

*arXiv:1505.07432*

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# Classical Verification of Quantum Proofs



Zhengfeng Ji  
IQC, Waterloo

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↓  
QCIS, UTS

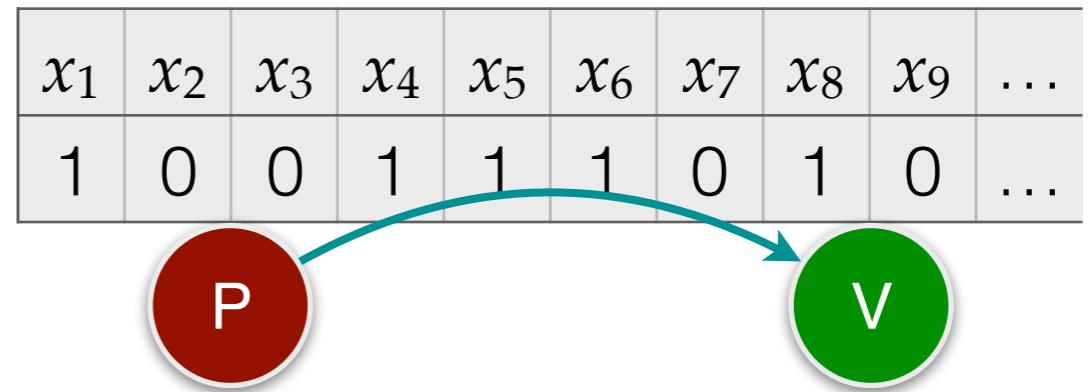
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- Proof verification is a central concept in computer science
  - **NP, IP, MIP, PCP, ...**

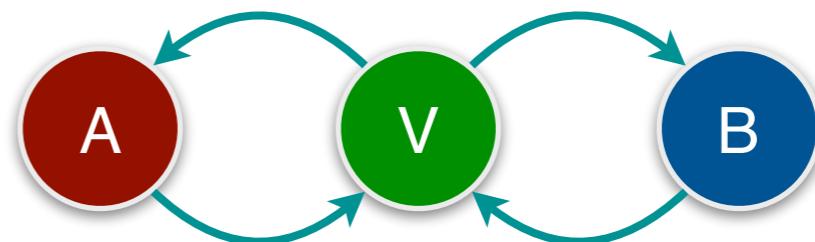
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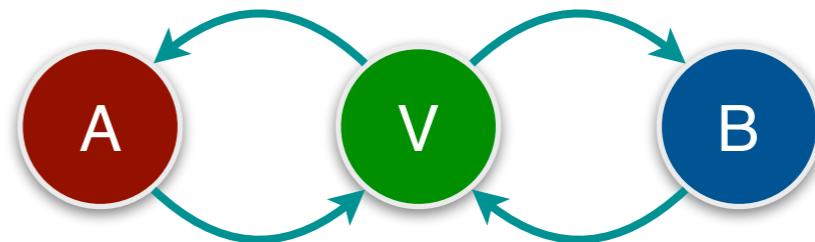
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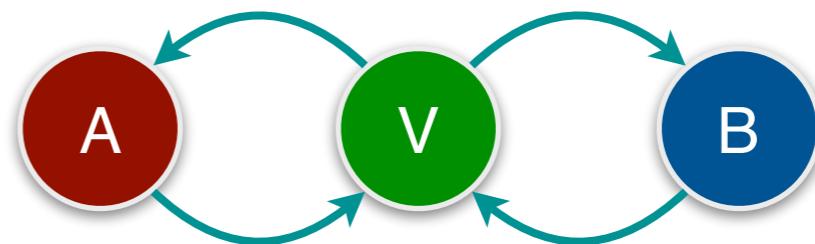
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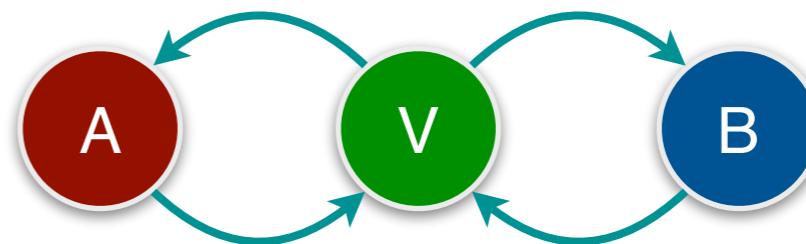
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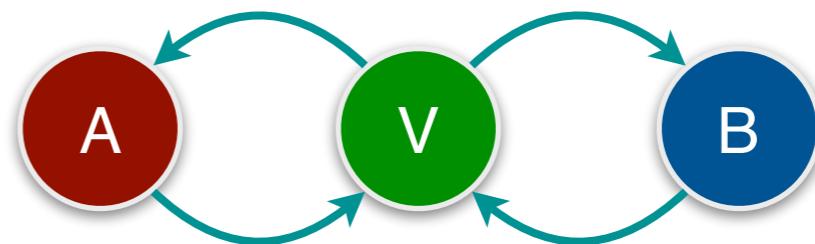
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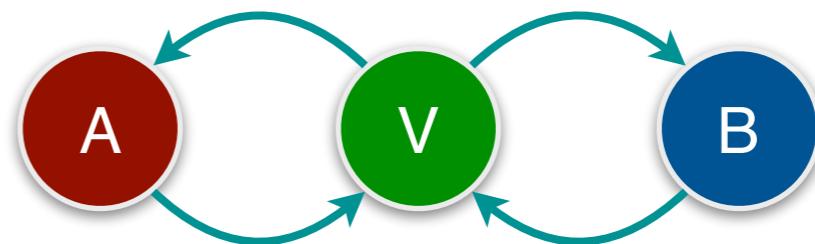
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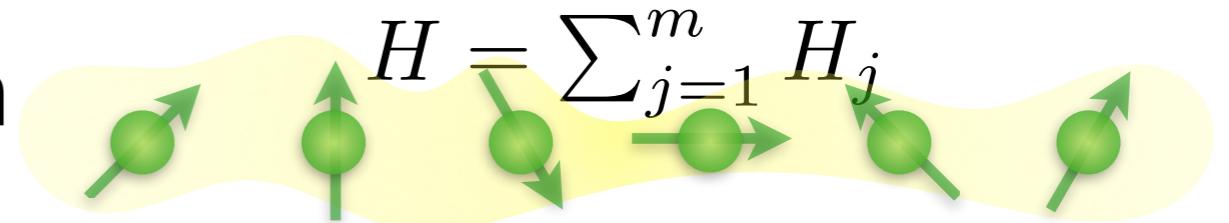
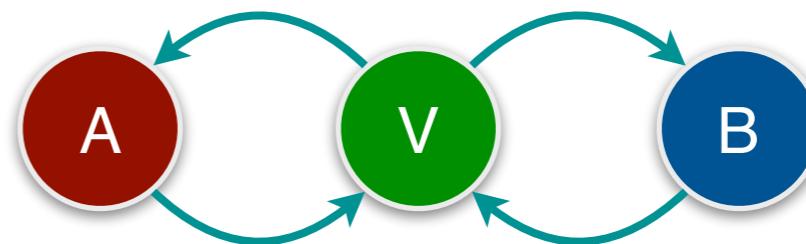
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$$H = \sum_{j=1}^m H_j$$

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Multi-player one-round  
games for NP

# Multi-player one-round games for NP

- Proof verification without seeing the whole proof

$$(x_1 \vee x_3 \vee x_5) \wedge (x_2 \vee \neg x_3 \vee \neg x_5) \wedge \dots$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	...
-	0	0	-	1	-	-	-	-	...

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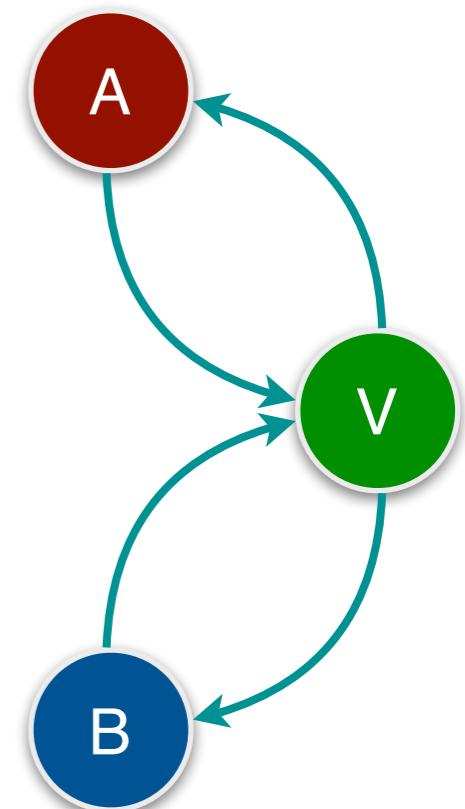
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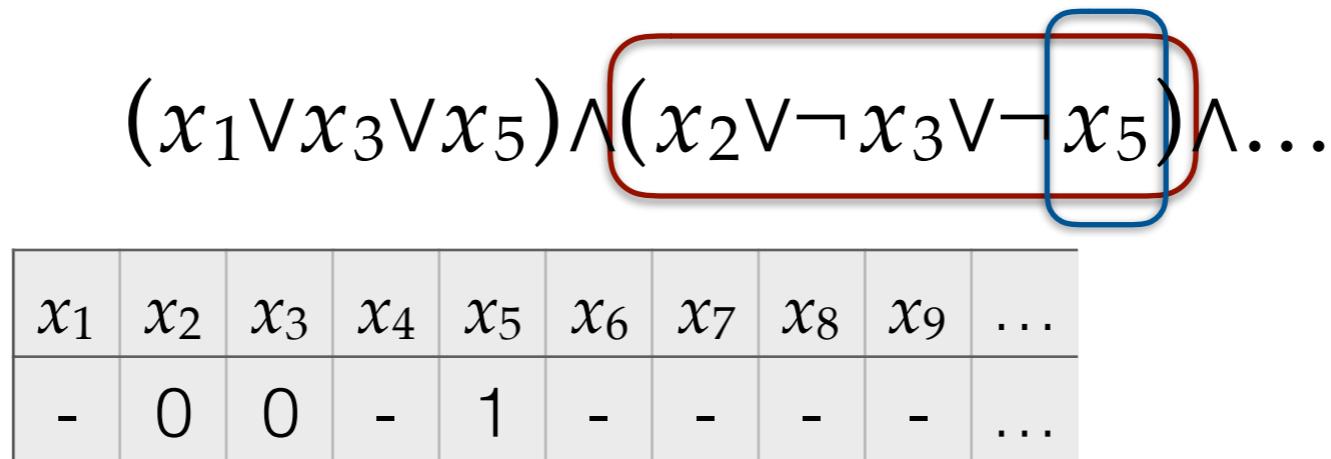
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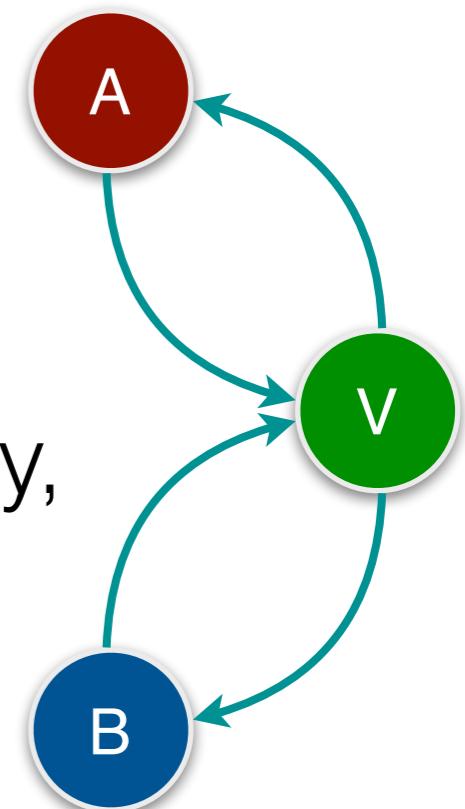
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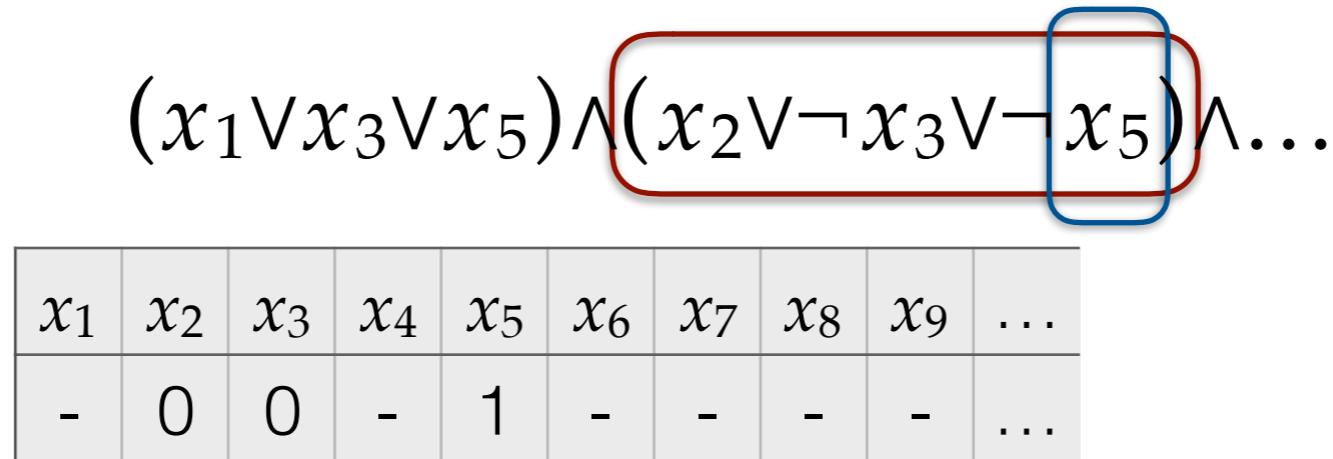
- The power of the **second** prover

- Query a variable in the clause randomly, check consistency (**oracularization**)

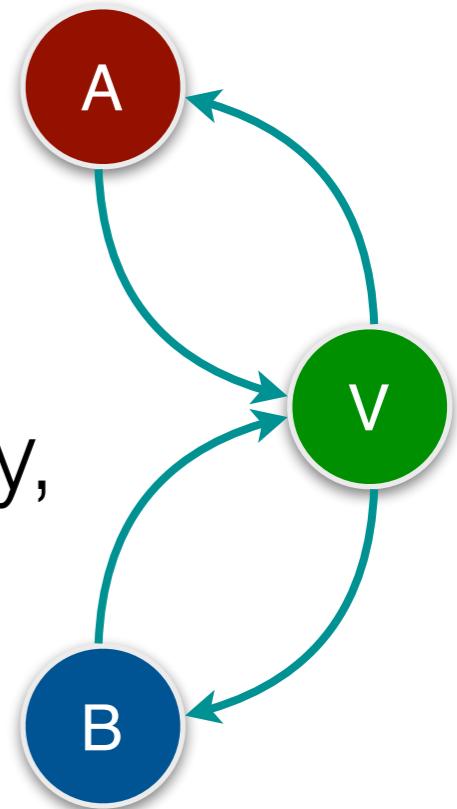


# Multi-player one-round games for NP

- Proof verification without seeing the whole proof



- The power of the **second** prover
  - Query a variable in the clause randomly, check consistency (**oracularization**)
  - **NP**-hardness of multi-player games

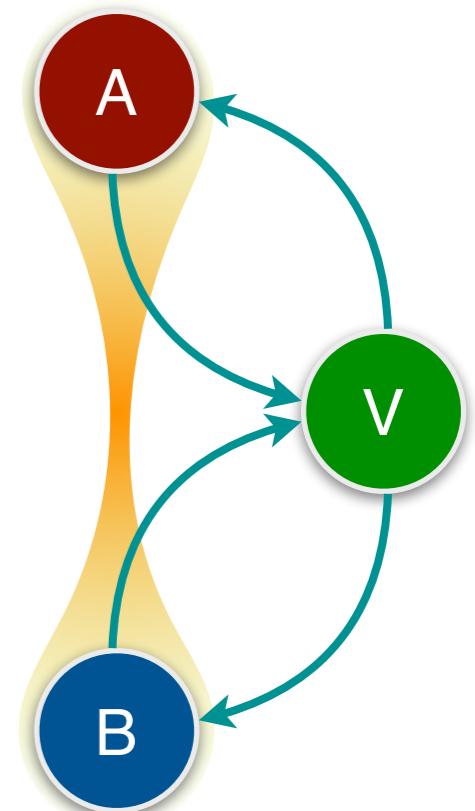


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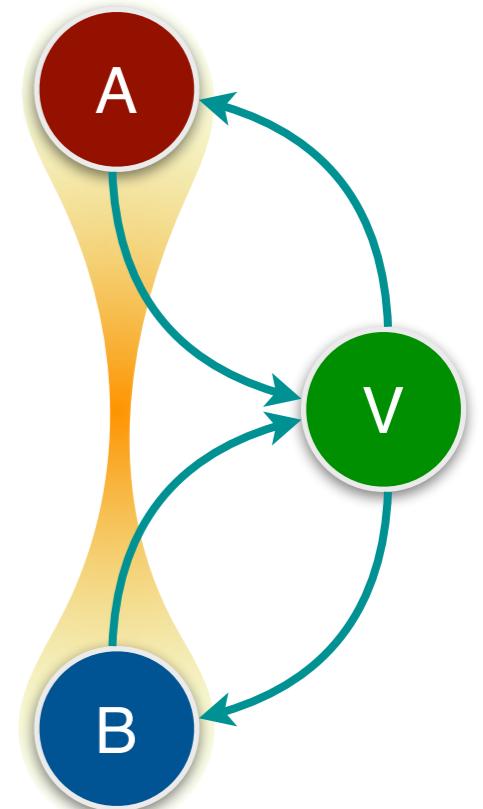
- Bell inequalities
- Entanglement can either **weaken** or strengthen the expressive power

[Cleve et al. 04]



