

# Superconducting qubits

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for ADVANCED RESEARCH

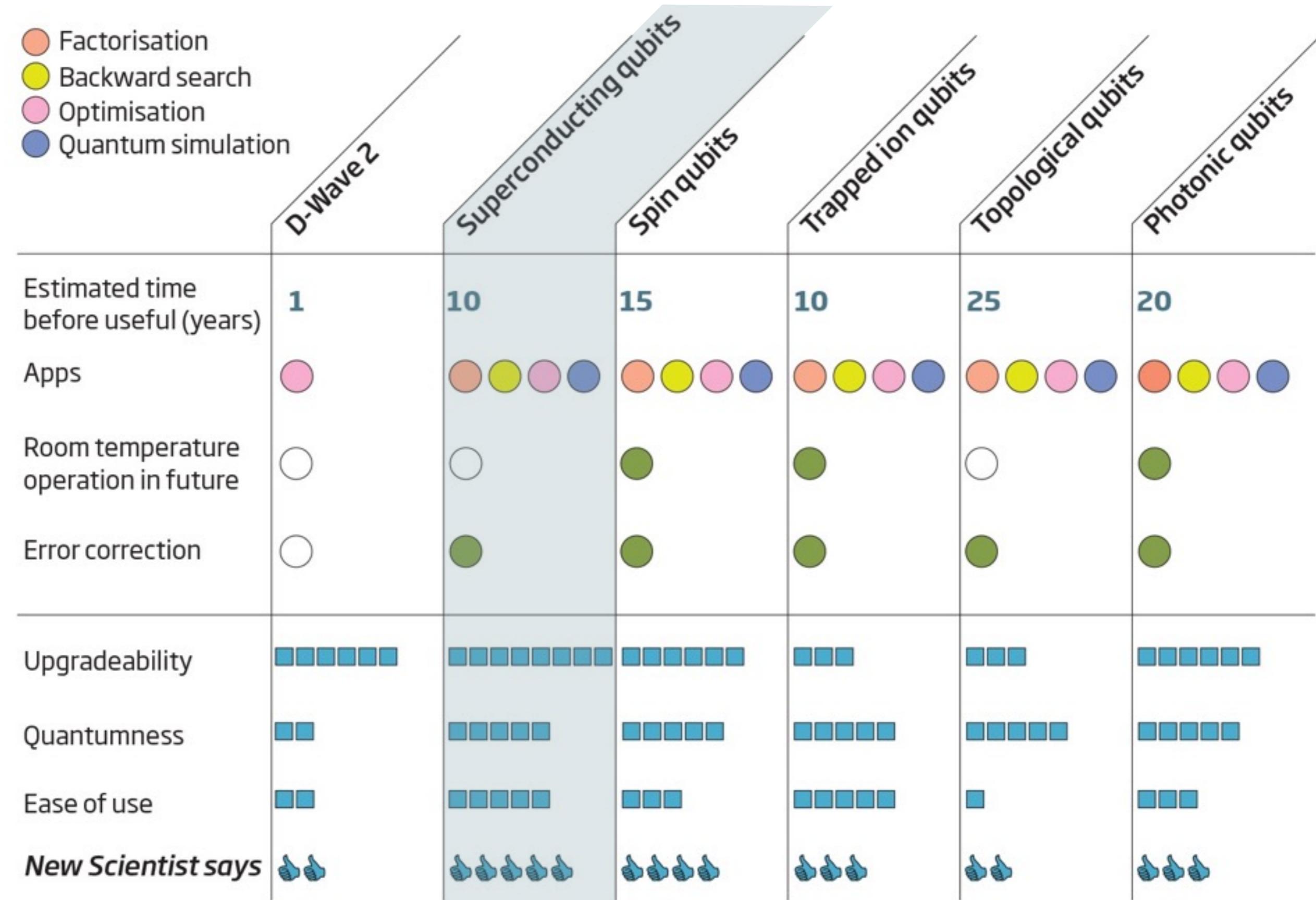


Which quantum computer  
is right for you?

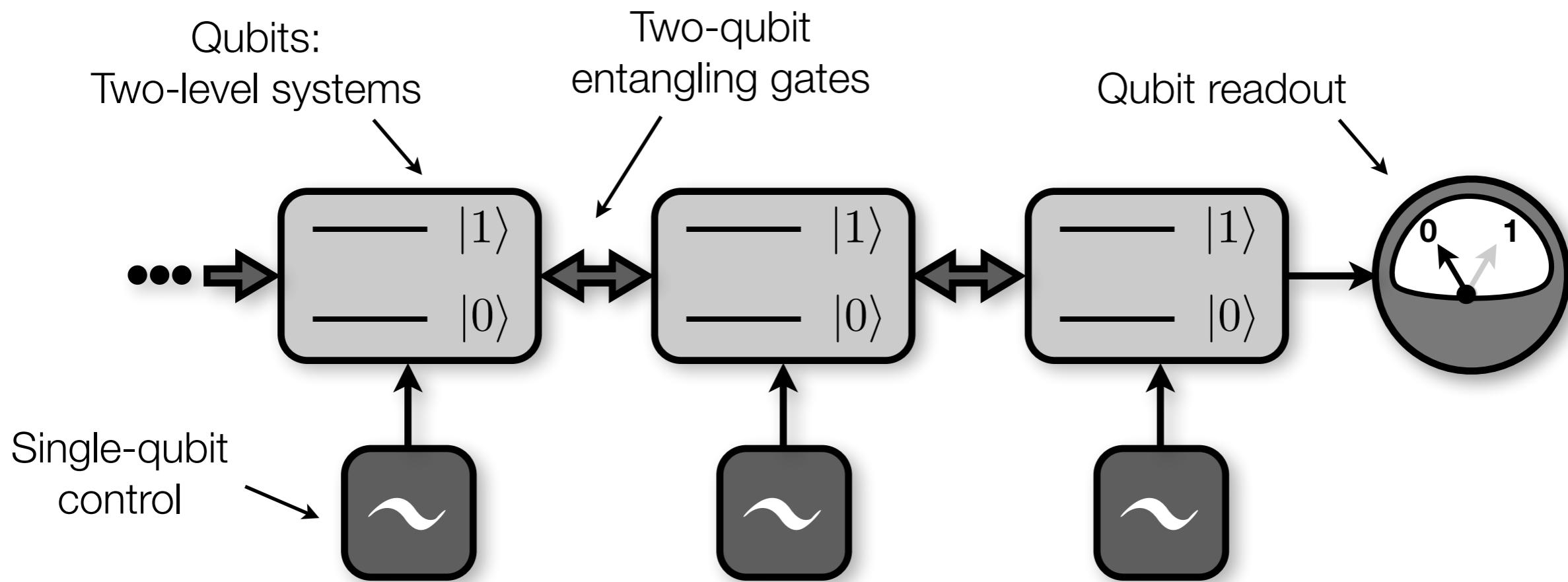
# Which quantum computer is right for you?

# NewScientist

There are many types to choose from. Here's how they compare and our all-important verdict



# Quantum information processing: the challenge



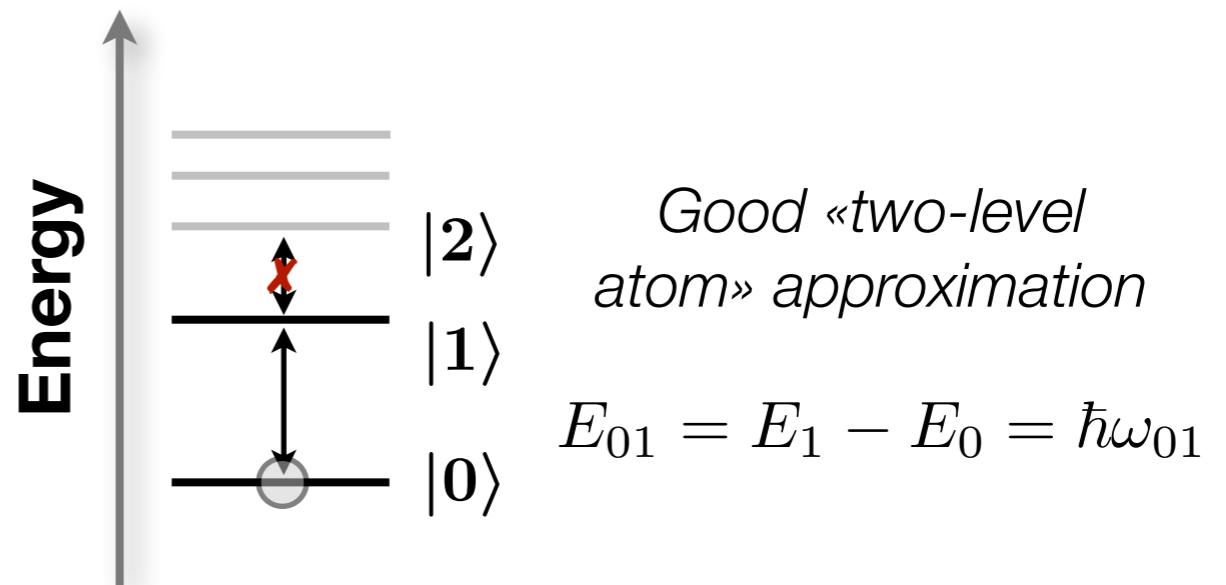
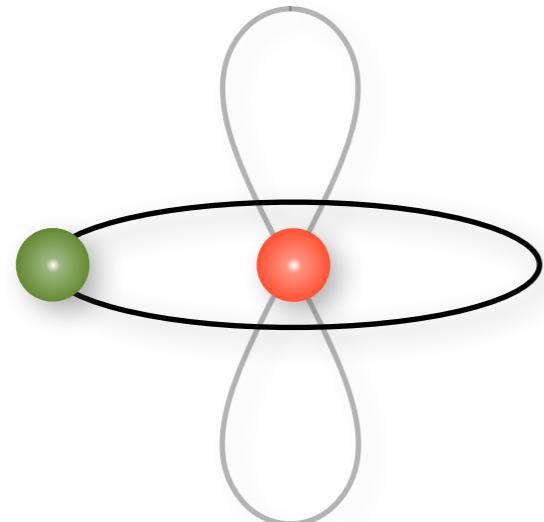
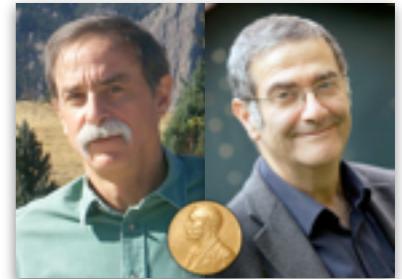
- Conflicting requirements: *long-lived* quantum effects, *fast* control and readout

# Outline

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- **Artificial atoms**
  - Physics 101: Harmonic oscillators and basic electrical circuits
  - Superconductivity and Josephson junctions
- **Circuit QED: a possible QC architecture**
- **Recent realizations and challenges**

# ‘Atomic atoms’

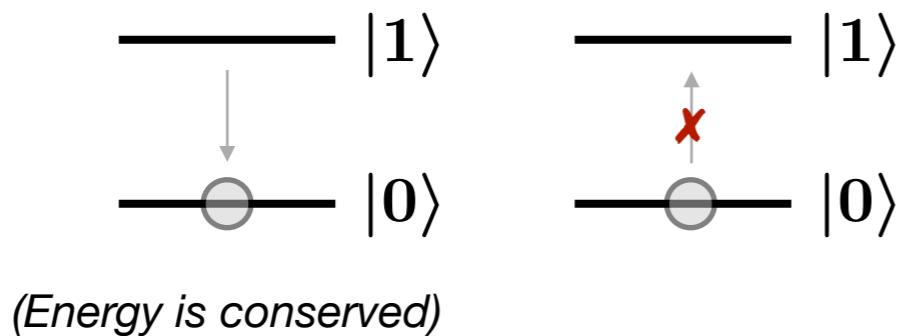


- Control by shining laser tuned at the desired transition frequency
- Hyperfine levels of  ${}^9\text{Be}_+$  have long relaxation and dephasing times

$$T_1 \sim \text{a few years} \quad T_2 \gtrsim 10 \text{ seconds}$$

# Relaxation and dephasing times

- $T_1$ : Relaxation = amplitude damping channel  $\neq$  bit flip channel



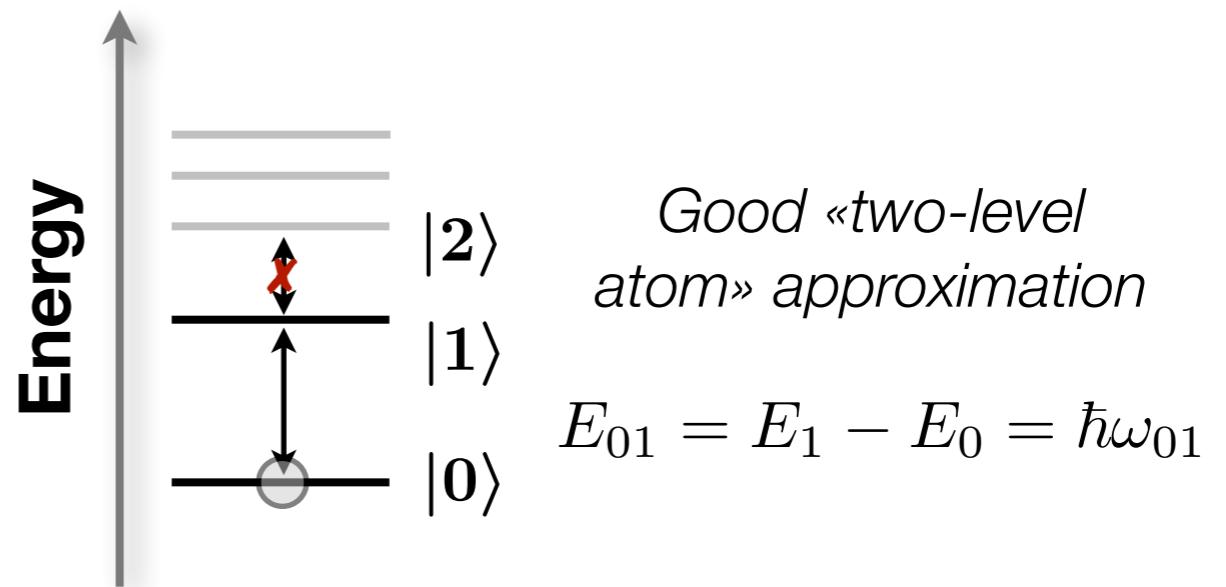
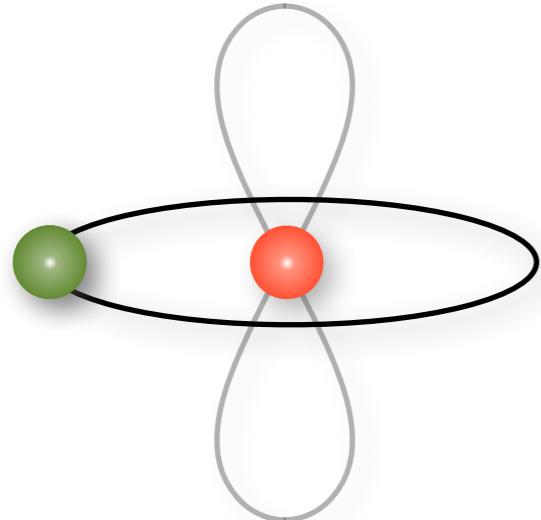
Probability of qubit having relaxed at time t:  
 $e^{-t/T_1} = e^{-\gamma_1 t}$

