        if(root!=0) clear(root);
    }
};

```

## 1.4 zero-matrix

```

int zero_matrix(vector<vector<int>>> a) {
    int n = a.size();
    int m = a[0].size();
    int ans = 0;
    vector<int> d(m, -1), d1(m), d2(m);
    stack<int> st;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            if (a[i][j] == 1)
                d[j] = i;
        }
        for (int j = 0; j < m; ++j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d1[j] = st.empty() ? -1 : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = m - 1; j >= 0; --j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d2[j] = st.empty() ? m : st.top();
        }
    }
}

```

```

        st.push(j);
    }
    while (!st.empty())
        st.pop();
    for (int j = 0; j < m; ++j)
        ans = max(ans, (i - d[j]) * (d2[j] - d1[j] - 1));
    }
    return ans;
}

```

## 2 DS

### 2.1 BIT

```

#include<bits/stdc++.h>
using namespace std;
const int MaxIdx=1e+5;
int tree[MaxIdx+1];
int read(int idx) {int sum = 0;
    while (idx > 0) {sum += tree[idx];idx -= (idx & -idx);}
    return sum;}
void update(int idx, int val) {while (idx <= MaxIdx) {
    tree[idx] += val;idx += (idx & -idx);}}
int readSingle(int idx) {
    int sum = tree[idx];
    if (idx > 0) { int z = idx - (idx & -idx);idx--;
        while (idx != z) { sum -= tree[idx];idx -= (idx & -idx);}}
    return sum;}
int findG(int cumFre) {int idx = 0;int bitMask=(1<<16);
    while (bitMask != 0) {int tIdx = idx + bitMask;bitMask >>=
        1;
        if (tIdx > MaxIdx)continue;
        if (cumFre >= tree[tIdx]) {idx = tIdx;cumFre -=
            tree[tIdx];}}
    if (cumFre != 0)return -1;else return idx;}

```

### 2.2 Heavy-Light-Decomposition

```

namespace HLD{
    struct Node{
        int mn=INT_MAX,unp=INT_MAX;
        Node(){
            Node(int mn,int unp): mn(mn),unp(unp){}};
        inline Node combine(Node a,Node b){
            return {min(a.mn,b.mn),INT_MAX};}
        inline Node propagate(Node to,Node from,int len){
            if(from.unp==INT_MAX)
                return to;
            to.mn=min(to.mn,from.unp);
            to.unp=min(to.unp,from.unp);
            return to;}
#define MAX_SIZE 100001
        vector<vector<int>> >G;
        vector<int> parent,depth, heavy,head,pos;
        vector<int> node_val;
        vector<int> node_val_order;
        SegTree<Node> sgt( MAX_SIZE,combine,propagate);
        int cur_pos;
        int dfs(int node){
            int sz=1;
            int max_c_size=0;
            for(auto c:G[node]){
                if(c!=parent[node]){
                    parent[c]=node;
                    depth[c]=depth[node]+1;
                    int c_size=dfs(c);
                    sz+=c_size;
                    if(c_size>max_c_size){
                        max_c_size=c_size;
                        heavy[node]=c; }}}
            return sz;}
        void decompose(int node,int h){
            pos[node]=cur_pos++;
            head[node]=h;
            if(heavy[node]!=-1)
                decompose(heavy[node],h);
            for(int c:G[node]){
                if(c!=parent[node]&& c!=heavy[node])
                    decompose(c,c);}
            return;}
}

```

```

//for query on path the node_val of a node is the cost of the
//edge to parent
//exclude=true for query on path,it excludes the value stored
//in lca(a,b)
int query(int a,int b,int n,bool exclude=false){//n number
//of node in the tree
    Node res;//not really generalized,for min max update
    accordingly
    while(head[a]!=head[b]){
        if(depth[head[a]]> depth[head[b]])swap(a,b);
        res=combine(res, sgt.query(pos[head[b]],pos[b]));
        b=parent[head[b]];}
    if(depth[a]>depth[b])swap(a,b);
    res=combine(res,exclude? sgt.query(pos[a]+1,pos[b])
        :sgt.query(pos[a],pos[b]));
    return res.mn;}
// void update(int node,int val)
// {
//     sgt.update(pos[node],val);
// }
void update(int a,int b,int val){
    while(head[a]!=head[b]){
        if(depth[head[a]]> depth[head[b]])
            swap(a,b);
        //res=combine(res,sgt.query(pos[head[b]],pos[b]));
        sgt.update(pos[head[b]], pos[b],{0,val});
        b=parent[head[b]];
    }
    if(depth[a]>depth[b])swap(a,b);
    //res=combine(res,exclude? sgt.query(pos[a]+1, pos[b]):
    //sgt.query(pos[a],pos[b]));
    sgt.update(pos[a],pos[b],{0,val});
    //return res.mn;}
void init(int n){
    parent.resize(n);
    depth.resize(n);
    heavy.assign(n,-1);
    head.resize(n);
    pos.resize(n);
    parent[0]=0;//might change later
    dfs(0);
    cur_pos=0;
    decompose(0,0);
    node_val_order.resize(n);
    for(int i=0;i<n;++i){
        node_val_order[pos[i]]= node_val[i];}
    sgt.n=n;
    sgt.build(node_val_order, 1,0,n-1); }
}

```

### 2.3 Lazy Propagation

```

struct node{
    int sum, prop;
    node(){sum=0, prop=0;}
} st[500000];
void propagate(int p, int c, int len){
    st[c].prop+=st[p].prop;
    st[c].sum+=len*st[p].prop;}
void combine(int v, int l, int r){
    st[v].prop=0;
    st[v].sum=st[l].sum+st[r].sum;}
void update(int v, int vl, int vr, int l, int r, int u){
    if(l>r) return;
    if(vl==l and vr==r){
        st[v].sum+=(vr-vl+1)*u;
        st[v].prop+=u;
        return;}
    int mid=(vl+vr)/2;
    propagate(v, 2*v, mid-vl+1);
    propagate(v, 2*v+1, vr-mid);
    if(r<=mid) update(2*v, vl, mid, l, r, u);
    else if(l>mid) update(2*v+1, mid+1, vr, l, r, u);
    else {
        update(2*v, vl, mid, l, mid, u);
        update(2*v+1, mid+1, vr, mid+1, r, u);}
    combine(v, 2*v, 2*v+1);}
int query(int v, int vl, int vr, int l, int r){
}

```

```

if(l>r) return 0;
if(vl==l and vr==r) return st[v].sum;
int mid=(vl+vr)/2;
propagate(v, 2*v, mid-vl+1);
propagate(v, 2*v+1, vr-mid);
int qres;
if(r<=mid) qres=query(2*v, vl, mid, l, r);
else if(l>mid) qres=query(2*v+1, mid+1, vr, l, r);
else qres=query(2*v, vl, mid, l, mid)+query(2*v+1, mid+1,
vr, mid+1, r);
combine(v, 2*v, 2*v+1);
return qres;
}

```

## 2.4 MO with update

```

const int N = 1e5 + 5;
const int P = 2000; // block size = (2*n^2)^(1/3)
struct query{
    int t, l, r, k, i;
};
vector<query> q;
vector<array<int, 3>> upd;
vector<int> ans, a;
void add(int x); void rem(int x); int get_answer();
void mos_algorithm(){
    sort(q.begin(), q.end(), [](const query &a, const query &b){
        if (a.t / P != b.t / P) return a.t < b.t;
        if (a.l / P != b.l / P) return a.l < b.l;
        if ((a.l / P) & 1) return a.r < b.r;
        return a.r > b.r;
    });
    for(int i=upd.size()-1; i>=0; --i) a[upd[i][0]] = upd[i][1];
    int L = 0, R = -1, T = 0;
    auto apply = [&](int i, int fl){
        int p = upd[i][0], x = upd[i][fl + 1];
        if (L <= p && p <= R){ rem(a[p]); add(x); }
        a[p] = x;
    };
    ans.clear(); ans.resize(q.size());
    for (auto qr : q){
        int t = qr.t, l = qr.l, r = qr.r, k = qr.k;
        while (T < t) apply(T++, 1);
        while (T > t) apply(--T, 0);
        while (R < r) add(a[+R]);
        while (L > l) add(a[--L]);
        while (R > r) rem(a[R--]);
        while (L < l) rem(a[L++]);
        ans[qr.i] = get_answer();
    }
}
void TEST_CASES(int cas){
    cin>>n>>m; a.resize(n); for(int i=0; i<n; i++) cin>>a[i];
    for(int i=0; i<m; i++){ int tp; scanf("%d", &tp);
        if (tp == 1){ int l, r, k; cin>>l>>r>>k;
            q.push_back({upd.size(), l - 1, r - 1, k, q.size()}); }
        else{ int p; cin>>p>>x; --p;
            upd.push_back({p, a[p], x}); a[p] = x;
        }
    }
    mos_algorithm();
}

```

## 2.5 bipartite-disjoint-set-union

```

void make_set(int v) {
    parent[v] = make_pair(v, 0); rank[v] = 0; bipartite[v] = true; }
pair<int, int> find_set(int v) { if (v != parent[v].first) {
    int parity = parent[v].second; parent[v] = find_set(
    parent[v].first); parent[v].second ^= parity; }
    return parent[v]; }
void add_edge(int a, int b) {
    pair<int, int> pa = find_set(a);
    a = pa.first; int x = pa.second;
    pair<int, int> pb = find_set(b); b = pb.first;
    int y = pb.second;
    if (a == b) {
        if (x == y) bipartite[a] = false;
    } else {
        if (rank[a] < rank[b]) swap (a, b);

```

```

        parent[b] = make_pair(a, x^y^1);
        bipartite[a] ^= bipartite[b];
        if (rank[a] == rank[b]) ++rank[a];
    }
}
bool is_bipartite(int v){ return bipartite[find_set(v).first]; }

```

## 2.6 centroid decomposition

```

set<int> g[N];
int par[N], sub[N], level[N], ans[N]; int DP[LOGN][N];
int n, m; int nn;
void dfs1(int u, int p){
    sub[u]=1; nn++;
    for(auto it=g[u].begin(); it!=g[u].end(); it++) if(*it!=p){
        dfs1(*it, u); sub[u]+=sub[*it]; }
}
int dfs2(int u, int p){
    for(auto it=g[u].begin(); it!=g[u].end(); it++)
        if(*it!=p && sub[*it]>nn/2)
            return dfs2(*it, u);
    return u;
}
void decompose(int root, int p){
    nn=0; dfs1(root, root); int centroid = dfs2(root, root);
    if(p==-1)p=centroid; par[centroid]=p;
    for(auto it=g[centroid].begin(); it!=g[centroid].end(); it++){
        g[*it].erase(centroid); decompose(*it, centroid); }
    g[centroid].clear();
}

```

## 2.7 dsu-rollback

```

struct dsu_save {
    int v, rnk, u, rnk; dsu_save() {}
    dsu_save(int _v, int _rnk, int _u, int _rnk)
        : v(_v), rnk(_rnk), u(_u), rnk(_rnk) {}
};
struct dsu_with_rollback {
    vector<int> p, rnk; int comps; stack<dsu_save> op;
    dsu_with_rollback() {}
    dsu_with_rollback(int n) { p.resize(n); rnk.resize(n);
        for (int i = 0; i < n; i++) { p[i] = i; rnk[i] = 0; }
        comps = n;
    }
    int find_set(int v){return (v == p[v])?v:find_set(p[v]);}
    bool unite(int v, int u) { v = find_set(v); u = find_set(u);
        if (v == u) return false; comps--;
        if (rnk[v] > rnk[u]) swap(v, u);
        op.push(dsu_save(v, rnk[v], u, rnk[u])); p[v] = u;
        if (rnk[u] == rnk[v]) rnk[u]++; return true;
    }
    void rollback() { if (op.empty()) return;
        dsu_save x = op.top(); op.pop(); comps++; p[x.v] = x.v;
        rnk[x.v] = x.rnk; p[x.u] = x.u; rnk[x.u] = x.rnk;
    }
};
struct query {
    int v, u; bool united;
    query(int _v, int _u) : v(_v), u(_u) {}
};
struct QueryTree {
    vector<vector<query>> t; dsu_with_rollback dsu; int T;
    QueryTree() {}
    QueryTree(int _T, int n) : T(_T) {
        dsu = dsu_with_rollback(n); t.resize(4 * T + 4); }
    void add_to_tree(int v, int l, int r, int ul, int ur, query& q){
        if (ul > ur) return;
        if (l == ul && r == ur) { t[v].push_back(q); return; }
        int mid = (l + r) / 2;
        add_to_tree(2 * v, l, mid, ul, min(ur, mid), q);
        add_to_tree(2*v+1, mid+1, r, max(ul, mid + 1), ur, q);
    }
    void add_query(query q, int l, int r) {
        add_to_tree(1, 0, T - 1, l, r, q); }
    void dfs(int v, int l, int r, vector<int>& ans) {
        for (query& q : t[v]) q.united = dsu.unite(q.v, q.u);
        if (l == r) ans[l] = dsu.comps;
    }
};

```

```

        else { int mid = (l + r) / 2;
        dfs(2 * v, l, mid, ans); dfs(2 * v + 1, mid + 1, r, ans); }
        for (query q : t[v]) { if (q.united) dsu.rollback(); }
    }
    vector<int> solve() {
        vector<int> ans(T); dfs(1, 0, T - 1, ans); return ans;
    }
};

```

## 2.8 fenwick-tree-2d

```

struct FenwickTree2D {
    vector<vector<int>> bit;
    int n, m;
    // init(...) { ... }
    int sum(int x, int y) {
        int ret = 0;
        for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
            for (int j = y; j >= 0; j = (j & (j + 1)) - 1)
                ret += bit[i][j];
        return ret;
    }
    void add(int x, int y, int delta) {
        for (int i = x; i < n; i = i | (i + 1))
            for (int j = y; j < m; j = j | (j + 1))
                bit[i][j] += delta;
    }
};

```

## 2.9 link cut tree

```

const int MOD = 998244353;
int sum(int a, int b) {
    return a+b >= MOD ? a+b-MOD : a+b;
}
int mul(int a, int b) {
    return (a*1LL*b)%MOD;
}
typedef pair< int , int > Linear;
Linear compose(const Linear &p, const Linear &q) {
    return Linear(mul(p.first, q.first), sum(mul(q.second,
    p.first), p.second));
}
struct SplayTree {
    struct Node {
        int ch[2] = {0, 0}, p = 0;
        long long self = 0, path = 0; // Path aggregates
        long long sub = 0, vir = 0; // Subtree aggregates
        bool flip = 0; // Lazy tags
        int size = 1;
        Linear _self{1, 0}, _path_shoja{1, 0}, _path_ultra{1, 0};
    };
    vector<Node> T;
    SplayTree(int n) : T(n + 1) {
        T[0].size = 0;
    }
    void push(int x) {
        if (!x || !T[x].flip) return;
        int l = T[x].ch[0], r = T[x].ch[1];
        T[l].flip ^= 1, T[r].flip ^= 1;
        swap(T[x].ch[0], T[x].ch[1]);
        T[x].flip = 0;
        swap(T[x]._path_shoja, T[x]._path_ultra);
    }
    void pull(int x) {
        int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);
        T[x].size = T[l].size + T[r].size + 1;
        T[x].path = T[l].path + T[x].self + T[r].path;
        T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
        T[x]._path_shoja = compose(T[r]._path_shoja,
        compose(T[x]._self, T[l]._path_shoja));
        T[x]._path_ultra = compose(T[l]._path_ultra,
        compose(T[x]._self, T[r]._path_ultra));
    }
    void set(int x, int d, int y) {
        T[x].ch[d] = y; T[y].p = x; pull(x);
    }
};

```

```

void splay(int x) {
    auto dir = [&](int x) {
        int p = T[x].p; if (!p) return -1;
        return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;
    };
    auto rotate = [&](int x) {
        int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);
        set(y, dx, T[x].ch[!dx]);
        set(x, !dx, y);
        if (~dy) set(z, dy, x);
        T[x].p = z;
    };
    for (push(x); ~dir(x); ) {
        int y = T[x].p, z = T[y].p;
        push(z); push(y); push(x);
        int dx = dir(x), dy = dir(y);
        if (~dy) rotate(dx != dy ? x : y);
        rotate(x);
    }
}

int KthNext(int x, int k) {
    assert(k > 0);
    splay(x);
    x = T[x].ch[1];
    if (T[x].size < k) return -1;
    while (true) {
        push(x);
        int l = T[x].ch[0], r = T[x].ch[1];
        if (T[l].size+1 == k) return x;
        if (k <= T[l].size) x = l;
        else k -= T[l].size+1, x = r;
    }
}

struct LinkCut : SplayTree {
    LinkCut(int n) : SplayTree(n) {}
    int access(int x) {
        int u = x, v = 0;
        for (; u; v = u, u = T[u].p) {
            splay(u);
            int& ov = T[u].ch[1];
            T[u].vir += T[ov].sub;
            T[u].vir -= T[v].sub;
            ov = v; pull(u);
        }
        splay(x);
        return v;
    }
    void reroot(int x) {
        access(x); T[x].flip ^= 1; push(x);
    }
    ///makes v parent of u (optional: u must be a root)
    void Link(int u, int v) {
        reroot(u); access(v);
        T[v].vir += T[u].sub;
        T[u].p = v; pull(v);
    }
    ///removes edge between u and v
    void Cut(int u, int v) {
        int _u = FindRoot(u);
        reroot(u); access(v);
        T[v].ch[0] = T[u].p = 0; pull(v);
        reroot(_u);
    }
    // Rooted tree LCA. Returns 0 if u and v arent connected.
    int LCA(int u, int v) {
        if (u == v) return u;
        access(u); int ret = access(v);
        return T[u].p ? ret : 0;
    }
    /// Query subtree of u where v is outside the subtree.
    long long Subtree(int u, int v) {
        int _v = FindRoot(v);
        reroot(v); access(u);
        long long ans = T[u].vir + T[u].self;
        reroot(_v);
    }
}

```

```

        return ans;
    }
    /// Query path [u..v]
    long long Path(int u, int v) {
        int _u = FindRoot(u);
        reroot(u); access(v);
        long long ans = T[v].path;
        reroot(_u);
        return ans;
    }
    Linear_Path(int u, int v) {
        reroot(u); access(v); return T[v]._path_shoja;
    }
    /// Update vertex u with value v
    void Update(int u, long long v) {
        access(u); T[u].self = v; pull(u);
    }
    /// Update vertex u with value v
    void _Update(int u, Linear v) {
        access(u); T[u]._self = v; pull(u);
    }
    int FindRoot(int u) {
        access(u);
        while (T[u].ch[0]) {
            u = T[u].ch[0];
            push(u);
        }
        access(u);
        return u;
    }
    ///k-th node (0-indexed) on the path from u to v
    int KthOnPath(int u, int v, int k) {
        if (u == v) return k == 0 ? u : -1;
        int _u = FindRoot(u);
        reroot(u); access(v);
        int ans = KthNext(u, k);
        reroot(_u);
        return ans;
    }
}

int main() {
    cin >> n >> q;
    LinkCut lct(n);
    for (int i = 1; i <= n; i++) {
        Linear l;
        cin >> l.first >> l.second;
        lct._Update(i, l);
    }
    for (int i = 1; i < n; i++) {
        int u, v;
        cin >> u >> v;
        lct.Link(u+1, v+1);
    }
    while (q--) {
        int op;
        cin >> op;
        if (op == 0) {
            int u, v, w, x;
            cin >> u >> v >> w >> x;
            lct.Cut(u+1, v+1);
            lct.Link(w+1, x+1);
        } else if (op == 1) {
            int p; Linear l;
            cin >> p >> l.first >> l.second;
            lct._Update(p+1, l);
        } else {
            int u, v, x;
            cin >> u >> v >> x;
            Linear l = lct._Path(u+1, v+1);
            cout << sum(mul(l.first, x), l.second) << "\n";
        }
    }
    return 0;
}

```

## 2.10 sparse table 2d

```

const int N=500; const int K = 8 ; /// k >= ceil(lg22(n)) +1
int arr[N][N]; int st[K+1][K+1][N][N]; int lg2[N+1];
void ini(){ lg2[1] = 0;

```

```

    for (int i = 2; i <= N; i++) lg2[i] = lg2[i/2] + 1; }
int f(int i,int j){ return max(i,j); }
void pre( int n,int m){
    for(int x=0;x<=K;x++){
        for(int y=0;y<=K;y++){
            for(int i=0;i<n;i++){
                for(int j=0;j<m;j++){
                    if(i+(1<<x)>n or j+(1<<y) > m) continue;
                    if(x>0) st[x][y][i][j] = f(st[x-1][y][i][j] ,
                                                st[x-1][y][i+(1<<(x-1))][j]);
                    else if(y>0) st[x][y][i][j] = f(st[x][y-1][i][j] ,
                                                st[x][y-1][i+(1<<(y-1))][j]);
                    else st[x][y][i][j] = f(arr[i][j] , arr[i][j]);
                } } } }
    int getf( int R1 ,int C1 , int R2 , int C2){
        if(R1>R2) swap(R1,R2); if(C1>C2) swap(C1,C2);
        int x = lg2[R2 - R1 + 1]; int y = lg2[C2 - C1 + 1];
        return f(f(f(st[x][y][R1][C1],st[x][y][R2-(1<<x)+1][C1]),
                    st[x][y][R1][C2-(1<<y)+1]),st[x][y][R2-(1<<x)+1][C2-(1<<y)+1]);
    }
}

```

## 2.11 treap

```

template <class T>
class treap{
    struct item{
        int prior, cnt;
        T key;
        item *l,*r;
        item(T v) {
            key=v;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root,*node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){
        if (it) it->cnt = cnt(it->l) + cnt(it->r) + 1;
    }
    void split (item * t, T key, item * & l, item * & r){
        if (!t)
            l = r = NULL;
        else if (key < t->key)
            split (t->l, key, l, t->l), r = t;
        else
            split (t->r, key, t->r, r), l = t;
        upd_cnt(t);
    }
    void insert (item * & t, item * it){
        if (!t)
            t = it;
        else if (it->prior > t->prior)
            split (t, it->key, it->l, it->r), t = it;
        else
            insert (it->key < t->key ? t->l : t->r, it);
        upd_cnt(t);
    }
    /// keys(l) < keys(r)
    void merge (item * & t, item * l, item * r){
        if (!l || !r)
            t = l ? l : r;
        else if (l->prior > r->prior)
            merge (l->r, l->r, r), t = l;
        else
            merge (r->l, l, r->l), t = r;
        upd_cnt(t);
    }
    void erase (item * & t, T key){
        if (t->key == key)
            merge (t, t->l, t->r);
        else
            erase (key < t->key ? t->l : t->r, key);
        upd_cnt(t);
    }
}

```