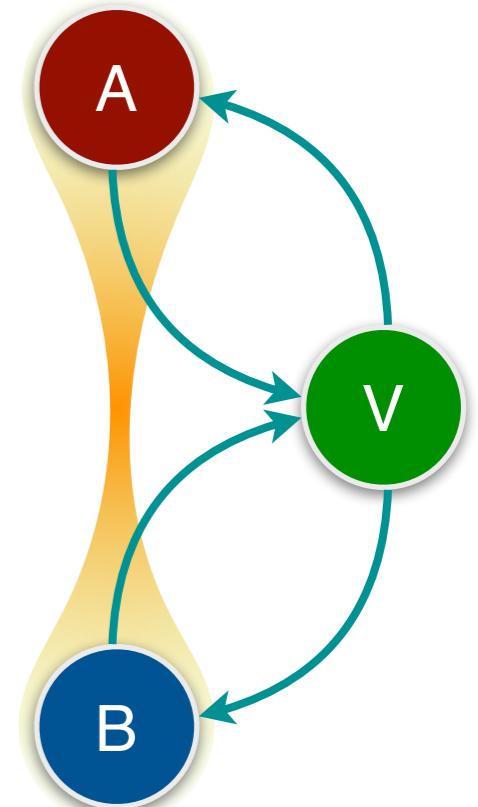
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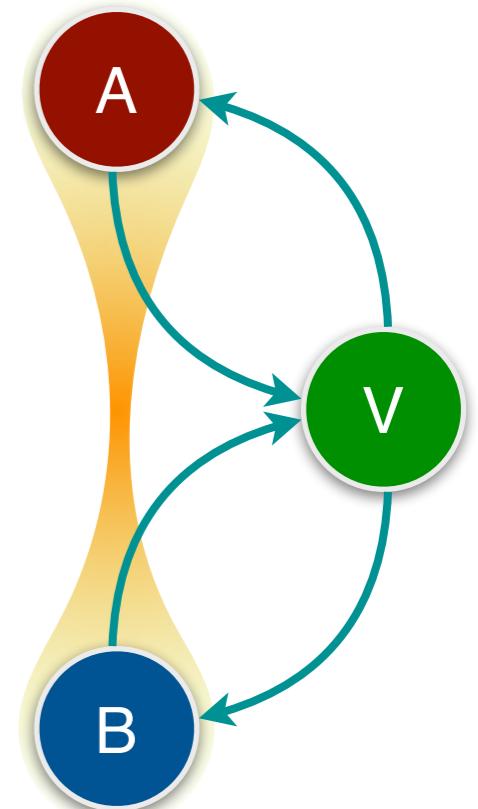
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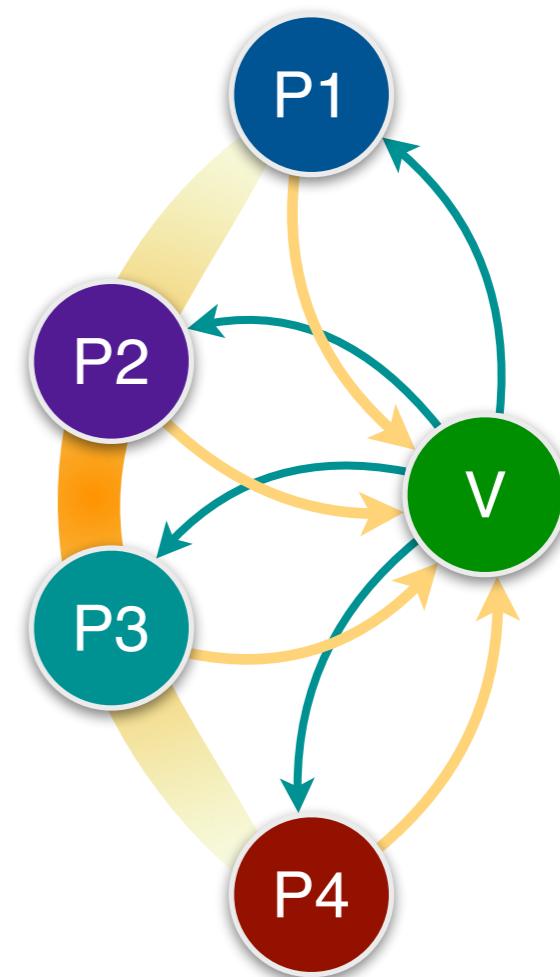
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# Multi-player games for QMA

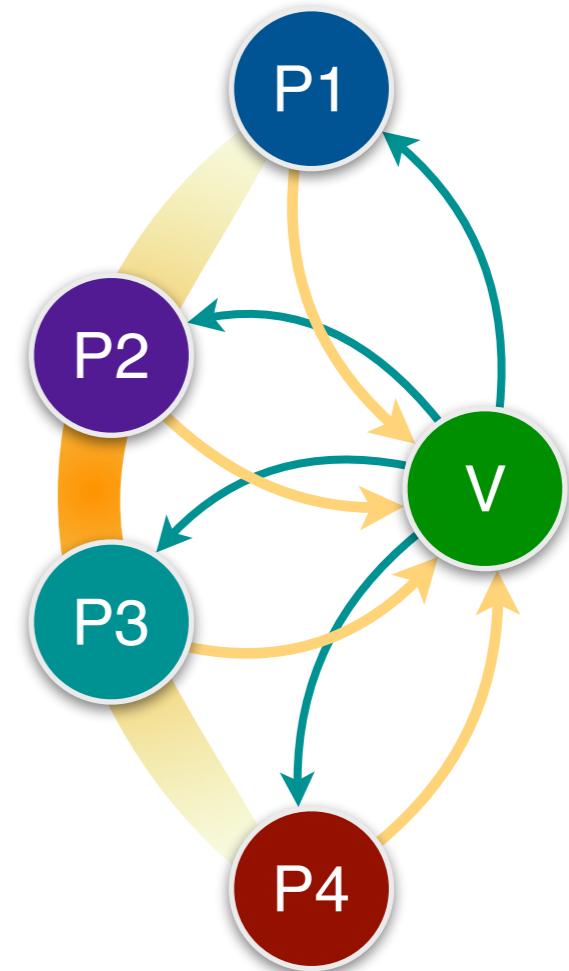
# Multi-player games for QMA

- Fitzsimons-Vidick protocol



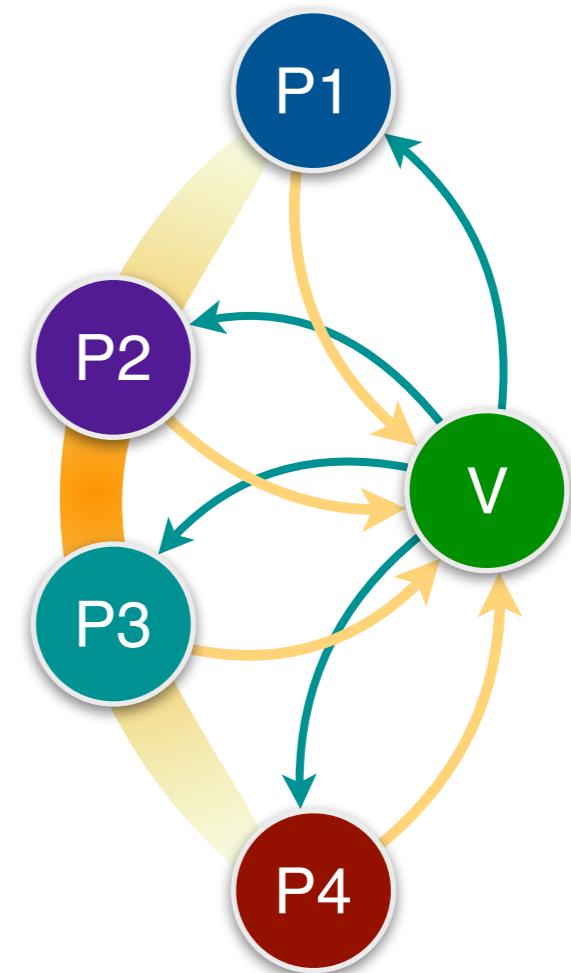
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- Encode the proof using the 4-qubit quantum error detecting code and do the following with equal probability:
  - Perform the **encoding** check
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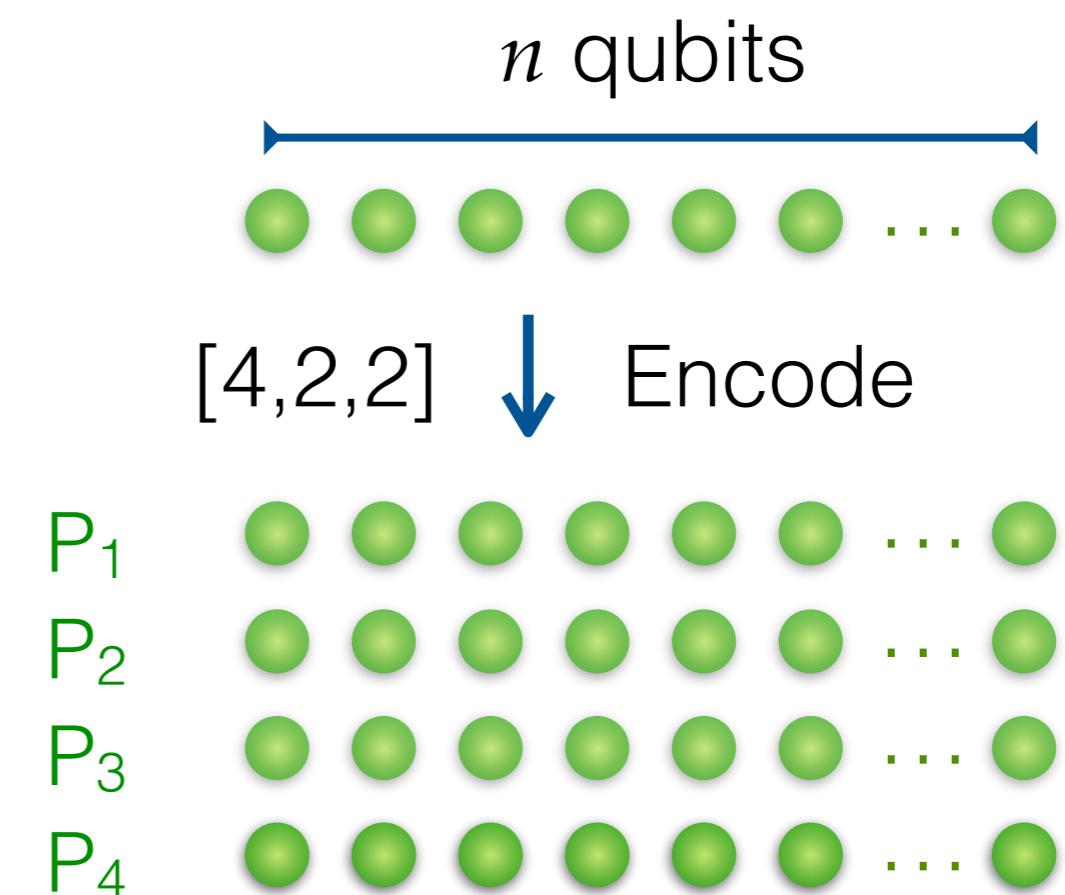
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- Quantum **oracularization**
  - Classical oracularization as an error detecting code



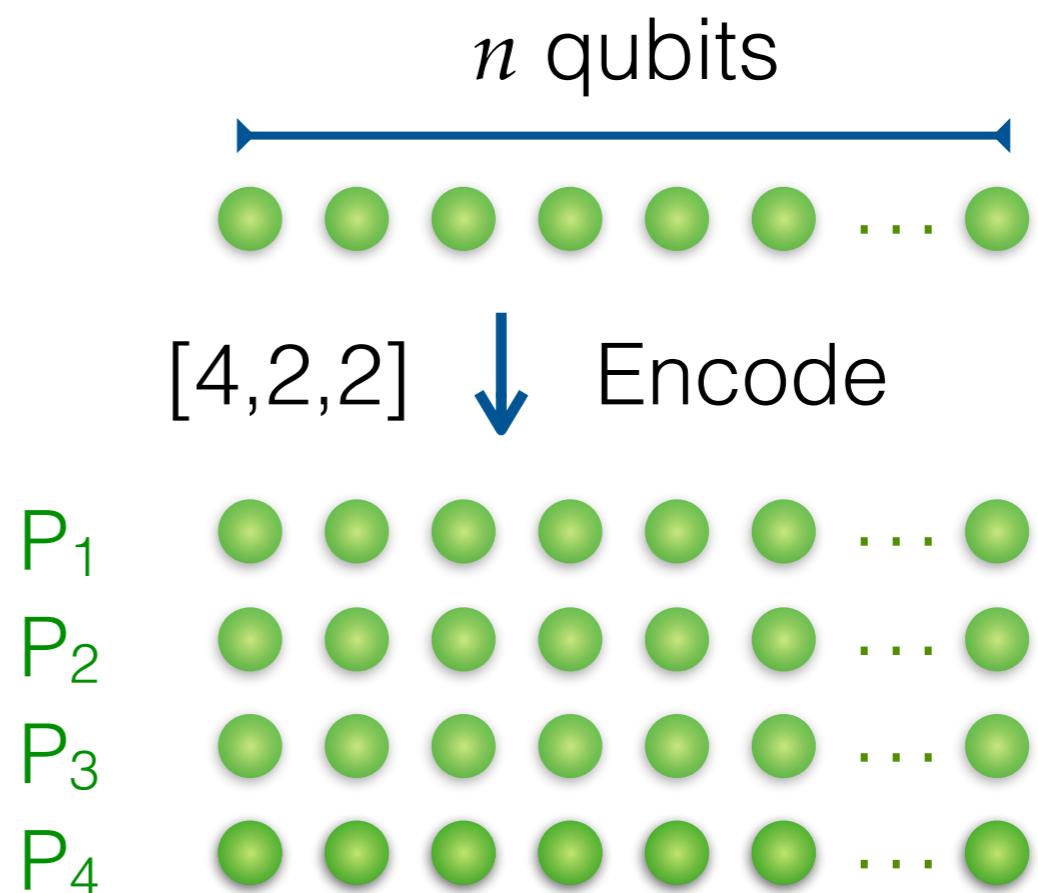
$0 \leftrightarrow 00, 1 \leftrightarrow 11$