- $T_2$ : Dephasing = phase damping channel = phase flip channel

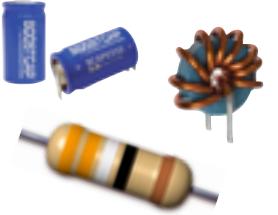
$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle \rightarrow \rho = \begin{pmatrix} |c_0|^2 & c_0 c_1^* e^{-t/T_2} \\ c_0^* c_1 e^{-t/T_2} & |c_1|^2 \end{pmatrix}$$

Probability of phase decay at time t:  
 $e^{-t/T_2} = e^{-\gamma_2 t}$

# ‘Atomic atoms’

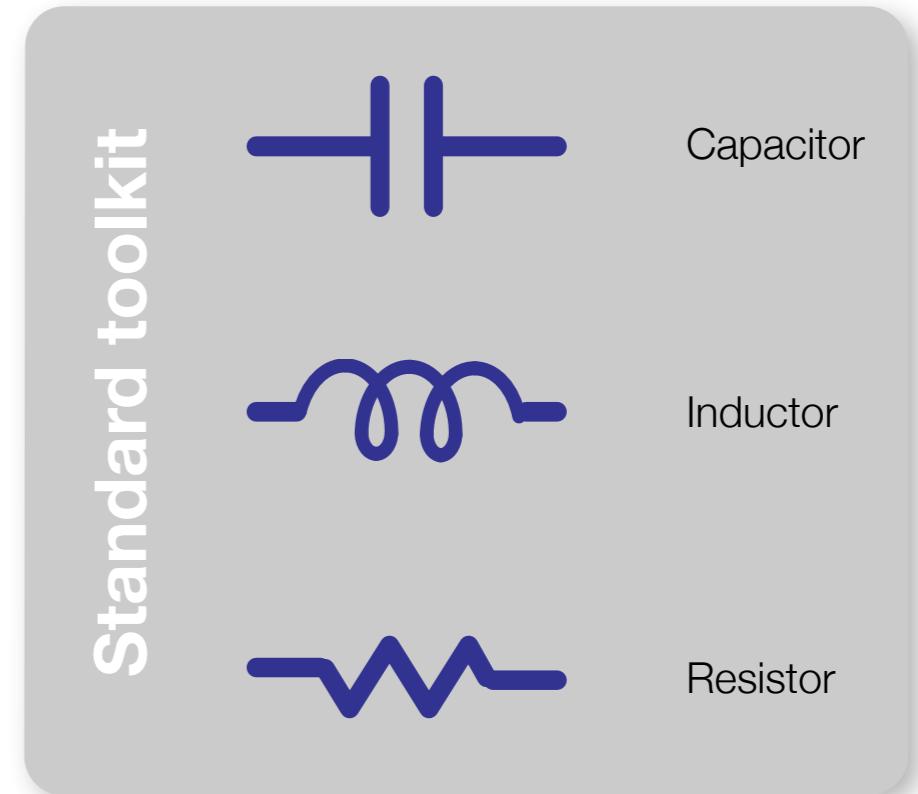


- Control by shining laser tuned at the desired transition frequency
- Hyperfine levels of  ${}^9\text{Be}_+$  have long relaxation and dephasing times  
 $T_1 \sim \text{a few years}$      $T_2 \gtrsim 10 \text{ seconds}$
- Reasonably short gate time  
 $T_{\text{not}} \sim 5 \mu\text{s}$
- Low error per gates: ~ 0.48%



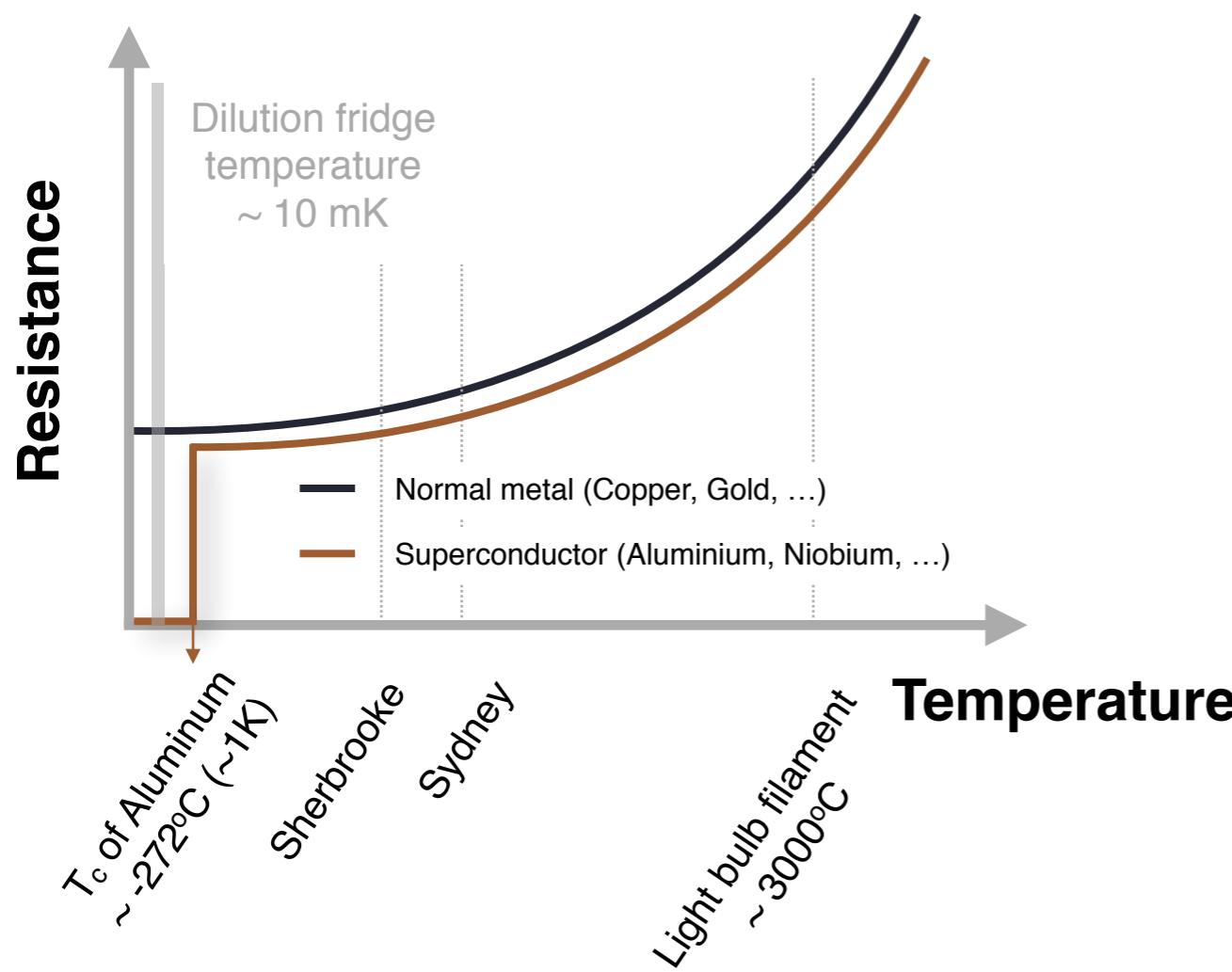
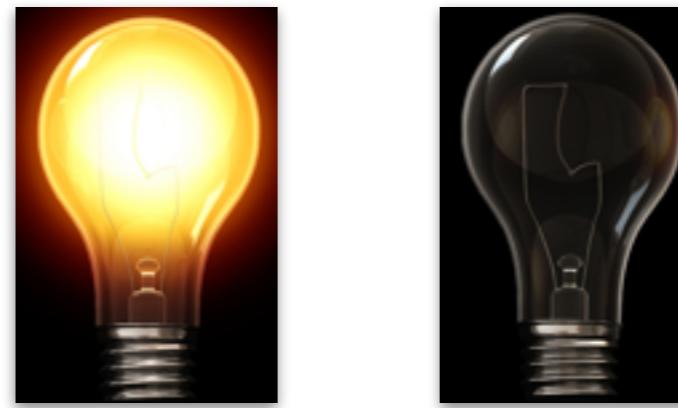
# Artificial atoms

- Based on microfabricated circuit elements
- Well defined energy levels
- Nonlinear distribution of energy levels
- Maximize numbers of thumbs up!

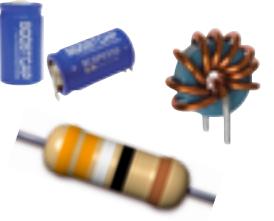


# Avoiding dissipation: superconductivity

- Normal metals dissipate energy

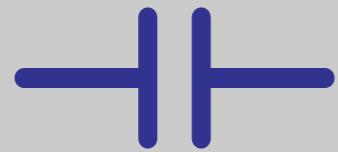


- No resistance in superconducting state  $\Rightarrow$  no dissipation
- Superconductivity is a (macroscopic) quantum effect
- A good starting point for a quantum device...

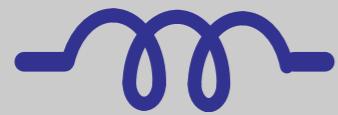


# Basic circuit elements (classical version)

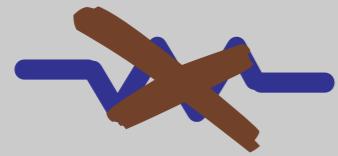
## Standard toolkit



Capacitor (C)



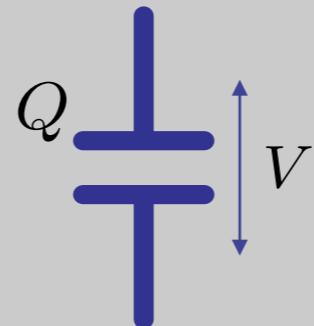
Inductor (L)



Resistor (R)

### Capacitor:

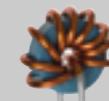
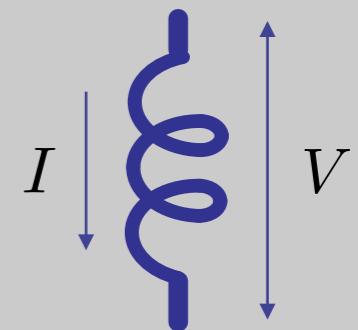
- Two metal plates separated by an insulator
- Relates voltage to charge



$$Q = CV$$

### Inductor:

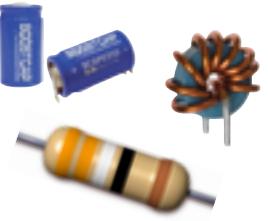
- A non-resistive wire
- Relates voltage to change of current



$$V = L \frac{dI}{dt}$$

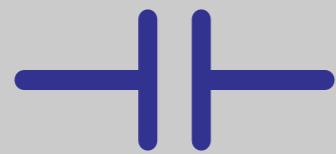
$$\Phi = LI$$

$$\Phi = \int_{-\infty}^t dt' V(t')$$

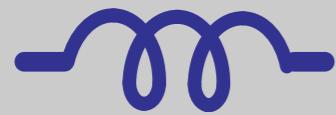


# Basic circuit elements (classical version)

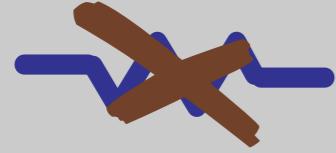
## Standard toolkit



Capacitor (C)



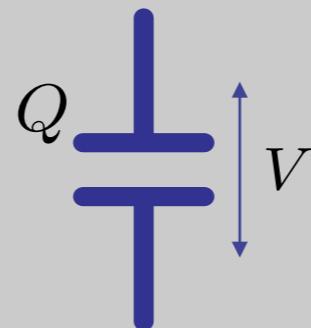
Inductor (L)



Resistor (R)

### Capacitor:

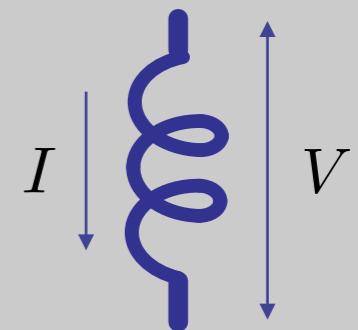
- Two metal plates separated by an insulator
- Relates voltage to charge



$$Q = CV$$

### Inductor:

- A non-resistive wire
- Relates voltage to change of current

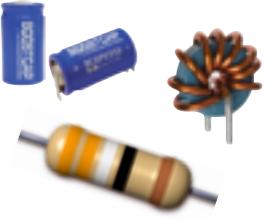


$$V = L \frac{dI}{dt}$$

### Current:

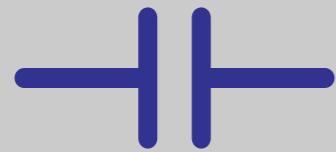
Change of charge in time

$$I = \frac{dQ}{dt}$$



# Basic circuit elements (classical version)

## Standard toolkit



Capacitor (C)



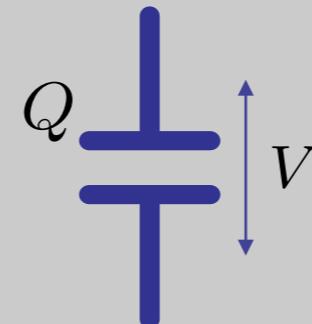
Inductor (L)



Resistor (R)