```

T elementAt(item * &t,int key){
    T ans;
    if(cnt(t->l)==key) ans=t->key;
    else if(cnt(t->l)>key) ans=elementAt(t->l,key);
    else ans=elementAt(t->r,key-1-cnt(t->l));
    upd_cnt(t);
    return ans;
}

item * unite (item * l, item * r){
    if (!l || !r) return l ? l : r;
    if (l->prior < r->prior) swap (l, r);
    item * lt, * rt;
    split (r, l->key, lt, rt);
    l->l = unite (l->l, lt);
    l->r = unite (l->r, rt);
    upd_cnt(l);
    upd_cnt(r);
    return l;
}

void heapify (item * t){
    if (!t) return;
    item * max = t;
    if (t->l != NULL && t->l->prior > max->prior)
        max = t->l;
    if (t->r != NULL && t->r->prior > max->prior)
        max = t->r;
    if (max != t)
    {
        swap (t->prior, max->prior);
        heapify (max);
    }
}

item * build (T * a, int n){
    if (n == 0) return NULL;
    int mid = n / 2;
    item * t = new item (a[mid], rand ());
    t->l = build (a, mid);
    t->r = build (a + mid + 1, n - mid - 1);
    heapify (t);
    return t;
}

void output (item * t,vector<T> &arr){
    if (!t) return;
    output (t->l,arr);
    arr.push_back(t->key);
    output (t->r,arr);
}

public:
    treap(){
        root=NULL;
    }
    treap(T *a,int n){
        build(a,n);
    }
    void insert(T value){
        node=new item(value);
        insert(root,node);
    }
    void erase(T value){
        erase(root,value);
    }
    T elementAt(int position){
        return elementAt(root,position);
    }
    int size(){
        return cnt(root);
    }
    void output(vector<T> &arr){
        output(root,arr);
    }
    int range_query(T l,T r){ // [l,r]
        item *previous,*next,*current;
        split(root,l,previous,current);
        split(current,r,current,next);
        int ans=cnt(current);
        merge(root,previous,current);
        merge(root,root,next);
        previous=NULL;
        current=NULL;

```

```

        next=NULL;
        return ans;
    }
};

template <class T>
class implicit_treap{
    struct item{
        int prior, cnt;
        T value;
        bool rev;
        item *l,*r;
        item(T v){
            value=v;
            rev=false;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root,*node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){
        if (it)
            it->cnt = cnt(it->l) + cnt(it->r) + 1;
    }
    void push (item * it){
        if (it && it->rev){
            it->rev = false;
            swap (it->l, it->r);
            if (it->l) it->l->rev ^= true;
            if (it->r) it->r->rev ^= true;
        }
    }
    void merge (item * &t, item * l, item * r){
        push (l);
        push (r);
        if (!l || !r)
            t = l ? l : r;
        else if (l->prior > r->prior)
            merge (l->r, l->r, r), t = l;
        else
            merge (r->l, l, r->l), t = r;
        upd_cnt (t);
    }
    void split (item * t, item * &l, item * &r, int key, int
        add = 0){
        if (!t)
            return void( l = r = 0 );
        push (t);
        int cur_key = add + cnt(t->l);
        if (key <= cur_key)
            split (t->l, l, t->l, key, add), r = t;
        else
            split (t->r, t->r, r, key, add + 1 + cnt(t->l)), l
                = t;
        upd_cnt (t);
    }
    void insert(item * &t,item * element,int key){
        item *l,*r;
        split(t,l,r,key);
        merge(l,l,element);
        merge(t,l,r);
        l=NULL;
        r=NULL;
    }
    T elementAt(item * &t,int key){
        push(t);
        T ans;
        if(cnt(t->l)==key) ans=t->value;
        else if(cnt(t->l)>key) ans=elementAt(t->l,key);
        else ans=elementAt(t->r,key-1-cnt(t->l));
        return ans;
    }
    void erase (item * &t, int key){
        push(t);
        if(!t) return;
        if (key == cnt(t->l))
            merge (t, t->l, t->r);

```

```

        else if(key<cnt(t->l))
            erase(t->l,key);
        else
            erase(t->r,key-cnt(t->l)-1);
        upd_cnt(t);
    }
    void reverse (item * &t, int l, int r){
        item *t1, *t2, *t3;
        split (t, t1, t2, l);
        split (t2, t2, t3, r-l+1);
        t2->rev ^= true;
        merge (t, t1, t2);
        merge (t, t, t3);
    }
    void cyclic_shift(item * &t,int L,int R){
        if(L==R) return;
        item *l,*r,*m;
        split(t,t,l,L);
        split(l,l,m,R-L+1);
        split(l,l,r,R-L);
        merge(t,t,r);
        merge(t,t,l);
        merge(t,t,m);
        l=NULL;
        r=NULL;
        m=NULL;
    }
    void output (item * t,vector<T> &arr){
        if (!t) return;
        push (t);
        output (t->l,arr);
        arr.push_back(t->value);
        output (t->r,arr);
    }
    public:
        implicit_treap(){
            root=NULL;
        }
        void insert(T value,int position){
            node=new item(value);
            insert(root,node,position);
        }
        void erase(int position){
            erase(root,position);
        }
        void reverse(int l,int r){
            reverse(root,l,r);
        }
        T elementAt(int position){
            return elementAt(root,position);
        }
        void cyclic_shift(int L,int R){
            cyclic_shift(root,L,R);
        }
        int size(){
            return cnt(root);
        }
        void output(vector<T> &arr){
            output(root,arr);
        }
    };

```

## 2.12 wavelet tree

```

#include <bits/stdc++.h>
using namespace std;
#define fo(i,n) for(i=0;i<n;i++)
#define ll long long
#define pb push_back
#define mp make_pair
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef vector<int> vi;
const int N = 3e5, M = N;
//=====
const int MAX = 1e6;
int a[N];
struct wavelet_tree{
#define vi vector<int>

```

```

#define pb push_back
int lo, hi;
wavelet_tree *l=0, *r=0;
vi b;
vi c; // c holds the prefix sum of elements
//nos are in range [x,y]
//array indices are [from, to)
wavelet_tree(int *from, int *to, int x, int y){
    lo = x, hi = y;
    if( from >= to)
        return;
    if( hi == lo ){
        b.reserve(to-from+1);
        b.pb(0);
        c.reserve(to-from+1);
        c.pb(0);
        for(auto it = from; it != to; it++){
            b.pb(b.back() + 1);
            c.pb(c.back() + *it);
        }
        return ;
    }
    int mid = (lo+hi)/2;
    auto f = [mid](int x){
        return x <= mid;
    };
    b.reserve(to-from+1);
    b.pb(0);
    c.reserve(to-from+1);
    c.pb(0);
    for(auto it = from; it != to; it++){
        b.pb(b.back() + f(*it));
        c.pb(c.back() + *it);
    }
    //see how lambda function is used here
    auto pivot = stable_partition(from, to, f);
    l = new wavelet_tree(from, pivot, lo, mid);
    r = new wavelet_tree(pivot, to, mid+1, hi);
}
// swap a[i] with a[i+1] , if a[i]!=a[i+1] call
//swapadjacent(i)
void swapadjacent(int i){
    if(lo == hi)
        return ;
    b[i] = b[i-1] + b[i+1] - b[i];
    c[i] = c[i-1] + c[i+1] - c[i];
    if( b[i+1]-b[i] == b[i] - b[i-1]){
        if(b[i]-b[i-1])
            return this->l->swapadjacent(b[i]);
        else
            return this->r->swapadjacent(i-b[i]);
    }
    else
        return ;
}
//kth smallest element in [l, r]
int kth(int l, int r, int k){
    if(l > r)
        return 0;
    if(lo == hi)
        return lo;
    int inLeft = b[r] - b[l-1];
    int lb = b[l-1]; //amt of nos in first (l-1) nos that
    go in left
    int rb = b[r]; //amt of nos in first (r) nos that go in
    left
    if(k <= inLeft)
        return this->l->kth(lb+1, rb, k);
    return this->r->kth(l-lb, r-rb, k-inLeft);
}
//count of nos in [l, r] Less than or equal to k
int LTE(int l, int r, int k){
    if(l > r or k < lo)
        return 0;
    if(hi <= k)
        return r - l + 1;
    int lb = b[l-1], rb = b[r];
    return this->l->LTE(lb+1, rb, k) + this->r->LTE(l-lb,
        r-rb, k);
}

```

```

//count of nos in [l, r] equal to k
int count(int l, int r, int k){
    if(l > r or k < lo or k > hi)
        return 0;
    if(lo == hi)
        return r - l + 1;
    int lb = b[l-1], rb = b[r], mid = (lo+hi)/2;
    if(k <= mid)
        return this->l->count(lb+1, rb, k);
    return this->r->count(l-lb, r-rb, k);
}
//sum of nos in [l, r] less than or equal to k
int sumk(int l, int r, int k){
    if(l > r or k < lo)
        return 0;
    if(hi <= k)
        return c[r] - c[l-1];
    int lb = b[l-1], rb = b[r];
    return this->l->sumk(lb+1, rb, k) + this->r->sumk(l-lb,
        r-rb, k);
}
wavelet_tree(){
    if(l)
        delete l;
    if(r)
        delete r;
}
int main(){
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    srand(time(NULL));
    int i,n,k,j,q,l,r;
    cin >> n;
    fo(i, n) cin >> a[i+1];
    wavelet_tree T(a+1, a+n+1, 1, MAX);
    cin >> q;
    while(q--){
        int x;
        cin >> x;
        cin >> l >> r >> k;
        if(x == 0){
            //kth smallest
            cout << "Kth smallest: ";
            cout << T.kth(l, r, k) << endl;
        }
        if(x == 1){
            //less than or equal to K
            cout << "LTE: ";
            cout << T.LTE(l, r, k) << endl;
        }
        if(x == 2){
            //count occurrence of K in [l, r]
            cout << "Occurrence of K: ";
            cout << T.count(l, r, k) << endl;
        }
        if(x == 3){
            //sum of elements less than or equal to K in [l, r]
            cout << "Sum: ";
            cout << T.sumk(l, r, k) << endl;
        }
    }
    return 0;
}

```

### 3 Extra

#### 3.1 Header

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace std;
typedef long long ll;
typedef pair <int, int> pii;
typedef pair <ll, ll> pll;
typedef double ftype;
typedef pair<ftype,ftype> pff;
#define all(a) a.begin(), a.end()
#define some(a,l,r) a.begin()+l,a.begin()+(r+1)

```

```

#define csort(a) sort(all(a))
#define pb push_back
#define pf push_front
#define pob pop_back
#define pof pop_front
#define fi first
#define se second
#define fastio ios_base::sync_with_stdio(false);cin.tie(NULL)
#ifdef COMEDIANS
#define infile ;
#define outfile ;
#define Gene template< class
#define Rics printer& operator,
Gene c> struct rge{c b, e};
Gene c> rge<c> range(c i, c j){ return {i, j};}
struct printer{
    ~printer(){cerr<<endl;}
    Gene c >Rics(c x){ cerr<<boolalpha<<x; return *this;}
    Rics(string x){cerr<<x;return *this;}
    Gene c, class d >Rics(pair<c, d> x){ return
        *this,("x.first:", "x.second,");}
    Gene ... d, Gene ...> class c >Rics(c<d...> x){ return
        *this, range(begin(x), end(x));}
    Gene c >Rics(rge<c> x){
        *this,["; for(auto it = x.b; it != x.e; ++it)
            *this,(it==x.b?"":", ")*it; return *this,"];}
};
#define stop getchar()
#define debug() cerr<<"LINE "<<__LINE__<<" >> ", printer()
#define dbg(x) debug(), "["#x,": ",(x),"] "
#define test_handle(T) cin>>T
#else
#define dbg(x) ;
#define infile ;
#define outfile ;
#define test_handle(T) T = 1
#endif
//Use -DCOMEDIANS in compiler flag in others tab, or remove
#ifdef
mt19937
rng(chrono::steady_clock::now().time_since_epoch().count());
const ftype EPS = 1e-10;
const ftype PI = acos(-1);
const int MAX = 3e5+5;
const int BMAX = 18;
const int MOD = 1e9+7;
using namespace __gnu_pbds;
/*
find_by_order(k) --> returns iterator to the kth largest
element counting from 0
order_of_key(val) --> returns the number of items in a set that
are strictly smaller than our item
*/
typedef tree<
int,
null_type,
less<int>,
rb_tree_tag,
tree_order_statistics_node_update>
ordered_set;
//#pragma GCC optimize("O3,unroll-loops")
//#pragma GCC target("avx2,bmi,bmi2,lzcnt")
//mt19937
rng(chrono::system_clock::now().time_since_epoch().count());
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15; //Random
        x=(x^(x>>30))*0xbf58476d1ce4e5b9; //Random
        x=(x^(x>>27))*0x94d049bb133111eb; //Random
        return x^(x>>31);
    }
    const uint64_t FIXED_RANDOM = chrono::
        steady_clock::now().time_since_epoch().count();
    size_t operator()(uint64_t x) const {
        return splitmix64(x + FIXED_RANDOM);
    }
    size_t operator()(pair<int, int> x) const {
        return splitmix64((uint64_t(x.first)<<32) +

```

```

        x.second + FIXED_RANDOM);
    }
};
gp_hash_table<pair<int,int>,int,custom_hash> ht;
namespace my_gcc_ints {
#pragma GCC diagnostic push
#pragma GCC diagnostic ignored "-Wpedantic"
    using int128 = __int128;
#pragma GCC diagnostic pop
}
# stresstester GENERATOR SOL1 SOL2 ITERATIONS
for i in $(seq 1 "$4"); do
    echo -en "\rAttempt $i/$4"
    $1 > in.txt
    $2 < in.txt > out1.txt
    $3 < in.txt > out2.txt
    diff -y out1.txt out2.txt > diff.txt
    if [ $? -ne 0 ]; then
        echo -e "\nTestcase Found: "; cat in.txt
        echo -e "\nOutputs: "; cat diff.txt
        exit
    fi
done

```

## 4 Game

### 4.1 HackenBush

```

/* tree case: g[u] = for all v : XOR[ g[v] + 1]
lose if no moves available
1. Colon Principle: Grundy number of a tree is the
xor of Grundy number of child subtrees.
2. Fusion Principle: Consider a pair of adjacent
vertices u, v that has another path (i.e.,
they are in a cycle). Then, we can contract u
and v without changing Grundy number.
We first decompose graph into two-edge connected
components. Then, by contracting each components by
using Fusion Principle, we obtain a tree (and many
self loops) that has the same Grundy number to the
original graph. By using Colon Principle, we can
compute the Grundy number. O(m + n). */
struct hackenbush {
    int n; vector<vector<int>> adj;
    hackenbush(int n) : n(n), adj(n) {}
    void add_edge(int u, int v) {
        adj[u].push_back(v);
        if(u!=v) adj[v].push_back(u);
    }
    int Grundy(int r) {
        vector<int> num(n), low(n); int t = 0;
        function<int(int, int)> dfs=[&](int p,int u) {
            num[u] = low[u] = ++t; int ans = 0;
            for (int v : adj[u]) {
                if (v == p) { p += 2 * n; continue; }
                if (num[v] == 0) {
                    int res = dfs(u, v);
                    low[u] = min(low[u], low[v]);
                    if (low[v] > num[u]) ans ^= (1+res)^1;
                    else ans ^= res; // non bridge
                }
                else low[u] = min(low[u], num[v]);
            }
            if (p > n) p -= 2 * n;
            for (int v : adj[u])
                if (v != p && num[u] <= num[v]) ans ^= 1;
            return ans;
        };
        return dfs(-1, r);
    }
};

```

## 5 Geo

### 5.1 3dGeo

```

int dcmp(double x) { return abs(x) < EPS ? 0 : (x<0 ? -1 : 1); }
double degreeToRadian(double rad) { return rad*PI/180; }
struct Point {
    double x, y, z; Point() : x(0), y(0), z(0) {}

```

```

    Point(double X, double Y, double Z) : x(X), y(Y), z(Z) {}
    Point operator + (const Point& u) const {
        return Point(x + u.x, y + u.y, z + u.z); }
    Point operator - (const Point& u) const {
        return Point(x - u.x, y - u.y, z - u.z); }
    Point operator * (const double u) const {
        return Point(x * u, y * u, z * u); }
    Point operator / (const double u) const {
        return Point(x / u, y / u, z / u); }
};
double dot(Point a, Point b){ return a.x*b.x+a.y*b.y+a.z*b.z; }
Point cross(Point a, Point b) { return
Point(a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y - a.y*b.x); }
double length(Point a) { return sqrt(dot(a, a)); }
double distance(Point a, Point b) { return length(a-b); }
Point unit(const Point &p) { return p/length(p); }
// Rotate p around axis x, with angle radians.
Point rotate(Point p, Point axis, double angle) {
    axis = unit(axis); Point comp1 = p * cos(angle);
    Point comp2 = axis * (1-cos(angle)) * dot(axis, p);
    Point comp3 = cross(axis, p) * sin(angle);
    return comp1 + comp2 + comp3;
}
struct Line {Point a, v;}; //attv
// returns the distance from point a to line l
double distancePointLine(Point p, Line l) {
    return length(cross(l.v, p - l.a)) / length(l.v); }
// distance from Line ab to Line cd
double distanceLineLine(Line a, Line b) {
    Point cr = cross(a.v, b.v); double crl = length(cr);
    if (dcmp(crl) == 0) return distancePointLine(a.a, b);
    return abs(dot(cr, a.a-b.a))/crl; }
struct Plane {
    Point normal; // Normal = (A, B, C)
    double d; // dot(Normal) = d <--> Ax + By + Cz = d
    Point P; // anyPoint on the plane, optional
    Plane(Point normal, double d) {
        double len = length(normal); assert(dcmp(len) > 0);
        normal = normal / len; d = d / len;
        if (dcmp(normal.x)) P = Point(d/normal.x, 0, 0);
        else if (dcmp(normal.y)) P = Point(0, d/normal.y, 0);
        else P = Point(0, 0, d/normal.z);
    }
    //Plane given by three Non-Collinear Points
    Plane(Point a, Point b, Point c) {
        normal = unit(cross(b-a, c-a)); d = dot(normal,a); P=a;
    }
    bool onPlane(Point a) { return dcmp(dot(normal,a)-d)== 0;}
    double distance(Point a) { return abs(dot(normal, a) - d);}
    double isParallel(Line l){return dcmp(dot(l.v,normal))==0;}
//return t st l.a + t*l.v is a point on the plane, check
//parallel first
    double intersectLine(Line l) {
        return dot(P-l.a, normal)/dot(l.v, normal); } };

```

### 5.2 Circle Cover

```

//Check if the all of the area of circ(O, R) in
//Circ(O0, RR) is covered by some other circle
bool CoverCircle(PT O, double R, vector<PT> &cen,
vector<double> &rad, PT O0, double RR) {
    int n = cen.size();
    vector<pair<double, double>> arcs;
    for (int i=0; i<n; i++) {
        PT P = cen[i]; double r = rad[i];
        if (i!=0 && R + sqrt(dist2(O, P))<r) return 1;
        if (i==0 && r + sqrt(dist2(O, P))<R) return 1;
        vector<PT> inter =
            CircleCircleIntersection(O, P, R, r);
        if (inter.size() <= 1) continue;
        PT X = inter[0], Y = inter[1];
        if (cross(O, X, Y) < 0) swap(X, Y);
        if (!(cross(O, X, P) >= 0 &&
            cross(O, Y, P) <= 0)) swap(X, Y);
        if (i==0) swap(X, Y);
        X = X-O; Y=Y-O;
        double ll = atan2(X.y, X.x);

```

```

        double rr = atan2(Y.y, Y.x);
        if (rr < ll) rr += 2*PI;
        arcs.emplace_back(ll, rr);
    }
    if (arcs.empty()) return false;
    sort(arcs.begin(), arcs.end());
    double st = arcs[0].ff, en = arcs[0].ss,ans = 0;
    for (int i=1; i<arcs.size(); i++) {
        if (arcs[i].first <= en + EPS)
            en = max(en, arcs[i].second);
        else st = arcs[i].first, en = arcs[i].second;
        ans = max(ans, en-st);
    }
    return ans >= 2*PI;
}

```

### 5.3 Circle Union Area

```

struct Point {
    LD x,y;
    LD operator*(const Point &a)const {
        return x*a.y-y*a.x;}
    LD operator/(const Point &a)const {
        return sqrt((a.x-x)*(a.x-x)+(a.y-y)*(a.y-y));}
}po[N];
LD r[N];
int sgn(LD x) {return fabs(x)<EPS?0:(x>0.0?1:-1);}
pair<LD,bool> ARG[2*N];
LD cir_union(Point c[],LD r[],int n) {
    LD sum = 0.0 , sum1 = 0.0 ,d,p1,p2,p3 ;
    for(int i = 0 ; i < n ; i++) {
        bool f = 1 ;
        for(int j = 0 ; f&&j<n ; j++)
            if(i!=j && sgn(r[j]-r[i]-c[i]/c[j])!=-1)f=0;
        if(!f) swap(r[i],r[--n]),swap(c[i--],c[n]);
    }
    for(int i = 0; i < n; i++) {
        int k = 0, cnt = 0;
        for(int j = 0; j < n; j++) {
            if(i!=j&&sgn((d=c[i]/c[j]-r[i]-r[j])<=0){
                p3=acos((r[i]*r[i]+d*d-r[j]*r[j])/
                    (2.0*r[i]*d));
                p2=atan2(c[j].y-c[i].y,c[j].x-c[i].x);
                p1 = p2-p3; p2 = p2+p3;
                if(sgn(p1+PI)==-1) p1+=2*PI,cnt++;
                if(sgn(p2-PI)==1) p2-=2*PI,cnt++;
                ARG[k++] = make_pair(p1,0);
                ARG[k++] = make_pair(p2,1);
            }
        }
        if(k) {
            sort(ARG,ARG+k) ;
            p1 = ARG[k-1].first-2*PI;
            p3 = r[i]*r[i] ;
            for(int j = 0 ; j < k ; j++) {
                p2 = ARG[j].first;
                if(cnt==0) {
                    sum+=(p2-p1-sin(p2-p1))*p3 ;
                    sum1+=(c[i]+Point(cos(p1),sin(p1))*
                        r[i])*(c[i]+
                        Point(cos(p2),sin(p2))*r[i]);
                }
                p1 = p2;
                ARG[j].second ? cnt--:cnt++;
            }
        }
        else sum += 2*PI*r[i]*r[i];
    }
    return (sum+fabs(sum1))*0.5 ;
}

```

### 5.4 basic-area-geometry

```

struct point2d {
    ftype x, y; point2d() {}
    point2d(ftype x, ftype y): x(x), y(y) {}
    point2d& operator+=(const point2d &t) {
        x += t.x; y += t.y; return *this;