# Fitzsimons-Vidick protocol



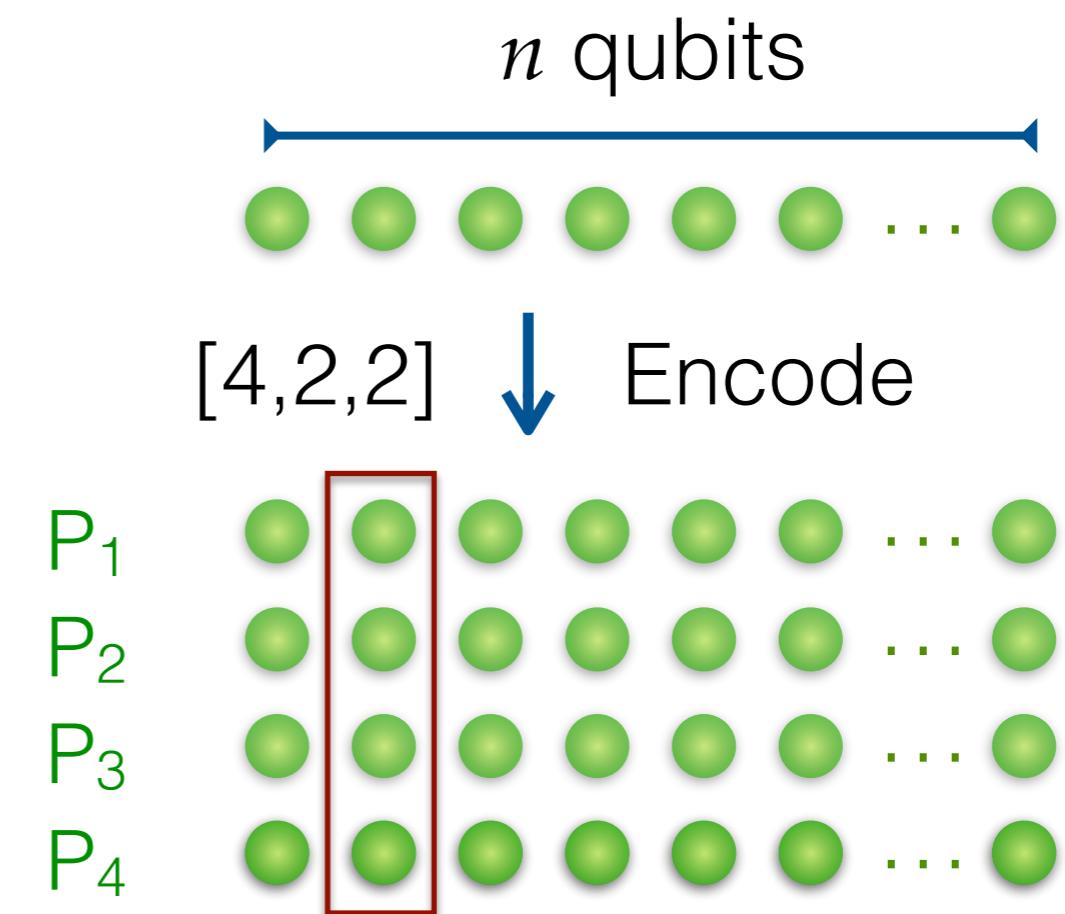
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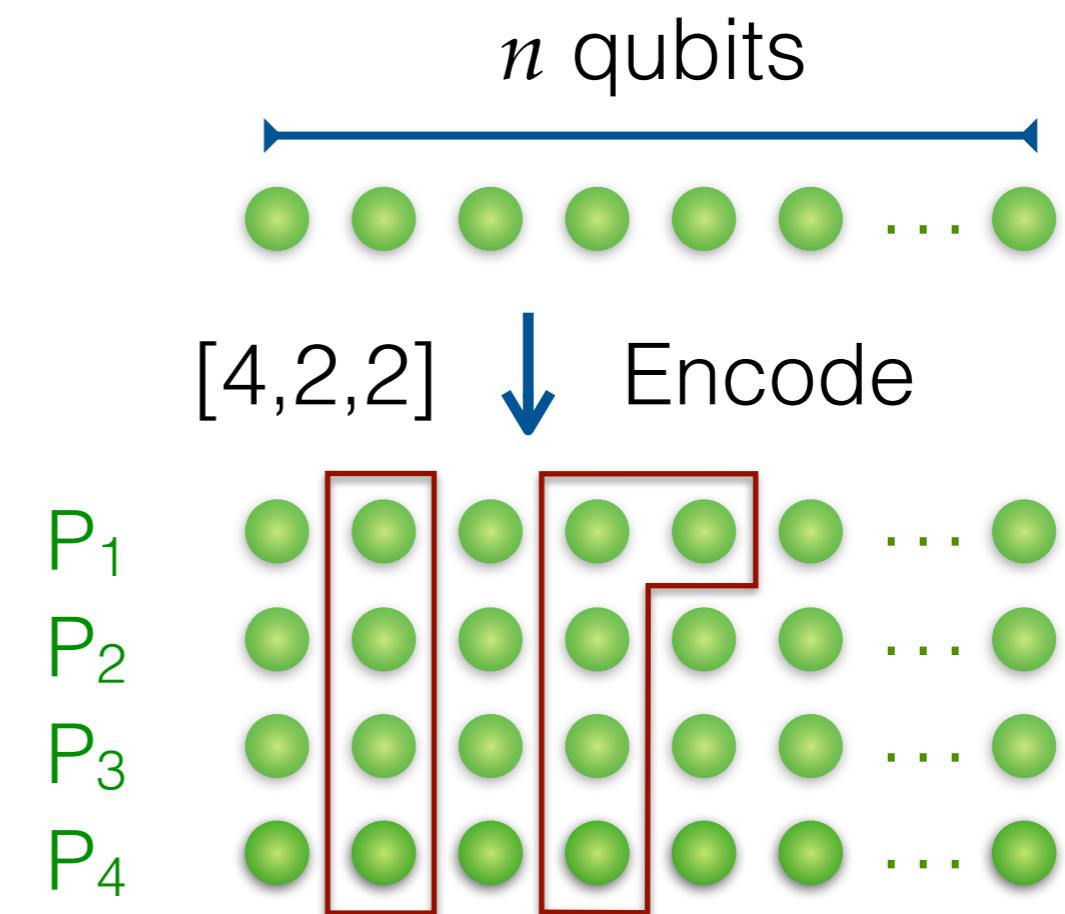
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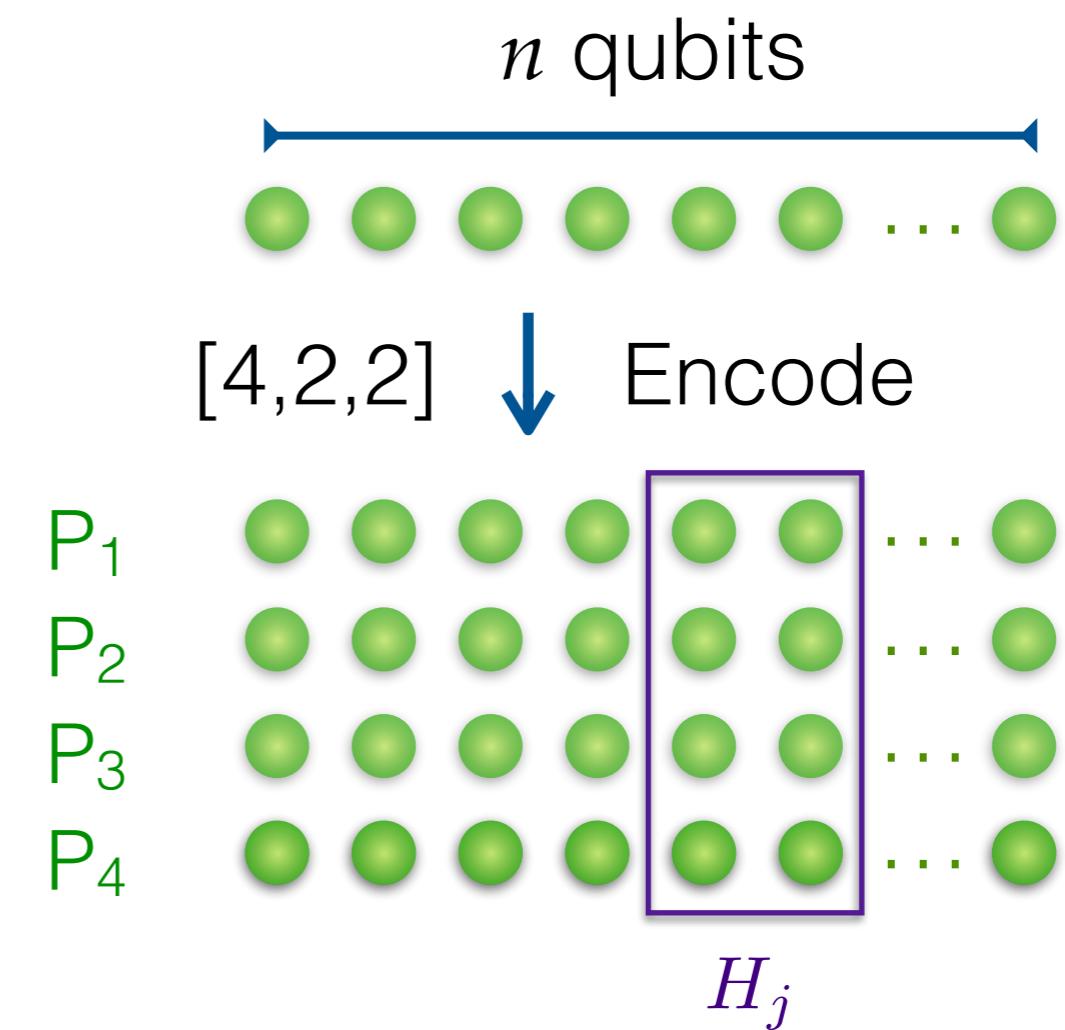
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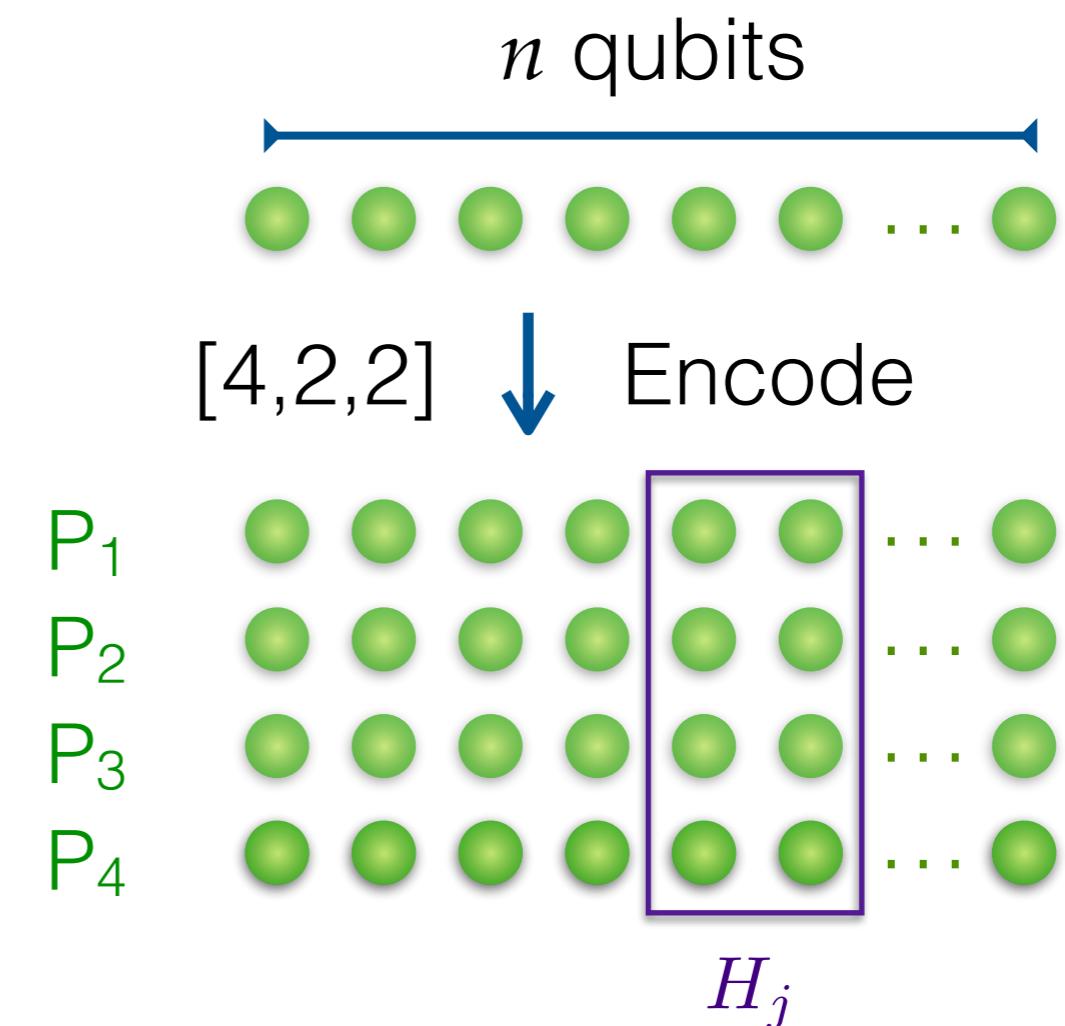
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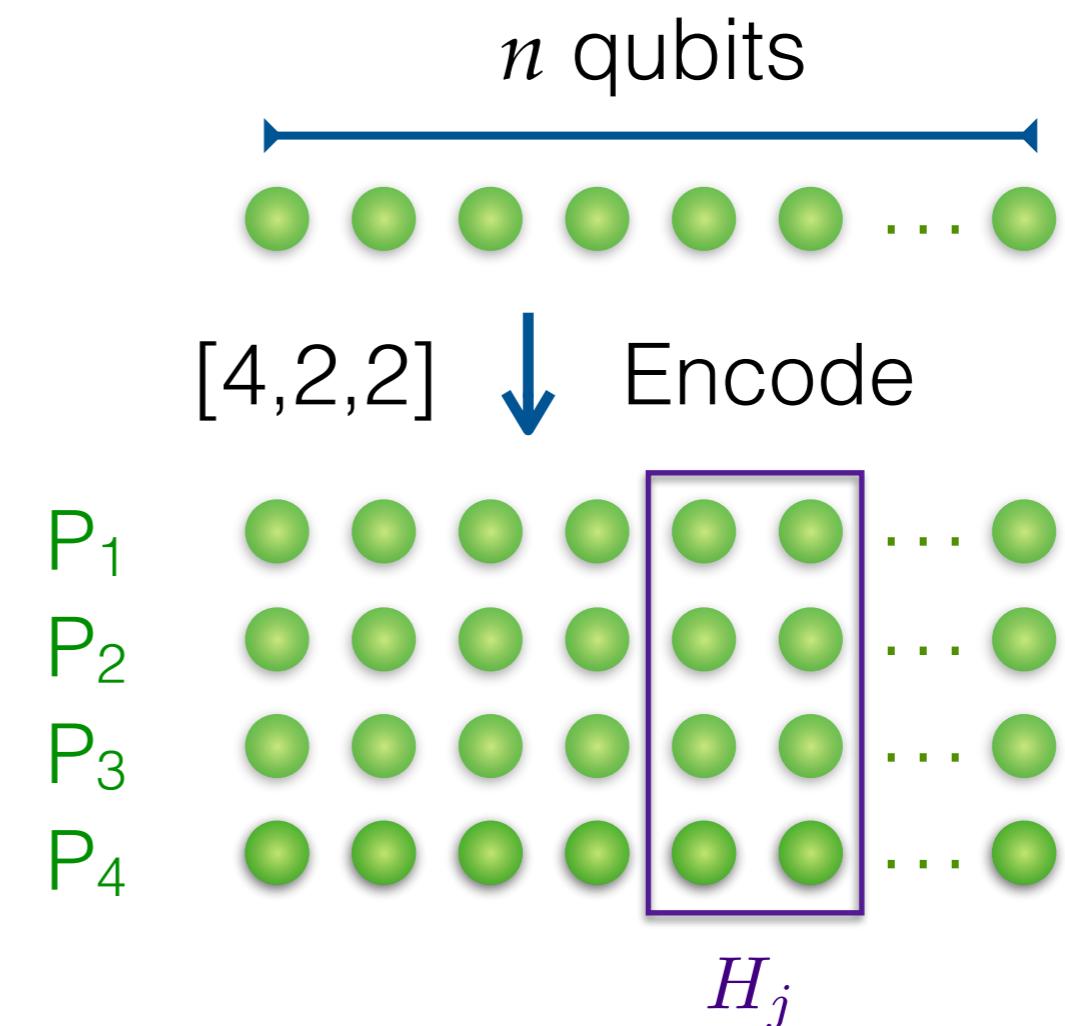
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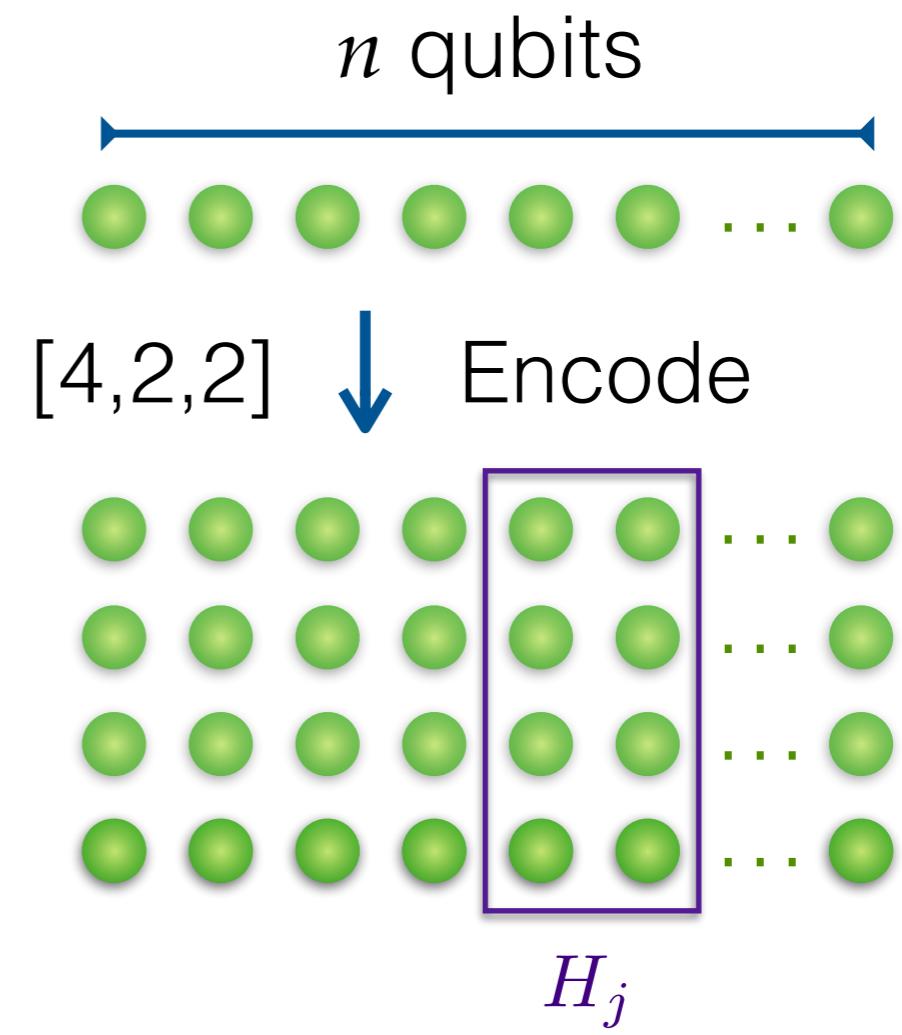
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- De-quantization of both the answer messages and verifier

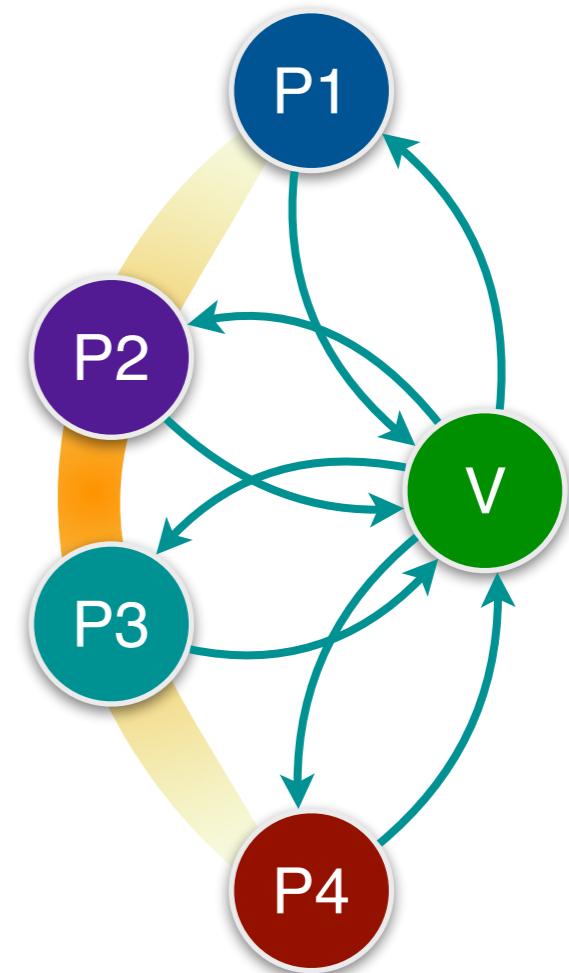


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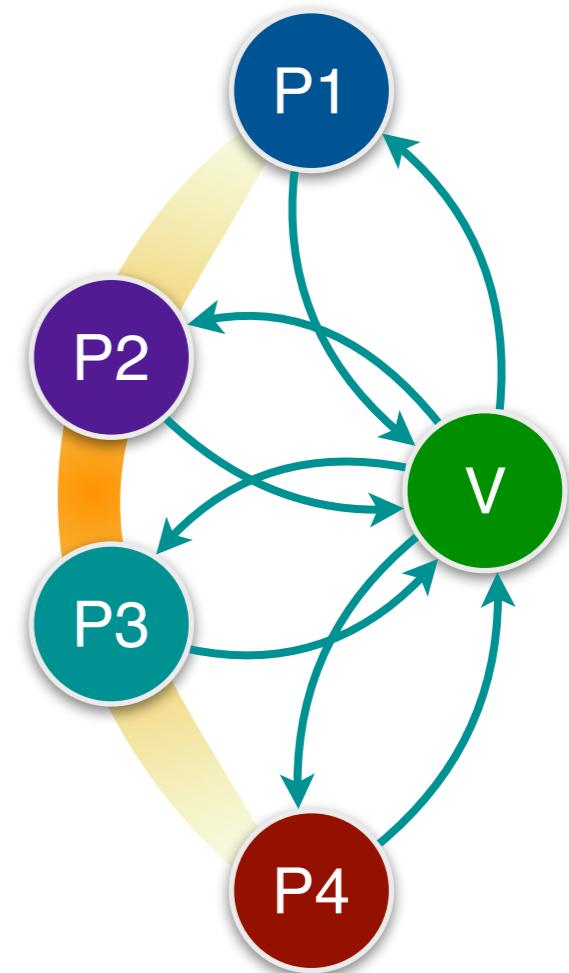
- A 4-player protocol for the local Hamiltonian problem