### Capacitor:

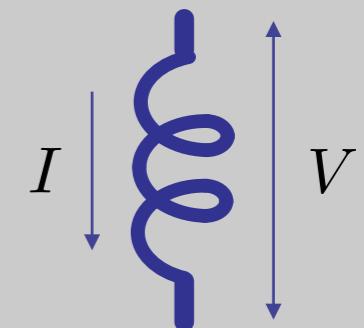
- Two metal plates separated by an insulator
- Relates voltage to charge



$$Q = CV$$

### Inductor:

- A non-resistive wire
- Relates voltage to change of current

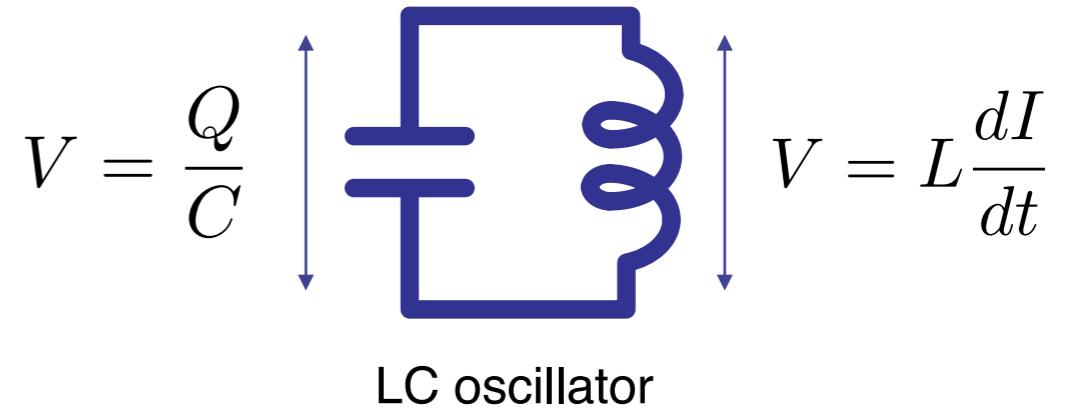


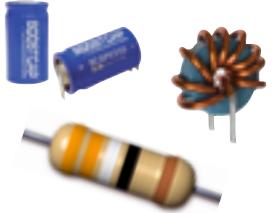
$$V = L \frac{dI}{dt}$$

Voltage is the same across L and C:

$$\frac{Q}{C} = L \frac{d^2Q}{dt^2}$$

$$Q(t) = Q(0) \cos(\omega_{LC}t) \quad \omega_{LC} = \frac{1}{\sqrt{LC}}$$



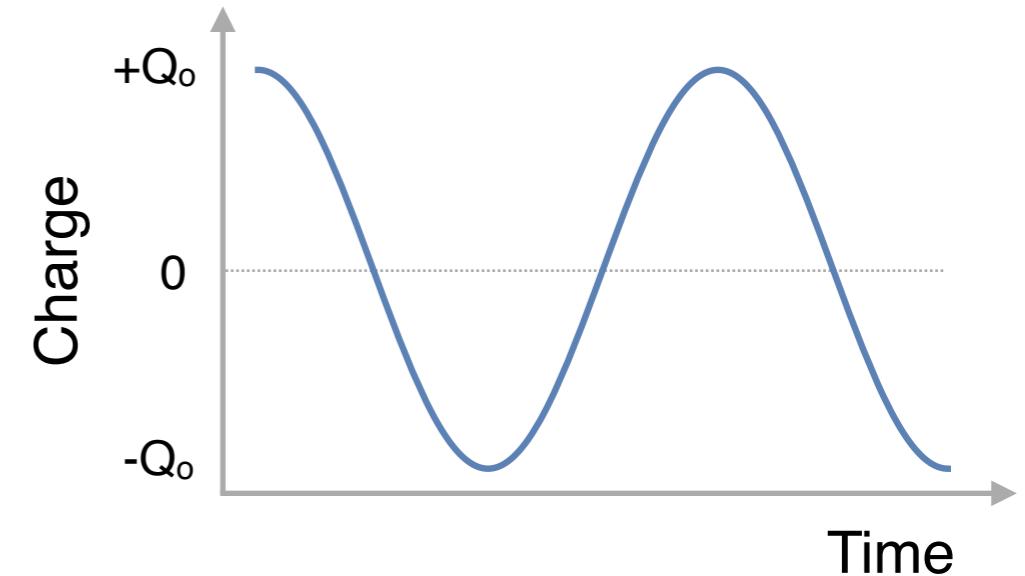
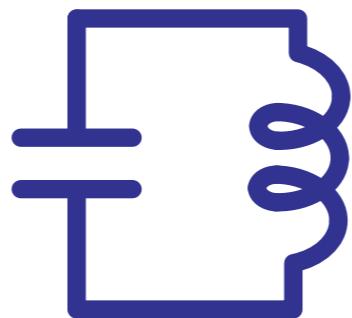


# Basic circuit elements (classical version)

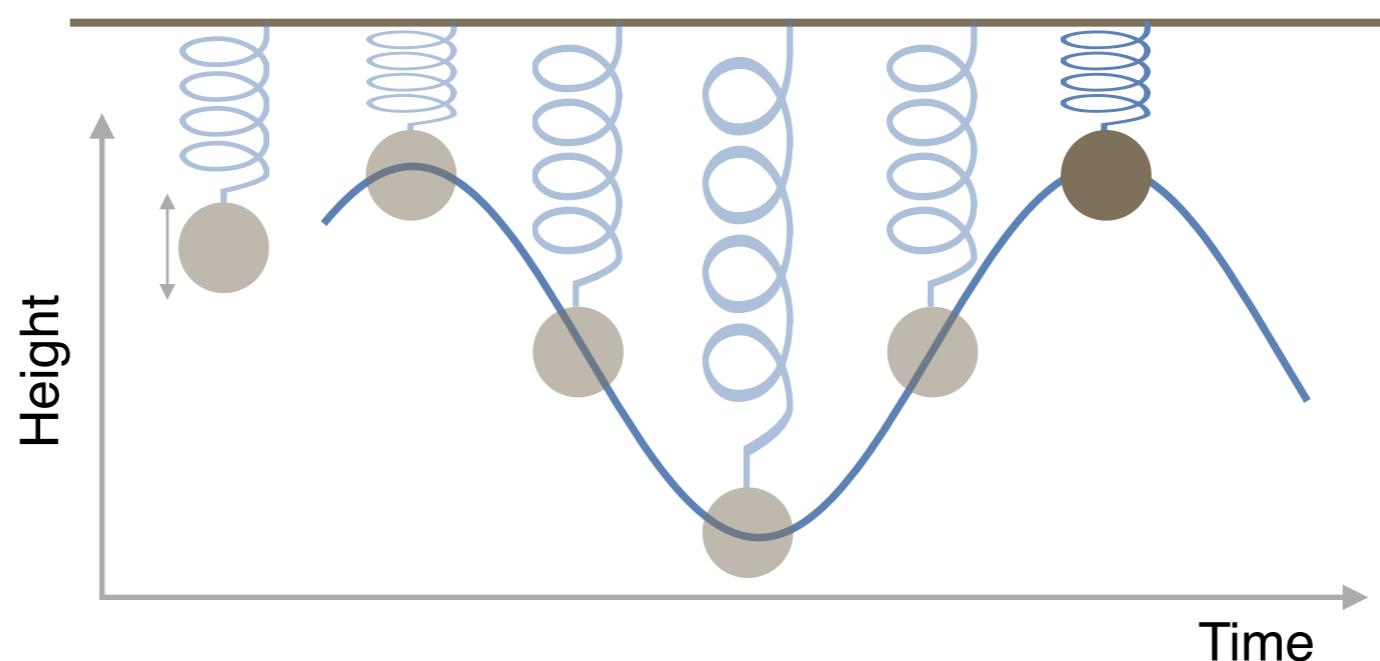
Oscillations of the charge:

$$Q(t) = Q(0) \cos(\omega_{LC} t)$$

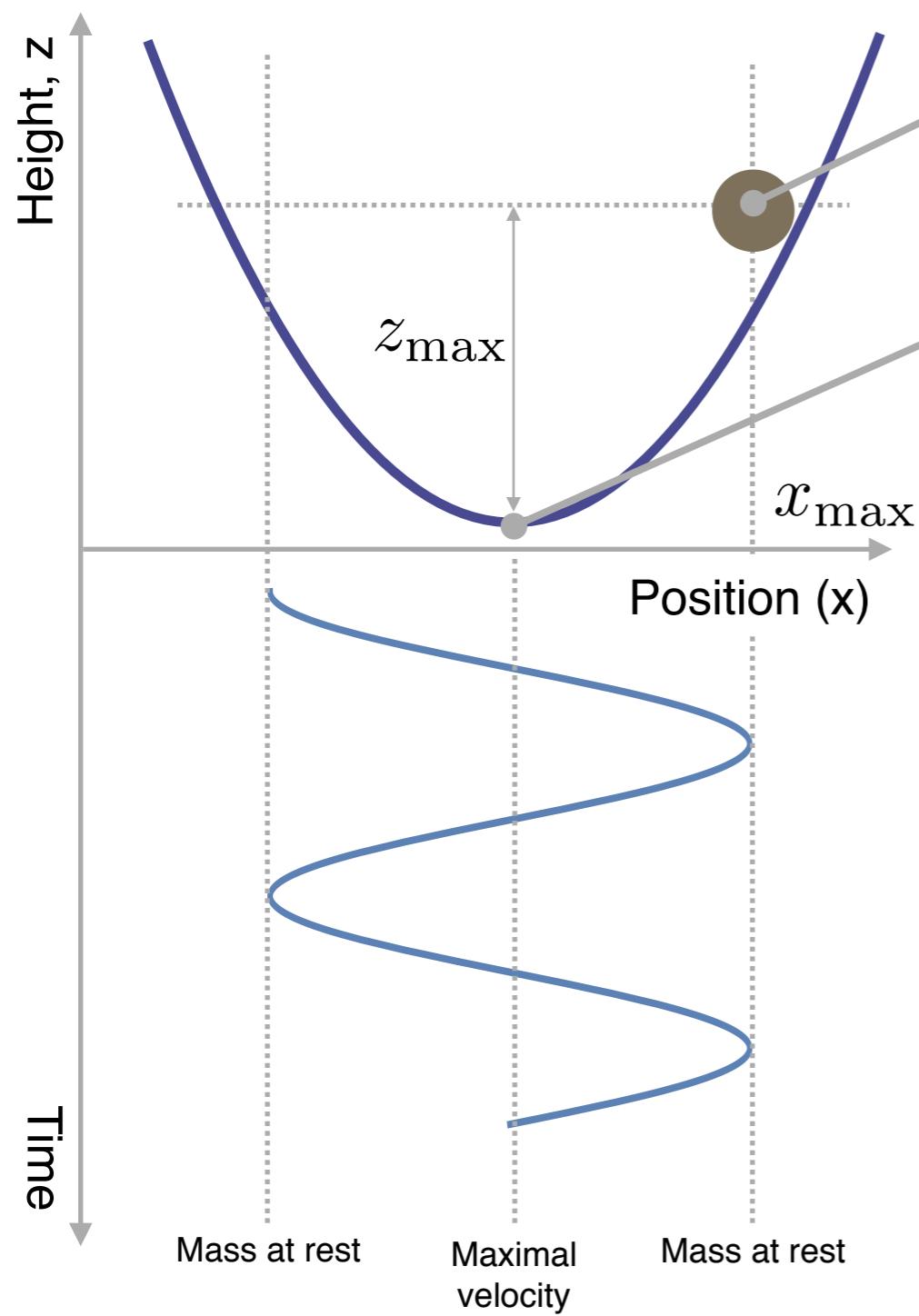
$$\omega_{LC} = \frac{1}{\sqrt{LC}}$$



One out of countless examples of *harmonic oscillator*



# Classical harmonic oscillator



$$E = \frac{1}{2}kx_{\max}^2$$

$$E = \frac{1}{2}mv_{\max}^2 = \frac{p_{\max}^2}{2m} \quad (p = mv)$$

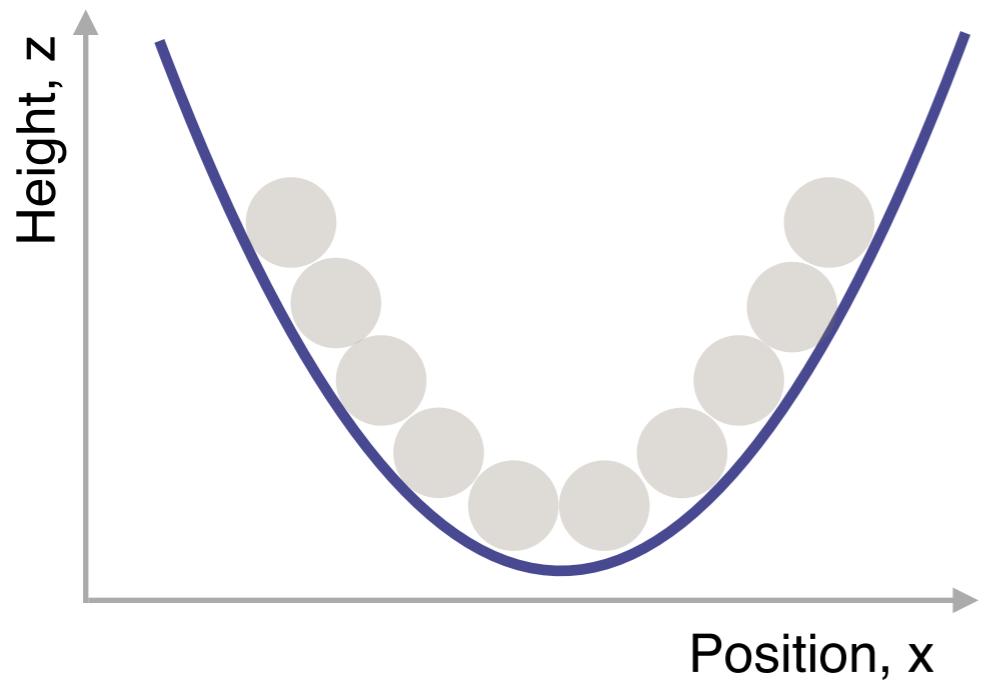
*Momentum*

Energy at arbitrary  $x$ :  
= Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

Frequency of oscillation:  $\omega = \sqrt{k/m}$

# Quantum harmonic oscillator



Energy at arbitrary  $x$ :  
= Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

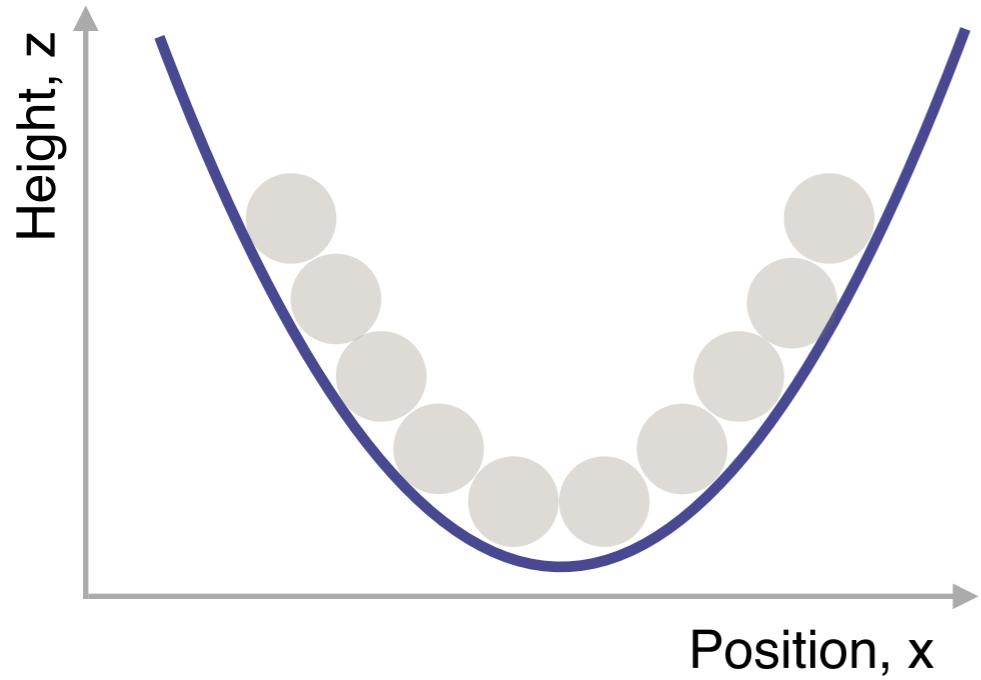
Heisenberg uncertainty principle:  
Impossible to know precisely both  $x$  and  $p$

Classical variables are promoted to hermitian operator acting on Hilbert space

$$x \rightarrow \hat{x} \quad p \rightarrow \hat{p}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

# Quantum harmonic oscillator



Energy at arbitrary  $x$ :

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

Classical variables are promoted to hermitian operator acting on Hilbert space

$$x \rightarrow \hat{x} \quad p \rightarrow \hat{p} \quad [\hat{x}, \hat{p}] = i\hbar$$

Useful to introduce:

$$\hat{a} = \left( \frac{mk}{4\hbar^2} \right)^{1/4} \left( \hat{x} + i \frac{\hat{p}}{\sqrt{mk}} \right)$$

$$\hat{a}^\dagger = \left( \frac{mk}{4\hbar^2} \right)^{1/4} \left( \hat{x} - i \frac{\hat{p}}{\sqrt{mk}} \right)$$

Commutation relation:

$$[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{H} = \hbar\omega \hat{a}^\dagger \hat{a} = \hbar\omega \hat{n}$$

$(\omega = \sqrt{k/m})$

# Quantum harmonic oscillator

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$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} = \hbar\omega\hat{n}$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{n}|n\rangle = n|n\rangle$$

$$\begin{aligned}\hat{a}|n\rangle &=? \\ \hat{a}^\dagger|n\rangle &=?\end{aligned}$$

What is the action of  $\hat{a}$  and  $\hat{a}^\dagger$  on the eigenstates of  $\hat{n}$ ?