```

```

    }
    point2d& operator==(const point2d &t) {
        x -= t.x; y -= t.y; return *this;
    }
    point2d& operator+=(ftype t) {
        x += t; y += t; return *this;
    }
    point2d& operator/=(ftype t) {
        x /= t; y /= t; return *this;
    }
    point2d operator+(const point2d &t) const {
        return point2d(*this) += t;
    }
    point2d operator-(const point2d &t) const {
        return point2d(*this) -= t;
    }
    point2d operator*(ftype t) const {
        return point2d(*this) *= t;
    }
    point2d operator/(ftype t) const {
        return point2d(*this) /= t;
    }
};
point2d operator*(ftype a, point2d b) {
    return b * a;
}

struct point3d {
    ftype x, y, z; point3d() {}
    point3d(ftype x, ftype y, ftype z): x(x), y(y), z(z) {}
    point3d& operator+=(const point3d &t) {
        x += t.x; y += t.y; z += t.z; return *this;
    }
    point3d& operator==(const point3d &t) {
        x -= t.x; y -= t.y; z -= t.z; return *this;
    }
    point3d& operator+=(ftype t) {
        x += t; y += t; z += t; return *this;
    }
    point3d& operator/=(ftype t) {
        x /= t; y /= t; z /= t; return *this;
    }
    point3d operator+(const point3d &t) const {
        return point3d(*this) += t;
    }
    point3d operator-(const point3d &t) const {
        return point3d(*this) -= t;
    }
    point3d operator*(ftype t) const {
        return point3d(*this) *= t;
    }
    point3d operator/(ftype t) const {
        return point3d(*this) /= t;
    }
};

point3d operator*(ftype a, point3d b) {
    return b * a;
}

ftype dot(point2d a, point2d b) {
    return a.x * b.x + a.y * b.y;
}

ftype dot(point3d a, point3d b) {
    return a.x * b.x + a.y * b.y + a.z * b.z;
}

ftype norm(point2d a) {
    return dot(a, a);
}

double abs(point2d a) {
    return sqrt(norm(a));
}

double proj(point2d a, point2d b) {
    return dot(a, b) / abs(b);
}

double angle(point2d a, point2d b) {
    return acos(dot(a, b) / abs(a) / abs(b));
}

point3d cross(point3d a, point3d b) {
    return point3d(a.y * b.z - a.z * b.y,
        a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
}

```

```

}
ftype triple(point3d a, point3d b, point3d c) {
    return dot(a, cross(b, c));
}

ftype cross(point2d a, point2d b) {
    return a.x * b.y - a.y * b.x;
}

point2d intersect(point2d a1, point2d d1,
    point2d a2, point2d d2) {
    return a1 + cross(a2 - a1, d2) / cross(d1, d2) * d1;
}

point3d intersect(point3d a1, point3d n1, point3d a2,
    point3d n2, point3d a3, point3d n3) {
    point3d x(n1.x, n2.x, n3.x); point3d y(n1.y, n2.y, n3.y);
    point3d z(n1.z, n2.z, n3.z);
    point3d d(dot(a1, n1), dot(a2, n2), dot(a3, n3));
    return point3d(triple(d, y, z), triple(x, d, z),
        triple(x, y, d)) / triple(n1, n2, n3);
}

int signed_area_parallelogram(point2d p1, point2d p2, point2d p3) {
    return cross(p2 - p1, p3 - p2);
}

double triangle_area(point2d p1, point2d p2, point2d p3) {
    return abs(signed_area_parallelogram(p1, p2, p3)) / 2.0;
}

bool clockwise(point2d p1, point2d p2, point2d p3) {
    return signed_area_parallelogram(p1, p2, p3) < 0;
}

bool counter_clockwise(point2d p1, point2d p2, point2d p3) {
    return signed_area_parallelogram(p1, p2, p3) > 0;
}

double area(const vector<point>& fig) {
    double res = 0;
    for (unsigned i = 0; i < fig.size(); i++) {
        point p = i? fig[i - 1] : fig.back(); point q = fig[i];
        res += (p.x - q.x) * (p.y + q.y);
    }
    return fabs(res) / 2;
}

//Pick: S = I + B/2 - 1
int count_lattices(Fraction k, Fraction b, long long n) {
    auto fk = k.floor(); auto fb = b.floor(); auto cnt = 0LL;
    if (k >= 1 || b >= 1) {
        cnt += (fk * (n - 1) + 2 * fb) * n / 2; k -= fk; b -= fb;
        auto t = k * n + b; auto ft = t.floor();
        if (ft >= 1)
            cnt += count_lattices(1 / k, (t - t.floor()) / k, t.floor());
        return cnt;
    }
}

```

## 5.5 geo-formulae

### Triangle Centers and Radii

- Incenter:  $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$ .
- Inradius:  $\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$ .
- Excenter:  $\left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$ .
- Exradius:  $\sqrt{\frac{s(s-b)(s-c)}{(s-a)}}$ .
- Circumcenter:  $\left(\frac{x_1\sin 2A+x_2\sin 2B+x_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}, \frac{y_1\sin 2A+y_2\sin 2B+y_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}\right)$ .
- Circumradius:  $\frac{abc}{\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}}$ .

## 5.6 half-plane-intersection

```

class HalfPlaneIntersection{
    static double eps, inf;
public:
    struct Point{
        double x, y;
        explicit Point(double x = 0, double y = 0) : x(x), y(y) {}
        // Addition, subtraction, multiply by constant, cross product.
        friend Point operator + (const Point& p, const Point& q){
            return Point(p.x + q.x, p.y + q.y);
        }
        friend Point operator - (const Point& p, const Point& q){
            return Point(p.x - q.x, p.y - q.y);
        }
        friend Point operator * (const Point& p, const double& k){
            return Point(p.x * k, p.y * k);
        }
        friend double cross(const Point& p, const Point& q){
            return p.x * q.y - p.y * q.x;
        }
    };
    // Basic half-plane struct.
    struct Halfplane{
        // 'p' is a passing point of the line and 'pq' is the direction vector of the line.
        Point p, pq;
        double angle;
        Halfplane() {}
        Halfplane(const Point& a, const Point& b) : p(a), pq(b - a){
            angle = atan2(pq.y, pq.x);
        }
        // Check if point 'r' is outside this half-plane.
        // Every half-plane allows the region to the LEFT of its line.
        bool out(const Point& r){
            return cross(pq, r - p) < -eps;
        }
        // Comparator for sorting.
        // If the angle of both half-planes is equal, the leftmost one should go first.
        bool operator < (const Halfplane& e) const{
            if (fabs(angle - e.angle) < eps) return cross(pq, e.p - p) < 0;
            return angle < e.angle;
        }
        // We use equal comparator for std::unique to easily remove parallel half-planes.
        bool operator == (const Halfplane& e) const{
            return fabs(angle - e.angle) < eps;
        }
        // Intersection point of the lines of two half-planes.
        // It is assumed they're never parallel.
        friend Point inter(const Halfplane& s, const Halfplane& t){
            double alpha = cross((t.p - s.p), t.pq) / cross(s.pq, t.pq);
            return s.p + (s.pq * alpha);
        }
    };
    static vector<Point> hp_intersect(vector<Halfplane>& H){
        Point box[4] = // Bounding box in CCW order{
            Point(inf, inf),
            Point(-inf, inf),
            Point(-inf, -inf),
            Point(inf, -inf)
        };
        for(int i = 0; i<4; i++) // Add bounding box half-planes.
            Halfplane aux(box[i], box[(i+1) % 4]);
        H.push_back(aux);
    }
    // Sort and remove duplicates
}

```



```

sort(H.begin(), H.end());
H.erase(unique(H.begin(), H.end()), H.end());
deque<Halfplane> dq;
int len = 0;
for(int i = 0; i < int(H.size()); i++){
    // Remove from the back of the deque while last
    // half-plane is redundant
    while (len > 1 && H[i].out(inter(dq[len-1],
        dq[len-2]))){
        dq.pop_back();
        --len;
    }
    // Remove from the front of the deque while first
    // half-plane is redundant
    while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
        dq.pop_front();
        --len;
    }
    // Add new half-plane
    dq.push_back(H[i]);
    ++len;
}
// Final cleanup: Check half-planes at the front
// against the back and vice-versa
while (len > 2 && dq[0].out(inter(dq[len-1],
    dq[len-2]))){
    dq.pop_back();
    --len;
}
while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
    dq.pop_front();
    --len;
}
// Report empty intersection if necessary
if (len < 3) return vector<Point>();
// Reconstruct the convex polygon from the remaining
// half-planes.
vector<Point> ret(len);
for(int i = 0; i+1 < len; i++){
    ret[i] = inter(dq[i], dq[i+1]);
}
ret.back() = inter(dq[len-1], dq[0]);
return ret;
}
};
double HalfPlaneIntersection::eps=1e-9;
double HalfPlaneIntersection::inf=1e9;

```

## 5.7 heart-of-geometry-2d

```

typedef double ftype;
const double EPS = 1E-9;
struct pt{
    ftype x, y;
    int id;
    pt() {}
    pt(ftype _x, ftype _y):x(_x), y(_y) {}
    pt operator+(const pt & p) const{
        return pt(x + p.x, y + p.y);
    }
    pt operator-(const pt & p) const{
        return pt(x - p.x, y - p.y);
    }
    ftype cross(const pt & p) const{
        return x * p.y - y * p.x;
    }
    ftype dot(const pt & p) const{
        return x * p.x + y * p.y;
    }
    ftype cross(const pt & a, const pt & b) const{
        return (a - *this).cross(b - *this);
    }
    ftype dot(const pt & a, const pt & b) const{
        return (a - *this).dot(b - *this);
    }
    ftype sqrLen() const{
        return this->dot(*this);
    }
    bool operator<(const pt& p) const{

```

```

        return x < p.x - EPS || (abs(x - p.x) < EPS && y < p.y
            - EPS);
    }
    bool operator==(const pt& p) const{
        return abs(x-p.x)<EPS && abs(y-p.y)<EPS;
    }
};
int sign(double x) { return (x > EPS) - (x < -EPS); }
inline int orientation(pt a, pt b, pt c) { return
    sign(a.cross(b,c)); }
bool is_point_on_seg(pt a, pt b, pt p) {
    if (fabs(b.cross(p,a)) < EPS) {
        if (p.x < min(a.x, b.x) - EPS || p.x > max(a.x, b.x) +
            EPS) return false;
        if (p.y < min(a.y, b.y) - EPS || p.y > max(a.y, b.y) +
            EPS) return false;
        return true;
    }
    return false;
}
bool is_point_on_polygon(vector<pt> &p, const pt& z) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        if (is_point_on_seg(p[i], p[(i + 1) % n], z)) return 1;
    }
    return 0;
}
int winding_number(vector<pt> &p, const pt& z) { // O(n)
    if (is_point_on_polygon(p, z)) return 1e9;
    int n = p.size(), ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) % n;
        bool below = p[i].y < z.y;
        if (below != (p[j].y < z.y)) {
            auto orient = orientation(z, p[j], p[i]);
            if (orient == 0) return 0;
            if (below == (orient > 0)) ans += below ? -1 : 1;
        }
    }
    return ans;
}
double dist_sqr(pt a, pt b){
    return ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
double dist(pt a, pt b){
    return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
double angle(pt a, pt b, pt c){
    if(b==a || b==c) return 0;
    double A2 = dist_sqr(b,c);
    double C2 = dist_sqr(a,b);
    double B2 = dist_sqr(c,a);
    double A = sqrt(A2), C = sqrt(C2);
    double ans = (A2 + C2 - B2)/(A*C*2);
    if(ans<-1) ans=acos(-1);
    else if(ans>1) ans=acos(1);
    else ans = acos(ans);
    return ans;
}
bool cmp(pt a, pt b){
    return a.x < b.x || (a.x == b.x && a.y < b.y);
}
bool ccw(pt a, pt b, pt c, bool include_collinear=false) {
    int o = orientation(a, b, c);
    return o > 0 || (include_collinear && o == 0);
}
bool cw(pt a, pt b, pt c, bool include_collinear=false) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c)
    == 0; }
double area(pt a, pt b, pt c){
    return (a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y))/2;
}
struct cmp_x{
    bool operator()(const pt & a, const pt & b) const{
        return a.x < b.x || (a.x == b.x && a.y < b.y);
    }
}

```

```

    }
};
struct cmp_y{
    bool operator()(const pt & a, const pt & b) const{
        return a.y < b.y || (a.y == b.y && a.x < b.x);
    }
};
struct circle : pt {
    ftype r;
};
bool insideCircle(circle c, pt p){
    return dist_sqr(c,p) <= c.r*c.r + EPS;
}
struct line {
    ftype a, b, c;
    line() {}
    line(pt p, pt q){
        a = p.y - q.y;
        b = q.x - p.x;
        c = -a * p.x - b * p.y;
        norm();
    }
    void norm(){
        double z = sqrt(a * a + b * b);
        if (abs(z) > EPS)
            a /= z, b /= z, c /= z;
    }
    line getParallel(pt p){
        line ans = *this;
        ans.c = -(ans.a*p.x+ans.b*p.y);
        return ans;
    }
    ftype getValue(pt p){
        return a*p.x+b*p.y+c;
    }
    line getPerpend(pt p){
        line ans;
        ans.a=this->b;
        ans.b=-(this->a);
        ans.c = -(ans.a*p.x+ans.b*p.y);
        return ans;
    }
    //dist formula is wrong but don't change
    double dist(pt p) const { return a * p.x + b * p.y + c; }
};
double sqr (double a) {
    return a * a;
}
double det(double a, double b, double c, double d) {
    return a*d - b*c;
}
bool intersect(line m, line n, pt & res) {
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS)
        return false;
    res.x = -det(m.c, m.b, n.c, n.b) / zn;
    res.y = -det(m.a, m.c, n.a, n.c) / zn;
    return true;
}
bool parallel(line m, line n) {
    return abs(det(m.a, m.b, n.a, n.b)) < EPS;
}
bool equivalent(line m, line n) {
    return abs(det(m.a, m.b, n.a, n.b)) < EPS
        && abs(det(m.a, m.c, n.a, n.c)) < EPS
        && abs(det(m.b, m.c, n.b, n.c)) < EPS;
}
double det(double a, double b, double c, double d){
    return a * d - b * c;
}
inline bool betw(double l, double r, double x){
    return min(l, r) <= x + EPS && x <= max(l, r) + EPS;
}
inline bool intersect_1d(double a, double b, double c, double
    d){
    if (a > b)
        swap(a, b);
    if (c > d)
        swap(c, d);
    return max(a, c) <= min(b, d) + EPS;
}

```

```

}
bool intersect_segment(pt a, pt b, pt c, pt d, pt& left, pt& right){
    if (!intersect_1d(a.x, b.x, c.x, d.x) || !intersect_1d(a.y, b.y, c.y, d.y))
        return false;
    line m(a, b);
    line n(c, d);
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS) {
        if (abs(m.dist(c)) > EPS || abs(n.dist(a)) > EPS)
            return false;
        if (b < a)
            swap(a, b);
        if (d < c)
            swap(c, d);
        left = max(a, c);
        right = min(b, d);
        return true;
    } else {
        left.x = right.x = -det(m.c, m.b, n.c, n.b) / zn;
        left.y = right.y = -det(m.a, m.c, n.a, n.c) / zn;
        return betw(a.x, b.x, left.x) && betw(a.y, b.y, left.y)
            && betw(c.x, d.x, left.x) && betw(c.y, d.y, left.y);
    }
}

void tangents (pt c, double r1, double r2, vector<line> &ans)
{
    double r = r2 - r1;
    double z = sqr(c.x) + sqr(c.y);
    double d = z - sqr(r);
    if (d < -EPS) return;
    d = sqrt (abs (d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back (l);
}

vector<line> tangents (circle a, circle b) {
    vector<line> ans;
    for (int i=-1; i<=1; i+=2)
        for (int j=-1; j<=1; j+=2)
            tangents (b-a, a.r*i, b.r*j, ans);
    for (size_t i=0; i<ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}

class pointLocationInPolygon{
    bool lexComp(const pt &l, const pt &r){
        return l.x < r.x || (l.x == r.x && l.y < r.y);
    }
    int sgn(ftype val){
        return val > 0 ? 1 : (val == 0 ? 0 : -1);
    }
    vector<pt> seq;
    int n;
    pt translate;
    bool pointInTriangle(pt a, pt b, pt c, pt point){
        ftype s1 = abs(a.cross(b, c));
        ftype s2 = abs(point.cross(a, b)) + abs(point.cross(b, c)) + abs(point.cross(c, a));
        return s1 == s2;
    }
public:
    pointLocationInPolygon(){
    }
    pointLocationInPolygon(vector<pt> & points){
        prepare(points);
    }
    void prepare(vector<pt> & points){
        seq.clear();
        n = points.size();
        int pos = 0;
        for(int i = 1; i < n; i++){
            if(lexComp(points[i], points[pos]))
                pos = i;
        }
    }
}

```

```

        translate.x=points[pos].x;
        translate.y=points[pos].y;
        rotate(points.begin(), points.begin() + pos, points.end());
        n--;
        seq.resize(n);
        for(int i = 0; i < n; i++)
            seq[i] = points[i + 1] - points[0];
    }
    bool pointInConvexPolygon(pt point){
        point.x-=translate.x;
        point.y-=translate.y;
        if(seq[0].cross(point) != 0 && sgn(seq[0].cross(point)) != sgn(seq[0].cross(seq[n - 1])))
            return false;
        if(seq[n - 1].cross(point) != 0 && sgn(seq[n - 1].cross(point)) != sgn(seq[n - 1].cross(seq[0])))
            return false;
        if(seq[0].cross(point) == 0)
            return seq[0].sqrLen() >= point.sqrLen();
        int l = 0, r = n - 1;
        while(r - l > 1){
            int mid = (l + r)/2;
            int pos = mid;
            if(seq[pos].cross(point) >= 0) l = mid;
            else r = mid;
        }
        int pos = l;
        return pointInTriangle(seq[pos], seq[pos + 1], pt(0, 0), point);
    }
    ~pointLocationInPolygon(){
        seq.clear();
    }
}

class Minkowski{
    static void reorder_polygon(vector<pt> & P){
        size_t pos = 0;
        for(size_t i = 1; i < P.size(); i++){
            if(P[i].y < P[pos].y || (P[i].y == P[pos].y && P[i].x < P[pos].x))
                pos = i;
        }
        rotate(P.begin(), P.begin() + pos, P.end());
    }
public:
    static vector<pt> minkowski(vector<pt> P, vector<pt> Q){
        // the first vertex must be the lowest
        reorder_polygon(P);
        reorder_polygon(Q);
        // we must ensure cyclic indexing
        P.push_back(P[0]);
        P.push_back(P[1]);
        Q.push_back(Q[0]);
        Q.push_back(Q[1]);
        // main part
        vector<pt> result;
        size_t i = 0, j = 0;
        while(i < P.size() - 2 || j < Q.size() - 2){
            result.push_back(P[i] + Q[j]);
            auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j]);
            if(cross >= 0)
                ++i;
            if(cross <= 0)
                ++j;
        }
        return result;
    }
}

vector<pt> circle_line_intersections(circle cir, line l){
    double r = cir.r, a = l.a, b = l.b, c = l.c + l.a*cir.x + l.b*cir.y;
    vector<pt> ans;
    double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
    if (c*c > r*r*(a*a+b*b)+EPS);
    else if (abs (c*c - r*r*(a*a+b*b)) < EPS){
        pt p;
        p.x=x0;

```

```

        p.y=y0;
        ans.push_back(p);
    }
    else{
        double d = r*r - c*c/(a*a+b*b);
        double mult = sqrt (d / (a*a+b*b));
        double ax, ay, bx, by;
        ax = x0 + b * mult;
        bx = x0 - b * mult;
        ay = y0 - a * mult;
        by = y0 + a * mult;
        pt p;
        p.x = ax;
        p.y = ay;
        ans.push_back(p);
        p.x = bx;
        p.y = by;
        ans.push_back(p);
    }
    for(int i=0; i<ans.size(); i++){
        ans[i] = ans[i] + cir;
    }
    return ans;
}

double circle_polygon_intersection(circle c, vector<pt> &V){
    int n = V.size();
    double ans = 0;
    for(int i=0; i<n; i++){
        line l(V[i], V[(i+1)%n]);
        vector<pt> lpts = circle_line_intersections(c, l);
        int sz=lpts.size();
        for(int j=sz-1; j>=0; j--){
            if(!is_point_on_seg(V[i], V[(i+1)%n], lpts[j])){
                swap(lpts.back(), lpts[j]);
                lpts.pop_back();
            }
        }
        lpts.push_back(V[i]);
        lpts.push_back(V[(i+1)%n]);
        sort(lpts.begin(), lpts.end());
        sz=lpts.size();
        if(V[(i+1)%n]<V[i])
            reverse(lpts.begin(), lpts.end());
        for(int j=1; j<sz; j++){
            if(insideCircle(c, lpts[j-1]) && insideCircle(c, lpts[j]))
                ans = ans + area(lpts[j-1], lpts[j], c);
            else{
                double ang = angle(lpts[j-1], c, lpts[j]);
                double aa = c.r*c.r*ang/2;
                if(ccw(lpts[j-1], lpts[j], c))
                    ans = ans+aa;
                else
                    ans = ans-aa;
            }
        }
    }
    ans = abs(ans);
    return ans;
}

void convex_hull(vector<pt>& a, bool include_collinear = false) {
    pt p0 = *min_element(a.begin(), a.end(), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x-a.x)*(p0.x-a.x) + (p0.y-a.y)*(p0.y-a.y) < (p0.x-b.x)*(p0.x-b.x) + (p0.y-b.y)*(p0.y-b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = (int)a.size()-1;
        while (i >= 0 && collinear(p0, a[i], a.back())) i--;
        reverse(a.begin()+i+1, a.end());
    }
    vector<pt> st;
    for (int i = 0; i < (int)a.size(); i++) {