Questions: logarithmic number of bits,  
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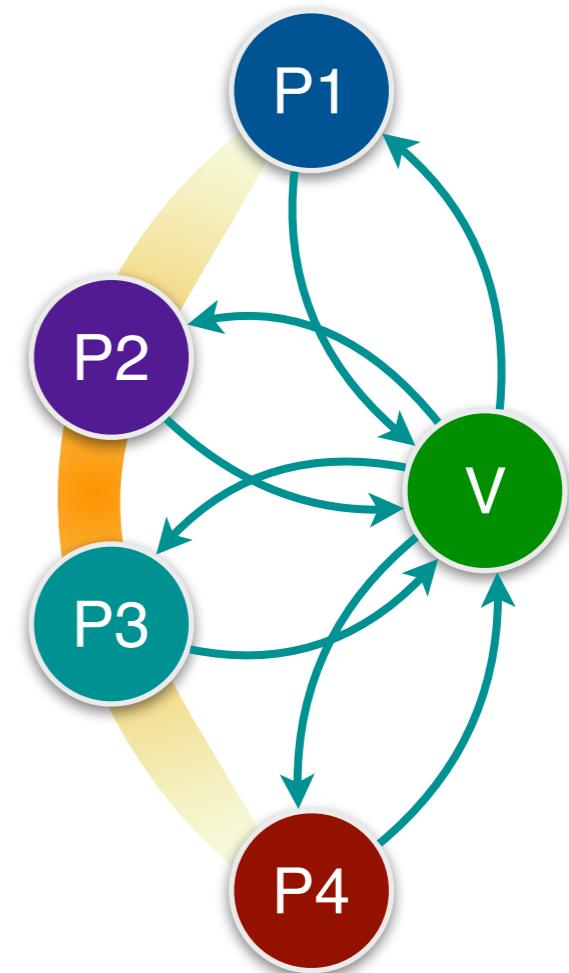
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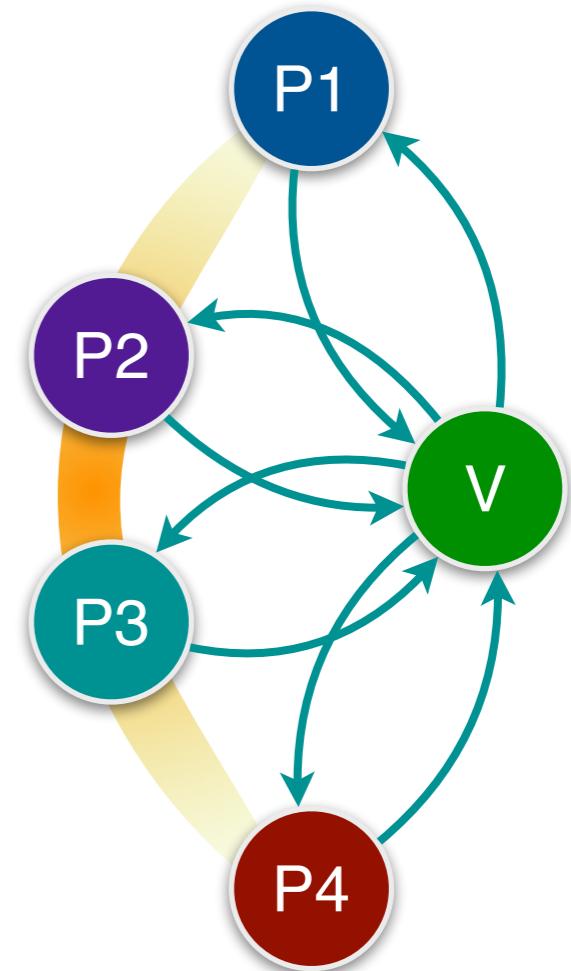
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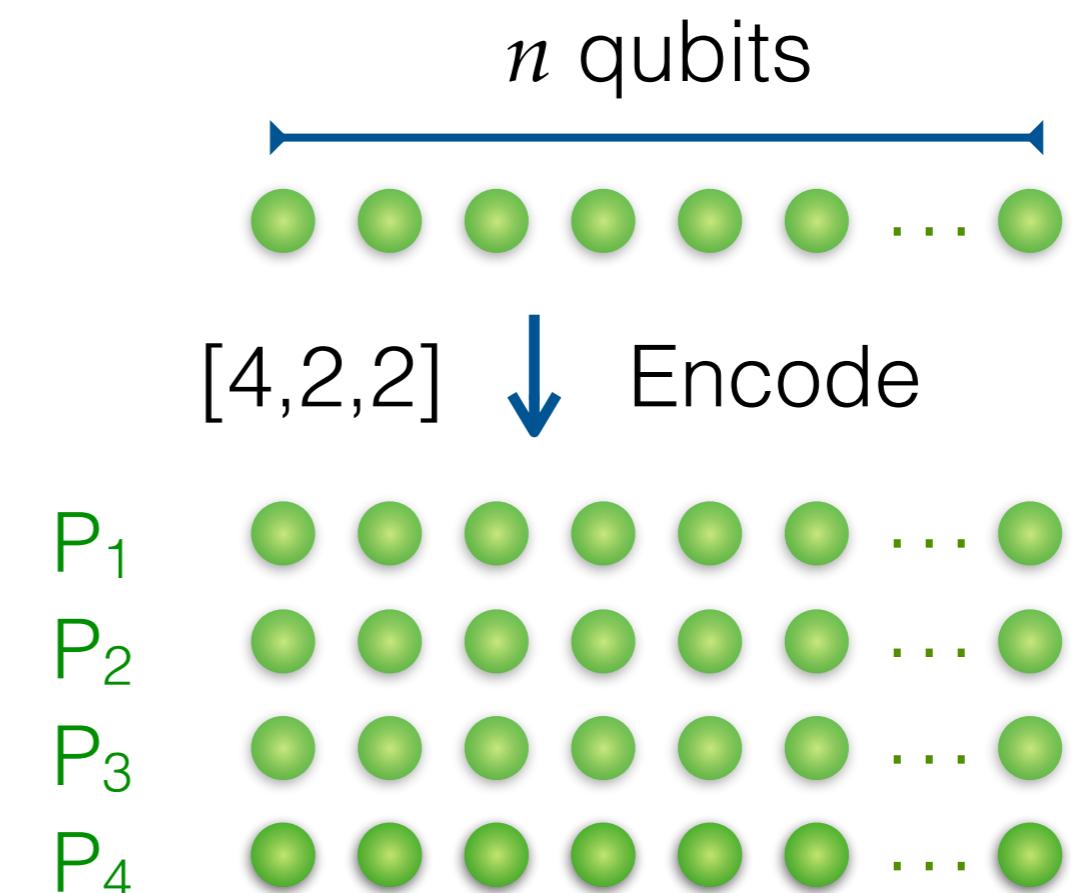


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- For exponentially small gapped c,s, **MIP**  $\subsetneq$  **MIP**<sup>\*</sup>(4,1,c,s) under assumptions

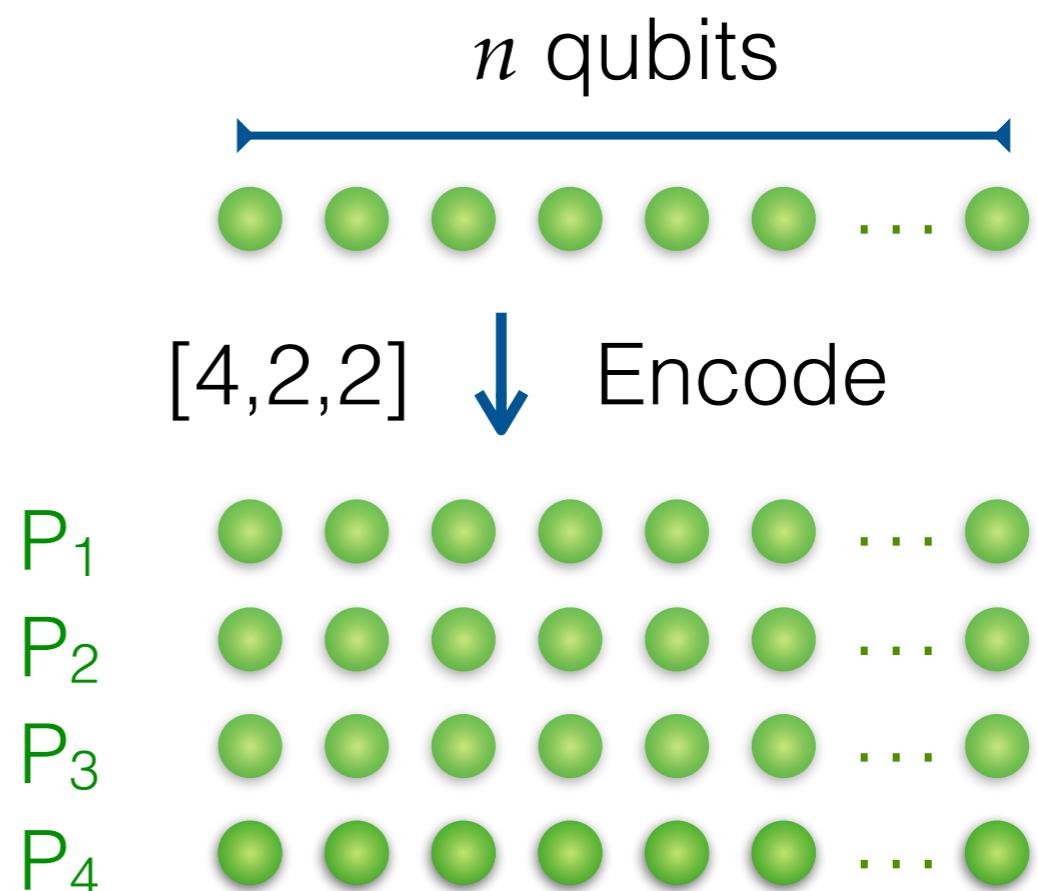


# Overview of the protocol



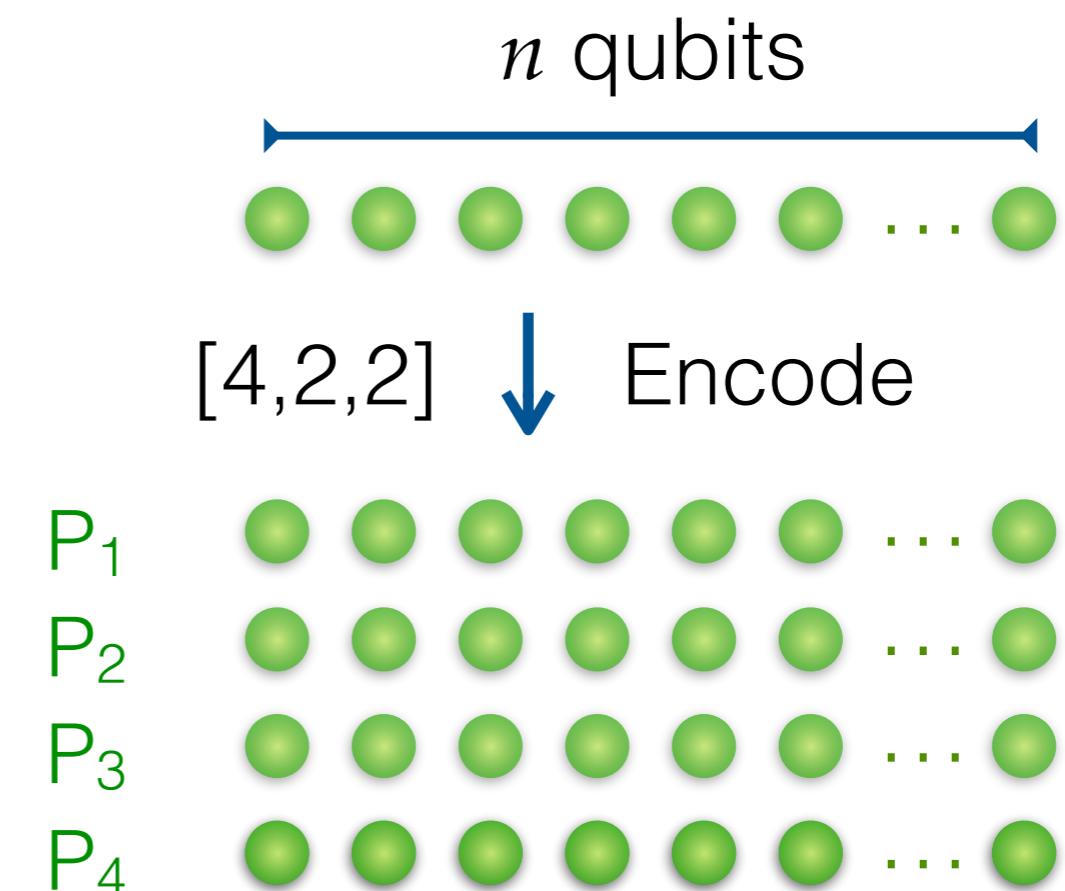
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- Follows Fitzsimons-Vidick protocol very closely