First observation:  $[\hat{n}, \hat{a}] = -\hat{a}$  and  $[\hat{n}, \hat{a}^\dagger] = \hat{a}^\dagger$

$$\Rightarrow \hat{n}(\hat{a}|n\rangle) = \hat{a}\hat{n}|n\rangle - \hat{a}|n\rangle = (n-1)\hat{a}|n\rangle$$

$$\hat{a}|n\rangle \propto |n-1\rangle$$

Second observation:  $||\hat{a}|n\rangle||^2 = \langle n|\hat{a}^\dagger\hat{a}|n\rangle = \langle n|\hat{n}|n\rangle = n \Rightarrow n \in \mathbb{N}_0$

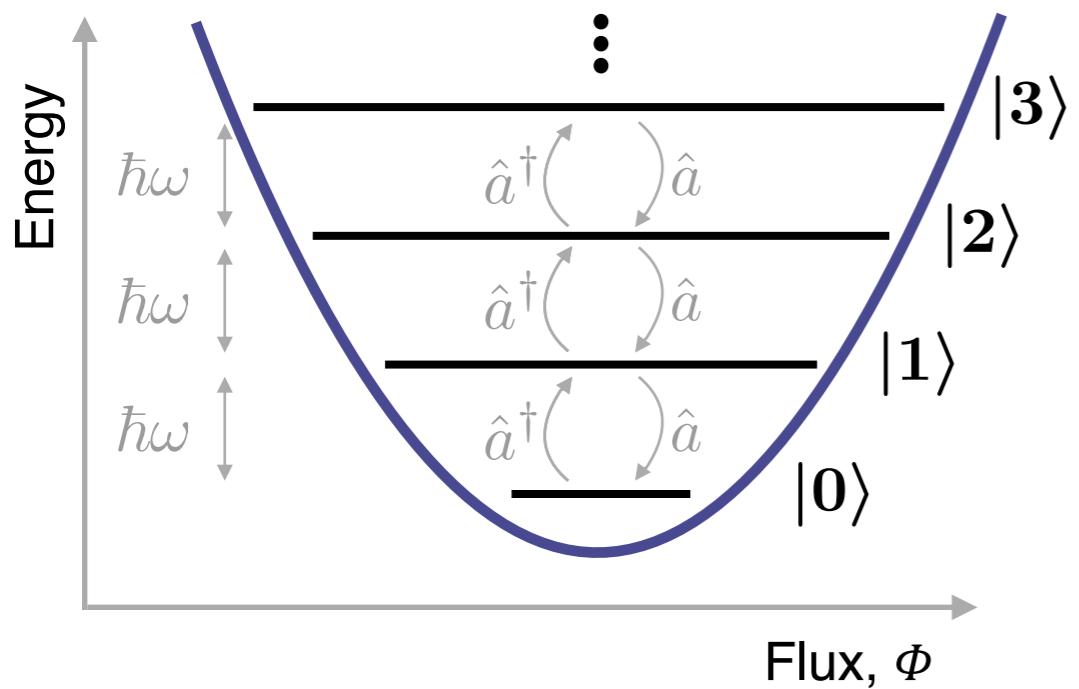
$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

# Quantum harmonic oscillator

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} = \hbar\omega\hat{n} \quad \hat{n}|n\rangle = n|n\rangle \quad n \geq 0$$

$$\begin{aligned}\hat{a}|n\rangle &= \sqrt{n}|n-1\rangle \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle\end{aligned}$$



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

$$\omega = \sqrt{k/m}$$

$$\omega_{LC} = \frac{1}{\sqrt{LC}}$$



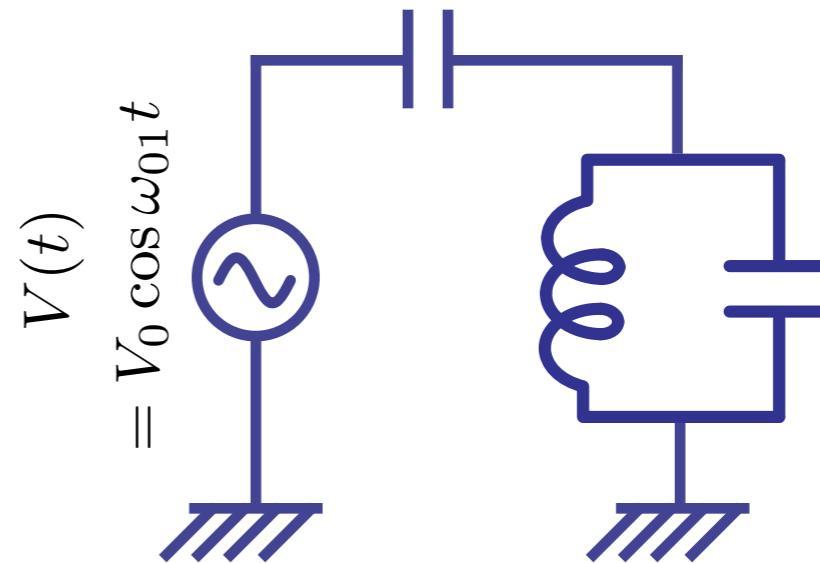
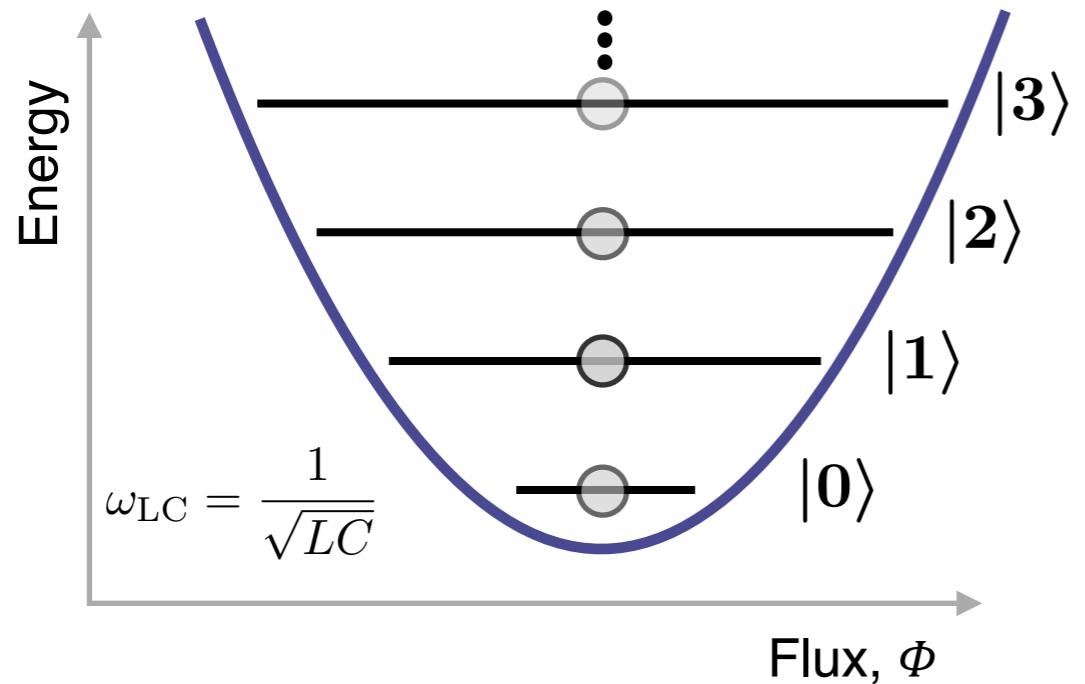
Flux:  $\Phi = \int dt V(t)$

$$\hat{a}^\dagger = \left( \frac{C}{4L\hbar^2} \right)^{1/4} \left( \hat{\Phi} - i \frac{\hat{Q}}{\sqrt{C/L}} \right)$$

(Magnetic field)      (Electric field)

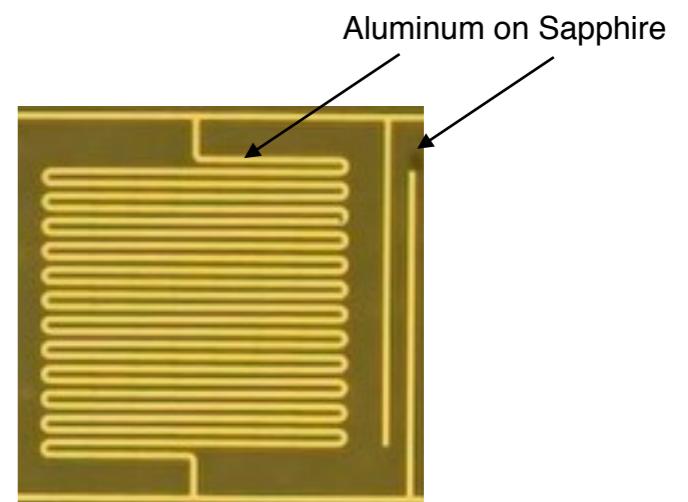
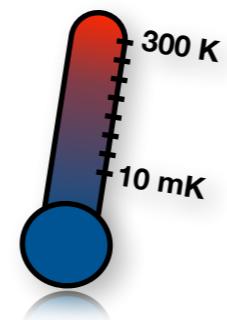
$\hat{a}^\dagger$  adds a photon to the LC circuit  
 $n$  = number of photons stored in the LC circuit

# Artificial atom



- Initialization to ground state is simple

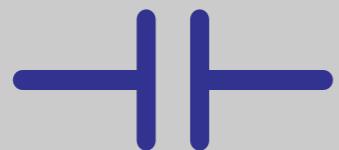
$$\omega_{01} = 1/\sqrt{LC} \sim 10 \text{ GHz}$$
$$\sim 0.5 \text{ K}$$



- Not a good «two-level» atom, not a qubit...

# Josephson junction

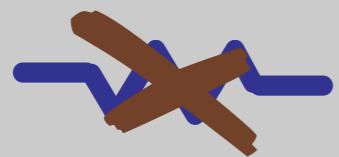
## Standard toolkit



Capacitor (C)



Inductor (L)



Resistor (R)

## Josephson junctions



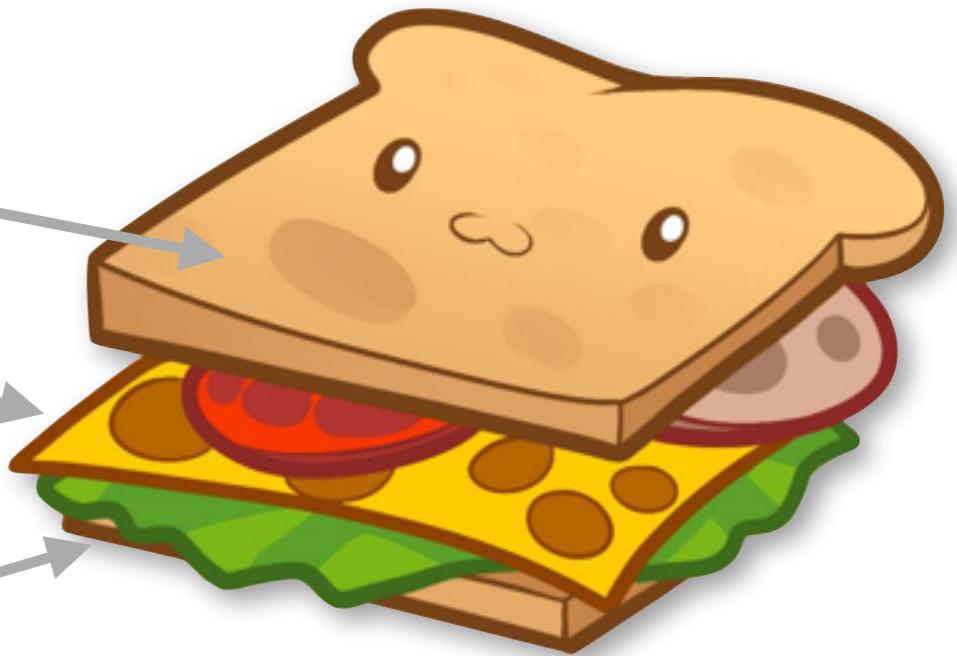
**Superconductor (Al)**



**Insulator ( $\text{AlO}_x$ )**

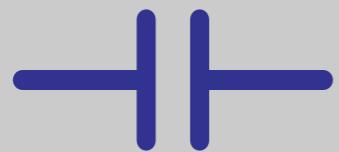


**Superconductor (Al)**



# Josephson junction

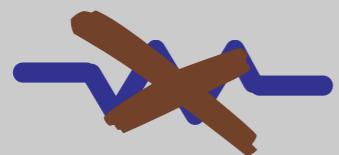
## Standard toolkit



Capacitor (C)