```

```

    while (st.size() > 1 && !cw(st[st.size()-2], st.back(),
        a[i], include_collinear))
        st.pop_back();
    st.push_back(a[i]);
}
a = st;
int m = a.size();
for(int i = 0; i < m-1; i++){
    swap(a[i], a[m-1-i]);
}
}
double mindist;
pair<int, pair<int, int>> best_pair;
void upd_ans(const pt & a, const pt & b, const pt & c){
    double distC = sqrt((a.x - b.x)*(a.x - b.x) + (a.y -
        b.y)*(a.y - b.y));
    double distA = sqrt((c.x - b.x)*(c.x - b.x) + (c.y -
        b.y)*(c.y - b.y));
    double distB = sqrt((a.x - c.x)*(a.x - c.x) + (a.y -
        c.y)*(a.y - c.y));
    if (distA + distB + distC < mindist){
        mindist = distA + distB + distC;
        best_pair = make_pair(a.id, make_pair(b.id, c.id));
    }
}
vector<pt> t;
//Min possible triplet distance
void rec(int l, int r){
    if (r - l <= 3 && r - l >= 2){
        for (int i = l; i < r; ++i){
            for (int j = i + 1; j < r; ++j){
                for (int k = j + 1; k < r; ++k){
                    upd_ans(a[i], a[j], a[k]);
                }
            }
        }
        sort(a.begin() + l, a.begin() + r, cmp_y());
        return;
    }
    int m = (l + r) >> 1;
    int midx = a[m-1].x;
    /*
    * Got WA in a team contest
    * for putting midx = a[m].x;
    * Don't know why. Maybe due to
    * floating point numbers.
    */
    rec(l, m);
    rec(m, r);
    merge(a.begin() + l, a.begin() + m, a.begin() + m,
        a.begin() + r, t.begin(), cmp_y());
    copy(t.begin(), t.begin() + r - l, a.begin() + l);
    int tsz = 0;
    for (int i = l; i < r; ++i){
        if (abs(a[i].x - midx) < mindist/2){
            for (int j = tsz - 1; j >= 0 && a[i].y - t[j].y <
                mindist/2; --j){
                if (i+1 < r) upd_ans(a[i], a[i+1], t[j]);
                if (j > 0) upd_ans(a[i], t[j-1], t[j]);
            }
            t[tsz++] = a[i];
        }
    }
}
}

```

## 5.8 intersecting-segments-pair

```

const double EPS = 1E-9;
struct pt {
    double x, y;
};
struct seg {
    pt p, q;
    int id;
    double get_y(double x) const {
        if (abs(p.x - q.x) < EPS)
            return p.y;
        return p.y + (q.y - p.y) * (x - p.x) / (q.x - p.x);
    }
}

```

```

};
bool intersect1d(double l1, double r1, double l2, double r2) {
    if (l1 > r1)
        swap(l1, r1);
    if (l2 > r2)
        swap(l2, r2);
    return max(l1, l2) <= min(r1, r2) + EPS;
}
int vec(const pt& a, const pt& b, const pt& c) {
    double s = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x
        - a.x);
    return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
}
bool intersect(const seg& a, const seg& b){
    return intersect1d(a.p.x, a.q.x, b.p.x, b.q.x) &&
        intersect1d(a.p.y, a.q.y, b.p.y, b.q.y) &&
        vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
        vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
}
bool operator<(const seg& a, const seg& b){
    double x = max(min(a.p.x, a.q.x), min(b.p.x, b.q.x));
    return a.get_y(x) < b.get_y(x) - EPS;
}
struct event {
    double x;
    int tp, id;
    event() {}
    event(double x, int tp, int id) : x(x), tp(tp), id(id) {}
    bool operator<(const event& e) const {
        if (abs(x - e.x) > EPS)
            return x < e.x;
        return tp > e.tp;
    }
};
set<seg> s;
vector<set<seg>::iterator> where;
set<seg>::iterator prev(set<seg>::iterator it) {
    return it == s.begin() ? s.end() : --it;
}
set<seg>::iterator next(set<seg>::iterator it) {
    return ++it;
}
pair<int, int> solve(const vector<seg>& a) {
    int n = (int)a.size();
    vector<event> e;
    for (int i = 0; i < n; ++i) {
        e.push_back(event(min(a[i].p.x, a[i].q.x), +1, i));
        e.push_back(event(max(a[i].p.x, a[i].q.x), -1, i));
    }
    sort(e.begin(), e.end());
    s.clear();
    where.resize(a.size());
    for (size_t i = 0; i < e.size(); ++i) {
        int id = e[i].id;
        if (e[i].tp == +1) {
            set<seg>::iterator nxt = s.lower_bound(a[id]), prv
                = prev(nxt);
            if (nxt != s.end() && intersect(*nxt, a[id]))
                return make_pair(nxt->id, id);
            if (prv != s.end() && intersect(*prv, a[id]))
                return make_pair(prv->id, id);
            where[id] = s.insert(nxt, a[id]);
        } else {
            set<seg>::iterator nxt = next(where[id]), prv =
                prev(where[id]);
            if (nxt != s.end() && prv != s.end() &&
                intersect(*nxt, *prv))
                return make_pair(prv->id, nxt->id);
            s.erase(where[id]);
        }
    }
    return make_pair(-1, -1);
}

```

## 5.9 radiant-geo

```

typedef double Tf; typedef double Ti;
const Tf PI = acos(-1), EPS = 1e-9;
int dcmp(Tf x) {return abs(x) < EPS ? 0 : (x < 0 ? -1 : 1);}

```

```

struct PT {
    Ti x, y;
    bool operator == (const PT& u) const
        { return dcmp(x-u.x)==0 && dcmp(y-u.y)==0; }
    bool operator != (const PT& u) const
        { return !(*this == u); }
    friend istream &operator >> (istream &is, PT &p)
        { return is >> p.x >> p.y; }
    friend ostream &operator << (ostream &os,
        const PT &p) {return os << p.x << " " << p.y; }
};
Ti dot(PT a, PT b) { return a.x*b.x + a.y*b.y; }
Ti cross(PT a, PT b) { return a.x*b.y - a.y*b.x; }
Tf length(PT a) { return sqrt(dot(a, a)); }
Ti sqLength(PT a) { return dot(a, a); }
Tf distance(PT a, PT b) {return length(a-b);}
Tf angle(PT u) { return atan2(u.y, u.x); }
Tf angleBetween(PT a, PT b) { //in range [-PI, PI]
    Tf ans = angle(b) - angle(a);
    return ans <= -PI ? ans + 2*PI :
        (ans > PI ? ans - 2*PI : ans);
}
PT rotate(PT a, Tf rad) {
    static_assert(is_same<Tf, Ti>::value);
    return PT(a.x * cos(rad) - a.y * sin(rad),
        a.x * sin(rad) + a.y * cos(rad));
}
// Rotate(a, rad) where cos(rad)=co, sin(rad)=si
PT rotatePrecise(PT a, Tf co, Tf si) {
    static_assert(is_same<Tf, Ti>::value);
    return PT(a.x*co - a.y*si, a.y*co + a.x*si);
}
PT rotate90(PT a) { return PT(-a.y, a.x); }
PT scale(PT a, Tf s) {
    static_assert(is_same<Tf, Ti>::value);
    return a / length(a) * s;
}
PT normal(PT a) {
    static_assert(is_same<Tf, Ti>::value);
    Tf l = length(a); return PT(-a.y / l, a.x / l);
}
// returns 1/0/-1 if c is left/on/right of ab
int orient(PT a, PT b, PT c) {
    return dcmp(cross(b - a, c - a));
}
//sort(v.begin(), v.end(), polarComp(0, dir))
struct polarComp {
    PT o, dir;
    polarComp(PT o = PT(0, 0), PT dir = PT(1, 0))
        : o(o), dir(dir) {}
    bool half(PT p) {
        return dcmp(cross(dir, p)) < 0 ||
            (dcmp(cross(dir, p))==0 && dcmp(dot(dir, p))>0);
    }
    bool operator()(PT p, PT q) {
        return make_tuple(half(p-o), 0) <
            make_tuple(half(q-o), cross(p-o, q-o));
    }
};
struct Segment {
    PT a, b;
    Segment() {}
    Segment(PT aa, PT bb) : a(aa), b(bb) {}
}; typedef Segment Line;
struct Circle {
    PT o; Tf r;
    Circle(PT o = PT(0, 0), Tf r = 0) : o(o), r(r) {}
    bool contains(PT p) {
        return dcmp(sqLength(p - o) - r * r) <= 0; }
    PT point(Tf rad) {
        static_assert(is_same<Tf, Ti>::value);
        return PT(o.x+cos(rad)*r, o.y+sin(rad)*r);
    }
}
Tf area(Tf rad = PI + PI) { return rad * r * r/2; }
Tf sector(Tf alpha) {
    return r*r*0.5*(alpha-sin(alpha)); }
};
#####

```

```

namespace Linear {
bool onSegment(PT p, Segment s) { //Is p on S?
    return dcmp(cross(s.a - p, s.b - p)) == 0 &&
        dcmp(dot(s.a - p, s.b - p)) <= 0;
}
bool segmentsIntersect(Segment p, Segment q) {
    if(onSegment(p.a,q)||onSegment(p.b,q))return 1;
    if(onSegment(q.a,p)||onSegment(q.b,p))return 1;
    Ti c1 = cross(p.b - p.a, q.a - p.a);
    Ti c2 = cross(p.b - p.a, q.b - p.a);
    Ti c3 = cross(q.b - q.a, p.a - q.a);
    Ti c4 = cross(q.b - q.a, p.b - q.a);
    return dcmp(c1)*dcmp(c2)<0&&dcmp(c3)*dcmp(c4)<0;
}
bool linesParallel(Line p, Line q) {
    return dcmp(cross(p.b - p.a, q.b - q.a)) == 0;
}
//returns if lines (p, p+w) && (q, q+w) intersect
bool lineLineIntersect(PT p, PT v, PT q, PT w, PT&o) {
    static_assert(is_same<Ti, Ti>::value);
    if(dcmp(cross(v, w)) == 0) return false;
    PT u = p - q; o = p + v*(cross(w,u)/cross(v,w));
    return true;
}
bool lineLineIntersect(Line p, Line q, PT& o) {
    return lineLineIntersect(p.a, p.b - p.a, q.a,
        q.b - q.a, o);
}
Tf distancePointLine(PT p, Line l) {
    return abs(cross(l.b-l.a, p-l.a)/length(l.b-l.a));
}
Tf distancePointSegment(PT p, Segment s) {
    if(s.a == s.b) return length(p - s.a);
    PT v1 = s.b - s.a, v2 = p - s.a, v3 = p - s.b;
    if(dcmp(dot(v1, v2)) < 0) return length(v2);
    else if(dcmp(dot(v1, v3)) > 0) return length(v3);
    else return abs(cross(v1, v2) / length(v1));
}
Tf distanceSegmentSegment(Segment p, Segment q) {
    if(segmentsIntersect(p, q)) return 0;
    Tf ans = distancePointSegment(p.a, q);
    ans = min(ans, distancePointSegment(p.b, q));
    ans = min(ans, distancePointSegment(q.a, p));
    ans = min(ans, distancePointSegment(q.b, p));
    return ans;
}
PT projectPointLine(PT p, Line l) {
    static_assert(is_same<Ti, Ti>::value);
    PT v = l.b - l.a;
    return l.a + v * ((Tf) dot(v, p-l.a)/dot(v, v));
} } // namespace Linear
#####
typedef vector<PT> Polygon;
namespace Polygonal {
// cannot be all collinear
Polygon RemoveCollinear(const Polygon& poly) {
    Polygon ret;
    int n = poly.size();
    for(int i = 0; i < n; i++) {
        PT a = poly[i];
        PT b = poly[(i + 1) % n];
        PT c = poly[(i + 2) % n];
        if(dcmp(cross(b-a, c-b))!=0 && (ret.empty() ||
            b != ret.back())) ret.push_back(b);
    }
    return ret;
}
Tf signedPolygonArea(const Polygon &p);
// returns inside = -1, on = 0, outside = 1
int pointInPolygon(const Polygon &p, PT o);
// returns (longest segment, total length)
pair<Tf, Tf> linePolygonIntersection(Line l,
    const Polygon &p) {
    using Linear::lineLineIntersect;
    int n = p.size(); vector<pair<Tf, int>> ev;
    for(int i=0; i<n; ++i) {
        PT a = p[i], b = p[(i+1)%n], z = p[(i-1+n)%n];

```

```

        int ora=orient(l.a,l.b,a), orb =
            orient(l.a,l.b,b), orz=orient(l.a,l.b,z);
        if(!ora) {
            Tf d = dot(a - l.a, l.b - l.a);
            if(orz && orb) {
                if(orz != orb) ev.emplace_back(d, 0);
                //else // PT Touch
            } else if(orz) ev.emplace_back(d, orz);
            else if(orb) ev.emplace_back(d, orb);
        }
        else if(ora == -orb) {
            PT ins;
            lineLineIntersect(l, Line(a, b), ins);
            ev.emplace_back(dot(ins-l.a, l.b-l.a),0);
        }
    }
    sort(ev.begin(), ev.end());
    Tf ans = 0, len = 0, last = 0, tot = 0;
    bool active = false; int sign = 0;
    for(auto &qq : ev) {
        int tp = qq.second;
        Tf d = qq.first; //current Seg is (last, d)
        if(sign) { //On Border
            len+=d-last; tot+=d-last; ans=max(ans,len);
            if(tp != sign) active = !active;
            sign = 0;
        }
        else {
            if(active) { //Strictly Inside
                len+=d-last; tot+=d-last; ans=max(ans,len);
            }
            if(tp == 0) active=!active; else sign = tp;
        }
        last = d; if(!active) len = 0;
    }
    ans /= length(l.b-l.a); tot /= length(l.b-l.a);
    return {ans, tot};
} } // namespace Polygonal
#####
namespace Convex {
//min area, min perimeter) rectangle containing p
pair<Tf,Tf>rotatingCalipersBBox(const Polygon &p){
    using Linear::distancePointLine;
    static_assert(is_same<Ti, Ti>::value);
    int n = p.size(); int l = 1, r = 1, j = 1;
    Tf area = 1e100; Tf perimeter = 1e100;
    for(int i = 0; i < n; i++) {
        PT v=(p[(i+1)%n]-p[i])/length(p[(i+1)%n]-p[i]);
        while(dcmp(dot(v, p[r%n] - p[i]) -
            dot(v, p[(r+1)%n] - p[i])) < 0) r++;
        while(j < r || dcmp(cross(v, p[j%n] - p[i]) -
            cross(v, p[(j+1)%n] - p[i])) < 0) j++;
        while(l < j || dcmp(dot(v, p[l%n] - p[i]) -
            dot(v, p[(l+1)%n] - p[i])) > 0) l++;
        Tf w = dot(v,p[r%n]-p[i])-dot(v,p[l%n]-p[i]);
        Tf h = distancePointLine(p[l%n],
            Line(p[i], p[(i+1)%n]));
        area = min(area, w * h);
        perimeter = min(perimeter, 2 * w + 2 * h);
    }
    return make_pair(area, perimeter);
}
// returns the left half of u on left on ray ab
Polygon cutPolygon(Polygon u, PT a, PT b) {
    using Linear::lineLineIntersect;
    using Linear::onSegment;
    Polygon ret; int n = u.size();
    for(int i = 0; i < n; i++) {
        PT c = u[i], d = u[(i + 1) % n];
        if(dcmp(cross(b-a, c-a))>=0) ret.push_back(c);
        if(dcmp(cross(b-a, d-c)) != 0) {
            PT t; lineLineIntersect(a, b-a, c, d-c, t);
            if(onSegment(t,Segment(c,d)))ret.push_back(t);
        }
    }
    return ret;
}
bool pointInTriangle(PT a, PT b, PT c, PT p) {
    return dcmp(cross(b - a, p - a)) >= 0
        && dcmp(cross(c - b, p - b)) >= 0

```

```

        && dcmp(cross(a - c, p - c)) >= 0;
    }
    int pointInConvexPolygon(const Polygon &pt, PT p);
// most extreme Point in the direction u
int extremePoint(const Polygon &poly, PT u) {
    int n = (int) poly.size();
    int a = 0, b = n;
    while(b - a > 1) {
        int c = (a + b) / 2;
        if(dcmp(dot(poly[c]-poly[(c+1)%n], u))>=0 &&
            dcmp(dot(poly[c]-poly[(c-1+n)%n], u))>=0) {
            return c;
        }
    }
    bool a_up=dcmp(dot(poly[(a+1)%n]-poly[a],u))>=0;
    bool c_up=dcmp(dot(poly[(c+1)%n]-poly[c],u))>=0;
    bool a_above_c=dcmp(dot(poly[a]-poly[c],u))>0;

    if(a_up && !c_up) b = c;
    else if(!a_up && c_up) a = c;
    else if(a_up && c_up) {
        if(a_above_c) b = c; else a = c;
    } else {
        if(!a_above_c) b = c; else a = c;
    }
    }
    if(dcmp(dot(poly[a]-poly[(a+1)%n],u))>0 &&
        dcmp(dot(poly[a]-poly[(a-1+n)%n],u))>0)
        return a;
    return b % n;
}
// return list of segs of p that touch/intersect l
// the i'th segment is (p[i], p[(i + 1)%p])
// #1 If a side is collinear, only that returned
// #2 If l goes through p[i], ith segment is added
vector<int> lineConvexPolyIntersection(
    const Polygon &p, Line l) {
    assert((int) p.size() >= 3); assert(l.a != l.b);
    int n = p.size(); vector<int> ret;
    PT v = l.b - l.a;
    int lf = extremePoint(p, rotate90(v));
    int rt = extremePoint(p, rotate90(v) * Ti(-1));
    int olf = orient(l.a, l.b, p[lf]);
    int ort = orient(l.a, l.b, p[rt]);
    if(!olf || !ort) {
        int idx = (!olf ? lf : rt);
        if(orient(l.a, l.b, p[(idx - 1 + n) % n])==0)
            ret.push_back((idx - 1 + n) % n);
        else ret.push_back(idx);
        return ret;
    }
    if(olf == ort) return ret;
    for(int i=0; i<2; ++i) {
        int lo = i ? rt : lf, hi = i ? lf : rt;
        int olo = i ? ort : olf;
        while(true) {
            int gap = (hi - lo + n) % n;
            if(gap < 2) break;
            int mid = (lo + gap / 2) % n;
            int omid = orient(l.a, l.b, p[mid]);
            if(!omid) {lo = mid; break;}
            if(omid == olo) lo = mid;
            else hi = mid;
        }
        ret.push_back(lo);
    }
    return ret;
}
// [ACW, CW] tangent pair from an external point
constexpr int CW = -1, ACW = 1;
bool isGood(PT u, PT v, PT q, int dir) {
    return orient(q, u, v) != -dir;
}
PT better(PT u, PT v, PT q, int dir) {
    return orient(q, u, v) == dir ? u : v;
}
PT pointPolyTangent(const Polygon &pt, PT Q,
    int dir, int lo, int hi) {
    while(hi - lo > 1) {
        int mid = (lo + hi) / 2;
        bool pvs = isGood(pt[mid], pt[mid-1], Q, dir);
        bool nxt = isGood(pt[mid], pt[mid+1], Q, dir);
        if(pvs && nxt) return pt[mid];
        if(!(pvs || nxt)) {

```

```

    PT p1 = pointPolyTangent(pt,Q,dir,mid+1,hi);
    PT p2 = pointPolyTangent(pt,Q,dir,lo,mid-1);
    return better(p1, p2, Q, dir);
}
if(!pvs) {
    if(orient(Q,pt[mid],pt[lo])==dir) hi=mid-1;
    else if(better(pt[lo],pt[hi],Q,dir)==pt[lo])
        hi = mid - 1; else lo = mid + 1;
}
if(!nxt) {
    if(orient(Q,pt[mid],pt[lo])==dir) lo=mid+1;
    else if(better(pt[lo],pt[hi],Q,dir)==pt[lo])
        hi = mid - 1; else lo = mid + 1;
}
}
PT ret = pt[lo];
for(int i = lo + 1; i <= hi; i++)
    ret = better(ret, pt[i], Q, dir);
return ret;
}
// [ACW, CW] Tangent
pair<PT,PT> pointPolyTangents(
    const Polygon &pt,PT Q) {
    int n = pt.size();
    PT acw_tan = pointPolyTangent(pt, Q, ACW,0,n-1);
    PT cw_tan = pointPolyTangent(pt, Q, CW, 0, n-1);
    return make_pair(acw_tan, cw_tan);
}
}
#####
namespace Circular {
// returns intersections in order of ray (l.a,l.b)
vector<PT> circleLineIntersection(Circle c,Line l){
    static_assert(is_same<Tf, Ti>::value);
    vector<PT> ret;
    PT b = l.b - l.a, a = l.a - c.o;
    Tf A = dot(b, b), B = dot(a, b);
    Tf C = dot(a, a) - c.r * c.r, D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(l.a + b * (-B-sqrt(D + EPS)) / A);
    if (D > EPS)
        ret.push_back(l.a + b * (-B + sqrt(D)) / A);
    return ret;
}
// circle(c.o, c.r) x triangle(c.o,s.a,s.b) (ccw)
Tf circleTriInterArea(Circle c, Segment s){
    using Linear::distancePointSegment;
    Tf OA = length(c.o-s.a), OB = length(c.o-s.b);
    if(dcmp(distancePointSegment(c.o, s) - c.r) >= 0)
        return angleBetween(s.a-c.o,s.b-c.o)*c.r*c.r/2;
    if(dcmp(OA - c.r) <= 0 && dcmp(OB - c.r) <= 0)
        return cross(c.o - s.b, s.a - s.b) / 2.0;
    vector<PT> Sect = circleLineIntersection(c, s);
    return circleTriInterArea(c,Segment(s.a,Sect[0]))
        +circleTriInterArea(c,Segment(Sect[0],Sect[1]))
        + circleTriInterArea(c,Segment(Sect[1],s.b));
}
}
Tf circlePolyIntersectionArea(Circle c, Polygon p);
// locates circle c2 relative to c1: intersect = 0
// inside = -2, inside touch = -1,
// outside touch = 1, outside = 2
int circleCirclePosition(Circle c1, Circle c2) {
    Tf d = length(c1.o - c2.o);
    int in = dcmp(d - abs(c1.r - c2.r)),
        ex = dcmp(d - (c1.r + c2.r));
    return in<0?-2:in==0?-1:ex==0?1:ex>0?2:0;
}
vector<PT> circleCircleInter(Circle c1, Circle c2){
    static_assert(is_same<Tf, Ti>::value);
    vector<PT> ret;
    Tf d = length(c1.o - c2.o);
    if(dcmp(d) == 0) return ret;
    if(dcmp(c1.r + c2.r - d) < 0) return ret;
    if(dcmp(abs(c1.r - c2.r) - d) > 0) return ret;
    PT v = c2.o - c1.o;
    Tf co = (c1.r * c1.r + sqLength(v) - c2.r*c2.r)
        / (2 * c1.r * length(v));
    Tf si = sqrt(abs(1.0 - co * co));
    PT p1 = scale(rotatePrecise(v,co,-si),c1.r)+c1.o;