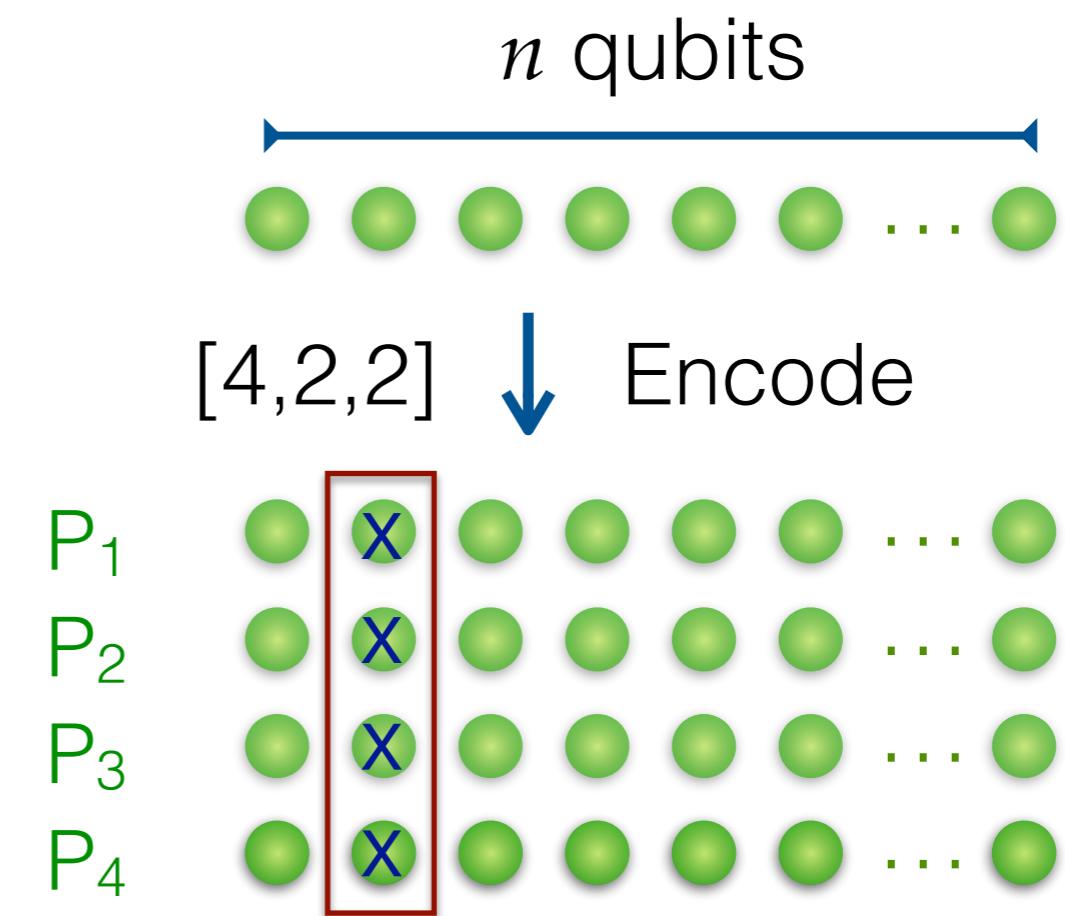
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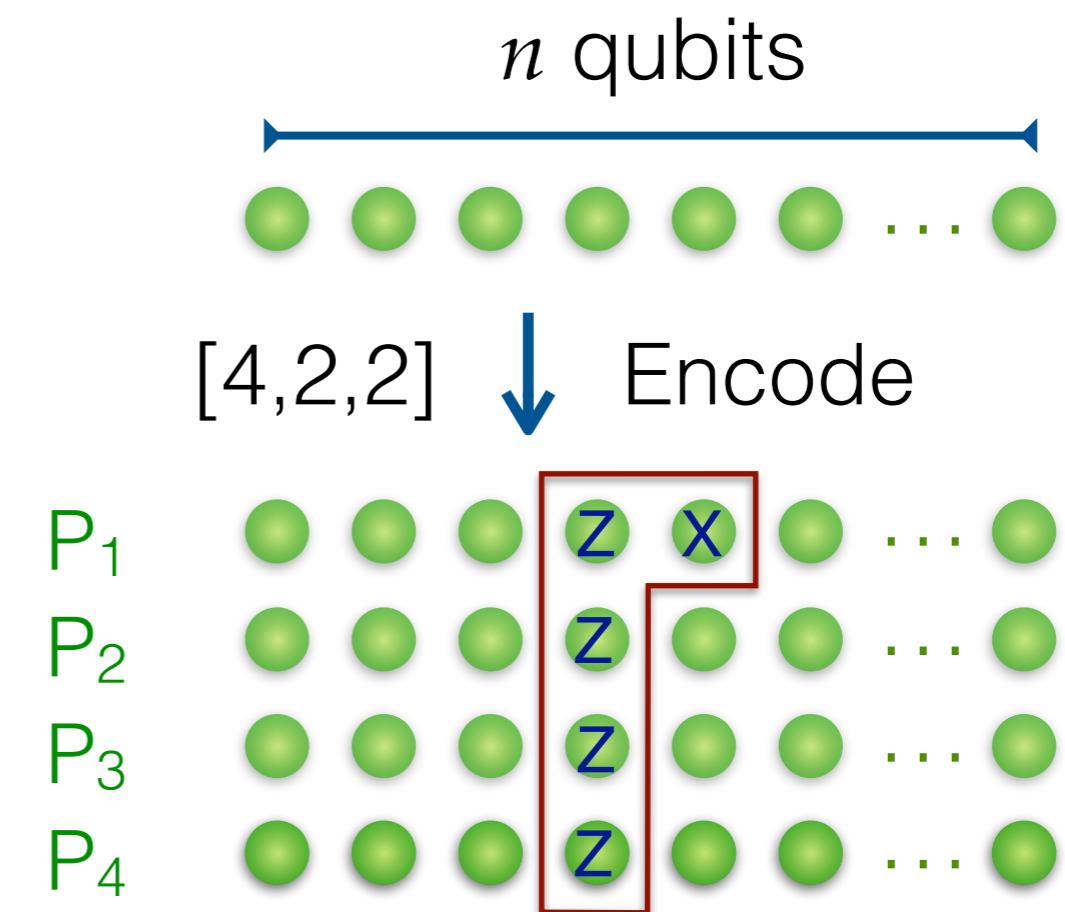
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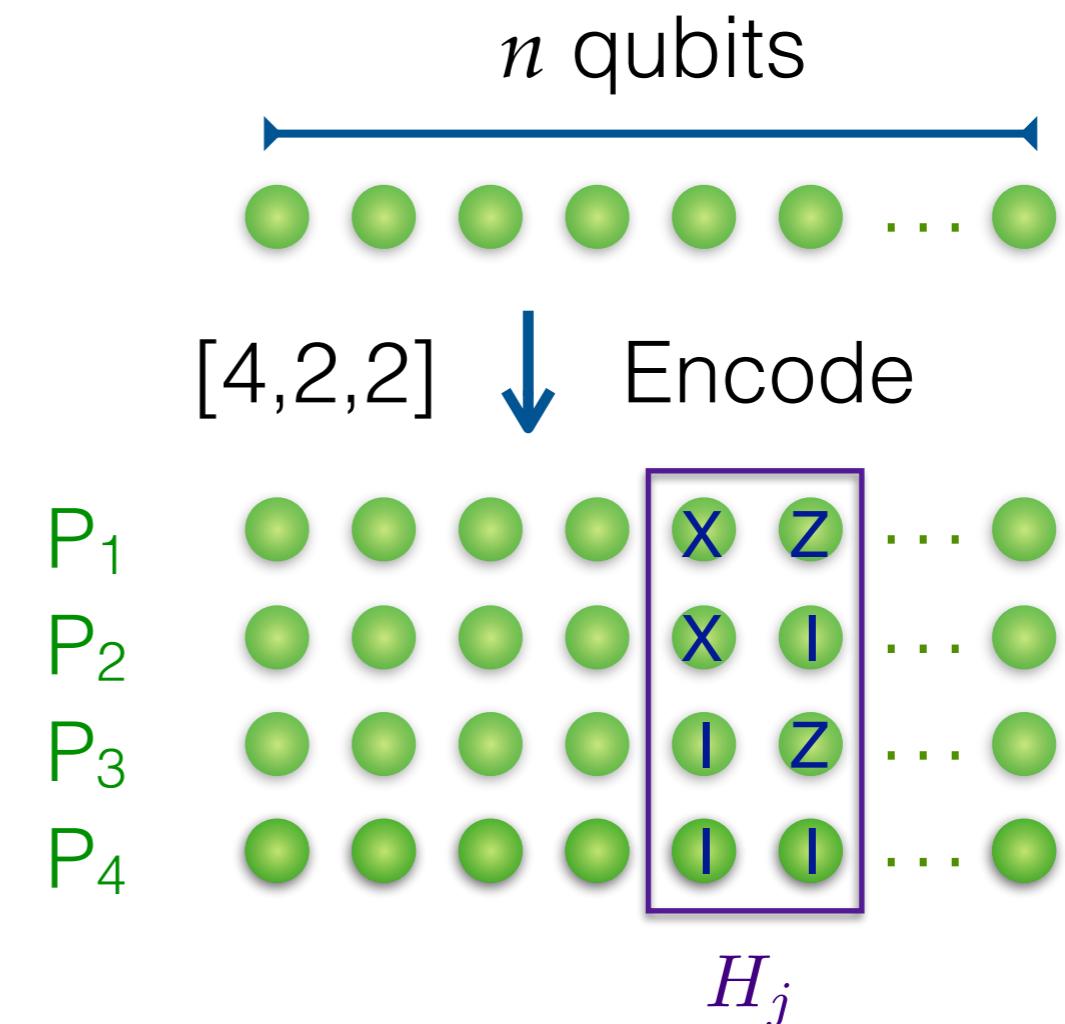
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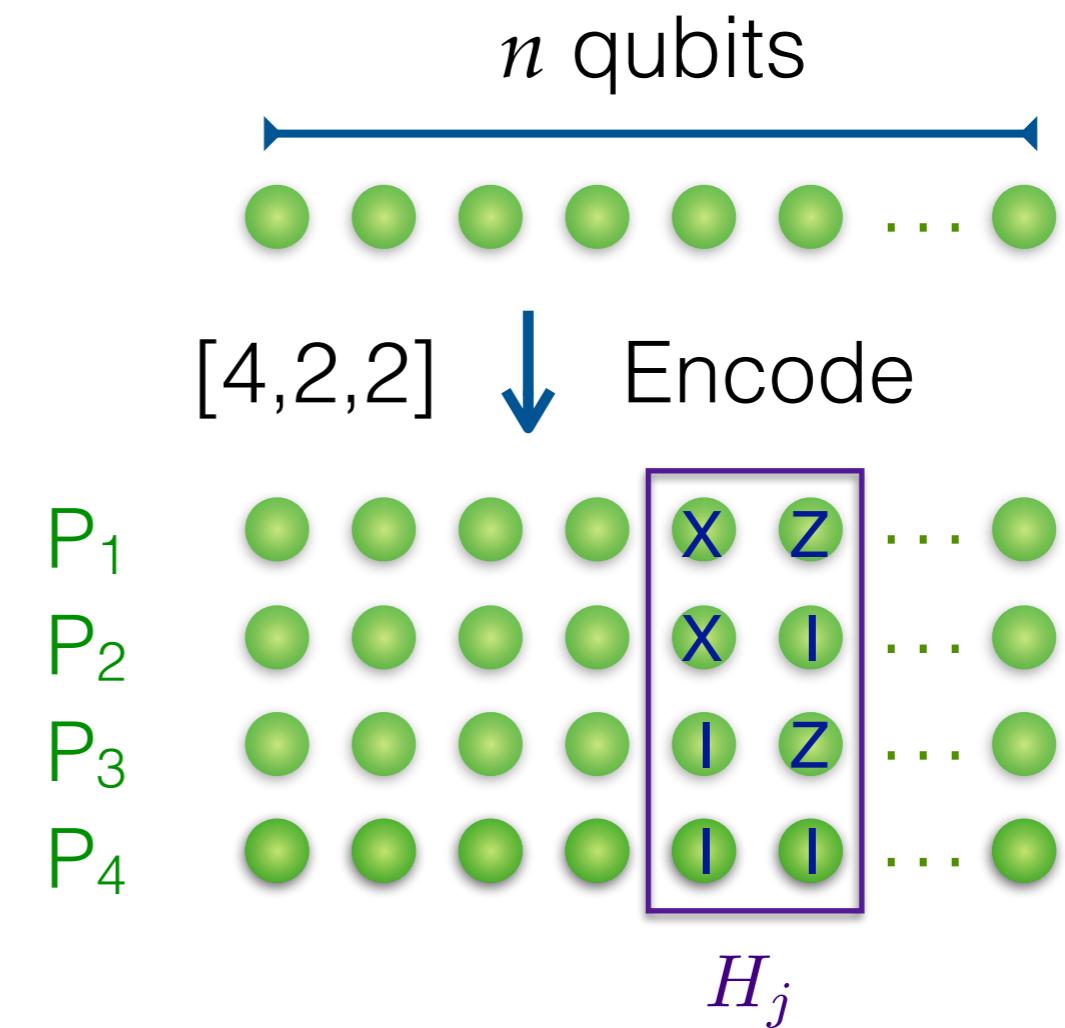
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- Follows Fitzsimons-Vidick protocol very closely
- Sends measurement specifications and asks for the outcome instead of asking for qubits from the provers
- How can we trust the provers?



# Where are the qubits?



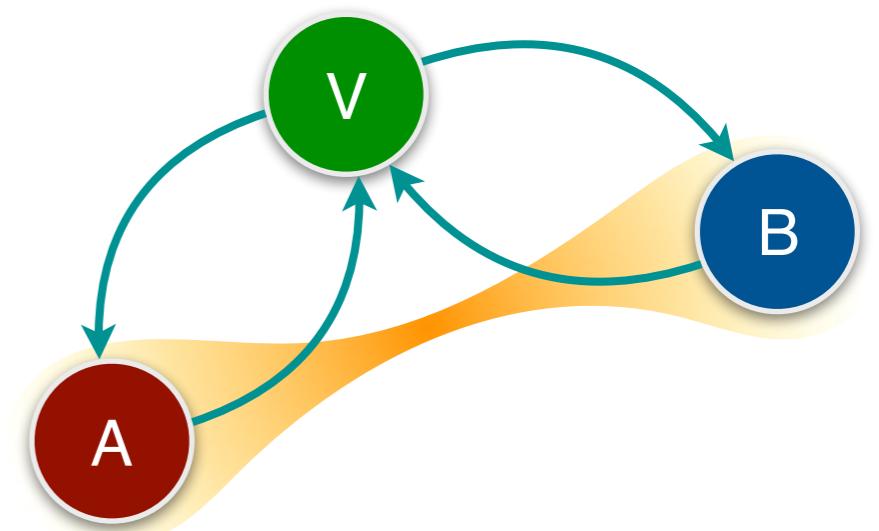
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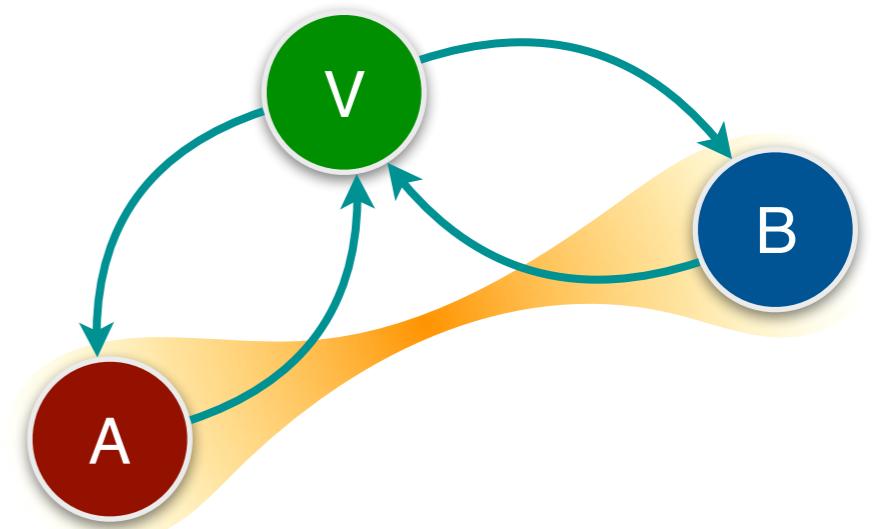
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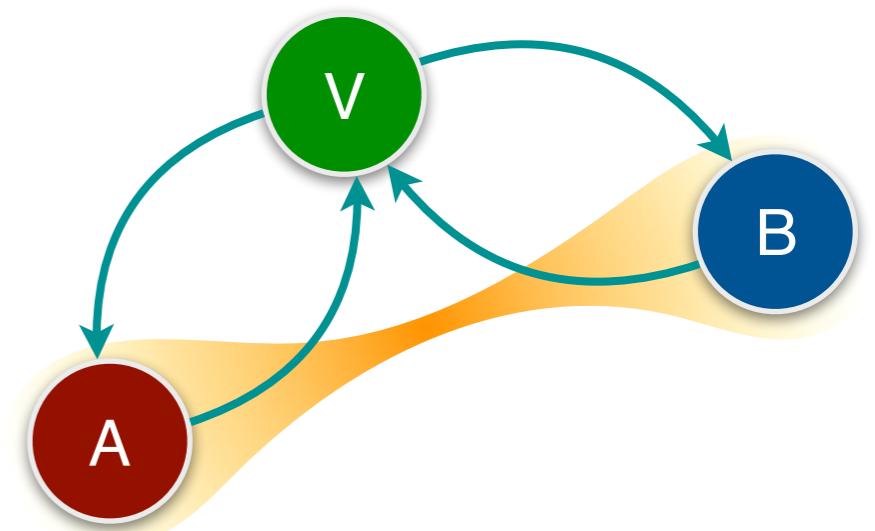


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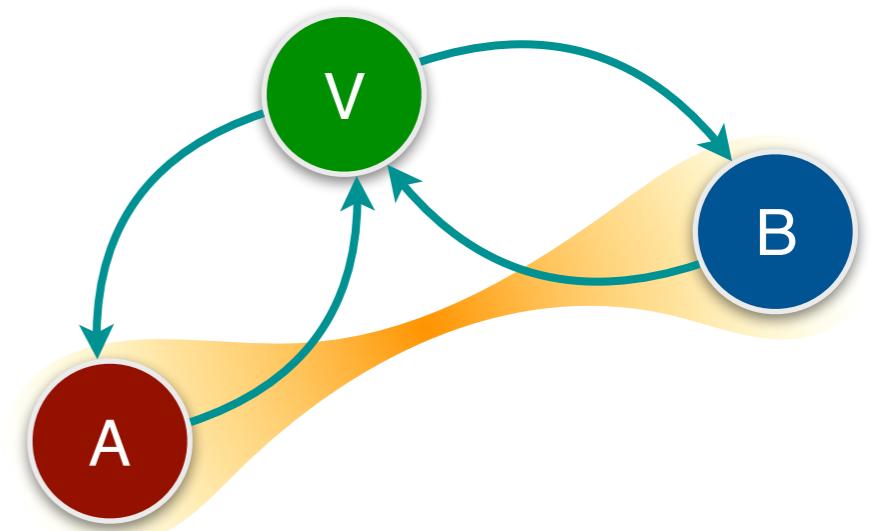
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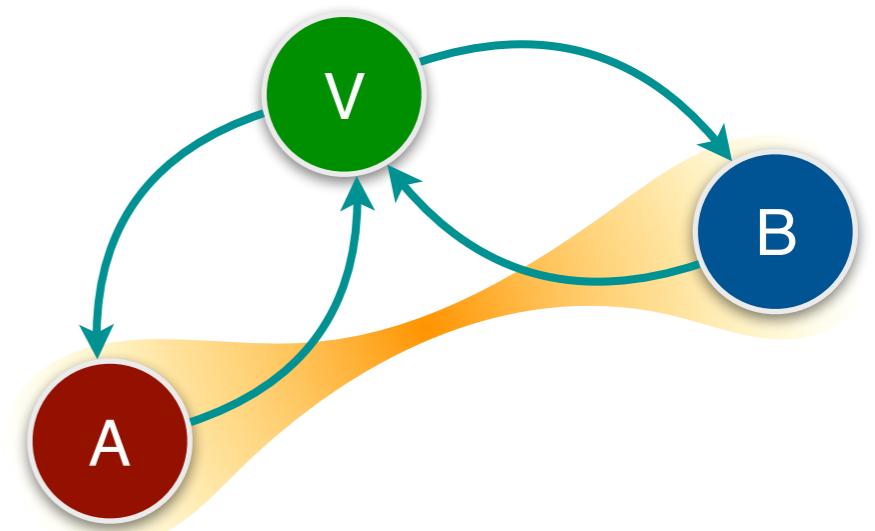


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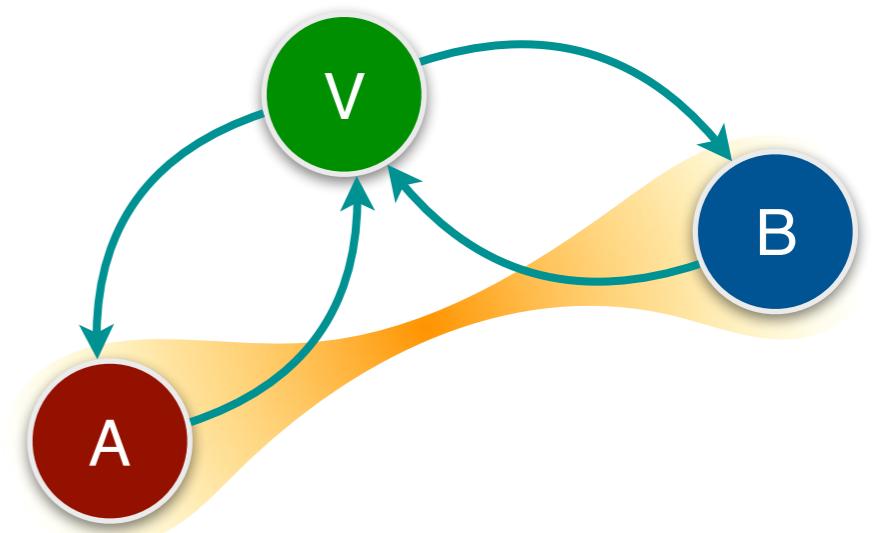
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  - Stabilizer games



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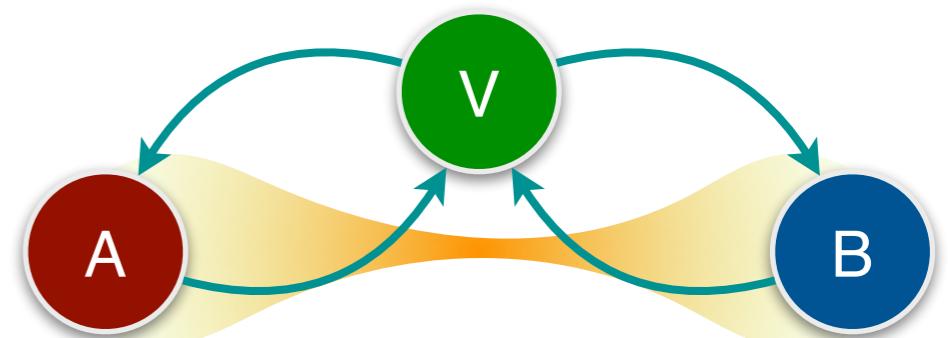
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CHSH game in terms of  
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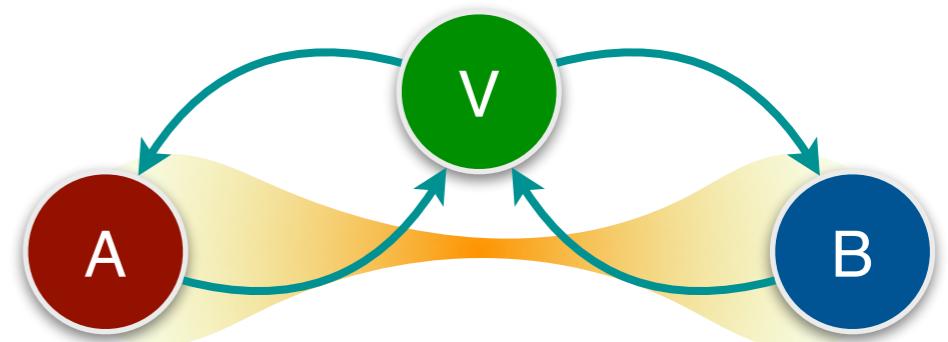