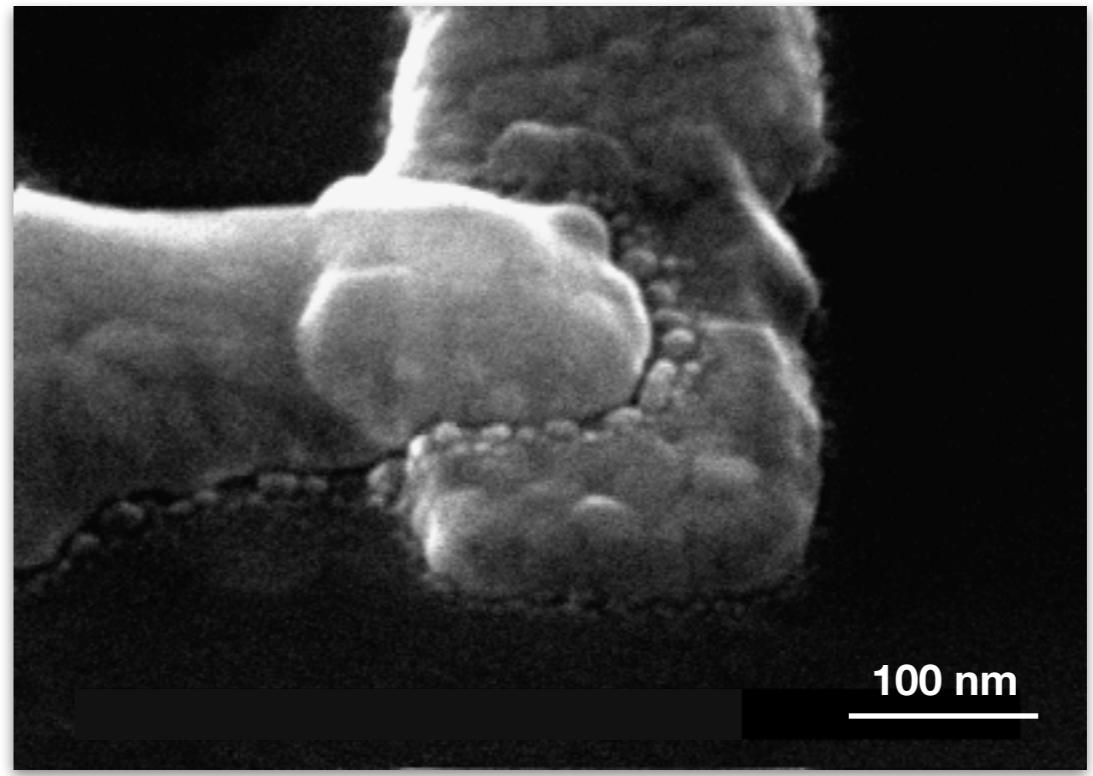
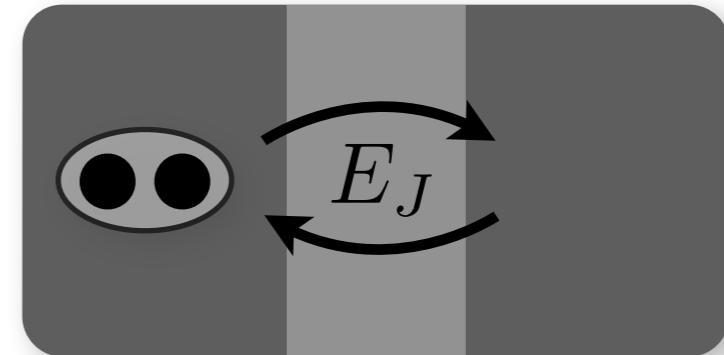


Inductor (L)

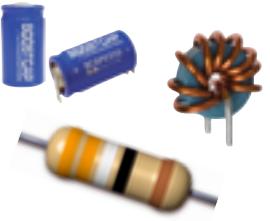


Resistor (R)

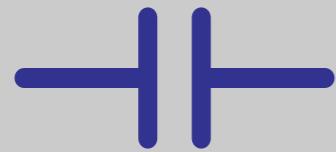
## Josephson junctions



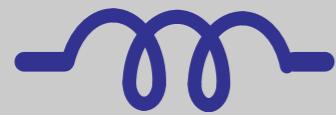
# Artificial atom toolkit



## Artificial atom toolkit



Capacitor (C)



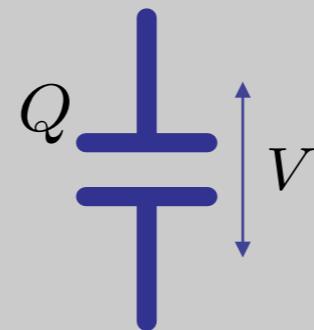
Inductor (L)



Josephson junction

### Capacitor:

- Two metal plates separated by an insulator
- Relates voltage to charge

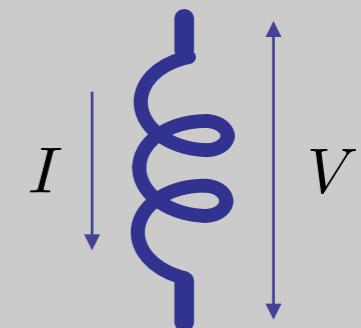


$$Q = CV$$



### Inductor:

- A non-resistive wire
- Relates current to flux

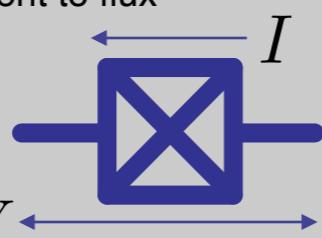


$$V = L \frac{dI}{dt}$$



$$\Phi = LI$$

$$\Phi = \int_{-\infty}^t dt' V(t')$$

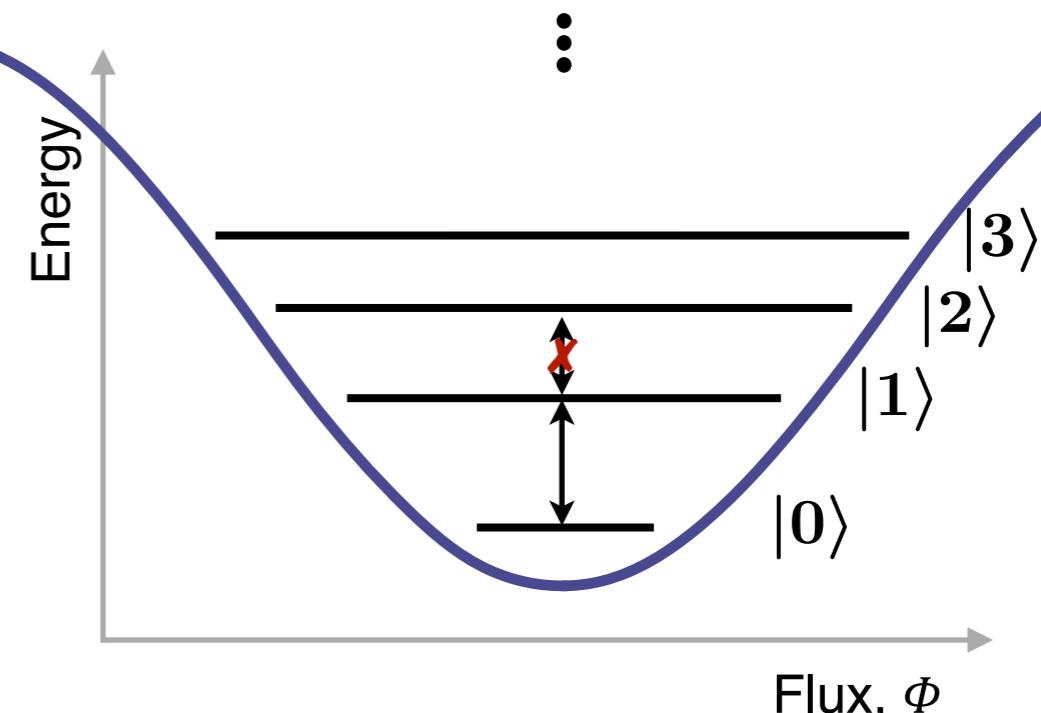
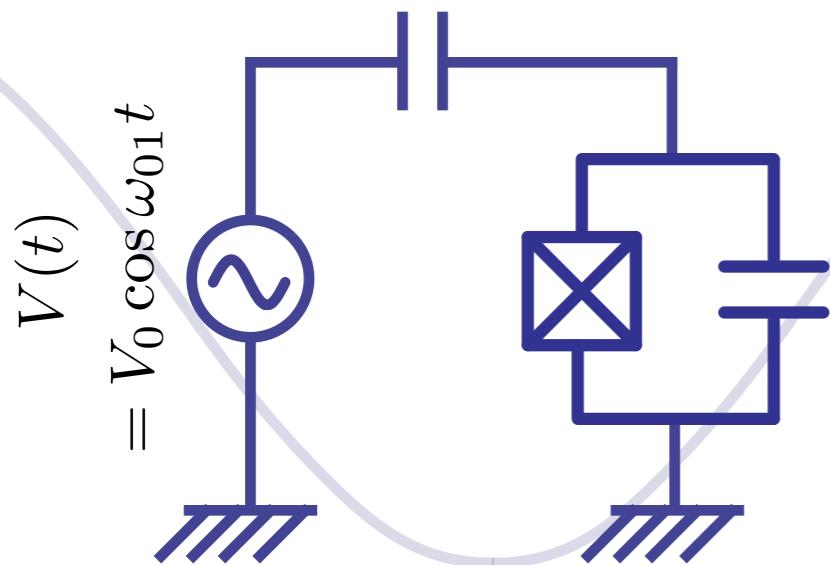


$$I = I_0 \sin(2\pi\Phi/\Phi_0)$$

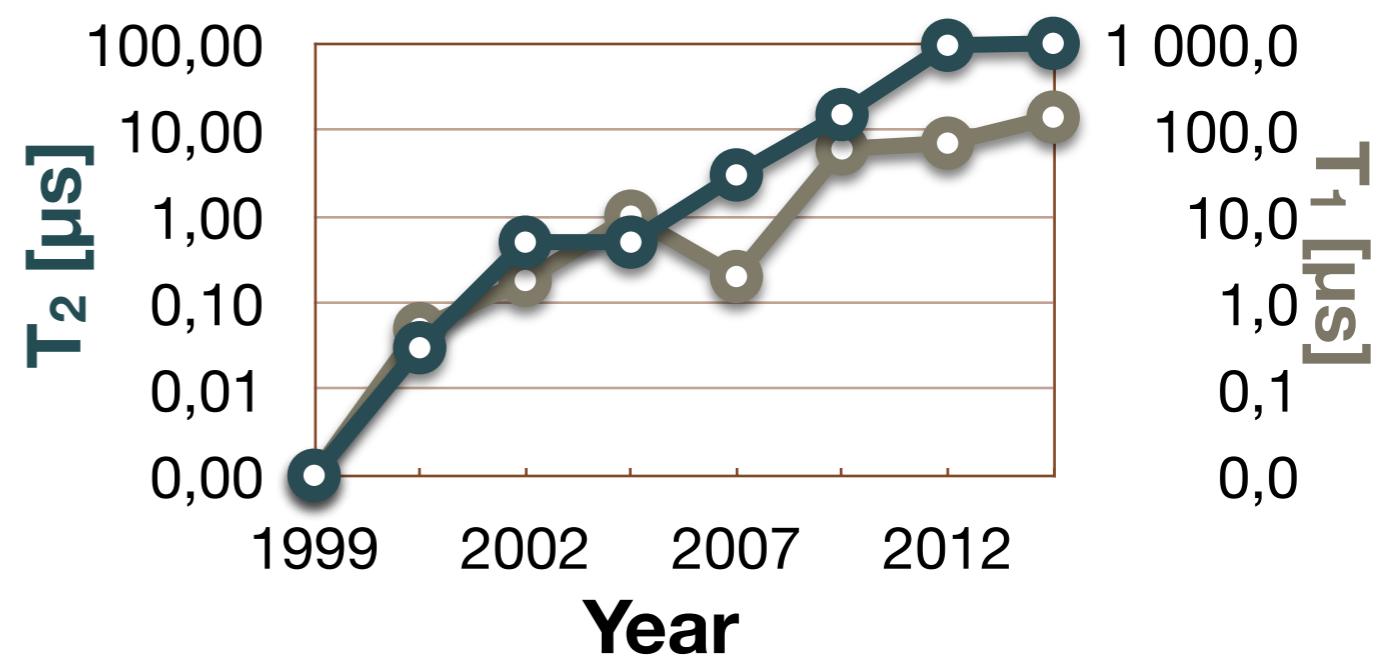
### Josephson junction:

- Two superconductors separated by an insulator
- Relates current to flux

# Superconducting artificial atom

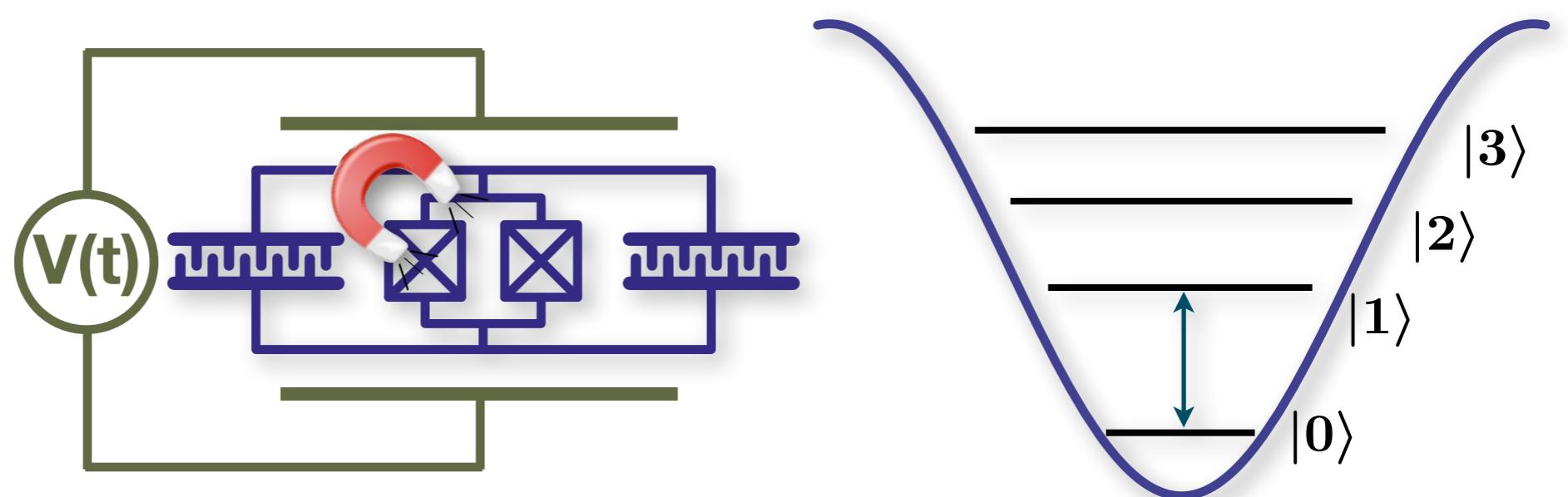
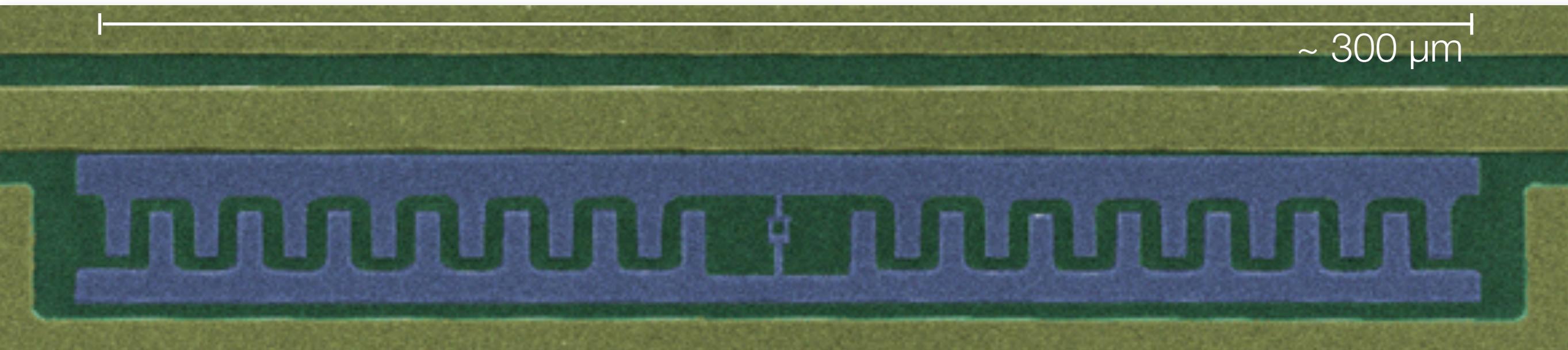