```

```

    PT p2 = scale(rotatePrecise(v,co,si),c1.r)+c1.o;
    ret.push_back(p1);
    if(p1 != p2) ret.push_back(p2); return ret;
}
Tf circleCircleInterArea(Circle c1, Circle c2) {
    PT AB = c2.o - c1.o; Tf d = length(AB);
    if(d >= c1.r + c2.r) return 0;
    if(d + c1.r <= c2.r) return PI * c1.r * c1.r;
    if(d + c2.r <= c1.r) return PI * c2.r * c2.r;
    Tf alpha1 = acos((c1.r*c1.r + d*d - c2.r*c2.r)
        / (2.0 * c1.r * d));
    Tf alpha2 = acos((c2.r*c2.r + d*d - c1.r*c1.r)
        / (2.0 * c2.r * d));
    return c1.sector(2*alpha1)+c2.sector(2*alpha2);
}
// returns tangents from a point p to circle c
vector<PT> pointCircleTangents(PT p, Circle c) {
    static_assert(is_same<Tf, Ti>::value);
    vector<PT> ret;PT u = c.o - p; Tf d = length(u);
    if(d < c.r);
    else if(dcmp(d - c.r) == 0) {
        ret = { rotate(u, PI / 2) }; }
    else {
        Tf ang = asin(c.r / d);
        ret = { rotate(u, -ang), rotate(u, ang) };
    } return ret;
}
//returns points on tangents that touches circle c
vector<PT>pointCircleTangencyPoints(PT p,Circle c){
    static_assert(is_same<Tf, Ti>::value);
    PT u = p - c.o; Tf d = length(u);
    if(d < c.r) return {};
    else if(dcmp(d - c.r) == 0) return {c.o + u};
    else {
        Tf ang = acos(c.r / d); u = u/length(u) * c.r;
        return{c.o+rotate(-ang), c.o+rotate(u,ang)};
    }
}
// finds a, b st a[i] on c1, b[i] on c2, Segment
// a[i], b[i] touches c1, c2. if c1, c2 touch at x
// (x, x) is also returned, -1 returned if c1 = c2
int circleCircleTangencyPoints(Circle c1,Circle c2,
    vector<PT> &a, vector<PT> &b) {
    a.clear(), b.clear(); int cnt = 0;
    if(dcmp(c1.r-c2.r)<0) {swap(c1, c2);swap(a, b);}
    Tf d2 = sqLength(c1.o - c2.o);
    Tf rdif = c1.r - c2.r, rsum = c1.r + c2.r;
    if(dcmp(d2 - rdif * rdif) < 0) return 0;
    if(dcmp(d2)==0 && dcmp(c1.r-c2.r)==0) return -1;
    Tf base = angle(c2.o - c1.o);
    if(dcmp(d2 - rdif * rdif) == 0) {
        a.push_back(c1.point(base));
        b.push_back(c2.point(base));
        cnt++; return cnt;
    }
    Tf ang = acos((c1.r - c2.r) / sqrt(d2));
    a.push_back(c1.point(base + ang));
    b.push_back(c2.point(base + ang)); cnt++;
    a.push_back(c1.point(base - ang));
    b.push_back(c2.point(base - ang)); cnt++;
    if(dcmp(d2 - rsum * rsum) == 0) {
        a.push_back(c1.point(base));
        b.push_back(c2.point(PI + base)); cnt++;
    }
    else if(dcmp(d2 - rsum * rsum) > 0) {
        Tf ang = acos((c1.r + c2.r) / sqrt(d2));
        a.push_back(c1.point(base + ang));
        b.push_back(c2.point(PI + base + ang)); cnt++;
        a.push_back(c1.point(base - ang));
        b.push_back(c2.point(PI + base - ang)); cnt++;
    } return cnt;
} } // namespace Circular
#####
namespace EnclosingCircle{
// returns false if points are collinear
bool inCircle(PT a, PT b, PT c, Circle &p) {
    using Linear::distancePointLine;

```

```

    static_assert(is_same<Tf, Ti>::value);
    if(orient(a, b, c) == 0) return false;
    Tf u=length(b-c), v=length(c-a), w=length(a-b);
    p.o = (a * u + b * v + c * w) / (u + v + w);
    p.r = distancePointLine(p.o, Line(a, b));
    return true;
}
// set of points A(x, y) st PA : QA = rp : rq
Circle apolloniusCircle(PT P, PT Q, Tf rp, Tf rq){
    static_assert(is_same<Tf, Ti>::value);
    rq == rp; rp == rp; Tf a=rq-rp; assert(dcmp(a));
    Tf g = (rq*P.x-rp*Q.x)/a, h = (rq*P.y-rp*Q.y)/a;
    Tf c = (rq*P.x*P.x - rp*Q.x*Q.x +
        rq*P.y*P.y - rp*Q.y*Q.y)/a;
    PT o(g, h); Tf R = sqrt(g * g + h * h - c);
    return Circle(o, R);
}
// returns false if points are collinear
bool circumCircle(PT a, PT b, PT c, Circle &p) {
    using Linear::lineLineIntersect;
    if(orient(a, b, c) == 0) return false;
    PT d = (a + b) / 2, e = (a + c) / 2;
    PT vd = rotate90(b - a), ve = rotate90(a - c);
    bool f = lineLineIntersect(d, vd, e, ve, p.o);
    if(f) p.r = length(a - p.o);
    return f;
}
// finds a circle that goes all of p, |p| <= 3.
Circle boundary(const vector<PT> &p) {
    Circle ret; int sz = p.size();
    if(sz == 0) ret.r = 0;
    else if(sz == 1) ret.o = p[0], ret.r = 0;
    else if(sz == 2) ret.o = (p[0] + p[1]) / 2,
        ret.r = length(p[0] - p[1]) / 2;
    else if(!circumCircle(p[0],p[1],p[2],ret))
        ret.r = 0;
    return ret;
}
// Min circle enclosing p[fr....n-1],
// with points in b on the boundary, |b| <= 3.
Circle welzl(const vector<PT> &p,
    int fr, vector<PT> &b) {
    if(fr >= (int) p.size() || b.size() == 3)
        return boundary(b);
    Circle c = welzl(p, fr + 1, b);
    if(!c.contains(p[fr])) {
        b.push_back(p[fr]); c = welzl(p, fr + 1, b);
        b.pop_back();
    } return c;
}
// MEC of p, using weizl's algo. amortized O(n).
Circle MEC(vector<PT> p) {
    random_shuffle(p.begin(), p.end());
    vector<PT> q; return welzl(p, 0, q);
}
}
#####
// Given list of segments v, finds a pair (i, j) st
// v[i],v[j] intersects. If none, returns {-1, -1}
namespace IntersectingSegments {
struct Event {
    Tf x; int tp, id;
    bool operator < (const Event &p) const {
        if(dcmp(x-p.x)) return x<p.x; return tp>p.tp;
    }
};
pair<int, int> anyInters(const vector<Segment> &v){
    using Linear::segmentsIntersect;
    static_assert(is_same<Tf, Ti>::value);
    vector<Event> ev;
    for(int i=0; i<v.size(); i++) {
        ev.push_back({min(v[i].a.x, v[i].b.x), +1, i});
        ev.push_back({max(v[i].a.x, v[i].b.x), -1, i});
    }
    sort(ev.begin(), ev.end());
    auto comp = [&v] (int i, int j) {
        Segment p = v[i], q = v[j];
        Tf x=max(min(p.a.x,p.b.x), min(q.a.x, q.b.x));
        auto yvalSegment = [&x](const Line &s) {

```

```

    if(dcmp(s.a.x - s.b.x) == 0) return s.a.y;
    return s.a.y + (s.b.y - s.a.y)
        * (x - s.a.x) / (s.b.x - s.a.x);
};
return dcmp(yvalSegment(p)-yvalSegment(q))<0;
};
multiset<int, decltype(comp)> st(comp);
typedef decltype(st)::iterator iter;
auto prev = [&st](iter it) {
    return it == st.begin() ? st.end() : --it;
};
auto next = [&st](iter it) {
    return it == st.end() ? st.end() : ++it;
};
vector<iter> pos(v.size());
for(auto &cur : ev) {
    int id = cur.id;
    if(cur.tp == 1) {
        iter nxt = st.lower_bound(id), pre=prev(nxt);
        if(pre != st.end() && segmentsIntersect
            (v[*pre], v[id])) return {*pre, id};
        if(nxt != st.end() && segmentsIntersect
            (v[*nxt], v[id])) return {*nxt, id};
        pos[id] = st.insert(nxt, id);
    }
    else {
        iter nxt=next(pos[id]), pre=prev(pos[id]);
        if(pre != st.end() && nxt != st.end() &&
            segmentsIntersect(v[*pre], v[*nxt]))
            return {*pre, *nxt};
        st.erase(pos[id]);
    }
} return {-1, -1};
}
}
#####

```

## 5.10 trianlge-ear-clipping

```

///O(n^3) v bad brute force implementation, implement better
algorithm later
template<class T>
int area(pair<T,T>& p1,pair<T,T>& p2,pair<T,T>& p3){
    return
        (p1.first*p2.second+p2.first*p3.second+p3.first*p1.second
        -p1.second*p2.first-p2.second*p3.first-p3.second*p1.first);
}
template<class T>
bool inside(pair<T,T>& a,pair<T,T>& b,pair<T,T>& c,pair<T,T>&
p)
{
    int ar=abs(area(a,b,c));
    int t=abs(area(a,b,p))+abs(area(b,c,p))+abs(area(c,a,p));
    return ar==t;
}
template<class T>
void triangulate(vector<pair<T,T> > p,vector<pair<T,T> >&out)
{
    int pindx=0;
    if((int)p.size()<=3)
    {
        out.resize(p.size());
        copy(p.begin(),p.end(),out.begin());
        return;
    }
    while(p.size()>3)
    {
        int n=(int)p.size();
        int i,j,k;
        for(i=0;i<n;i++)
        {
            j=i+1;
            k=i+2;
            j=j>=n?j-n:j;
            k=k>=n?k-n:k;
            if(area(p[i],p[j],p[k])<0)
            {
                continue;
            }
            bool chk=true;

```

```

        for(int l=0;l<n;l++)
        {
            if(l==i||l==j||l==k)
                continue;
            if(inside(p[i],p[j],p[k],p[l]))
            {
                chk=false;
                break;
            }
        }
        if(chk)
            break;
        out[pindx++]=p[i];
        out[pindx++]=p[j];
        out[pindx++]=p[k];
        p.erase(p.begin()+j);
    }
    for(auto e:p)
        out[pindx++]=e;
}

```

## 5.11 vertical-decomposition

```

typedef double dbl;
const dbl eps = 1e-9;
inline bool eq(dbl x, dbl y){
    return fabs(x - y) < eps;
}
inline bool lt(dbl x, dbl y){
    return x < y - eps;
}
inline bool gt(dbl x, dbl y){
    return x > y + eps;
}
inline bool le(dbl x, dbl y){
    return x < y + eps;
}
inline bool ge(dbl x, dbl y){
    return x > y - eps;
}
struct pt{
    dbl x, y;
    inline pt operator - (const pt & p)const{
        return pt{x - p.x, y - p.y};
    }
    inline pt operator + (const pt & p)const{
        return pt{x + p.x, y + p.y};
    }
    inline pt operator * (dbl a)const{
        return pt{x * a, y * a};
    }
    inline dbl cross(const pt & p)const{
        return x * p.y - y * p.x;
    }
    inline dbl dot(const pt & p)const{
        return x * p.x + y * p.y;
    }
    inline bool operator == (const pt & p)const{
        return eq(x, p.x) && eq(y, p.y);
    }
};
struct Line{
    pt p[2];
    Line(){
        Line(pt a, pt b):p{a, b}{}
    }
    pt vec()const{
        return p[1] - p[0];
    }
    pt& operator [] (size_t i){
        return p[i];
    }
};
inline bool lexComp(const pt & l, const pt & r){
    if(fabs(l.x - r.x) > eps){
        return l.x < r.x;
    }
    else return l.y < r.y;
}

```

```

vector<pt> interSegSeg(Line l1, Line l2){
    if(eq(l1.vec().cross(l2.vec(), 0))){
        if(!eq(l1.vec().cross(l2[0] - l1[0]), 0))
            return {};
        if(!lexComp(l1[0], l1[1]))
            swap(l1[0], l1[1]);
        if(!lexComp(l2[0], l2[1]))
            swap(l2[0], l2[1]);
        pt l = lexComp(l1[0], l2[0]) ? l2[0] : l1[0];
        pt r = lexComp(l1[1], l2[1]) ? l1[1] : l2[1];
        if(l == r)
            return {l};
        else return lexComp(l, r) ? vector<pt>{l, r} :
            vector<pt>{};
    }
    else{
        dbl s = (l2[0] - l1[0]).cross(l2.vec()) /
            l1.vec().cross(l2.vec());
        pt inter = l1[0] + l1.vec() * s;
        if(ge(s, 0) && le(s, 1) && le((l2[0] - inter).dot(l2[1]
            - inter), 0))
            return {inter};
        else
            return {};
    }
}
inline char get_segtype(Line segment, pt other_point){
    if(eq(segment[0].x, segment[1].x))
        return 0;
    if(!lexComp(segment[0], segment[1]))
        swap(segment[0], segment[1]);
    return (segment[1] - segment[0]).cross(other_point -
        segment[0]) > 0 ? 1 : -1;
}
dbl union_area(vector<tuple<pt, pt, pt> > triangles){
    vector<Line> segments(3 * triangles.size());
    vector<char> segtype(segments.size());
    for(size_t i = 0; i < triangles.size(); i++){
        pt a, b, c;
        tie(a, b, c) = triangles[i];
        segments[3 * i] = lexComp(a, b) ? Line(a, b) : Line(b, a);
        segtype[3 * i] = get_segtype(segments[3 * i], c);
        segments[3 * i + 1] = lexComp(b, c) ? Line(b, c) : Line(c, b);
        segtype[3 * i + 1] = get_segtype(segments[3 * i + 1], a);
        segments[3 * i + 2] = lexComp(c, a) ? Line(c, a) : Line(a, c);
        segtype[3 * i + 2] = get_segtype(segments[3 * i + 2], b);
    }
    vector<dbl> k(segments.size()), b(segments.size());
    for(size_t i = 0; i < segments.size(); i++){
        if(segtype[i]){
            k[i] = (segments[i][1].y - segments[i][0].y) /
                (segments[i][1].x - segments[i][0].x);
            b[i] = segments[i][0].y - k[i] * segments[i][0].x;
        }
    }
    dbl ans = 0;
    for(size_t i = 0; i < segments.size(); i++){
        if(!segtype[i])
            continue;
        dbl l = segments[i][0].x, r = segments[i][1].x;
        vector<pair<dbl, int> > evts;
        for(size_t j = 0; j < segments.size(); j++){
            if(!segtype[j] || i == j)
                continue;
            dbl l1 = segments[j][0].x, r1 = segments[j][1].x;
            if(ge(l1, r) || ge(l, r1))
                continue;
            dbl common_l = max(l, l1), common_r = min(r, r1);
            auto pts = interSegSeg(segments[i], segments[j]);
            if(pts.empty()){
                dbl y1 = k[j] * common_l + b[j];
                dbl y2 = k[i] * common_r + b[i];
                if(lt(y1, y2) == (segtype[i] == 1)){
                    int evt_type = -segtype[i] * segtype[j];
                    evts.emplace_back(common_l, evt_type);
                    evts.emplace_back(common_r, -evt_type);
                }
            }
        }
    }
}

```

```

    }
    else if(pts.size() == 1u){
dbl yl = k[i] * common_l + b[i], yl1 = k[j] * common_l + b[j];
    int evt_type = -segtype[i] * segtype[j];
    if(1t(yl1, yl) == (segtype[i] == 1)){
        evts.emplace_back(common_l, evt_type);
        evts.emplace_back(pts[0].x, -evt_type);
    }
    yl = k[i] * common_r + b[i], yl1 = k[j] * common_r + b[j];
    if(1t(yl1, yl) == (segtype[i] == 1)){
        evts.emplace_back(pts[0].x, evt_type);
        evts.emplace_back(common_r, -evt_type);
    }
    }
    else{
        if(segtype[j] != segtype[i] || j > i){
            evts.emplace_back(common_l, -2);
            evts.emplace_back(common_r, 2);
        }
    }
}
evts.emplace_back(1,0); sort(evts.begin(), evts.end());
size_t j = 0; int balance = 0;
while(j < evts.size()){
    size_t ptr = j;
    while(ptr < evts.size() && eq(evts[j].first,
        evts[ptr].first)){
        balance += evts[ptr].second;
        ++ptr;
    }
    if(!balance && !eq(evts[j].first, r)){
        dbl next_x = ptr == evts.size() ? r :
            evts[ptr].first;
        ans -= segtype[i] * (k[i] * (next_x +
            evts[j].first) + 2 * b[i]) * (next_x -
            evts[j].first);
    }
    j = ptr;
}
return ans/2;
}
pair<dbl,dbl> union_perimeter(vector<tuple<pt, pt> >
    triangles){
    //Same as before
    pair<dbl,dbl> ans = make_pair(0,0);
    for(size_t i = 0; i < segments.size(); i++){
        //Same as before
        double dist=(segments[i][1].x-segments[i][0].x)
            *(segments[i][1].x-segments[i][0].x)
            +(segments[i][1].y-segments[i][0].y)
            *(segments[i][1].y-segments[i][0].y);
        dist=sqrt(dist);
        while(j < evts.size()){
            size_t ptr = j;
            while(ptr < evts.size() && eq(evts[j].first,
                evts[ptr].first)){
                balance += evts[ptr].second; ++ptr;
            }
            if(!balance && !eq(evts[j].first, r)){
                dbl next_x = ptr == evts.size() ? r :
                    evts[ptr].first;
                ans.first += dist * (next_x - evts[j].first) /
                    (r-1);
                if(eq(segments[i][1].y,segments[i][0].y))
                    ans.second+=(next_x - evts[j].first);
            }
            j = ptr;
        }
    }
    return ans;
}

```

## 6 Graph

### 6.1 DMST with solution

// not tested yet

```

const int INF = 1029384756;
#define MAXN 1000
#define FOR(i,x) for(auto i : x)
struct edge_t {
    int u,v,w;
    set< pair<int,int> > add, sub;
    edge_t() : u(-1), v(-1), w(0) {}
    edge_t(int _u, int _v, int _w) {
        u = _u;
        v = _v;
        w = _w;
        add.insert({u, v});
    }
    edge_t& operator += (const edge_t& obj) {
        w += obj.w;
        for (auto it : obj.add) {
            if (!sub.count(it)) add.insert(it);
            else sub.erase(it);
        }
        for (auto it : obj.sub) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
    edge_t& operator -= (const edge_t& obj) {
        w -= obj.w;
        for (auto it : obj.sub) {
            if (!sub.count(it)) add.insert(it);
            else sub.erase(it);
        }
        for (auto it : obj.add) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
}
eg[MAXN*MAXN], prv[MAXN], EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MAXN],contracted[MAXN];
vector<int> E[MAXN];
edge_t dmst(int rt) {
    edge_t cost;
    for (int i=0; i<N; i++) {
        contracted[i] = incyc[i] = 0;
        prv[i] = EDGE_INF;
    }
    cid = 0;
    int u,v;
    while (true) {
        for (v=0; v<N; v++) {
            if (v != rt && !contracted[v] && prv[v].w == INF)
                break;
        }
        if (v >= N) break; // end
        for (int i=0; i<M; i++) {
            if (eg[i].v == v && eg[i].w < prv[v].w)
                prv[v] = eg[i];
        }
        if (prv[v].w == INF) // not connected
            return EDGE_INF;
        cost += prv[v];
        for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
        if (u == -1) continue;
        incyc[v] = ++cid;
        for (u=prv[v].u; u!=v; u=prv[u].u) {
            contracted[u] = 1;
            incyc[u] = cid;
        }
        for (int i=0; i<M; i++) {
            if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
                cid) {
                eg[i] -= prv[eg[i].v];
            }
        }
        for (int i=0; i<M; i++) {
            if (incyc[eg[i].u] == cid) eg[i].u = v;
            if (incyc[eg[i].v] == cid) eg[i].v = v;
            if (eg[i].u == eg[i].v) eg[i]-- = eg[--M];
        }
    }
}

```

```

    for (int i=0; i<N; i++) {
        if (contracted[i]) continue;
        if (prv[i].u==0 && incyc[prv[i].u] == cid)
            prv[i].u = v;
    }
    prv[v] = EDGE_INF;
}
return cost;
}
#define F first
#define S second
void solve() {
    edge_t cost = dmst(0);
    for (auto it : cost.add) { // find a solution
        E[it.F].push_back(it.S);
        prv[it.S] = edge_t(it.F,it.S,0);
    }
}
}

```

### 6.2 DMST

```

// tested on https://lightoj.com/problem/teleport
const int inf = 1e9;
struct edge {
    int u, v, w;
    edge() {}
    edge(int a,int b,int c) : u(a), v(b), w(c) {}
    bool operator < (const edge& o) const {
        if (u == o.u)
            if (v == o.v) return w < o.w;
            else return v < o.v;
        return u < o.u;
    }
};
int dmst(vector<edge> &edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }
        lo[root] = 0;
        for (int i = 0; i < lo.size(); ++i) {
            if (i == root) continue;
            if (lo[i] == inf) return -1;
        }
        int cur_id = 0;
        vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
        for (int i = 0; i < cur_nodes; ++i) {
            ans += lo[i];
            int u = i;
            while (u != root and id[u] < 0 and mark[u] != i) {
                mark[u] = i;
                u = pi[u];
            }
            if (u != root and id[u] < 0) { // Cycle
                for (int v = pi[u]; v != u; v = pi[v]) id[v] =
                    cur_id;
                id[u] = cur_id++;
            }
        }
        if (cur_id == 0) break;
        for (int i = 0; i < cur_nodes; ++i)
            if (id[i] < 0) id[i] = cur_id++;
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            edges[i].u = id[u];
            edges[i].v = id[v];
            if (id[u] != id[v]) edges[i].w -= lo[v];
        }
        cur_nodes = cur_id;
        root = id[root];
    }
    return ans;
}

```

### 6.3 Flow With Demands

**Finding an arbitrary flow** Consider flow networks, where we additionally require the flow of each edge to have a certain amount, i.e. we bound the flow from below by a **demand** function  $d(e)$ :

$$d(e) \leq f(e) \leq c(e)$$

So next each edge has a minimal flow value, that we have to pass along the edge.

We make the following changes in the network. We add a new source  $s'$  and a new sink  $t'$ , a new edge from the source  $s'$  to every other vertex, a new edge for every vertex to the sink  $t'$ , and one edge from  $t$  to  $s$ . Additionally we define the new capacity function  $c'$  as:

- $c'((s', v)) = \sum_{u \in V} d((u, v))$  for each edge  $(s', v)$ .
- $c'((v, t')) = \sum_{w \in V} d((v, w))$  for each edge  $(v, t')$ .
- $c'((u, v)) = c((u, v)) - d((u, v))$  for each edge  $(u, v)$  in the old network.
- $c'((t, s)) = \infty$

If the new network has a saturating flow (a flow where each edge outgoing from  $s'$  is completely filled, which is equivalent to every edge incoming to  $t'$  is completely filled), then the network with demands has a valid flow, and the actual flow can be easily reconstructed from the new network. Otherwise there doesn't exist a flow that satisfies all conditions. Since a saturating flow has to be a maximum flow, it can be found by any maximum flow algorithm.