# CHSH game in terms of stabilizers

- The EPR state as a stabilizer

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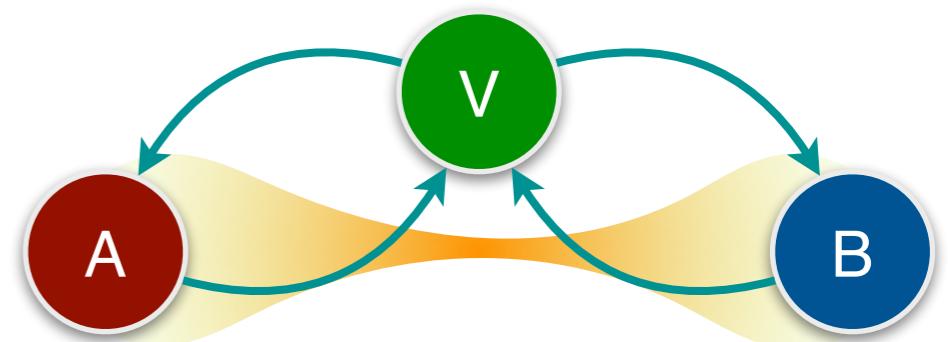
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$$\langle XX + ZZ \rangle = 2$$

CHSH:  $a \oplus b \stackrel{?}{=} s \wedge t$



# CHSH game in terms of stabilizers

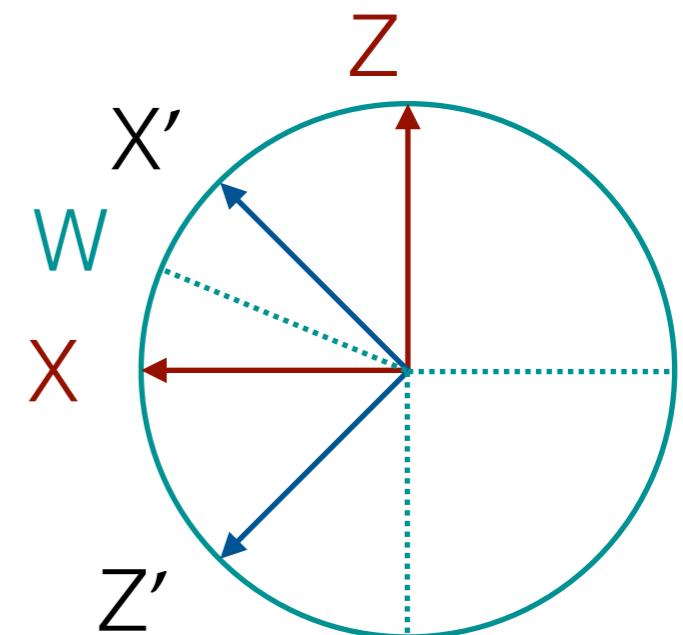
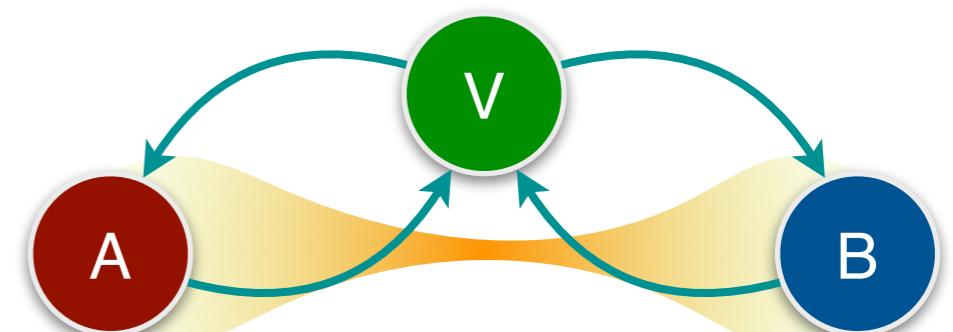
- The EPR state as a stabilizer

$$\begin{array}{c} X \quad X \\ \hline Z \quad Z \end{array}$$

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$$X = \frac{X' + Z'}{\sqrt{2}} \quad Z = \frac{X' - Z'}{\sqrt{2}}$$

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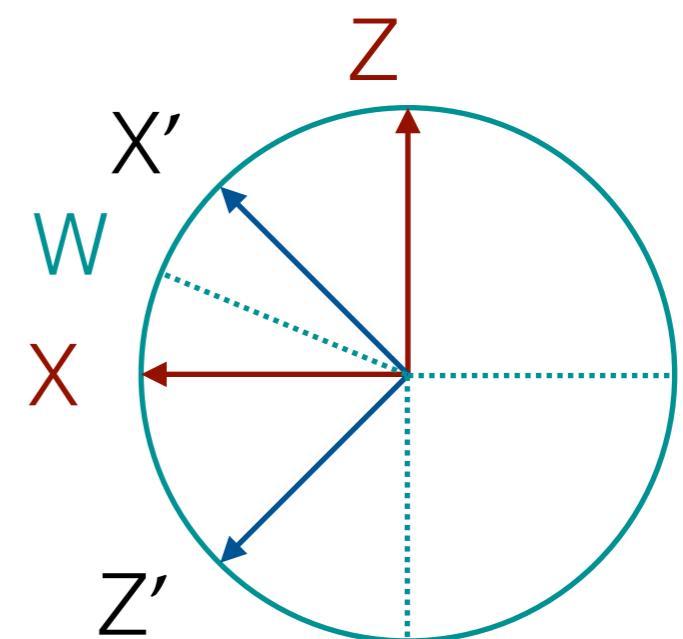
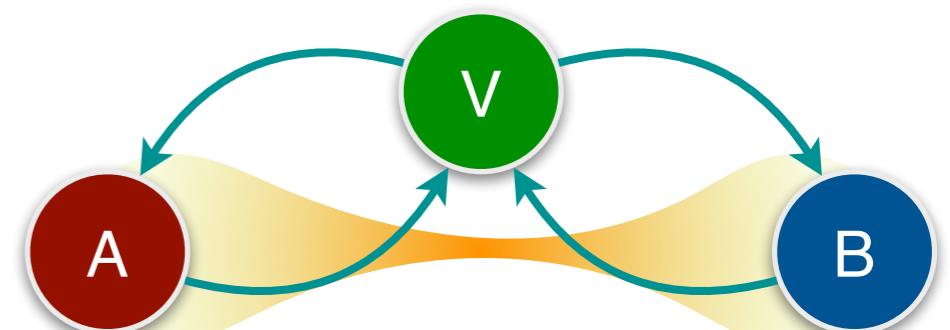
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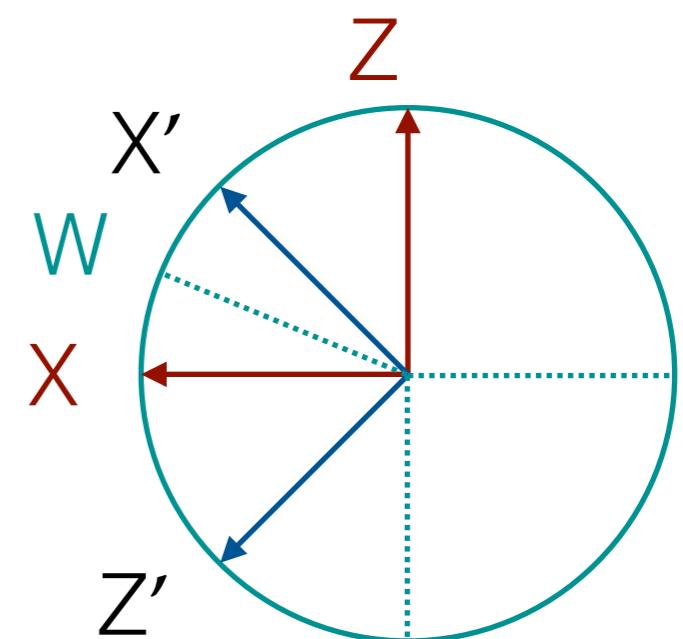
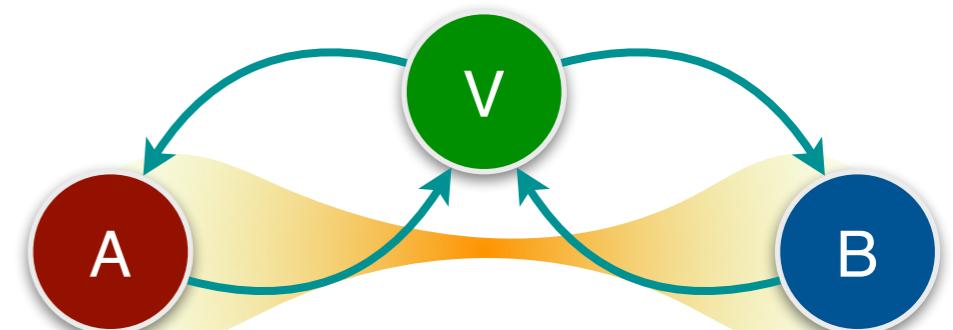
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$$\begin{array}{r} + \ X \ X' \\ + \ X \ Z' \\ + \ Z \ X' \\ - \ Z \ Z' \end{array}$$

$$\begin{array}{r} + \ 0 \ 0 \\ + \ 0 \ 1 \\ + \ 1 \ 0 \\ - \ 1 \ 1 \end{array}$$

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# Stabilizer games with a special player

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- Apply the 45-degree rotation trick to the stabilizers of the [4,2,2] code

$$\begin{array}{c} \begin{array}{cccc|c} \times & \times & \times & \times & \\ \hline z & z & z & z & \end{array} \end{array} \longrightarrow \begin{array}{c} \begin{array}{ccccc} + & x & x & x & x' \\ \hline + & x & x & x & z' \\ \hline + & z & z & z & x' \\ \hline - & z & z & z & z' \end{array} \end{array}$$

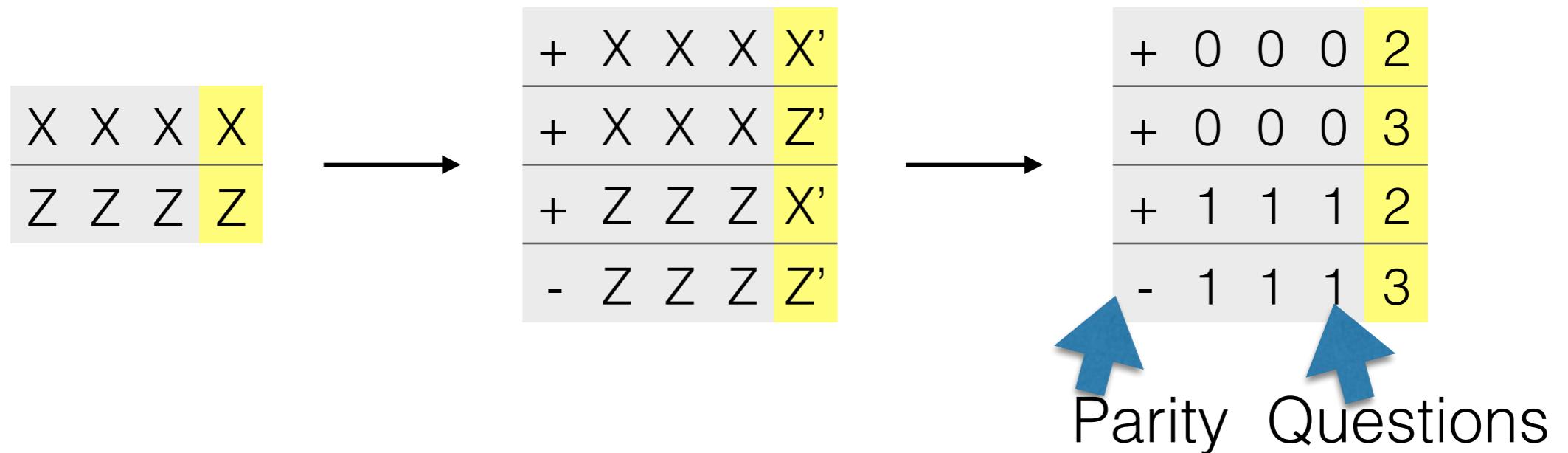
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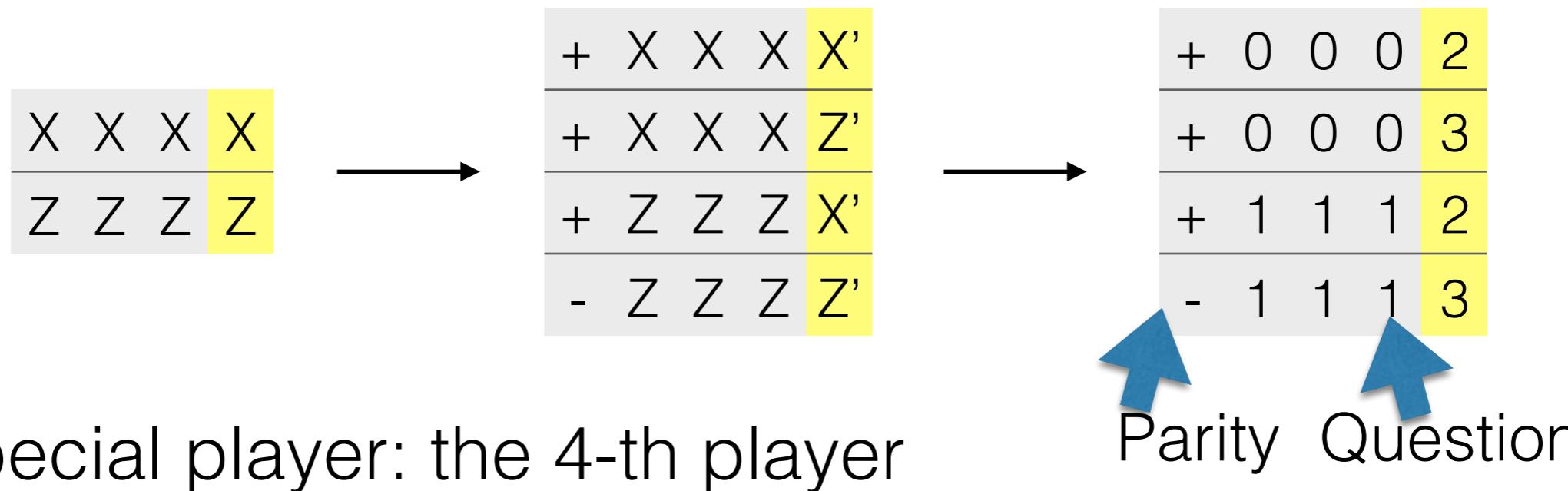
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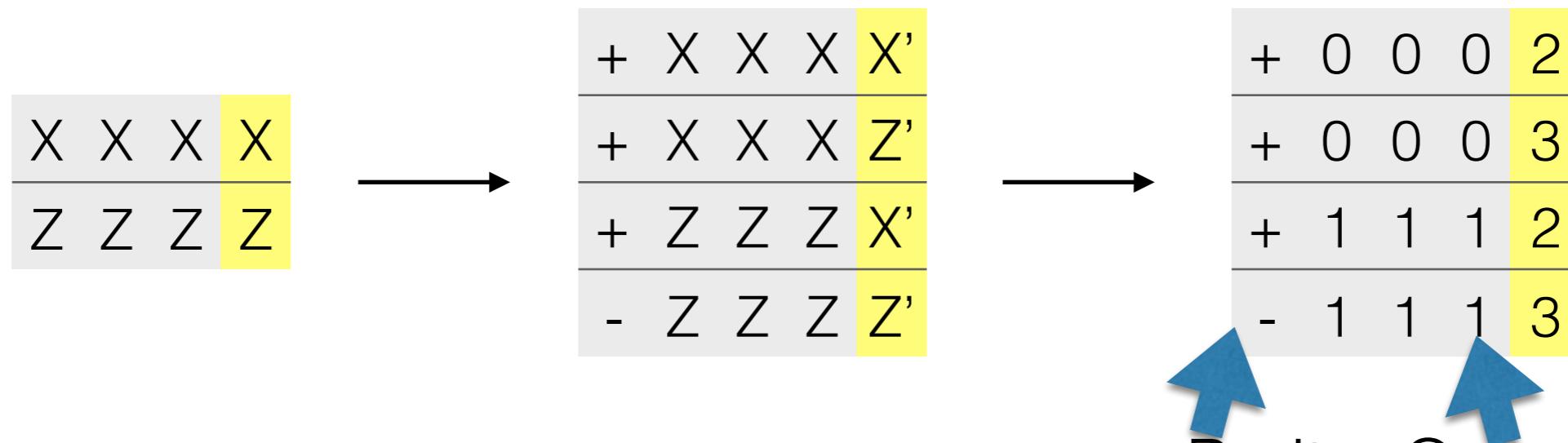
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# Stabilizer games with a special player

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- Special player: the 4-th player
- No full rigidity, but **partial rigidity**: the special player must measure honestly

# Partial rigidity of the special player stabilizer game

Lemma (Partial Rigidity). For any strategy  $\mathcal{S} = (\rho, \{R_w^{(i)}\})$  of the special player stabilizer game whose value is at least  $\omega_{\text{sps}}^* - \varepsilon$  there exists an isometry  $V : \mathcal{H}_4 \rightarrow \mathbb{C}^2 \otimes \hat{\mathcal{H}}_4$  such that

$$R_3^{(4)} = V^\dagger (Z' \otimes I) V,$$

$$R_2^{(4)} \approx_{\sqrt{\varepsilon}} V^\dagger (X' \otimes I) V.$$

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Proof of the lemma uses the Jordan's lemma and a proof technique for the CHSH rigidity from [Reichardt, Unger, Vazirani 13]

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- Encoded Werner states are certifiable!