- Very short  $\pi$ -pulse time  
 $T_\pi \sim 4 - 20$  ns
- Big improvements in relaxation and dephasing times in last 10 years
- Error per gates of 0.2%, similar to trapped ion results



Low error per gates: E. Magesan *et al*, Phys. Rev. Lett. **109**, 080505 (2012)  
Long  $T_1$  and  $T_2$ : H. Paik *et al*, Phys. Rev. Lett. **107**, 240501 (2011)

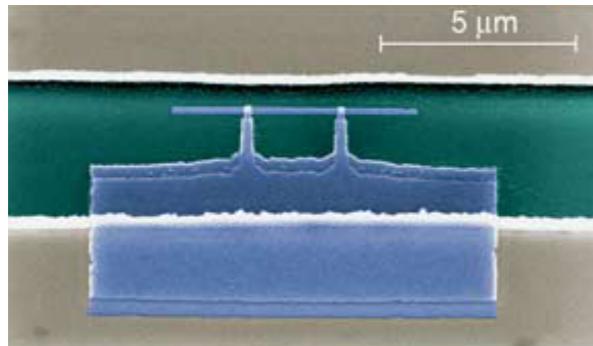
# Superconducting *transmon* qubits





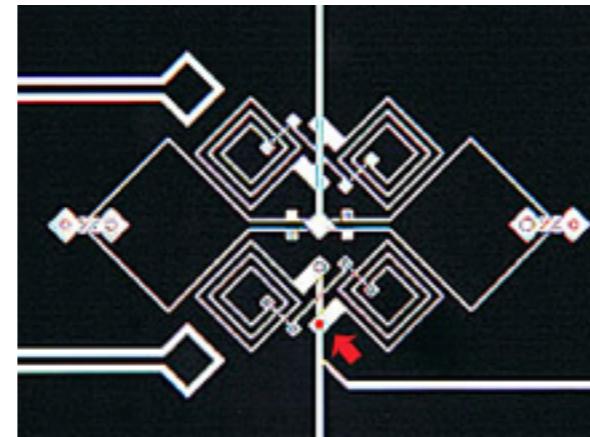
# Superconducting qubits, a family tree

**Charge**



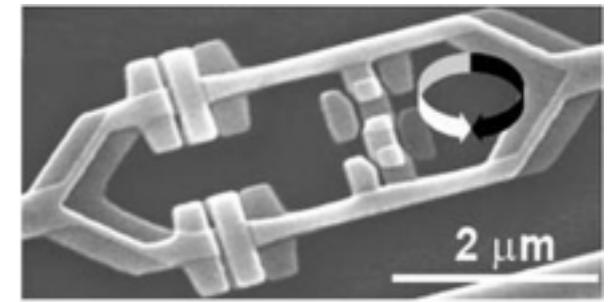
NEC, Saclay, 1999

**Phase**



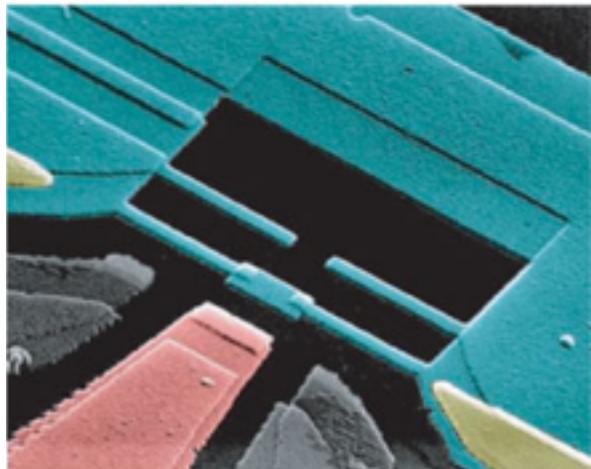
NIST 2002

**Flux**



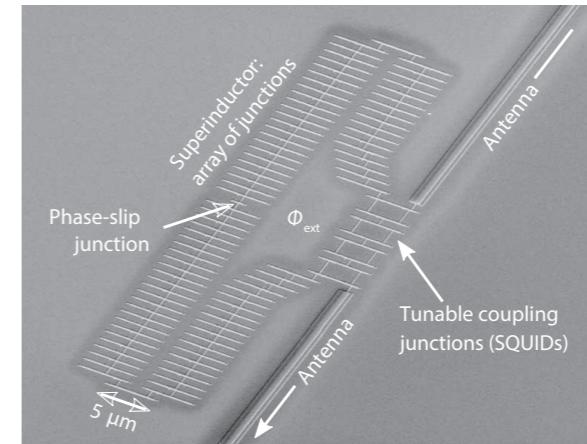
Delft, 1999

**Quantronium**



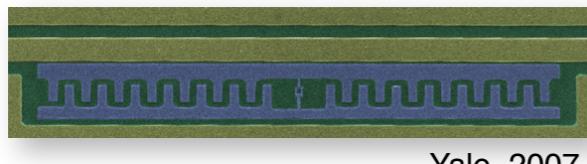
Saclay, 2002

**Fluxonium**



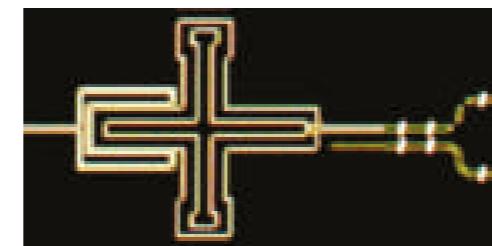
Yale, 2009

**Transmon**



Yale, 2007

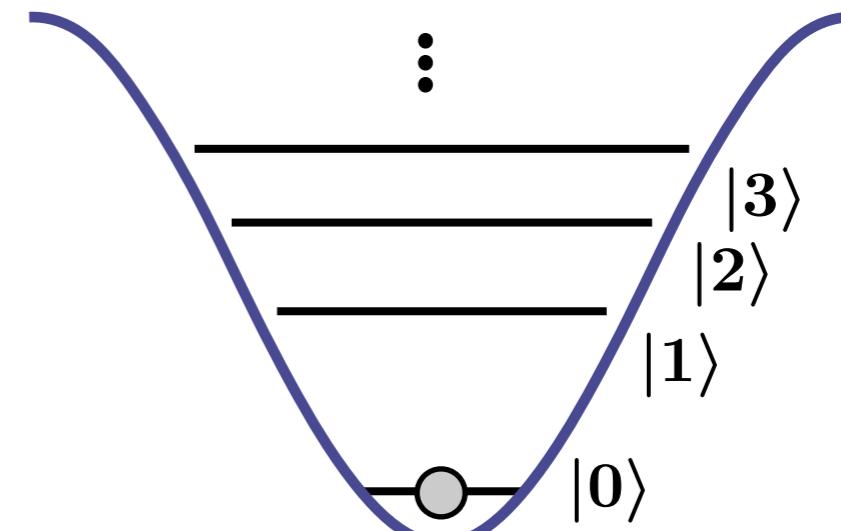
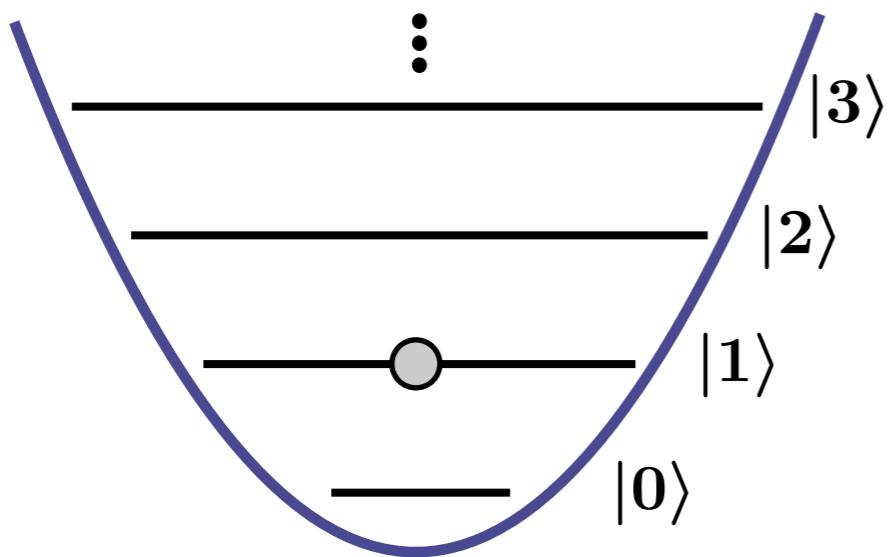
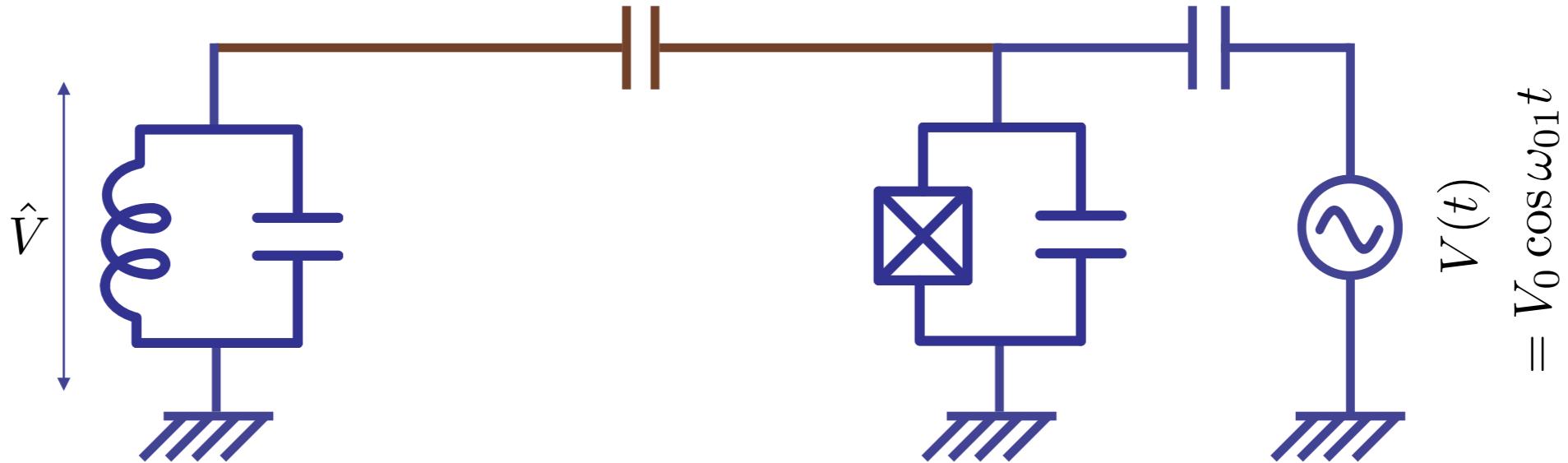
=



UCSB, 2013

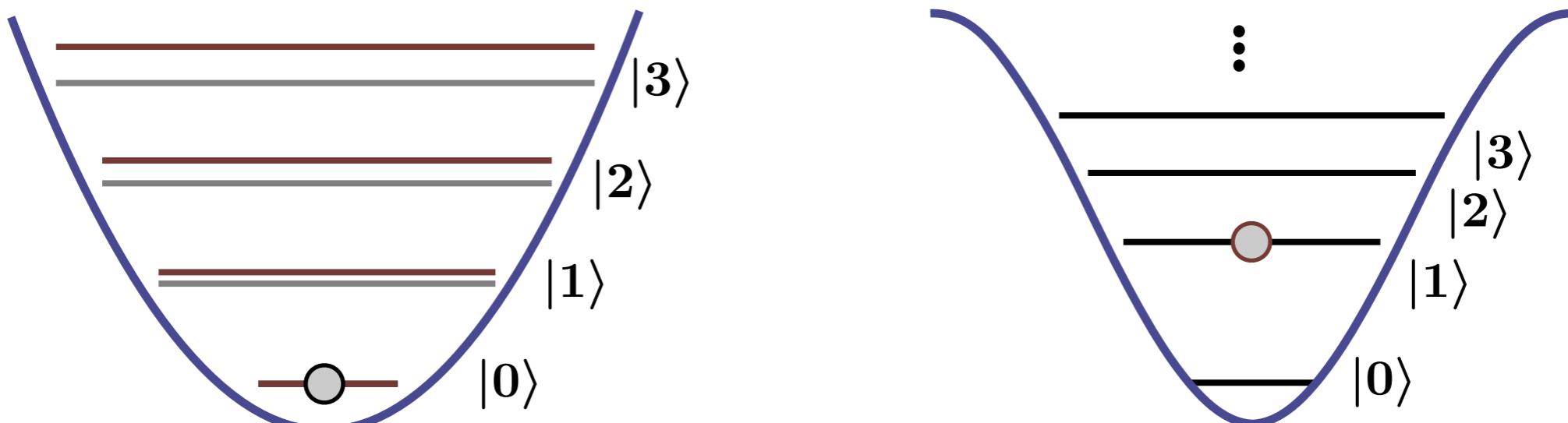
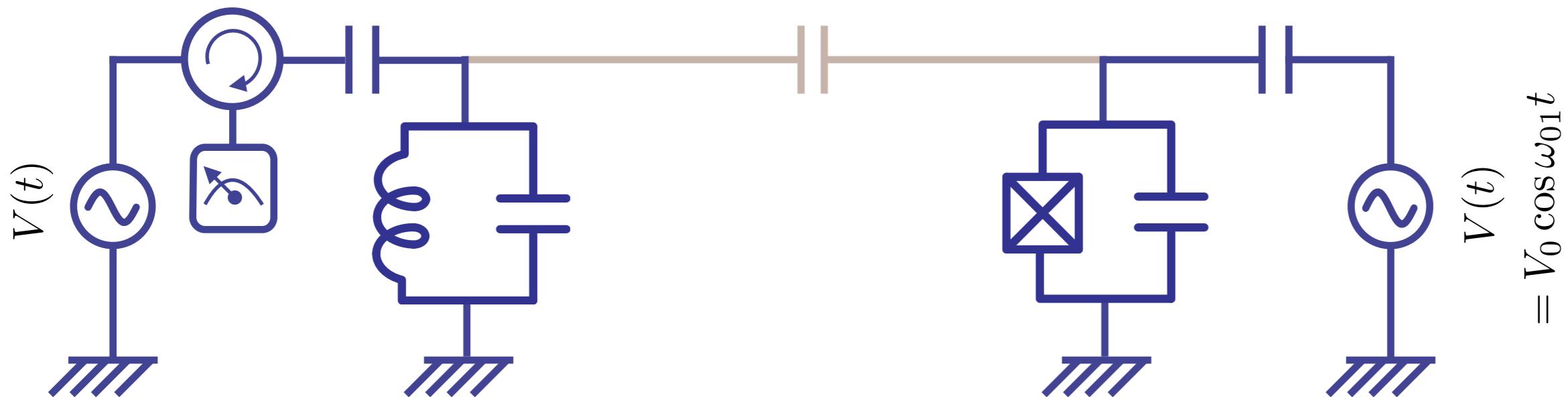
# Circuit QED