**Minimal flow** Note that along the edge  $(t, s)$  (from the old sink to the old source) with the capacity  $\infty$  flows the entire flow of the corresponding old network. I.e. the capacity of this edge effects the flow value of the old network. By giving this edge a sufficient large capacity (i.e.  $\infty$ ), the flow of the old network is unlimited. By limiting this edge by smaller capacities, the flow value will decrease. However if we limit this edge by a too small value, than the network will not have a saturated solution, e.g. the corresponding solution for the original network will not satisfy the demand of the edges. Obviously here can use a binary search to find the lowest value with which all constraints are still satisfied. This gives the minimal flow of the original network.

### 6.4 LCA

```
int n; //beware n is decalred global
int bparent[MAXN] [LOG], depth[MAXN];
bool vis[MAXN];
void dfs(int a){
    vis[a]=true;
    for(auto v: g[a]){
        if(!vis[v]){
            bparent[v][0]=a;
            depth[v]=1+depth[a];
            dfs(v);
        }
    }
}
void build_ancestor(){
```

```
    dfs(1);
    for(int i=1; (1<<i)<n; i++){
        for(int j=1; j<=n; j++){
            bparent[j][i]=bparent[bparent[j][i-1]][i-1];
        }
    }
int pth_ancestor(int a, int p){
    for(int i=0; (1<<i)<=p; i++){
        if((1<<i)&p) a=bparent[a][i];
    }
    return a;
}
int lca(int u, int v){
    if(depth[v]>depth[u])
        v=pth_ancestor(v, depth[v]-depth[u]);
    if(depth[u]>depth[v])
        u=pth_ancestor(u, depth[u]-depth[v]);
    if(u==v) return u;
    for(int i=log2(n-1); i>=0; i--){
        if(bparent[u][i]!=bparent[v][i]){
            u=bparent[u][i];
            v=bparent[v][i];
        }
    }
    return bparent[u][0];
}
```

### 6.5 articulation-vertex

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] >= tin[v] && p!=-1)
                IS_CUTPOINT(v);
            ++children;
        }
    }
    if(p == -1 && children > 1)
        IS_CUTPOINT(v);
}
void find_cutpoints() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs(i);
    }
}
```

### 6.6 bellman-ford

```
struct Edge {
    int a, b, cost;
};
int n, m;
vector<Edge> edges;
const int INF = 1000000000;
void solve(){
    vector<int> d(n);
    vector<int> p(n, -1);
    int x;
    for (int i = 0; i < n; ++i) {
        x = -1;
        for (Edge e : edges) {
            if (d[e.a] + e.cost < d[e.b]) {
                d[e.b] = d[e.a] + e.cost;
                p[e.b] = e.a;
                x = e.b;
            }
        }
    }
}
```

```
if (x == -1) {
    cout << "No negative cycle found.";
} else {
    for (int i = 0; i < n; ++i)
        x = p[x];
    vector<int> cycle;
    for (int v = x; v = p[v];) {
        cycle.push_back(v);
        if (v == x && cycle.size() > 1)
            break;
    }
    reverse(cycle.begin(), cycle.end());
    cout << "Negative cycle: ";
    for (int v : cycle)
        cout << v << ' ';
    cout << endl;
}
}
```

### 6.7 bridge

```
const int vmax = 2e5+10, emax = 2e5+10;
namespace Bridge { //edge, nodes, comps 1 indexed
    vector<int> adj[vmax]; // edge-id
    pair<int, int> edges[emax]; // (u, v)
    bool isBridge[emax];
    int visited[vmax]; // 0-unvis, 1-vising, 2-vis
    int st[vmax], low[vmax], clk = 0, edgeId = 0;
    // For bridge tree components
    int who[vmax], compId = 0;
    vector<int> stk;
    // For extra end time calc
    int en[vmax];
    void dfs(int u, int parEdge) {
        visited[u] = 1; low[u] = st[u] = ++clk;
        stk.push_back(u);
        for (auto e : adj[u]) {
            if (e == parEdge) continue;
            int v=edges[e].first^edges[e].second^u;
            if (visited[v] == 1) {
                low[u] = min(low[u], st[v]);
            } else if (visited[v] == 0) {
                dfs(v, e); low[u] = min(low[u], low[v]);
            }
        }
        visited[u] = 2;
        if(st[u] == low[u]){ // found
            ++compId; int cur;
            do{
                cur = stk.back(); stk.pop_back();
                who[cur] = compId;
            }while(cur != u);
            if(parEdge != -1){isBridge[parEdge] = true;}
        }
        en[u] = clk;
    }
    void clearAll(int n){
        for(int i = 0; i<=n; i++) {
            adj[i].clear(); visited[i] = st[i] = 0;
        }
        for(int i = 0; i<=edgeId; i++) isBridge[i]=0;
        clk = compId = edgeId = 0;
    }
    void findBridges(int n){
        for(int i = 1; i<=n; i++){
            if(visited[i] == 0) dfs(i, -1);
        }
    }
    bool isReplacable(int eid, int u, int v){
        if(!isBridge[eid]) return true;
        int a=edges[eid].first, b=edges[eid].second;
        if(st[a] > st[b]) swap(a, b);
        return (st[b] <= st[u] && st[u] <= en[b])
            != (st[b] <= st[v] && st[v] <= en[b]);
    }
    void addEdge(int u, int v){
        edgeId++; edges[edgeId] = {u, v};
        adj[u].emplace_back(edgeId);
        adj[v].emplace_back(edgeId);
    }
}
```



## 6.8 edmond-blossom

```

/**Copied from https://codeforces.com/blog/entry/49402**/
/*
GETS:
V->number of vertices
E->number of edges
pair of vertices as edges (vertices are 1..V)
GIVES:
output of edmonds() is the maximum matching
match[i] is matched pair of i (-1 if there isn't a matched pair)
*/
const int M=500;
struct struct_edge
{
    int v;
    struct_edge* n;
};
typedef struct_edge* edge;
struct_edge pool[M*M*2];
edge top=pool,adj[M];
int V,E,match[M],qh,qt,q[M],father[M],base[M];
bool inq[M],inb[M],ed[M][M];
void add_edge(int u,int v)
{
    top->v=v,top->n=adj[u],adj[u]=top++;
    top->v=u,top->n=adj[v],adj[v]=top++;
}
int LCA(int root,int u,int v)
{
    static bool inp[M];
    memset(inp,0,sizeof(inp));
    while(1)
    {
        inp[u=base[u]]=true;
        if (u==root) break;
        u=father[match[u]];
    }
    while(1)
    {
        if (inp[v=base[v]]) return v;
        else v=father[match[v]];
    }
}
void mark_blossom(int lca,int u)
{
    while (base[u]!=lca)
    {
        int v=match[u];
        inb[base[u]]=inb[base[v]]=true;
        u=father[v];
        if (base[u]!=lca) father[u]=v;
    }
}
void blossom_contraction(int s,int u,int v)
{
    int lca=LCA(s,u,v);
    memset(inb,0,sizeof(inb));
    mark_blossom(lca,u);
    mark_blossom(lca,v);
    if (base[u]!=lca)
        father[u]=v;
    if (base[v]!=lca)
        father[v]=u;
    for (int u=0; u<V; u++)
        if (inb[base[u]])
        {
            base[u]=lca;
            if (!inq[u])
                inq[q[++qt]=u]=true;
        }
}
int find_augmenting_path(int s)
{
    memset(inq,0,sizeof(inq));
    memset(father,-1,sizeof(father));
    for (int i=0; i<V; i++) base[i]=i;
}

```

```

inq[q[qh=qt=0]=s]=true;
while (qh<=qt)
{
    int u=q[qh++];
    for (edge e=adj[u]; e; e=e->n)
    {
        int v=e->v;
        if (base[u]!=base[v]&&match[u]!=v)
            if ((v==s)|| (match[v]!=-1 &&
                father[match[v]]!=-1))
                blossom_contraction(s,u,v);
            else if (father[v]==-1)
            {
                father[v]=u;
                if (match[v]==-1)
                    return v;
                else if (!inq[match[v]])
                    inq[q[++qt]=match[v]]=true;
            }
        }
    }
    return -1;
}
int augment_path(int s,int t)
{
    int u=t,v,w;
    while (u!=-1)
    {
        v=father[u];
        w=match[v];
        match[v]=u;
        match[u]=v;
        u=w;
    }
    return t!=-1;
}
int edmonds()//Gives number of matchings
{
    int matchc=0;
    memset(match,-1,sizeof(match));
    for (int u=0; u<V; u++)
        if (match[u]==-1)
            matchc+=augment_path(u,find_augmenting_path(u));
    return matchc;
}
//To add edge add_edge(u-1,v-1); ed[u-1][v-1]=ed[v-1][u-1]=true;

```

## 6.9 euler-path

```

int main() {
    int n;
    vector<vector<int>> g(n, vector<int>(n));
    // reading the graph in the adjacency matrix
    vector<int> deg(n);
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j)
            deg[i] += g[i][j];
    }
    int first = 0;
    while (first < n && !deg[first])
        ++first;
    if (first == n) {
        cout << -1;
        return 0;
    }
    int v1 = -1, v2 = -1;
    bool bad = false;
    for (int i = 0; i < n; ++i) {
        if (deg[i] & 1) {
            if (v1 == -1)
                v1 = i;
            else if (v2 == -1)
                v2 = i;
            else
                bad = true;
        }
    }
    if (v1 != -1)
        ++g[v1][v2], ++g[v2][v1];
    stack<int> st;
    st.push(first);
}

```

```

vector<int> res;
while (!st.empty()) {
    int v = st.top();
    int i;
    for (i = 0; i < n; ++i)
        if (g[v][i])
            break;
    if (i == n) {
        res.push_back(v);
        st.pop();
    } else {
        --g[v][i];
        --g[i][v];
        st.push(i);
    }
}
if (v1 != -1) {
    for (size_t i = 0; i + 1 < res.size(); ++i) {
        if ((res[i] == v1 && res[i + 1] == v2) ||
            (res[i] == v2 && res[i + 1] == v1)) {
            vector<int> res2;
            for (size_t j = i + 1; j < res.size(); ++j)
                res2.push_back(res[j]);
            for (size_t j = 1; j <= i; ++j)
                res2.push_back(res[j]);
            res = res2;
            break;
        }
    }
}
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
        if (g[i][j])
            bad = true;
    }
}
if (bad) {
    cout << -1;
} else {
    for (int x : res)
        cout << x << " ";
}
}

```

## 6.10 hopcraft-karp

```

/** Source: https://iq.opengenus.org/hopcroft-karp-algorithm/
**/
// A class to represent Bipartite graph for
// Hopcroft Karp implementation
class BGraph{
    // m and n are number of vertices on left
    // and right sides of Bipartite Graph
    int m, n;
    // adj[u] stores adjacents of left side
    // vertex 'u'. The value of u ranges from 1 to m.
    // 0 is used for dummy vertex
    std::list<int> *adj;
    // pointers for hopcroftKarp()
    int *pair_u, *pair_v, *dist;
public:
    BGraph(int m, int n); // Constructor
    void addEdge(int u, int v); // To add edge
    // Returns true if there is an augmenting path
    bool bfs();
    // Adds augmenting path if there is one beginning
    // with u
    bool dfs(int u);
    // Returns size of maximum matching
    int hopcroftKarpAlgorithm();
};
// Returns size of maximum matching
int BGraph::hopcroftKarpAlgorithm(){
    // pair_u[u] stores pair of u in matching on left side of
    // Bipartite Graph.
    // If u doesn't have any pair, then pair_u[u] is NIL
    pair_u = new int[m + 1];
    // pair_v[v] stores pair of v in matching on right side of
    // Bipartite Graph.
}

```

```

int v = *it;
// Follow the distances set by BFS search
if (dist[pair_v[v]] == dist[u] + 1){
    // If dfs for pair of v also return true then
    if (dfs(pair_v[v]) == true){ // new matching
        possible, store the matching
        pair_v[v] = u;
        pair_u[u] = v;
        return true;
    }
}
}
// If there is no augmenting path beginning with u then
dist[u] = INF;
return false;
}
return true;
}
// Constructor for initialization
BGraph::BGraph(int m, int n){
    this->m = m;
    this->n = n;
    adj = new std::list<int>[m + 1];
}
// function to add edge from u to v
void BGraph::addEdge(int u, int v){
    adj[u].push_back(v); // Add v to us list.
}
}

```

---

## 6.11 hungarian-algorithm

---

```

class HungarianAlgorithm{
    int N,inf,n,max_match;
    int *lx,*ly,*xy,*yx,*slack,*slackx,*prev;
    int **cost;
    bool *S,*T;
    void init_labels(){
        for(int x=0;x<n;x++) lx[x]=0;
        for(int y=0;y<n;y++) ly[y]=0;
        for (int x = 0; x < n; x++){
            for (int y = 0; y < n; y++){
                lx[x] = max(lx[x], cost[x][y]);
            }
        }
        void update_labels(){
            int x, y, delta = inf; //init delta as infinity
            for (y = 0; y < n; y++) //calculate delta using slack
                if (!T[y])
                    delta = min(delta, slack[y]);
            for (x = 0; x < n; x++) //update X labels
                if (S[x]) lx[x] -= delta;
            for (y = 0; y < n; y++) //update Y labels
                if (T[y]) ly[y] += delta;
            for (y = 0; y < n; y++) //update slack array
                if (!T[y])
                    slack[y] -= delta;
        }
        void add_to_tree(int x, int prevx)
        //x - current vertex,prevx - vertex from X before x in the
        //alternating path,
        //so we add edges (prevx, xy[x]), (xy[x], x){
            S[x] = true; //add x to S
            prev[x] = prevx; //we need this when augmenting
            for (int y = 0; y < n; y++) //update slacks, because we
            //add new vertex to S
                if (lx[x] + ly[y] - cost[x][y] < slack[y]){
                    slack[y] = lx[x] + ly[y] - cost[x][y];
                    slackx[y] = x;
                }
        }
        void augment() //main function of the algorithm{
            if (max_match == n) return; //check wether matching is
            //already perfect
            int x, y, root; //just counters and root vertex
            int q[N], wr = 0, rd = 0; //q - queue for bfs, wr,rd -
            //write and read
            //pos in queue
            //memset(S, false, sizeof(S)); //init set S
            for(int i=0;i<n;i++) S[i]=false;
            //memset(T, false, sizeof(T)); //init set T

```

```

for(int i=0;i<n;i++) T[i]=false;
//memset(prev,-1,sizeof(prev)); //init set prev - for
//the alternating tree
for(int i=0;i<n;i++) prev[i]=-1;
for (x = 0; x < n; x++) //finding root of the tree{
    if (xy[x] == -1){
        q[wr++] = root = x;
        prev[x] = -2;
        S[x] = true;
        break;
    }
}
for (y = 0; y < n; y++) //initializing slack array{
    slack[y] = lx[root] + ly[y] - cost[root][y];
    slackx[y] = root;
}
while (true) //main cycle{
    while (rd < wr) //building tree with bfs cycle{
        x = q[rd++]; //current vertex from X part
        for (y = 0; y < n; y++) //iterate through all
            edges in equality graph{
                if (cost[x][y] == lx[x] + ly[y] && !T[y]){
                    if (yx[y] == -1) break; //an exposed
                    vertex in Y found, so
//augmenting path exists!
                    T[y] = true; //else just add y to T,
                    q[wr++] = yx[y]; //add vertex yx[y],
                    which is matched
//with y, to the queue
                    add_to_tree(yx[y], x); //add edges (x,y)
                    and (y,yx[y]) to the tree
                }
            }
        if (y < n) break; //augmenting path found!
    }
    if (y < n) break; //augmenting path found!
    update_labels(); //augmenting path not found, so
    improve labeling
    wr = rd = 0;
    for (y = 0; y < n; y++){
        //in this cycle we add edges that were added to
        //the equality graph as a
//result of improving the labeling, we add edge (slackx[y], y)
//to the tree if
//and only if !T[y] && slack[y] == 0, also with this edge we
//add another one
//((y, yx[y]) or augment the matching, if y was exposed
        if (!T[y] && slack[y] == 0){
            if (yx[y] == -1) //exposed vertex in Y found
            - augmenting path exists!{
                x = slackx[y];
                break;
            }
            else{
                T[y] = true; //else just add y to T,
                if (!S[yx[y]]){
                    q[wr++] = yx[y]; //add vertex yx[y],
                    which is matched with
//y, to the queue
                    add_to_tree(yx[y], slackx[y]); //and
                    add edges (x,y) and (y,
//yx[y]) to the tree
                }
            }
        }
        if (y < n) break; //augmenting path found!
    }
    if (y < n) //we found augmenting path!{
        max_match++; //increment matching
//in this cycle we inverse edges along augmenting path
        for (int cx = x, cy = y, ty; cx != -2; cx =
            prev[cx], cy = ty){
            ty = xy[cx];
            yx[cy] = cx;
            xy[cx] = cy;
        }
    }
}

```

```

    augment(); //recall function, go to step 1 of the
               algorithm
}
} //end of augment() function
public:
HungarianAlgorithm(int vv,int inf=1000000000){
    N=vv;
    n=N;
    max_match=0;
    this->inf=inf;
    lx=new int[N];
    ly=new int[N]; //labels of X and Y parts
    xy=new int[N]; //xy[x] - vertex that is matched with x,
    yx=new int[N]; //yx[y] - vertex that is matched with y
    slack=new int[N]; //as in the algorithm description
    slackx=new int[N]; //slackx[y] such a vertex, that
    l(slackx[y]) + l(y) - w(slackx[y],y) = slack[y]
    prev=new int[N]; //array for memorizing alternating paths
    S=new bool[N];
    T=new bool[N]; //sets S and T in algorithm
    cost=new int*[N]; //cost matrix
    for(int i=0; i<N; i++){
        cost[i]=new int[N];
    }
}
~HungarianAlgorithm(){
    delete []lx;
    delete []ly;
    delete []xy;
    delete []yx;
    delete []slack;
    delete []slackx;
    delete []prev;
    delete []S;
    delete []T;
    int i;
    for(i=0; i<N; i++){
        delete []cost[i];
    }
    delete []cost;
}
void setCost(int i,int j,int c){
    cost[i][j]=c;
}
int* matching(bool first=true){
    int *ans;
    ans=new int[N];
    for(int i=0; i<N; i++){
        if(first) ans[i]=xy[i];
        else ans[i]=yx[i];
    }
    return ans;
}
int hungarian(){
    int ret = 0; //weight of the optimal matching
    max_match = 0; //number of vertices in current matching
    for(int x=0; x<n; x++) xy[x]=-1;
    for(int y=0; y<n; y++) yx[y]=-1;
    init_labels(); //step 0
    augment(); //steps 1-3
    for (int x = 0; x < n; x++) //forming answer there
        ret += cost[x][xy[x]];
    return ret;
}
};

```

## 6.12 max-flow-dinic

```

#include<bits/stdc++.h>
#include<vector>
using namespace std;
#define MAX 100
#define HUGE_FLOW 1000000000
#define BEGIN 1
#define DEFAULT_LEVEL 0
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u),
    cap(cap) {}
};

```

```

};
struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;
    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }
    void add_edge(int v, int u, long long cap) {
        edges.emplace_back(v, u, cap);
        edges.emplace_back(u, v, 0);
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }
    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }
    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size();
            cid++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap -
                edges[id].flow < 1)
                continue;
            long long tr = dfs(u, min(pushed, edges[id].cap -
                edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }
    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
};
int main(){
    return 0;
}

```

## 6.13 min-cost-max-flow

```

struct Edge{ int from, to, capacity, cost; };
vector<vector<int>>> adj, cost, capacity;
const int INF = 1e9;
void shortest_paths(int n,int v0,vector<int>&d,vector<int>&p){
    d.assign(n, INF); d[v0] = 0; vector<bool> inq(n, false);
    queue<int> q; q.push(v0); p.assign(n, -1);
    while (!q.empty()) {
        int u = q.front(); q.pop(); inq[u] = false;
        for (int v : adj[u]) {
            if (capacity[u][v] > 0 && d[v] > d[u]+cost[u][v]){
                d[v] = d[u] + cost[u][v]; p[v] = u;
                if (!inq[v]) { inq[v] = true; q.push(v); }
            }
        }
    }
}
int min_cost_flow(int N,vector<Edge> edges,int K,int s,int t) {
    adj.assign(N,vector<int>());cost.assign(N,vector<int>(N,0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to); adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost; cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity; }
    int flow = 0; int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p); if (d[t] == INF) break;
        // find max flow on that path
        int f = K - flow; int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]); cur = p[cur];
        }
        // apply flow
        flow += f; cost += f * d[t]; cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f; capacity[cur][p[cur]]+= f;
            cur = p[cur]; }
    }
    if (flow < K) return -1;
    else return cost;
}

```

## 6.14 online-bridge

```

vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
int bridges; int lca_iteration;
vector<int> last_visit;
void init(int n) {
    par.resize(n); dsu_2ecc.resize(n); dsu_cc.resize(n);
    dsu_cc_size.resize(n); lca_iteration=0; last_visit.assign(n,0);
    for (int i=0; i<n; ++i) {
        dsu_2ecc[i] = i; dsu_cc[i] = i; dsu_cc_size[i] = 1;
        par[i] = -1;
    }
    bridges = 0;
}
int find_2ecc(int v) {
    if (v == -1) return -1;
    return dsu_2ecc[v]==v?v:dsu_2ecc[v]=find_2ecc(dsu_2ecc[v]);
}
int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
}
void make_root(int v) {
    v = find_2ecc(v); int root = v; int child = -1;
    while (v != -1) {
        int p = find_2ecc(par[v]); par[v] = child;
        dsu_cc[v] = root; child = v; v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}
void merge_path (int a, int b) {
    ++lca_iteration; vector<int> path_a, path_b; int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a); path_a.push_back(a);
            if (last_visit[a] == lca_iteration){

```

```

    lca = a; break; }
    last_visit[a] = lca_iteration; a = par[a];
}
if (b != -1) {
    b = find_2ecc(b); path_b.push_back(b);
    if (last_visit[b] == lca_iteration){
        lca = b; break; }
    last_visit[b] = lca_iteration; b = par[b];
}
}
for (int v : path_a) {
    dsu_2ecc[v] = lca; if (v == lca) break; --bridges;
}
for (int v : path_b) {
    dsu_2ecc[v] = lca; if (v == lca) break; --bridges;
}
}
void add_edge(int a, int b) {
    a = find_2ecc(a); b = find_2ecc(b);
    if (a == b) return;
    int ca = find_cc(a); int cb = find_cc(b);
    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b); swap(ca, cb); }
        make_root(a); par[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}
}

```

## 6.15 scc + 2 Sat