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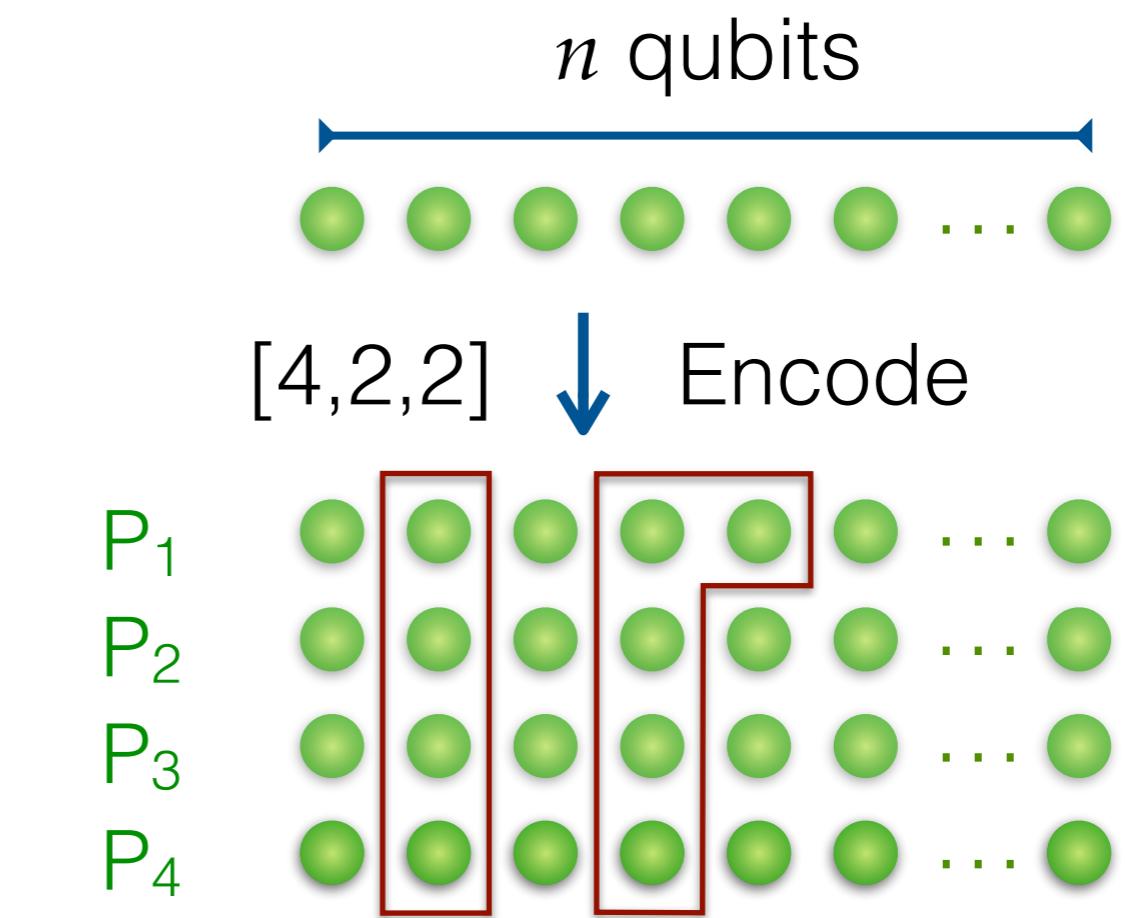
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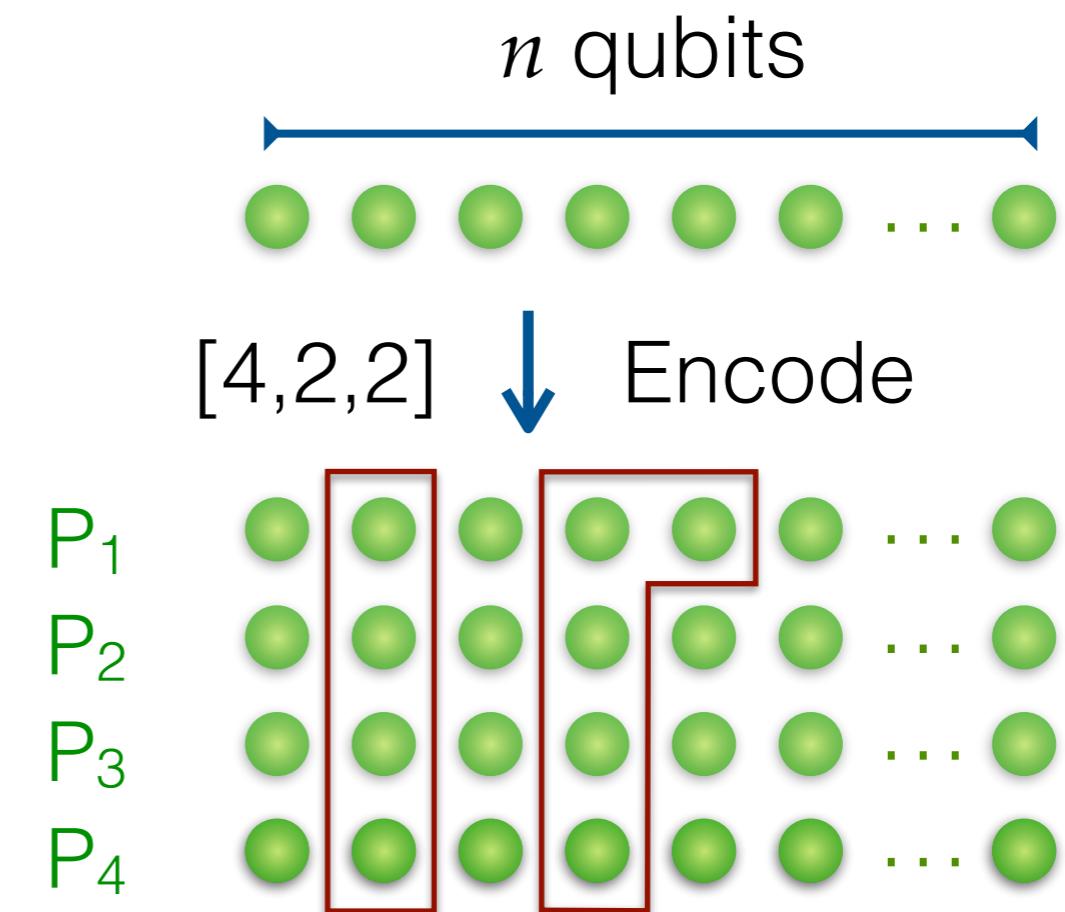
The proof uses the **consistency** properties of the game

# Multi-qubit stabilizer game



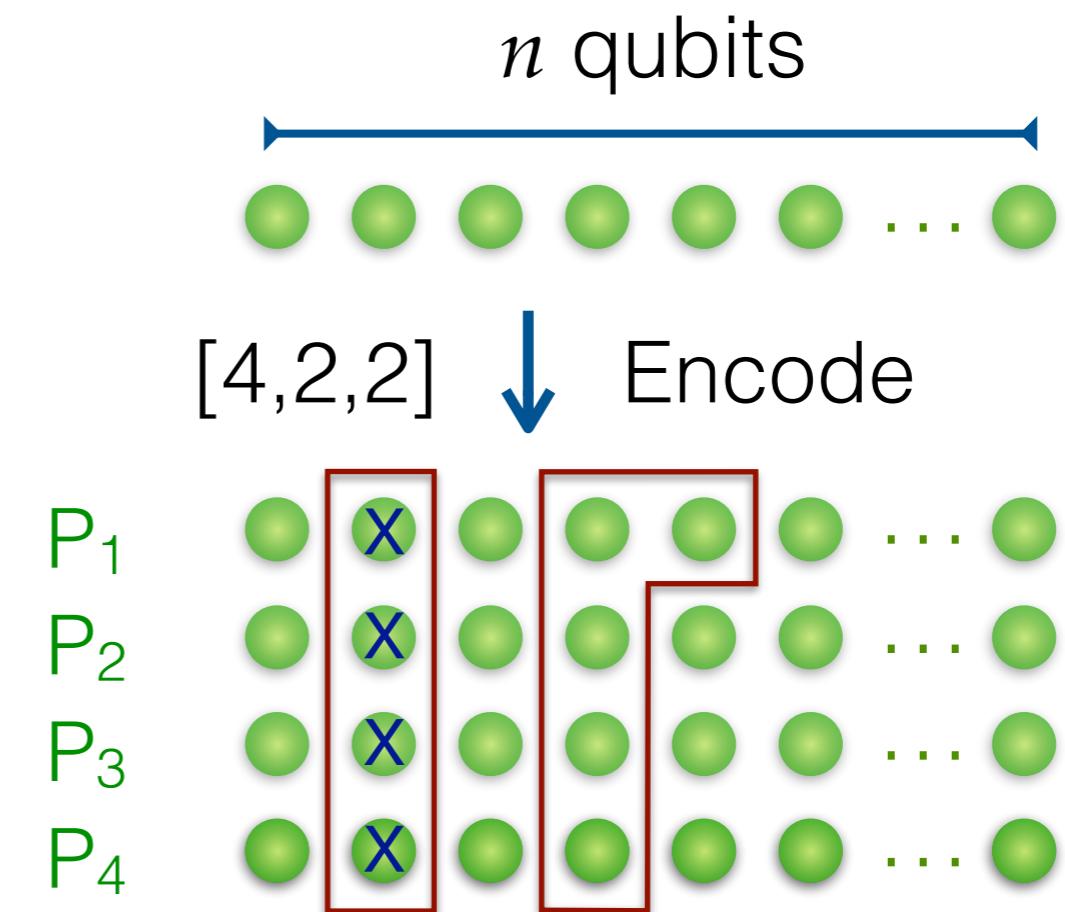
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- For both types of the encoding checks, the verifier plays the corresponding stabilizer game



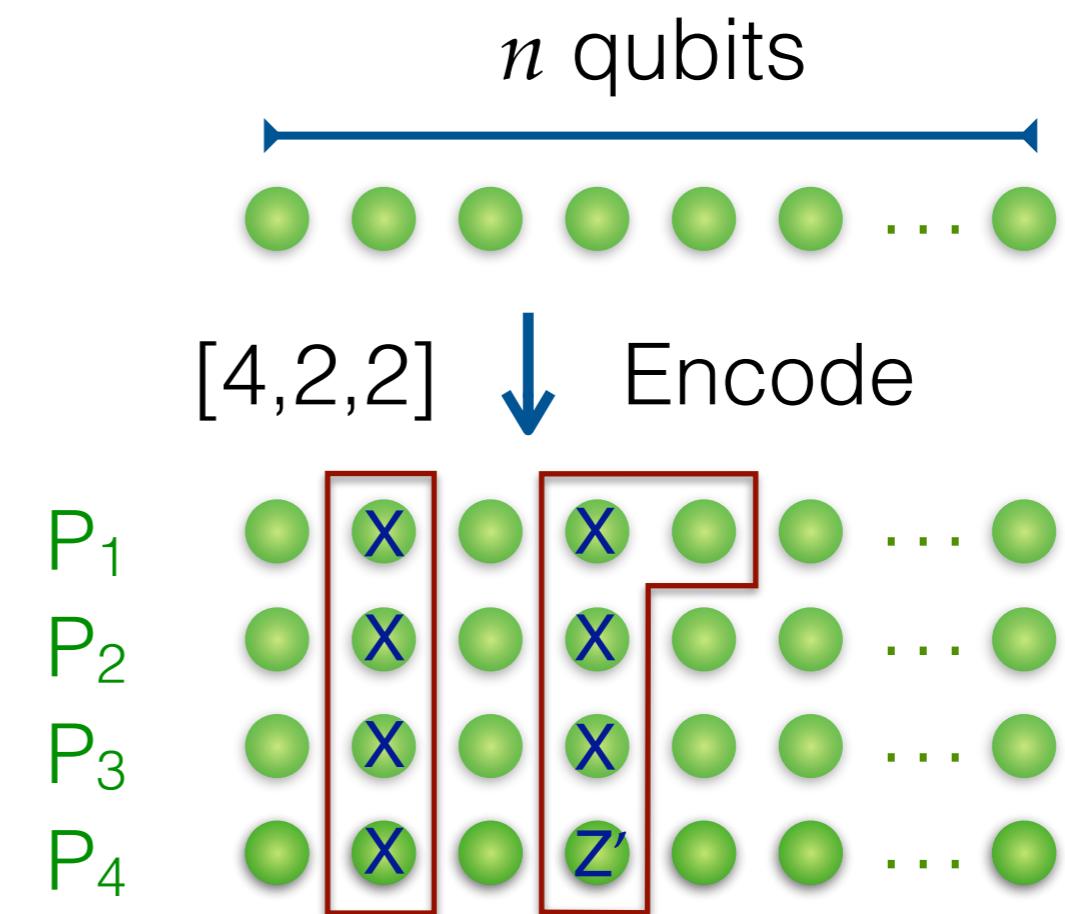
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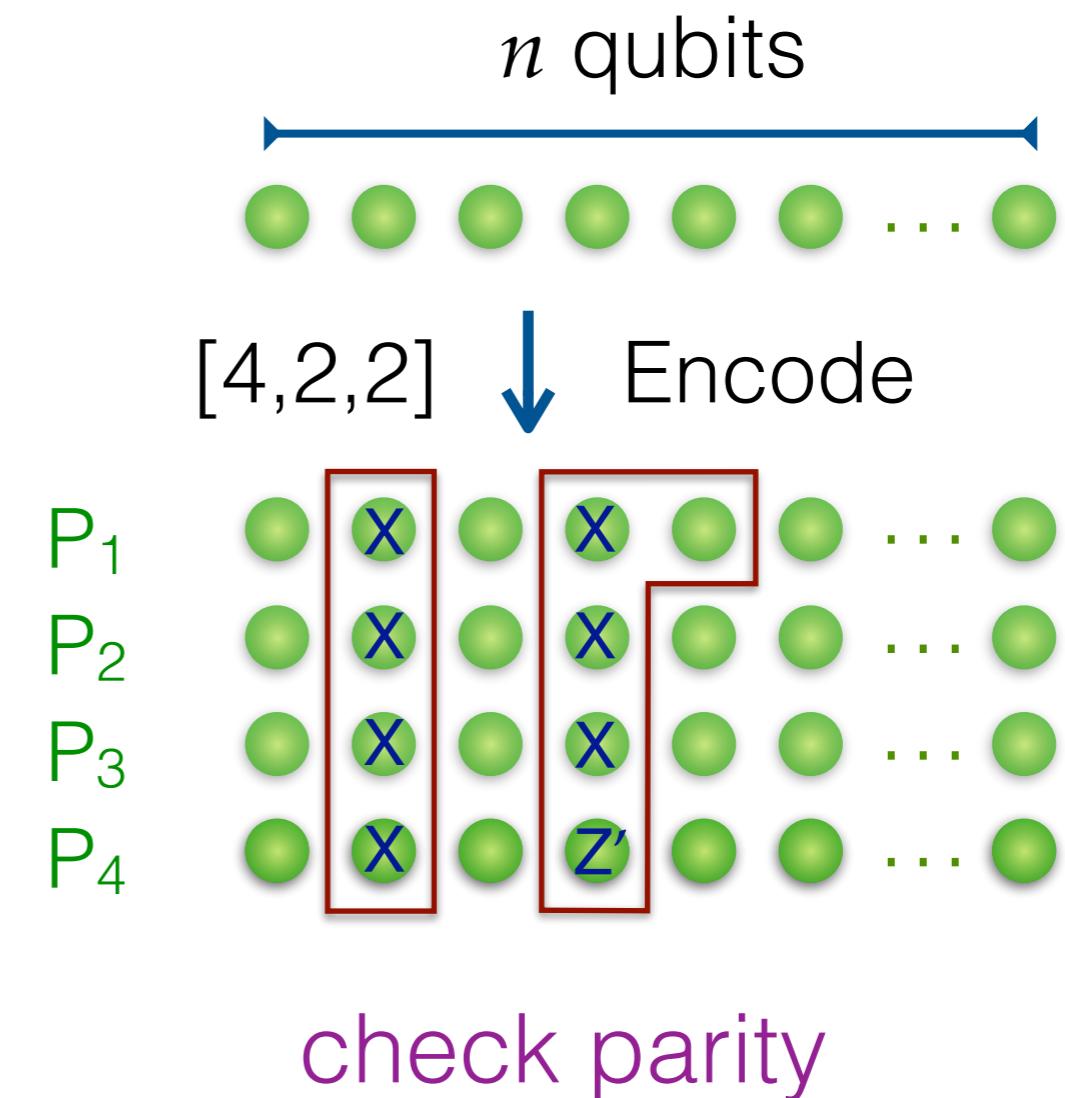
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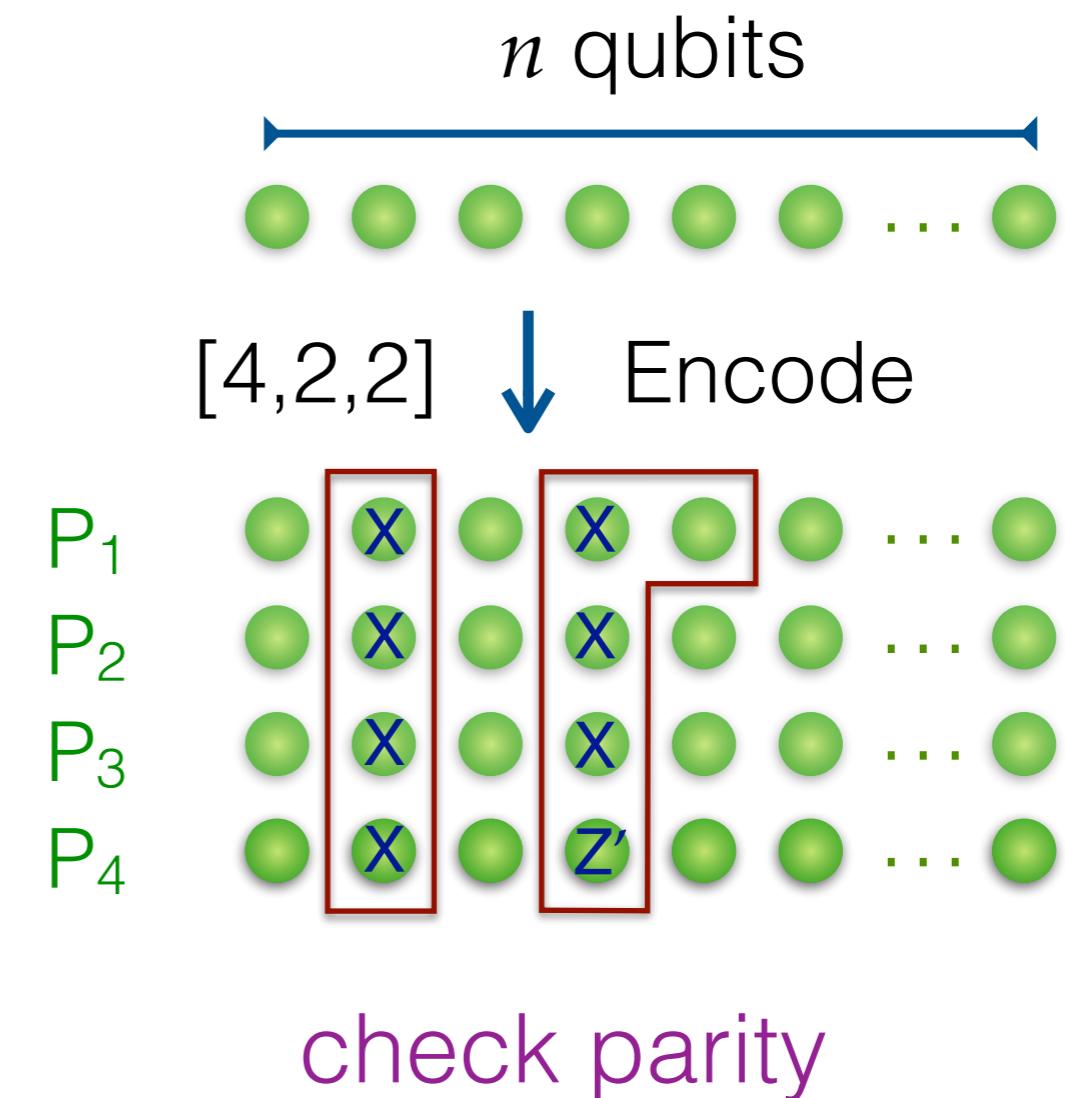
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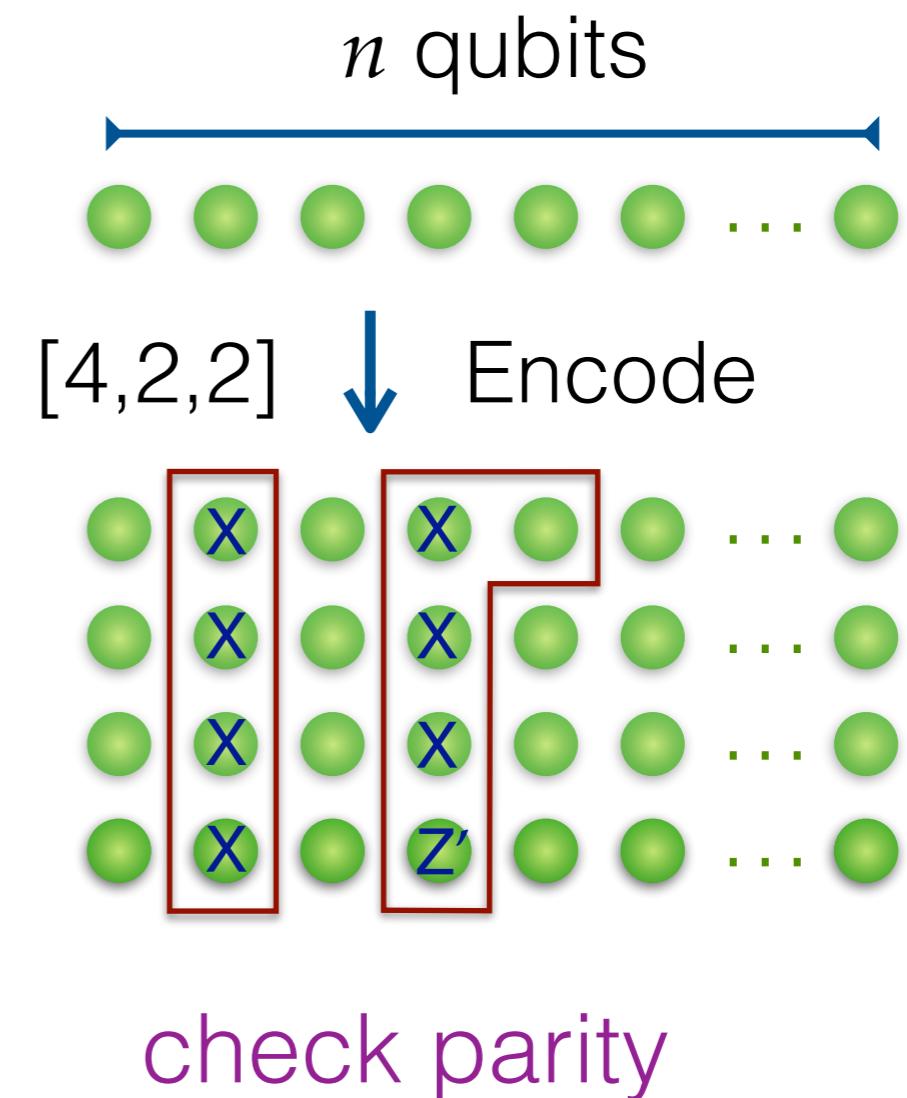
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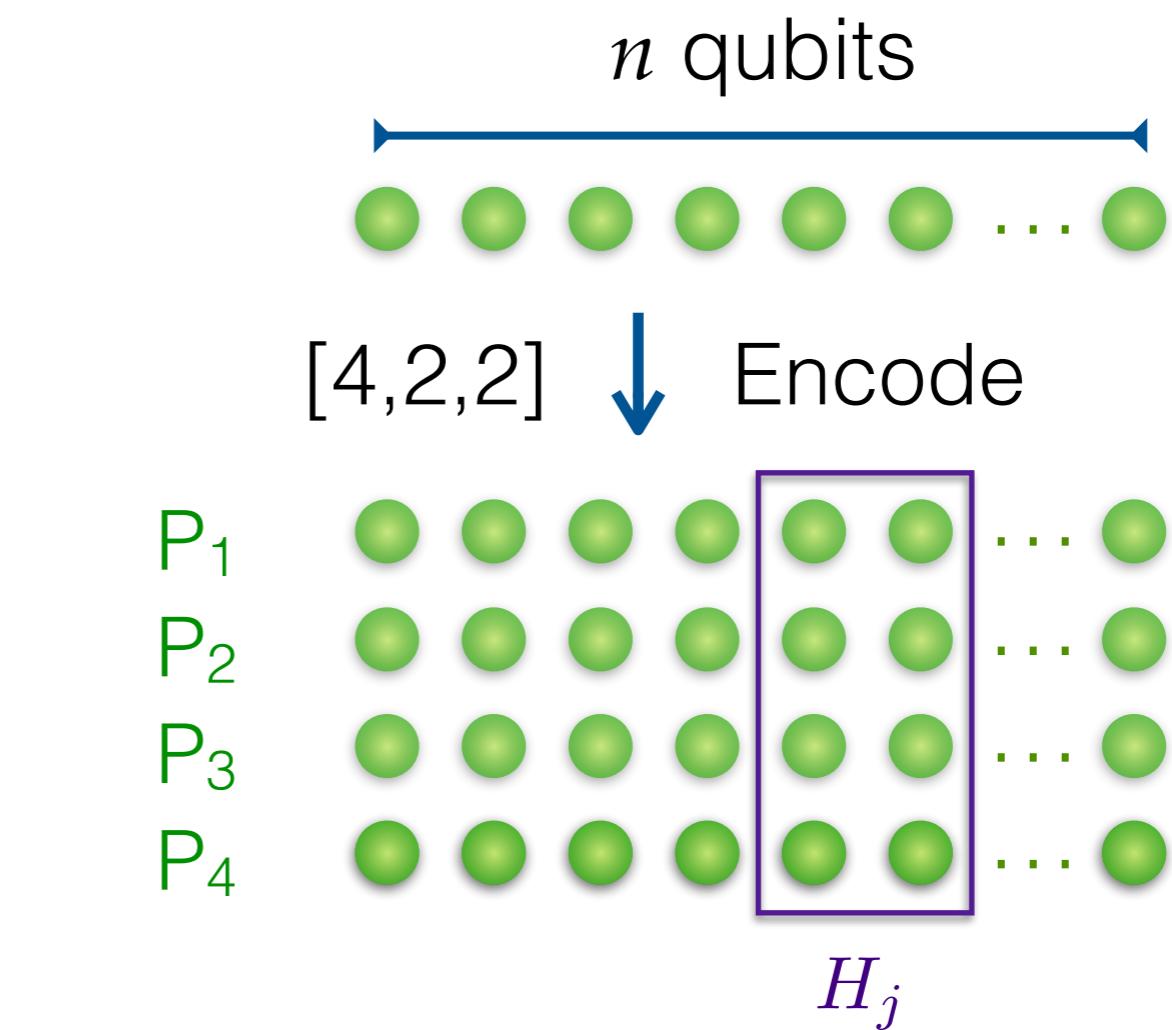