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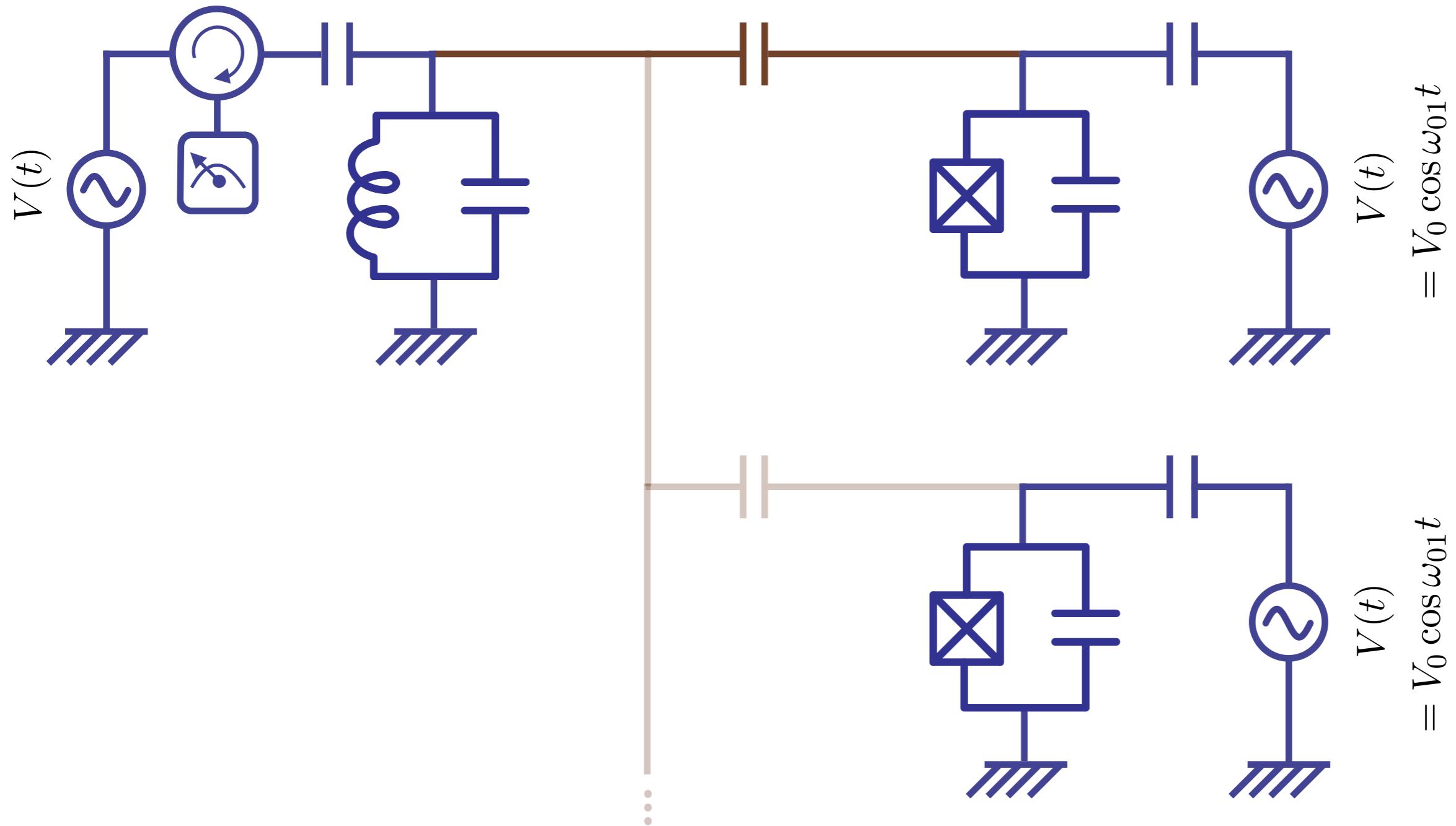
# Circuit QED

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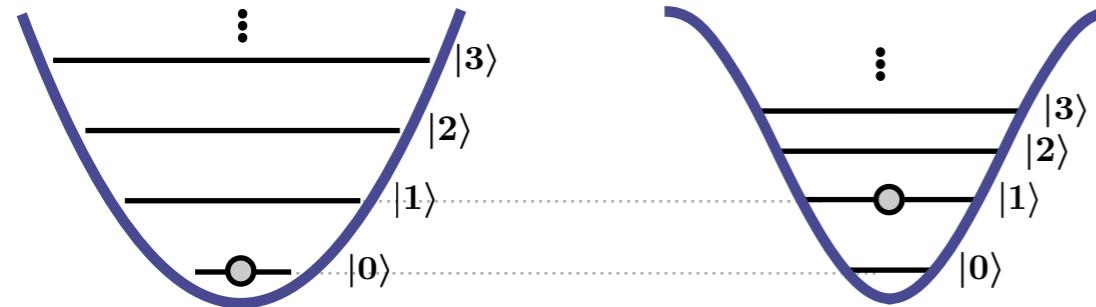


# Circuit QED: Multi-qubit architecture

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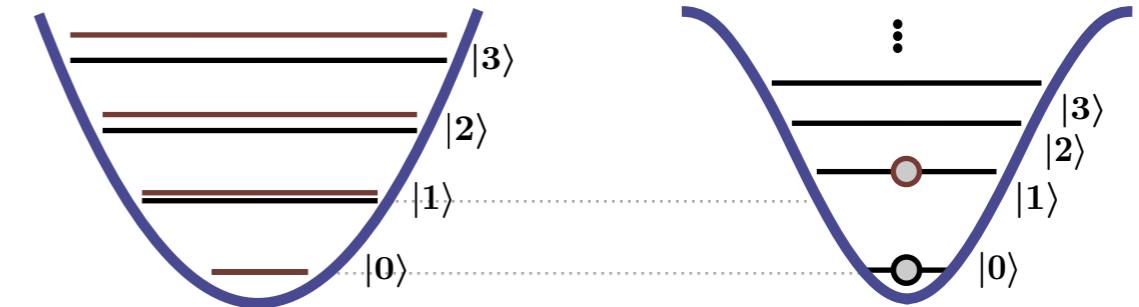


# Circuit QED: Resonant and dispersive regimes



## Resonant regime:

- Identical 0-1 transition frequencies
- Energy exchange between qubits and oscillator
- Oscillator acts as quantum bus for entangling qubits



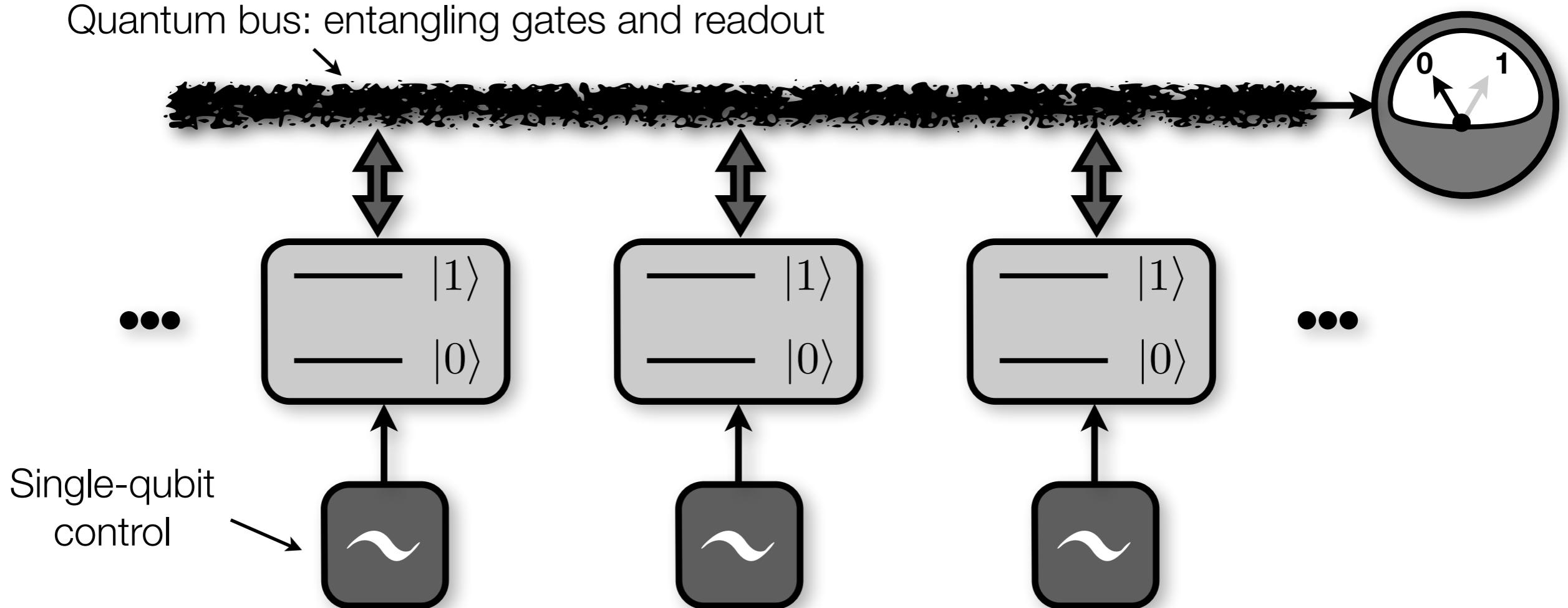
## Dispersive regime:

- Different 0-1 transition frequencies
- No energy exchange between qubits and oscillator
- Qubit-state dependent oscillator frequency leads allows qubit readout

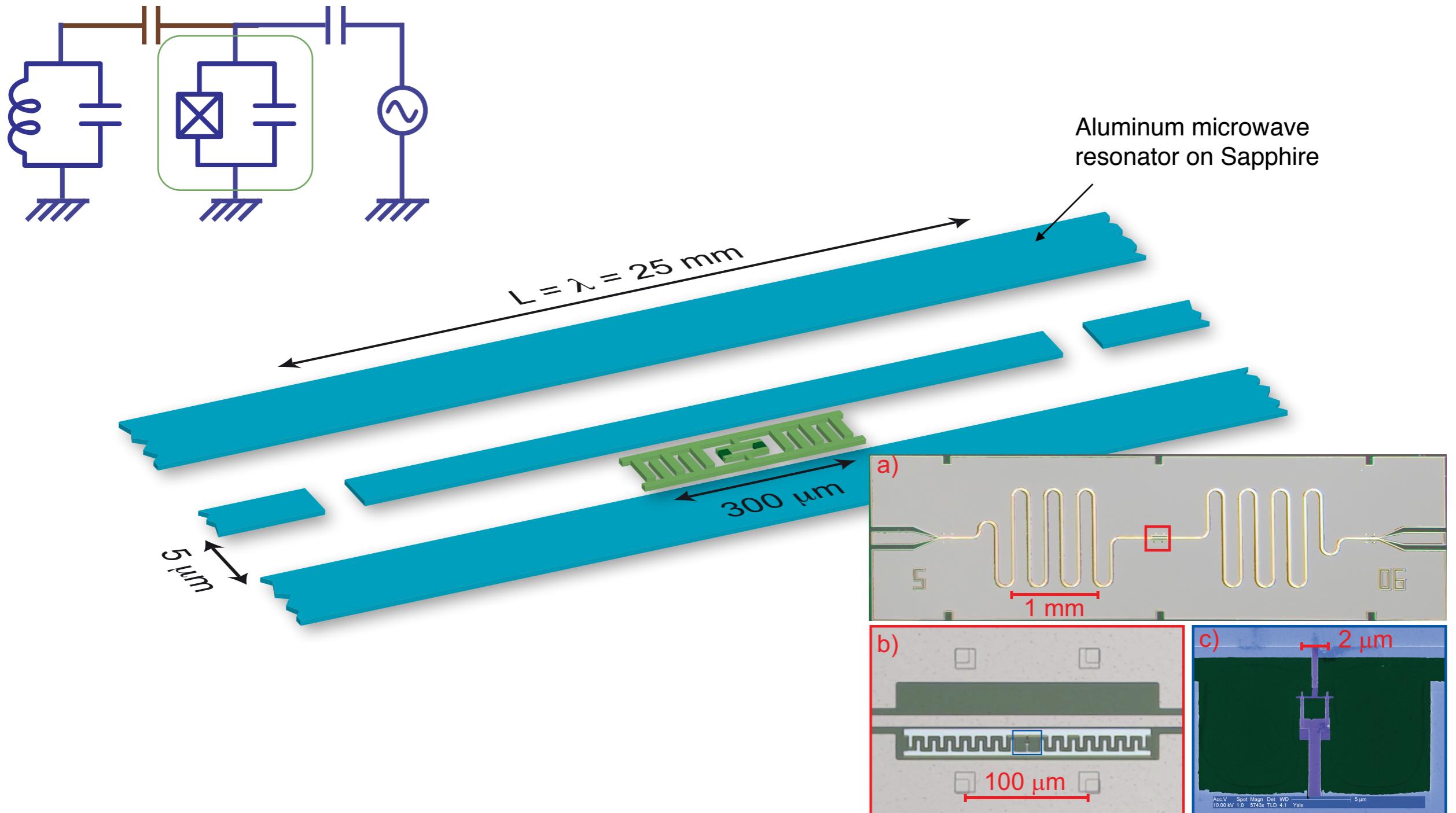
# Circuit QED: Multi-qubit architecture

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Quantum bus: entangling gates and readout