```

namespace SCC { //Everything 0-indexed.
const int N = 2e6+7; int which[N], vis[N], cc;
vector<int> adj[N], adjr[N]; vector<int> order;
void addEdge(int u, int v) {
    adj[u].push_back(v); adjr[v].push_back(u);
}
void dfs1(int u){
    if (vis[u]) return; vis[u] = true;
    for(int v: adj[u]) dfs1(v); order.push_back(u);
}
void dfs2(int u, int id) {
    if(vis[u]) return; vis[u] = true;
    for(int v: adjr[u]) dfs2(v, id); which[u] = id;
}
int last = 0;
void findSCC(int n) {
    cc=0,last=n; order.clear(); fill(vis, vis+n, 0);
    for(int i=0; i<n; i++) if(!vis[i]) dfs1(i);
    reverse(order.begin(), order.end());
    fill(vis, vis+n, 0);
    for (int u: order) {
        if (vis[u]) continue; dfs2(u, cc); ++cc;
    }
}
void clear() {
    for (int i=0; i<last; i++)
        adj[i].clear(), adjr[i].clear();
}
}
struct TwoSat {
    int n; int vars = 0; vector<bool> ans;
    TwoSat(int n) : n(n), ans(n) {
        SCC::clear(); vars = 2*n;
    }
    void implies(int x, int y) {
        SCC::addEdge(x, y); SCC::addEdge(y^1, x^1);
    }
    void OR(int x, int y) {
        SCC::addEdge(x^1, y); SCC::addEdge(y^1, x);
    }
    void XOR(int x, int y) {
        implies(x, y^1); implies(x^1, y);
    }
    void atmostOne(vector<int> v) {
        int k = v.size();
        for (int i=0; i<k; i++) {

```

```

        if (i+1<k) implies(vars+2*i, vars+2*i+2);
        implies(v[i], vars+2*i);
        if (i>0) implies(v[i], vars+2*i-1);
    }
    vars += 2*k;
}
bool solve() {
    SCC::findSCC(vars); ans.resize(vars/2);
    for (int i=0; i<vars; i+=2) {
        if (SCC::which[i]==SCC::which[i+1])return 0;
        if (i<2*n)
            ans[i/2] = SCC::which[i]>SCC::which[i+1];
    }
    return true;
}
}

```

## 7 Math

### 7.1 BerleKampMassey

```

const int MOD = 998244353;
vector<LL> berlekampMassey(vector<LL> s) {
    if (s.empty()) return {};
    int n = s.size(), L = 0, m = 0;
    vector<LL> C(n), B(n), T;
    C[0] = B[0] = 1; LL b = 1;
    for (int i = 0; i < n; ++i) {
        ++m; LL d = s[i] % MOD;
        for (int j = 1; j <= L; ++j) d = (d + C[j] * s[i - j]) % MOD;
        if (!d) continue;
        T = C; LL coeff = d * bigMod(b, -1) % MOD;
        for (int j = m; j < n; ++j) C[j] = (C[j] - coeff * B[j - m]) % MOD;
        if (2*L > i) continue;
        L = i+1-L, B = T, b = d, m = 0;
    }
    C.resize(L + 1), C.erase(C.begin());
    for (LL &x : C) x = (MOD - x) % MOD;
    return C;
}

```

### 7.2 FloorSum

```

LL mod(LL a, LL m) {
    LL ans = a%m;
    return ans < 0 ? ans+m : ans;
}
///Sum(floor((ax+b)/m)) for i=0 to n-1, (n,m >= 0)
LL floorSum(LL n, LL m, LL a, LL b) {
    LL ra = mod(a, m), rb = mod(b, m), k = (ra+n+rb);
    LL ans = ((a-ra)/m) * n*(n-1)/2 + ((b-rb)/m) * n;
    if (k < m) return ans;
    return ans + floorSum(k/m, ra, m, k%m);
}

```

### 7.3 On Bit Count

```

ll bitcount(ll num,int pos)
{
    ll mask=1;
    mask = mask<<pos;
    ll mval = mask&num;
    ll ans;
    if(mval==0)
    {
        ans = num;
        ans = ans>>(pos+1);
        ans = (ans*mask);
    }
    else
    {
        ll tmp = num;
        tmp = tmp>>(pos+1);
        tmp++;
        ans=(tmp*mask);
        tmp=tmp<<(pos+1);
        tmp--;
        tmp=tmp-num;
        ans=ans-tmp;
    }
}

```

```

}
return ans;
}

```

### 7.4 Stern Brocot Tree

```

//finds x/y with min y st: L <= (x/y) < R
pair<LL,LL>solve(LD L, LD R){
    pair<LL, LL> l(0, 1), r(1, 1);
    if(L==0.0) return l; // corner case
    while(true) {
        pair<int, int> m(l.x+r.x, l.y+r.y);
        if(m.x<L*m.y){ // move to the right
            LL kl=1, kr=1;
            while(l.x+kr*r.x <= L*(l.y+kr*r.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(l.x+km*r.x < L*(l.y+km*r.y)) kl=km+1;
                else kr=km;
            }
            l={l.x+(kl-1)*r.x,l.y+(kl-1)*r.y};
        }
        else if(m.x>R*m.y){//move to left
            LL kl=1, kr=1;
            while(r.x+kr*l.x>R*(r.y+kr*l.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(r.x+km*l.x>R*(r.y+km*l.y)) kl = km+1;
                else kr = km;
            }
            r={r.x+(kl-1)*l.x,r.y+(kl-1)*l.y};
        }
        else return m;
    }
}

```

### 7.5 Sum Of Kth Power

```

LL mod; LL S[105][105];
// Find 1^k+2^k+...+n^k % mod
void solve() {
    LL n, k;
    /* x^k = sum (i=1 to k) Stirling2(k, i) * i! * ncr(x, i)
    sum (x = 0 to n) x^k
    = sum (i=0 to k) Stirling2(k, i) * i! * sum (x=0 to n)ncr(x, i)
    = sum (i=0 to k) Stirling2(k, i) * i! * ncr(n+1, i+1)
    = sum (i=0 to k) Stirling2(k, i) * i! * (n+1)! / (i+1)!/(n-i)!
    = sum (i=0 to k) Stirling2(k, i) * (n-i+1) *
      (n-i+2) * ... (n+1) / (i+1) */
    S[0][0] = 1 % mod;
    for (int i = 1; i <= k; i++) {
        for (int j = 1; j <= i; j++) {
            if (i == j) S[i][j] = 1 % mod;
            else S[i][j] = (j * S[i-1][j] + S[i-1][j-1]) % mod;
        }
    }
    LL ans = 0;
    for (int i = 0; i <= k; i++) {
        LL fact = 1, z = i+1;
        for (LL j = n-i+1; j <= n+1; j++) {
            LL mul = j;
            if (mul % z == 0) {
                mul /= z; z /= z;
            }
            fact = (fact * mul) % mod;
        }
        ans = (ans + S[k][i] * fact) % mod;
    }
}

```

### 7.6 combination-generator

```

bool next_combination(vector<int>& a, int n) {
    int k = (int)a.size();
    for (int i = k-1; i >= 0; i--) {
        if (a[i] < n-k+i+1) {
            a[i]++;
            for (int j = i+1; j < k; j++)
                a[j] = a[j-1] + 1;
            return true;
        }
    }
}

```

```

    }
    return false;
}
vector<int> ans;
void gen(int n, int k, int idx, bool rev) {
    if (k > n || k < 0)
        return;
    if (!n) {
        for (int i = 0; i < idx; ++i) {
            if (ans[i])
                cout << i + 1;
        }
        cout << "\n";
        return;
    }
    ans[idx] = rev;
    gen(n - 1, k - rev, idx + 1, false);
    ans[idx] = !rev;
    gen(n - 1, k - !rev, idx + 1, true);
}
void all_combinations(int n, int k) {
    ans.resize(n);
    gen(n, k, 0, false);
}

```

## 7.7 continued-fractions

```

auto fraction(int p, int q) {
    vector<int> a;
    while(q) {
        a.push_back(p / q);
        tie(p, q) = make_pair(q, p % q);
    }
    return a;
}
auto convergents(vector<int> a) {
    vector<int> p = {0, 1};
    vector<int> q = {1, 0};
    for(auto it: a) {
        p.push_back(p[p.size() - 1] * it + p[p.size() - 2]);
        q.push_back(q[q.size() - 1] * it + q[q.size() - 2]);
    }
    return make_pair(p, q);
}

```

## 7.8 convolution

```

//zeta transform or sos dp
void zeta(vll& d, int m) {
    int n = 1 < m;
    for(int len=2; len<=n; len*=2) {
        for(int i=0; i<n; i+=len) {
            int l2=len>>1;
            for(int j=i; j<i+l2; ++j) {
                d[j+l2] += d[j];
            }
        }
    }
    //zeta inverse or mobius transform
    void zinv(vll& d, int m) {
        int n = 1 < m;
        for(int len=2; len<=n; len*=2) {
            for(int i=0; i<n; i+=len) {
                int l2=len>>1;
                for(int j=i; j<i+l2; ++j) {
                    d[j+l2] -= d[j];
                }
            }
        }
    }
    //subset sum convolution
    //not fully tested, got some error if used with mod
#define MAX_SIZE 1<<20
    ll f[MAX_SIZE];
    ll g[MAX_SIZE];
    ll res[MAX_SIZE];
    ll fhat[20][MAX_SIZE];
    ll ghat[20][MAX_SIZE];
    ll h[20][MAX_SIZE];
    void subsetConvolution(int m) {
        int n = 1 < m;
        memset(fhat, 0, sizeof(fhat));
        memset(ghat, 0, sizeof(ghat));
        memset(res, 0, sizeof(res));
        memset(h, 0, sizeof(h));
        for(int i=0; i<n; ++i) {

```

```

        fhat[__builtin_popcount(i)][i] = f[i];
        ghat[__builtin_popcount(i)][i] = g[i];
        for (int i=0; i<=m; i++) {
            zeta(fhat[i], m);
            zeta(ghat[i], m);
            for (int j=0; j<=i; j++) {
                for (int mask = 0; mask < n; mask++) {
                    h[i][mask] += fhat[j][mask] * ghat[i-j][mask];
                }
                zinv(h[i], m);
            }
            for(int i=0; i<n; ++i)
                res[i] = h[__builtin_popcount(i)][i];
        }
    }
}

```

## 7.9 crt anachor

```

/// Chinese remainder theorem (special case): find z st z%m1 =
r1, z%m2 = r2.
/// z is unique modulo M = lcm(m1, m2). Returns (z, M). On
failure, M = -1.
PLL CRT(LL m1, LL r1, LL m2, LL r2) {
    LL s, t;
    LL g = egcd(m1, m2, s, t);
    if (r1%g != r2%g) return PLL(0, -1);
    LL M = m1*m2;
    LL ss = ((s*r2)%m2)*m1;
    LL tt = ((t*r1)%m1)*m2;
    LL ans = ((ss+tt)%M+M)%M;
    return PLL(ans/g, M/g);
}
// expected: 23 105
// 11 12
PLL ans = CRT({3,5,7}, {2,3,2});
cout << ans.first << " " << ans.second << endl;
ans = CRT({4,6}, {3,5});
cout << ans.first << " " << ans.second << endl;

```

## 7.10 discrete-root

```

#define MAX 100000
int prime[MAX+1], Phi[MAX+1];
vector<int> pr;
void sieve() {
    for (int i=2; i<=N; ++i) {
        if (prime[i] == 0) {
            prime[i] = i;
            pr.push_back(i);
        }
        for (int j=0; j<(int)pr.size() && pr[j] <= prime[i]
            && i*pr[j] <= N; ++j) {
            prime[i * pr[j]] = pr[j];
        }
    }
}
void PhiWithSieve() {
    int i;
    for(i=2; i<=MAX; i++){
        if(prime[i]==i){
            Phi[i]=i-1;
        }
        else if((i/prime[i])%prime[i]==0){
            Phi[i]=Phi[i/prime[i]]*prime[i];
        }
        else{
            Phi[i]=Phi[i/prime[i]]*(prime[i]-1);
        }
    }
}
int powmod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 1ll * a % p), --b;
        else
            a = int (a * 1ll * a % p), b >>= 1;
    return res;
}
int PrimitiveRoot(int p){
    vector<int> fact;
    int phi=Phi[p];
    int n=phi;

```

```

while(n>1){
    if(prime[n]==n){
        fact.push_back(n);
        n=1;
    }
    else{
        int f=prime[n];
        while(n%f==0){
            n=n/f;
        }
        fact.push_back(f);
    }
}
int res;
for(res=p-1; res>1; --res){
    for(n=0; n<fact.size(); n++){
        if(powmod(res, phi/fact[n], p)==1){
            break;
        }
    }
    if(n==fact.size()) return res;
}
return -1;
}
ll DiscreteLog(ll a, ll b, ll mod){
    ll n, p, q;
    n = (ll) sqrt((double) mod) + 1ll;
    ll an = powmod(a, n, mod);
    map<ll, ll> map_;
    ll cur = an;
    for(p=1; p<=n; p++){
        if(map_.find(cur) == map_.end()){
            map_.insert({cur, p});
        }
        cur = (cur * an) % mod;
    }
    cur = b;
    ll mn = LLONG_MAX;
    for(q=0; q<=n; q++){
        if(map_.find(cur) != map_.end()){
            mn = min((n * map_[cur]) - q, mn);
        }
        cur = (cur * a) % mod;
    }
    if(mn == LLONG_MAX) return -1;
    else return mn;
}
vector<int> DiscreteRoot(int n, int a, int k){
    int g = PrimitiveRoot(n);
    vector<int> ans;
    int any_ans = DiscreteLog(powmod(g, k, n), a, n);
    if (any_ans == -1) {
        return ans;
    }
    int delta = (n-1) / gcd(k, n-1);
    for (int cur = any_ans % delta; cur < n-1; cur += delta)
        ans.push_back(powmod(g, cur, n));
    sort(ans.begin(), ans.end());
    return ans;
}

```

## 7.11 fast-walsh-hadamard

```

void FWHT(vector<LL> &p, bool inv) {
    int n = p.size(); assert((n&(n-1))==0);
    for (int len=1; 2*len<=n; len<=1) {
        for (int i=0; i<n; i+=len*len) {
            for (int j=0; j<len; j++) {
                LL u = p[i+j], v = p[i+len+j];
                //XOR p[i+j]=u+v, p[i+len+j]=u-v;
                //OR if(!inv) p[i+j]=v, p[i+len+j]=u+v;
                //OR else p[i+j]=-u+v, p[i+len+j]=u;
                //AND if(!inv) p[i+j]=u+v, p[i+len+j]=u;
                //AND else p[i+j]=v, p[i+len+j]=u-v;
            }
        }
    }
    //XOR if(inv) for(int i=0; i<n; i++) p[i]/=n;
}

```

```
vector<LL> convo(vector<LL> a, vector<LL> b) {
    int n = 1, sz = max(a.size(), b.size());
    while(n<sz) n*=2;
    a.resize(n); b.resize(n); vector<LL>res(n, 0);
    FWHT(a, 0); FWHT(b, 0);
    for(int i=0; i<n; i++) res[i] = a[i] * b[i];
    FWHT(res, 1);
    return res;
}
```

## 7.12 fft

```
struct CD {
    double x, y;
    CD(double x=0, double y=0) :x(x), y(y) {}
    CD operator+(const CD& o) { return {x+o.x, y+o.y};}
    CD operator-(const CD& o) { return {x-o.x, y-o.y};}
    CD operator*(const CD& o) { return {x*o.x-y*o.y,
        x*o.y+o.x*y};}
    void operator /= (double d) { x/=d; y/=d;}
    double real() {return x;}
    double imag() {return y;}
    CD conj(const CD &c) {return CD(c.x, -c.y);}
    const double PI = acos(-1.0L);
    namespace FFT {
        int N;
        vector<int> perm;
        vector<CD> wp[2];
        void precalculate(int n) {
            assert((n & (n-1)) == 0);
            N = n;
            perm = vector<int>(N, 0);
            for (int k=1; k<N; k<=1) {
                for (int i=0; i<k; i++) {
                    perm[i] <= 1;
                    perm[i+k] = 1 + perm[i];}
            }
            wp[0] = wp[1] = vector<CD>(N);
            for (int i=0; i<N; i++) {
                wp[0][i] = CD( cos(2*PI*i/N), sin(2*PI*i/N) );
                wp[1][i] = CD( cos(2*PI*i/N), -sin(2*PI*i/N) );}
            void fft(vector<CD> &v, bool invert = false) {
                if (v.size() != perm.size()) precalculate(v.size());
                for (int i=0; i<N; i++)
                    if (i < perm[i])
                        swap(v[i], v[perm[i]]);
                for (int len = 2; len <= N; len *= 2) {
                    for (int i=0; i<N; i+=len) {
                        for (int j=0, idx=0; j<len/2; j++, idx += len) {
                            CD x = v[i+j];
                            CD y = wp[invert][idx]*v[i+j+len/2];
                            v[i+j] = x+y;
                            v[i+j+len/2] = x-y;}}
                }
                if (invert) {
                    for (int i=0; i<N; i++) v[i]/=N;}}
            void pairfft(vector<CD> &a, vector<CD> &b, bool invert =
                false) {
                int N = a.size();
                vector<CD> p(N);
                for (int i=0; i<N; i++) p[i] = a[i] + b[i] * CD(0, 1);
                fft(p, invert);
                p.push_back(p[0]);

                for (int i=0; i<N; i++) {
                    if (invert) {
                        a[i] = CD(p[i].real(), 0);
                        b[i] = CD(p[i].imag(), 0);}
                    else {
                        a[i] = (p[i]+conj(p[N-i]))*CD(0.5, 0);
                        b[i] = (p[i]-conj(p[N-i]))*CD(0, -0.5);}}
            }
        }
    }
    vector<ll> multiply(const vector<ll> &a, const vector<ll>
        &b) {
        int n = 1;
        while (n < a.size() + b.size()) n<=1;
        vector<CD> fa(a.begin(), a.end()), fb(b.begin(), b.end());
        fa.resize(n); fb.resize(n);
        //    fft(fa); fft(fb);
        pairfft(fa, fb);
        for (int i=0; i<n; i++) fa[i] = fa[i] * fb[i];
    }
}
```

```
fft(fa, true);
vector<ll> ans(n);
for (int i=0; i<n; i++) ans[i] = round(fa[i].real());
return ans;}
const int M = 1e9+7, B = sqrt(M)+1;
vector<ll> anyMod(const vector<ll> &a, const vector<ll> &b) {
    int n = 1;
    while (n < a.size() + b.size()) n<=1;
    vector<CD> al(n), ar(n), bl(n), br(n);
    for (int i=0; i<a.size(); i++) al[i] = a[i]%M/B, ar[i] =
        a[i]%M%B;
    for (int i=0; i<b.size(); i++) bl[i] = b[i]%M/B, br[i] =
        b[i]%M%B;
    pairfft(al, ar); pairfft(bl, br);
    //    fft(al); fft(ar); fft(bl); fft(br);
    for (int i=0; i<n; i++) {
        CD l1 = (al[i] * bl[i]), lr = (al[i] * br[i]);
        CD r1 = (ar[i] * bl[i]), rr = (ar[i] * br[i]);
        al[i] = l1; ar[i] = lr;
        bl[i] = r1; br[i] = rr;}
    pairfft(al, ar, true); pairfft(bl, br, true);
    //    fft(al, true); fft(ar, true); fft(bl, true); fft(br,
        true);
    vector<ll> ans(n);
    for (int i=0; i<n; i++) {
        ll right = round(br[i].real()), left =
            round(al[i].real());
        ll mid = round(bl[i].real()) + round(ar[i].real());
        ans[i] = ((left%M)*B*B + (mid%M)*B + right)%M;
    }
    return ans;}}
```

## 7.13 formulas

### Binomial Coefficient List

- $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m+1}{m}.$
- $\binom{n}{0} + \binom{n-1}{1} + \dots + \binom{n-k}{k} + \dots + \binom{0}{n} = F_{n+1}$

### Catalan's Triangle

- $C(n, 0) = 1, n \geq 0.$
- $C(n, 1) = n, n \geq 1.$
- $C(n+1, k) = C(n+1, k-1) + C(n, k), 1 < k < n+1.$
- $C(n, k) = \binom{n+k}{k} - \binom{n+k}{k-1} = \frac{n-k+1}{n+1} \binom{n+k}{k}.$

### Fibonacci Numbers

- $F_{n-1}F_{n+1} - F_n^2 = (-1)^n.$
- $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n.$

## 7.14 integer-factorization

```
typedef long long LL;
typedef unsigned long long ULL;
namespace Rho {
    ULL mult(ULL a, ULL b, ULL mod) {
        LL ret = a * b - mod * (ULL)(1.0L/mod*a*b);
        return ret+mod*(ret<0) - mod*(ret>=(LL) mod);
    }
    ULL power(ULL x, ULL p, ULL mod){
        ULL s=1, m=x;
        while(p) {
            if(p&1) s = mult(s, m, mod);
            p>>=1; m = mult(m, m, mod);
        }
        return s;
    }
}
```

```
vector<LL> bases =
{2,325, 9375, 28178, 450775, 9780504, 1795265022};
bool isprime(LL n) {
    if (n<2) return 0;
    if (n%2==0) return n==2;
    ULL s = __builtin_ctzll(n-1), d = n>>s;
    for (ULL x: bases) {
        ULL p = power(x%n, d, n), t = s;
        while (p!=1&&p!=n-1&&x%n&&t-->0) p=mult(p,p,n);
        if (p!=n-1 && t != s) return 0;
    }
    return 1;
}
mt19937_64 rng(chrono::system_clock::
    now().time_since_epoch().count());
ULL FindFactor(ULL n) {
    if (n == 1 || isprime(n)) return n;
    ULL c=1, x=0, y=0, t=0, prod = 2, x0 = 1, q;
    auto f = [&](ULL X) { return mult(X, X, n) + c;};
    while (t++ % 128 or __gcd(prod, n) == 1) {
        if (x == y) c = rng()%(n-1)+1, x = x0, y=f(x);
        if ((q=mult(prod, max(x, y) - min(x, y), n)))
            prod = q;
        x = f(x), y = f(y));
    }
    return __gcd(prod, n);
}
```

## 7.15 integration-simpson

```
const int N = 1000 * 1000; // number of steps (already
    multiplied by 2)
double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final
        Simpson's formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}
```

## 7.16 linear-diophantine-equation-gray-code