# Multi-qubit stabilizer game

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- “Locates” the  $n$  qubits in a sequential way



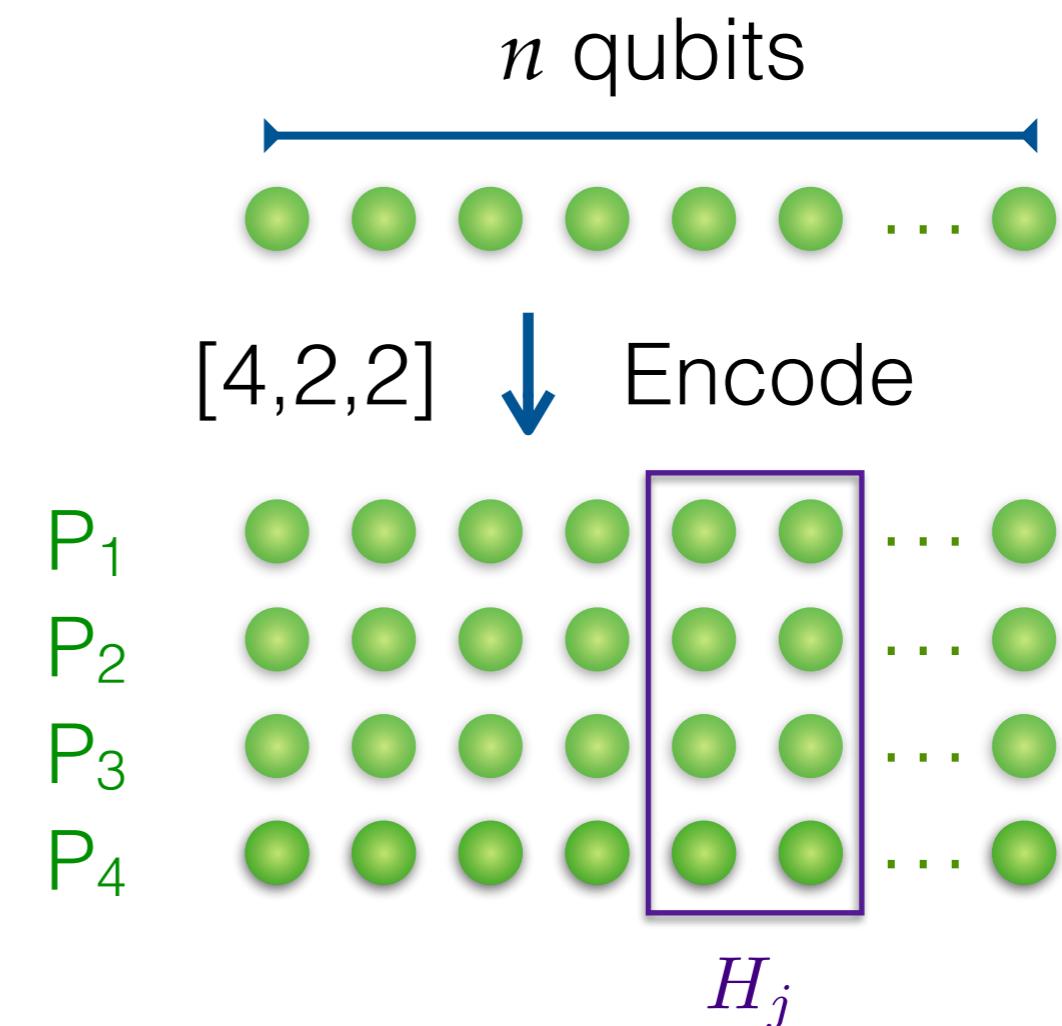
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- Hamiltonians with XZ interactions remain **QMA**-complete

[Cubitt, Montanaro 14]

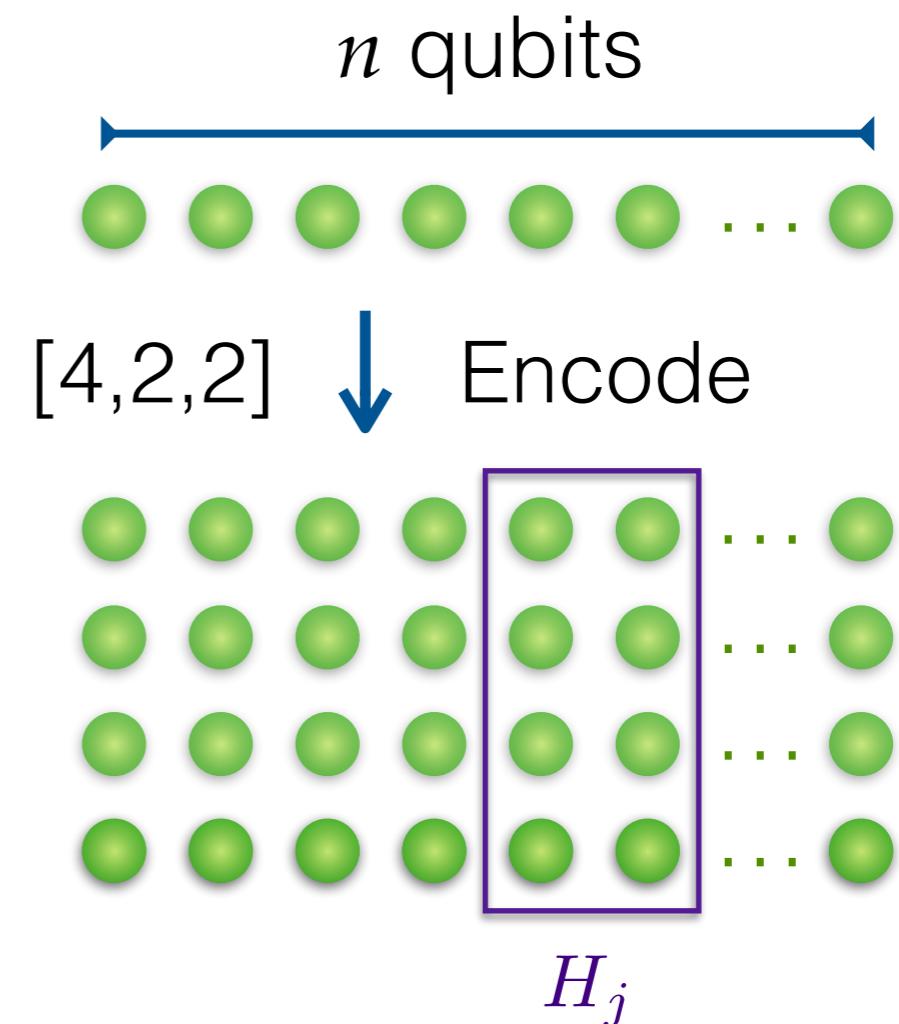


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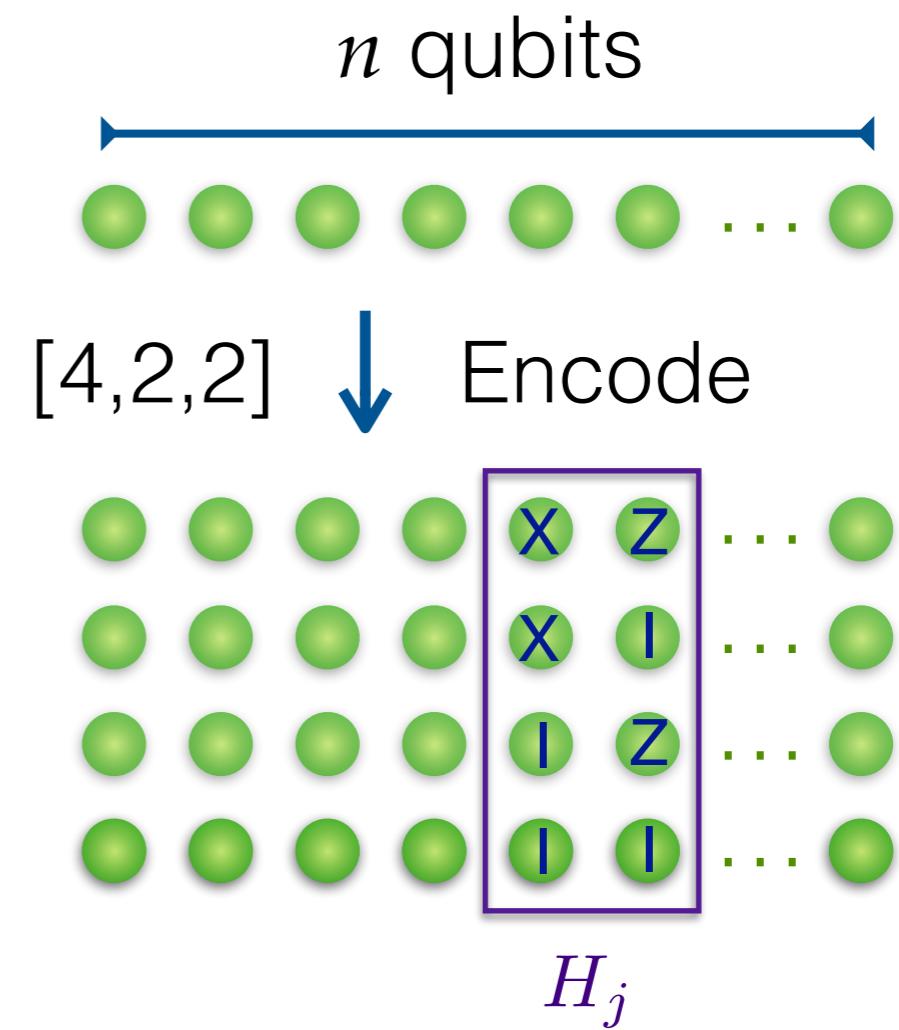
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X	X	I	I
<hr/>			
Z	I	Z	I



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001000120302012003021203012030300012012012012030  
1201200212031230302312310231230330233210203032  
10020303022010311101201201101032031012021201201030  
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01201020101012000012002031000001203020120030212030  
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001000120302012003021203012030300012012012012030  
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