```
int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
bool find_any_solution(int a, int b, int c, int &x0, int &y0,
    int &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
void shift_solution(int &x, int &y, int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}
int find_all_solutions(int a, int b, int c, int minx, int
    maxx, int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;
    int sign_a = a > 0 ? +1 : -1;
```

```

int sign_b = b > 0 ? +1 : -1;
shift_solution(x, y, a, b, (minx - x) / b);
if (x < minx)
    shift_solution(x, y, a, b, sign_b);
if (x > maxx)
    return 0;
int lx1 = x;
shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
int rx1 = x;
shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
if (y > maxy)
    return 0;
int lx2 = x;
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
int rx2 = x;
if (lx2 > rx2)
    swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx)
    return 0;
return (rx - lx) / abs(b) + 1;
}
int g (int n) {
    return n ^ (n >> 1);
}
int rev_g (int g) {
    int n = 0;
    for (; g; g >>= 1)
        n ^= g;
    return n;
}

```

## 7.17 linear-equation-system

```

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity
// or a big number
int gauss (vector< vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<=m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }
    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
}

```

```

    return 1;
}
7.18 math
/*****finding all factor below n in O(nlogn)*****/
int *pfactor;
void build(int n){//prime factor of every number below n
    pfactor=new int[n+1];
    memset(pfactor,0,sizeof(int)*(n+1));
    int i,j;
    for(i=2;i<=n;i++){
        if(pfactor[i]==0){
            for(j=i;j<=n;j+=i){
                pfactor[j]=i;}}
    }
    return;
}
int get_p_factor(vector<int>& pf,vector<int>& pfp,int n){//pf
    //and pfp must have size>log(n) returns number of prime
    //factor
    int i=0;
    int j,pw;
    while(n>1){
        j=pfactor[n];
        pw=0;
        while(!(n%j)){
            n/=j;
            pw++;}
        pf[i]=j;
        pfp[i]=pw;
        i++;}
    return i;
}
int get_all_factor(vector<int>& pf,vector<int>& pfp,int
    sz,vector<int> &vct){
    vct[0]=1;
    int i,j,k,l,r,s=1;
    for(i=0;i<sz;i++){
        l=0;
        for(j=0;j<pfp[i];j++){
            r=s;
            for(k=1;k<r;k++,s++){
                vct[s]=(vct[k]*pfp[i]);}
            l=r;}}
    return s;
}
/*****General multiplicative function*****/
int *mf;
int base_case(int p,int k){//base case for p^k
    return k+1;
}
void comp_mult_func(int n){
    mf=new int[n+1];
    memset(mf,-1,sizeof(int)*(n+1));
    int i,k,k2;l=1;
    mf[1]=1;
    for(i=2;i<=n;i++){
        if(mf[i]==-1){
            for(l=i+1;l<=n;l+=i)
                mf[l]=-i;
            l=i;k=1;
            while(l<=n){
                mf[l]=base_case(i,k);
                k++;
                l*=i;}}
    }
    for(i=2;i<=n;i++){
        if(mf[i]<0){
            mf[i]=-mf[i];
            k=i;
            while(!(k%mf[i])){
                k/=mf[i];
            }
            mf[i]=mf[k]*mf[i/k];}}
    return;
}
7.19 nCr mod p^a
LL F[1000009];
void pre(LL mod,LL pp){ // mod is pp^a, pp is prime
    F[0] = 1;
    REPL(i,1,mod){ // we keep in F factorial with
        // the terms which are coprime with pp
        if(i%pp!= 0) F[i]=(F[i-1]*i)%mod;
        else F[i]=F[i-1];
    }
}

```

```

}
LL fact2(LL nn,LL mod){
    LL cycle = nn/mod;
    LL n2=nn%mod;
    return (bigmod(F[mod],cycle,mod)*F[n2])%mod;
}
// returns highest power of pp that divides N and the coprime
// with pp part of N! %mod
PLL fact(LL N,LL pp,LL mod){
    LL nn = N; LL ord = 0;
    while(nn > 0){nn /= pp;ord += nn;}
    LL ans = 1; nn = N;
    while(nn > 0){ ans =(ans*fact2(nn,mod))%mod;
        nn/=pp;}
    return MP(ord,ans);
}
LL ncrp(ULL n,ULL r,LL prm,LL pr){ //ncr mod prm^pr
    LL mod=bigmod(prm,pr,INF),temp;
    pre(mod,prm);
    PLL x=fact(n,prm,mod),y=fact(r,prm,mod),z=fact(n-r,prm,mod);
    LL guun=x.second*modInverse(y.second,mod,prm);
    guun%=mod;guun*=modInverse(z.second,mod,prm);
    guun%=mod;
    LL guun2=x.first-y.first-z.first;
    guun*=bigmod(prm,guun2,mod);
    guun%=mod;
    return guun;
}

```

## 7.20 ntt

```

7340033 = 7 * 2^20, G = 3
645922817 = 77 * 2^23, G = 3
897581057 = 107 * 2^23, G = 3
998244353 = 119 * 2^23, G = 3
namespace NTT {
    vector<int> perm, wp[2];
    const int mod = 998244353, G = 3; ///G is the primitive root
    // of M
    int root, inv, N, invN;
    int power(int a, int p) {
        int ans = 1;
        while (p) {
            if (p & 1) ans = (1LL*ans*a)%mod;
            a = (1LL*a*a)%mod;
            p >>= 1;
        }
        return ans;
    }
    void precalculate(int n) {
        assert( (n&(n-1)) == 0 && (mod-1)%n==0);
        N = n;
        invN = power(N, mod-2);
        perm = wp[0] = wp[1] = vector<int>(N);

        perm[0] = 0;
        for (int k=1; k<N; k<=1)
            for (int i=0; i<k; i++) {
                perm[i] <= 1;
                perm[i+k] = 1 + perm[i];}
        root = power(G, (mod-1)/N);
        inv = power(root, mod-2);
        wp[0][0]=wp[1][0]=1;
        for (int i=1; i<N; i++) {
            wp[0][i] = (wp[0][i-1]*1LL*root)%mod;
            wp[1][i] = (wp[1][i-1]*1LL*inv)%mod;}}
    void fft(vector<int> &v, bool invert = false) {
        if (v.size() != perm.size()) precalculate(v.size());
        for (int i=0; i<N; i++)
            if (i < perm[i])
                swap(v[i], v[perm[i]]);
        for (int len = 2; len <= N; len *= 2) {
            for (int i=0, d = N/len; i<N; i+=len) {
                for (int j=0, idx=0; j<len/2; j++, idx += d) {
                    int x = v[i+j];
                    int y = (wp[invert][idx] * 1LL*v[i+j+len/2])%mod;
                    v[i+j] = (x+y)%mod ? x+y-mod : x+y;
                    v[i+j+len/2] = (x-y)%mod ? x-y : x-y+mod;}}
            if (invert) {
                for (int &x : v) x = (x*1LL*invN)%mod;}}
    }
}

```

```
vector<int> multiply(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < a.size() + b.size()) n<=<=1;
    a.resize(n);
    b.resize(n);
    fft(a);
    fft(b);
    for (int i=0; i<n; i++) a[i] = (a[i] * 1LL * b[i])%mod;
    fft(a, true);
    return a;};
```

## 7.21 primality-test

```
using u64 = uint64_t;
using u128 = __uint128_t;
u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1; base %= mod;
    while (e) {
        if (e & 1) result = (u128)result * base % mod;
        base = (u128)base * base % mod; e >>= 1;
    }
    return result;
}
bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1) return false;
    }
    return true;
};
// returns true if n is prime, else returns false.
bool MillerRabin(u64 n) {
    if (n < 2) return false;
    int r = 0; u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1; r++;
    }
    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (n == a) return true;
        if (check_composite(n, a, d, r)) return false;
    }
    return true;
}
```

## 7.22 prime counting function

```
#define MAXN 500
#define MAXM 100010
#define MAXP 666666
#define MAX 10000010
#define ll long long int
#define chkbit(ar,i) (((ar[(i)>>6])&(1<<(((i)>>1)&31))))
#define setbit(ar,i) (((ar[(i)>>6])|=(1<<(((i)>>1)&31))))
#define isprime(x) (((x)&&((x)&1)&&(!chkbit(ar,(x))))||((x)==2))
namespace pc{
    long long dp[MAXN][MAXM];
    unsigned int ar[(MAX>>6)+5] = {0};
    int len=0, primes[MAXP], counter[MAX];
    void Sieve(){ setbit(ar,0), setbit(ar,1);
        for (int i=3;(i*i)<MAX;i++){
            if(!chkbit(ar,i)){ int k=i<<1;
                for(int j=(i*i);j<MAX;j+=k) setbit(ar,j);
            }
        }
        for(int i=1;i<MAX;i++){ counter[i]=counter[i-1];
            if(isprime(i)) primes[len++]=i, counter[i]++;
        }
    }
    void init(){
        Sieve();
        for(int n=0;n<MAXN;n++){
            for(int m=0;m<MAXM;m++){
                if(!n) dp[n][m]=m;
                else dp[n][m]=dp[n-1][m]-dp[n-1][m/primes[n-1]];
            }
        }
    }
}
```

```
ll phi(ll m,int n){
    if(n==0) return m; if(primes[n-1]>=m) return 1;
    if(m<MAXM && n<MAXN) return dp[n][m];
    return phi(m,n-1) - phi(m/primes[n-1],n-1);
}
ll Lehmer(long long m){
    if(m<MAX) return counter[m];
    ll w,res=0; int i,a,s,c,x,y;
    s=sqrt(0.9+m), y=c=cbirt(0.9+m);
    a=counter[y], res=phi(m,a)+a-1;
    for(i=a;primes[i]<=s;i++){
        res=res-Lehmer(m/primes[i])+Lehmer(primes[i])-1;
    }
    return res;
}
```

## 8 String

### 8.1 Hashing

```
ll fmod(ll a, ll b, int md=mods[0]) {
    unsigned long long x = (long long) a * b;
    unsigned xh = (unsigned) (x >> 32), xl = (unsigned) x, d, m;
    asm(
        "div %4; \n\t"
        : "=a" (d), "=d" (m)
        : "d" (xh), "a" (xl), "r" (md)
    );
    return m;
}
void Build1(const string &str) {
    for(ll i = str.size() - 1; i >= 0; i--){
        hsh[i] = fmod(hsh[i+1],bases[j],mods[j])+str[i];
        if (hsh[i] > mods[j]) hsh[i] -= mods[j];
    }
}
ll getSingleHash(ll i, ll j) {
    assert(i <= j);
    ll tmp1 = (hsh[i] - fmod(hsh[j+1],pwbase[0][j-i+1]));
    if(tmp1 < 0) tmp1 += mods[0]; return tmp1;
}
```

### 8.2 aho-corasick

```
const int K = 26;
struct Vertex {
    int next[K]; bool leaf = false; int p = -1; char pch;
    int link = -1; int go[K];
    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> t(1);
void add_string(string const& s) { int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size(); t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf = true;
}
int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0) t[v].link = 0;
        else t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1) t[v].go[c] = t[v].next[c];
        else t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}
```

### 8.3 manacher

```
char s[MAX]; vector<int> d1(n); vector<int> d2(n);
for (int i = 0, l = 0, r = -1; i < n; i++){
    int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
    while (0 <= i - k && i + k < n && s[i - k] == s[i + k])
        k++;
    d1[i] = k--;
    if (i + k > r) { l = i - k; r = i + k; }
}
for (int i = 0, l = 0, r = -1; i < n; i++){
    int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
    while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i + k])
        k++;
    d2[i] = k--;
    if (i + k > r) { l = i - k - 1; r = i + k; }
}
```

### 8.4 palindromic tree

```
struct node {
    int next[26]; int len; int sufflink; int num; };
int len; char s[MAXN]; node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root with len 0
int suff; // max suffix palindrome
bool addLetter(int pos) {
    int cur = suff, curlen = 0; int let = s[pos] - 'a';
    while (true) {
        curlen = tree[cur].len;
        if (pos-1-curlen >= 0 && s[pos-1-curlen] == s[pos])
            break;
        cur = tree[cur].sufflink;
    }
    if (tree[cur].next[let]) {
        suff = tree[cur].next[let]; return false;
    }
    num++; suff = num; tree[num].len = tree[cur].len + 2;
    tree[cur].next[let] = num;
    if (tree[num].len == 1) { tree[num].sufflink = 2;
        tree[num].num = 1; return true;
    }
    while (true) {
        cur = tree[cur].sufflink; curlen = tree[cur].len;
        if (pos-1-curlen >= 0 && s[pos-1-curlen] == s[pos]) {
            tree[num].sufflink = tree[cur].next[let]; break;
        }
    }
    tree[num].num=1+tree[tree[num].sufflink].num; return true;
}
void initTree() {
    num = 2; suff = 2; // memset tree must
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}
int main() { gets(s); len = strlen(s); initTree();
    for (int i = 0; i < len; i++) { addLetter(i);
        ans += tree[suff].num; }
    cout << ans << endl; return 0;
}
```

### 8.5 suffix array da

```
/* sa => ith smallest suffix of the string
rak => rak[i] indicates the position of suffix(i) in the suffix
array; height => height[i] indicates the LCP of i-1 and i th
suffix; LCP of suffix(i) & suffix(j) = { L = rak[i], R = rak[j]
, min(height[L+1, R]);}*/
const int maxn = 5e5+5;
int wa[maxn],wb[maxn],wv[maxn],wc[maxn];
int r[maxn],sa[maxn],rak[maxn], height[maxn],dp[maxn][22],
jump[maxn], SIGMA = 0 ;
int cmp(int *r,int a,int b,int l)
{return r[a]==r[b]&&r[a+l]==r[b+l];}
void da(int *r,int *sa,int *sa,int n,int m){
    int i,j,p,*x=wa,*y=wb,*t;
    for(i=0;i<m;i++) wc[i]=0;
    for(i=0;i<n;i++) wc[x[i]=r[i]]++;
    for(i=1;i<m;i++) wc[i] += wc[i-1];
    for(i=n-1;i>=0;i--)sa[--wc[x[i]]] = i;
```



```

for( j= 1,p=1;p<n;j*=2,m=p){
    for(p=0,i=n-j;i<n;i++)y[p++] = i;
    for(i=0;i<n;i++)if(sa[i] >= j) y[p++] = sa[i] - j;
    for(i=0;i<n;i++)wv[i] = x[y[i]];
    for(i=0;i<m;i++) wc[i] = 0;
    for(i=0;i<n;i++) wc[wv[i]] ++;
    for(i=1;i<m;i++) wc[i] += wc[i-1];
    for(i=n-1;i>=0;i--) sa[--wc[wv[i]]] = y[i];
    for(t=x,y=t,p=1,x[sa[0]] = 0,i=1;i<n;i++)
        x[sa[i]] = cmp(y,sa[i-1],sa[i],j) ? p-1:p++;
}
}
void calheight(int *r,int *sa,int n){
    int i,j,k=0;
    for(i=1;i<=n;i++) rak[sa[i]] = i;
    for(i=0;i<n;height[rak[i+1]] = k) {
        for(k?k--:0, j=sa[rak[i]-1] ; r[i+k] == r[j+k] ; k ++);
    }
}
void initRMQ(int n){
    for(int i= 0;i<=n;i++) dp[i][0] = height[i];
    for(int j= 1; (1<<j) <= n; j ++ ){
        for(int i= 0; i + (1<<j) - 1 <= n ; i ++ ) {
            dp[i][j] = min(dp[i][j-1] ,dp[i+(1<<(j-1))][j-1]);
        }
    }
    for(int i= 1;i <= n; i ++ ) {
        int k= 0; while((1 << (k+1)) <= i) k++; jump[i] = k;
    }
}
int askRMQ(int L,int R){
    int k = jump[R-L+1];
    return min(dp[L][k] , dp[R - (1<<k) + 1][k]);
}
int main(){
    scanf("%s",s); int n = strlen(s);
    for(int i= 0; i < n; i ++ ) {
        r[i] = s[i]-'a' + 1; SIGMA = max(SIGMA, r[i]);
    }
    r[n] = 0; da(r,sa,n+1,SIGMA + 1);
    calheight(r,sa,n);
    /* don't forget SIGMA + 1. It will ruin you.*/ }

```

## 8.6 suffix-automaton

```

class SuffixAutomaton{
    bool complete; int last;
    set<char> alphabet;
    struct state{
        int len, link, endpos, first_pos,snas,height;
        long long substrings,sublen;
        bool is_clone;
        map<char, int> next;
        vector<int> inv_link;
        state(int leng=0,int li=0){
            len=leng; link=li;
            first_pos=-1; substrings=0;
            sublen=0; // length of all substrings
            snas=0; // shortest_non_appearing_string
            endpos=1; is_clone=false; height=0;
        }
    };
    vector<state> st;
    void process(int node){
        map<char, int> ::iterator mit;
        st[node].substrings=1;
        st[node].snas=st.size();
        if((int) st[node].next.size()<(int) alphabet.size())
            st[node].snas=1;
        for(mit=st[node].next.begin(); mit!=st[node].next.end();++mit){
            if(st[mit->second].substrings==0) process(mit->second);
            st[node].height=max(st[node].height,1+st[mit->second].height);
            st[node].substrings=
                st[node].substrings+st[mit->second].substrings;
            st[node].sublen=st[node].sublen
                +st[mit->second].sublen+st[mit->second].substrings;
            st[node].snas=min(st[node].snas,
                1+st[mit->second].snas);
        }
    }
}

```

```

if(st[node].link!=-1)
    st[st[node].link].inv_link.push_back(node);
}
void set_suffix_links(int node){
    int i;
    for(i=0; i<st[node].inv_link.size(); i++){
        set_suffix_links(st[node].inv_link[i]);
        st[node].endpos=
            st[node].endpos+st[st[node].inv_link[i]].endpos; }
}
void output_all_occurrences(int v,int P_length,vector<int>&pos){
    if (!st[v].is_clone)
        pos.push_back(st[v].first_pos - P_length + 1);
    for (int u : st[v].inv_link)
        output_all_occurrences(u, P_length, pos);
}
void kth_smallest(int node,int k,vector<char> &str){
    if(k==0) return;
    map<char, int> ::iterator mit;
    for(mit=st[node].next.begin(); mit!=st[node].next.end();++mit){
        if(st[mit->second].substrings<k)k=k-st[mit->second].substrings;
        else{
            str.push_back(mit->first);
            kth_smallest(mit->second,k-1,str);
            return;
        }
    }
}
int find_occurrence_index(int node,int index,vector<char>&str){
    if(index==str.size()) return node;
    if(!st[node].next.count(str[index])) return -1;
    else return find_occurrence_index(st[node].next[str[index]]
        ,index+1,str);
}
void klen_smallest(int node,int k,vector<char> &str){
    if(k==0) return;
    map<char, int> ::iterator mit;
    for(mit=st[node].next.begin(); mit!=st[node].next.end();
        ++mit){ if(st[mit->second].height>=k-1){
            str.push_back(mit->first);
            klen_smallest(mit->second,k-1,str);
            return;
        }
    }
}
void minimum_non_existing_string(int node,vector<char> &str){
    map<char, int> ::iterator mit;
    set<char>::iterator sit;
    for(mit=st[node].next.begin(),sit=alphabet.begin();
        sit!=alphabet.end(); ++sit,++mit){
        if(mit==st[node].next.end()||mit->first!=(*sit)){
            str.push_back(*sit);
            return;
        }
        else if(st[node].snas==1+st[mit->second].snas){
            str.push_back(*sit);
            minimum_non_existing_string(mit->second,str);
            return;
        }
    }
}
void find_substrings(int node,int index,vector<char> &str,
    vector<pair<long long,long long> > &sub_info){
    sub_info.push_back(make_pair(st[node].substrings,
        st[node].sublen+st[node].substrings*index));
    if(index==str.size()) return;
    if(st[node].next.count(str[index])){ find_substrings(
        st[node].next[str[index]],index+1,str,sub_info);return;
    }
    else{
        sub_info.push_back(make_pair(0,0));
    }
}
void check(){
    if(!complete){
        process(0);
        set_suffix_links(0);
        int i;
        complete=true;
    }
}

```

```

}
}
public:
    SuffixAutomaton(set<char> &alpha){
        st.push_back(state(0,-1));
        last=0;
        complete=false;
        set<char>::iterator sit;
        for(sit=alpha.begin(); sit!=alpha.end(); sit++)
            alphabet.insert(*sit);
        st[0].endpos=0;
    }
    void sa_extend(char c){
        int cur = st.size();
        st.push_back(state(st[last].len + 1));
        st[cur].first_pos=st[cur].len-1;
        int p = last;
        while (p != -1 && !st[p].next.count(c)){
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if (p == -1){
            st[cur].link = 0;
        }
        else{
            int q = st[p].next[c];
            if (st[p].len + 1 == st[q].len){
                st[cur].link = q;
            }
            else{
                int clone = st.size();
                st.push_back(state(st[p].len + 1,st[q].link));
                st[clone].next = st[q].next;
                st[clone].is_clone=true;
                st[clone].endpos=0;
                st[clone].first_pos=st[q].first_pos;
                while (p != -1 && st[p].next[c] == q){
                    st[p].next[c] = clone; p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
        complete=false;
    }
    ~SuffixAutomaton(){
        int i;
        for(i=0; i<st.size(); i++){
            st[i].next.clear();
            st[i].inv_link.clear();
        }
        st.clear();
        alphabet.clear();
    }
    void kth_smallest(int k,vector<char> &str){
        check();
        kth_smallest(0,k,str);
    }
    int FindFirstOccurrenceIndex(vector<char> &str){
        check();
        int ind=find_occurrence_index(0,0,str);
        if(ind==0) return -1;
        else if(ind==-1) return st.size();
        else return st[ind].first_pos+1-(int) str.size();
    }
    void FindAllOccurrenceIndex(vector<char> &str,vector<int>&pos){
        check();
        int ind=find_occurrence_index(0,0,str);
        if(ind!=-1) output_all_occurrences(ind,str.size(),pos);
    }
    int Occurrences(vector<char> &str){
        check();
        int ind=find_occurrence_index(0,0,str);
        if(ind==0) return 1;
        else if(ind==-1) return 0;
        else return st[ind].endpos;
    }
    void klen_smallest(int k,vector<char> &str){
        check();
    }
}

```

```
        if(st[0].height>=k) klen_smallest(0,k,str);
    }
    void minimum_non_existing_string(vector<char> &str){
        check();
        int ind=find_occurrence_index(0,0,str);
        if(ind!=-1) minimum_non_existing_string(ind,str);
    }
};
```

## 8.7 z-algorithm

```
vector<int> z_function(string s) {
    int n = (int) s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min (r - i + 1, z[i - l]);
```

```
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}
```