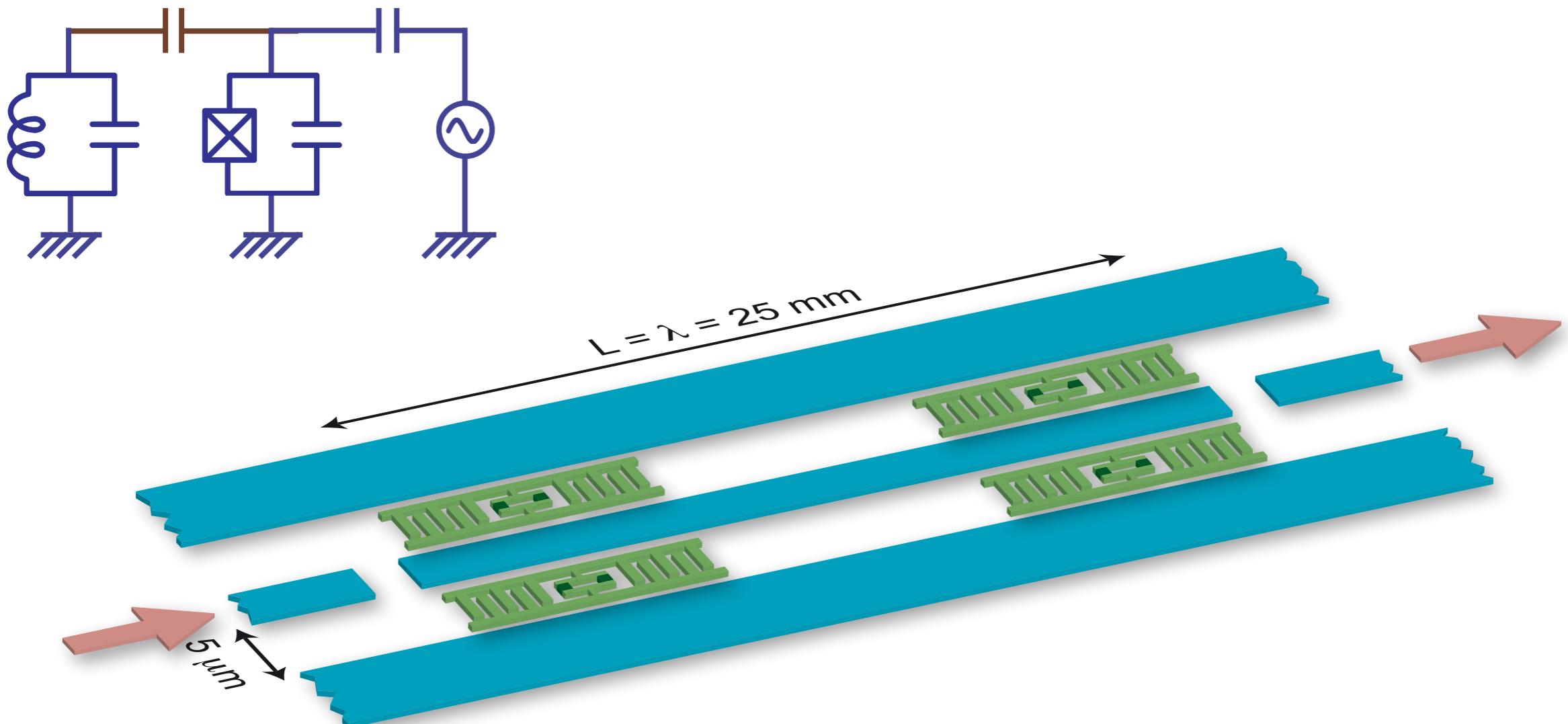


# Circuit QED: ‘1D’ realization



Proposal: Blais, Huang, Wallraff, Girvin & Schoelkopf, Phys. Rev. A **69**, 062320 (2004)  
First realization: Wallraff, Schuster, Blais, Frunzio, Huang, Majer, Kumar, Girvin & Schoelkopf. Nature **431**, 162 (2004)

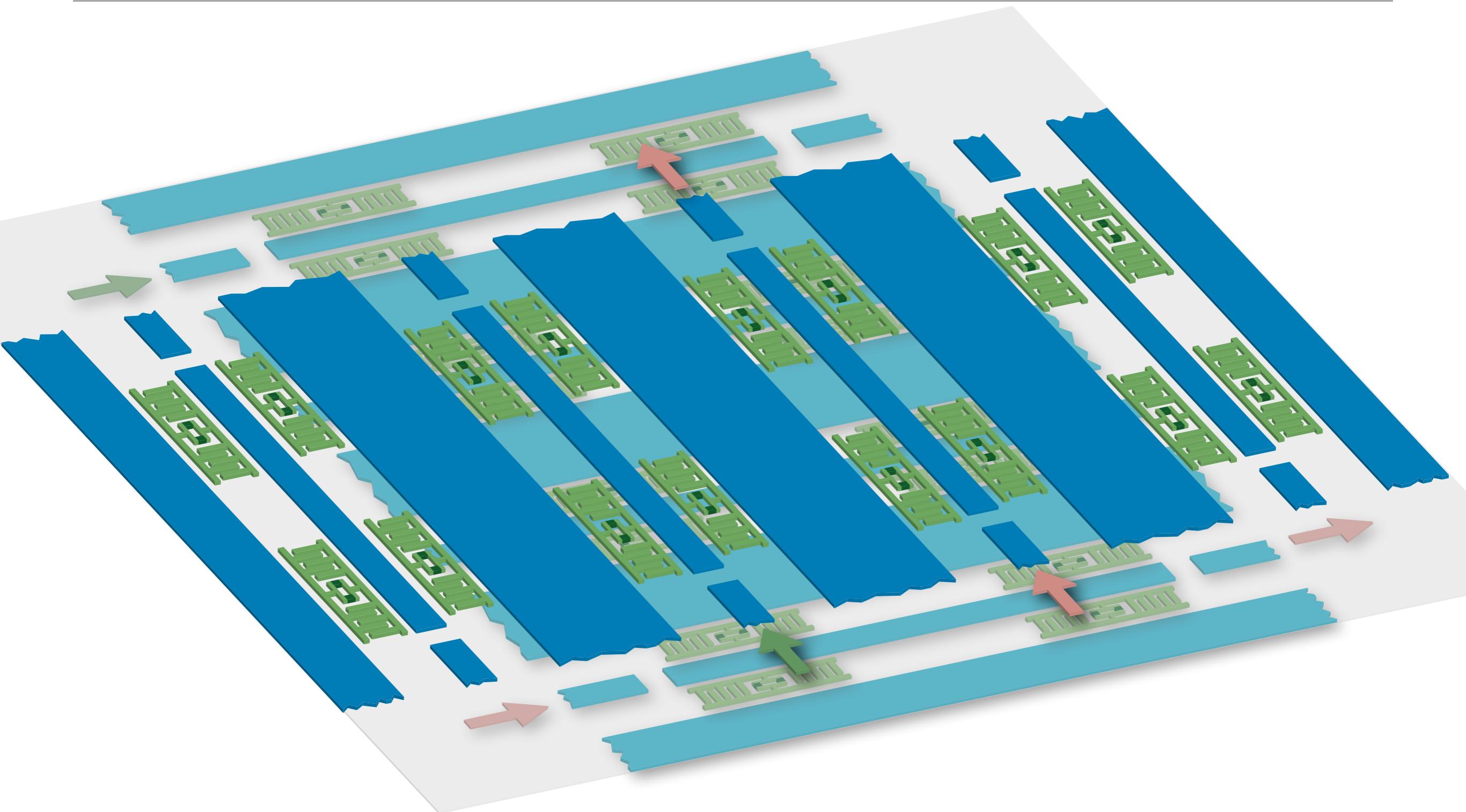
# Circuit QED: ‘1D’ realization



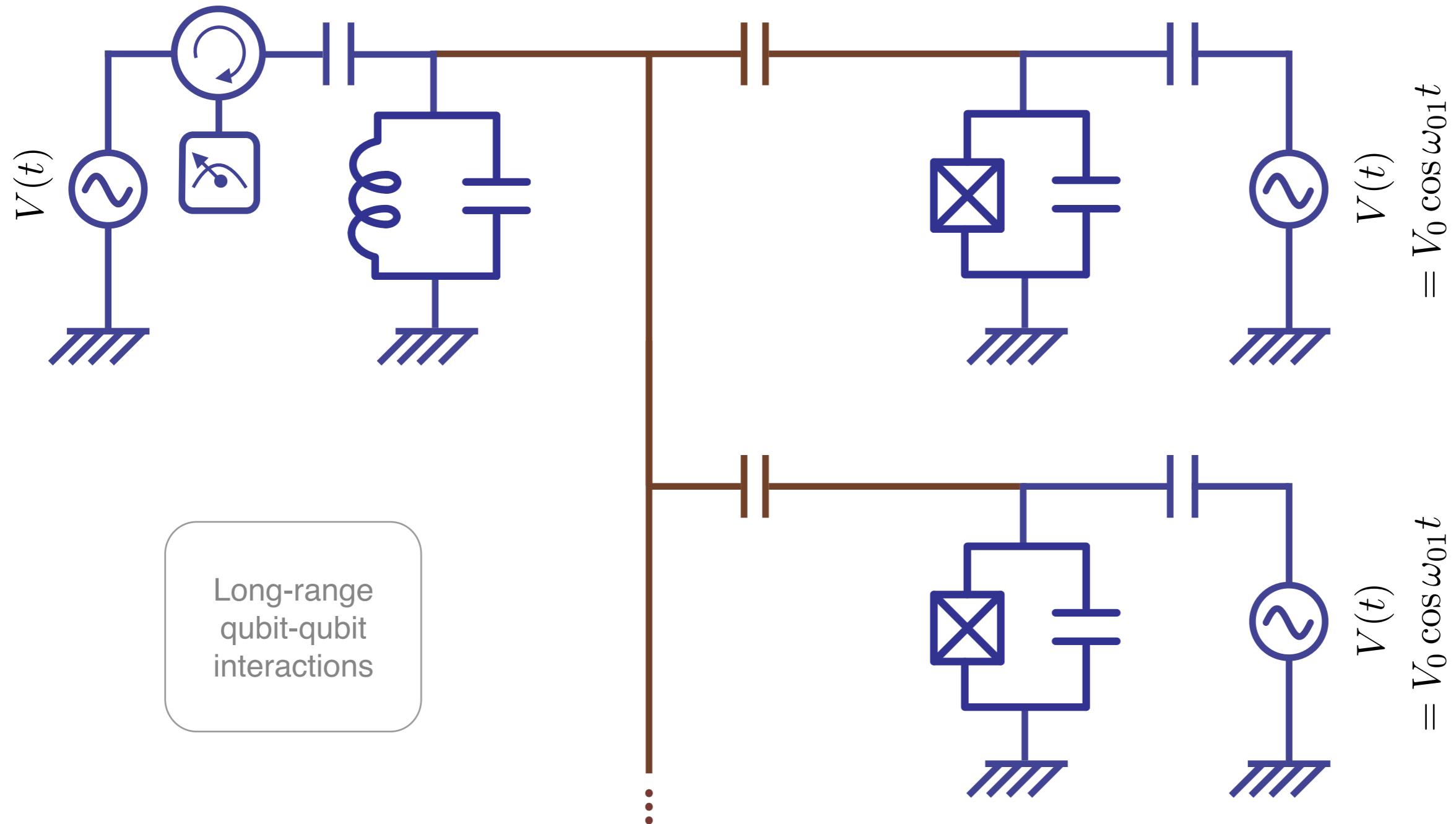
Proposal: Blais, Huang, Wallraff, Girvin & Schoelkopf, Phys. Rev. A **69**, 062320 (2004)

First realization: Wallraff, Schuster, Blais, Frunzio, Huang, Majer, Kumar, Girvin & Schoelkopf. Nature **431**, 162 (2004)

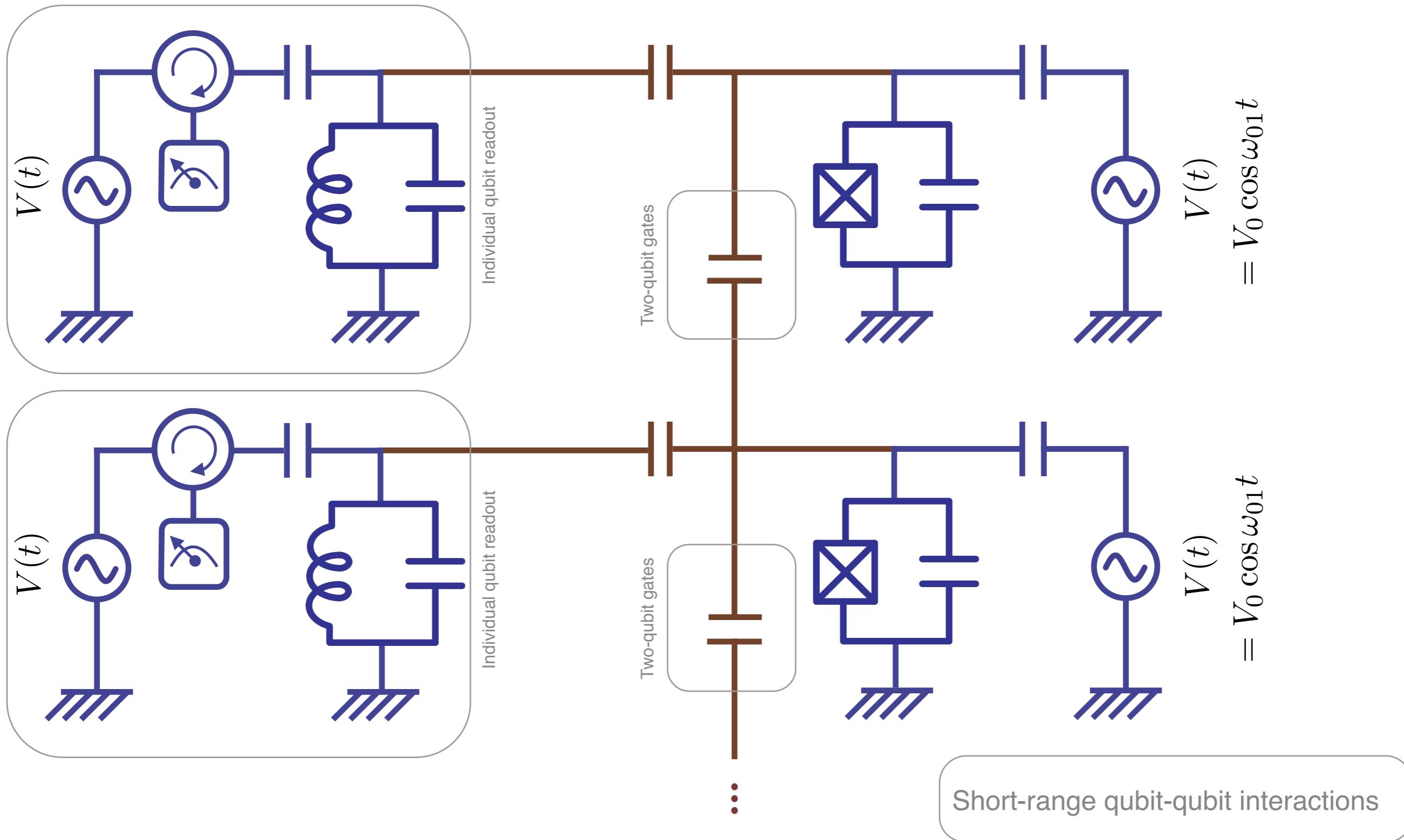
# Circuit QED: scaling up



# Circuit QED: Multi-qubit architecture



# Circuit QED: alternative architecture





# Circuit QED: recent realizations and challenges

# 10 years of circuit QED

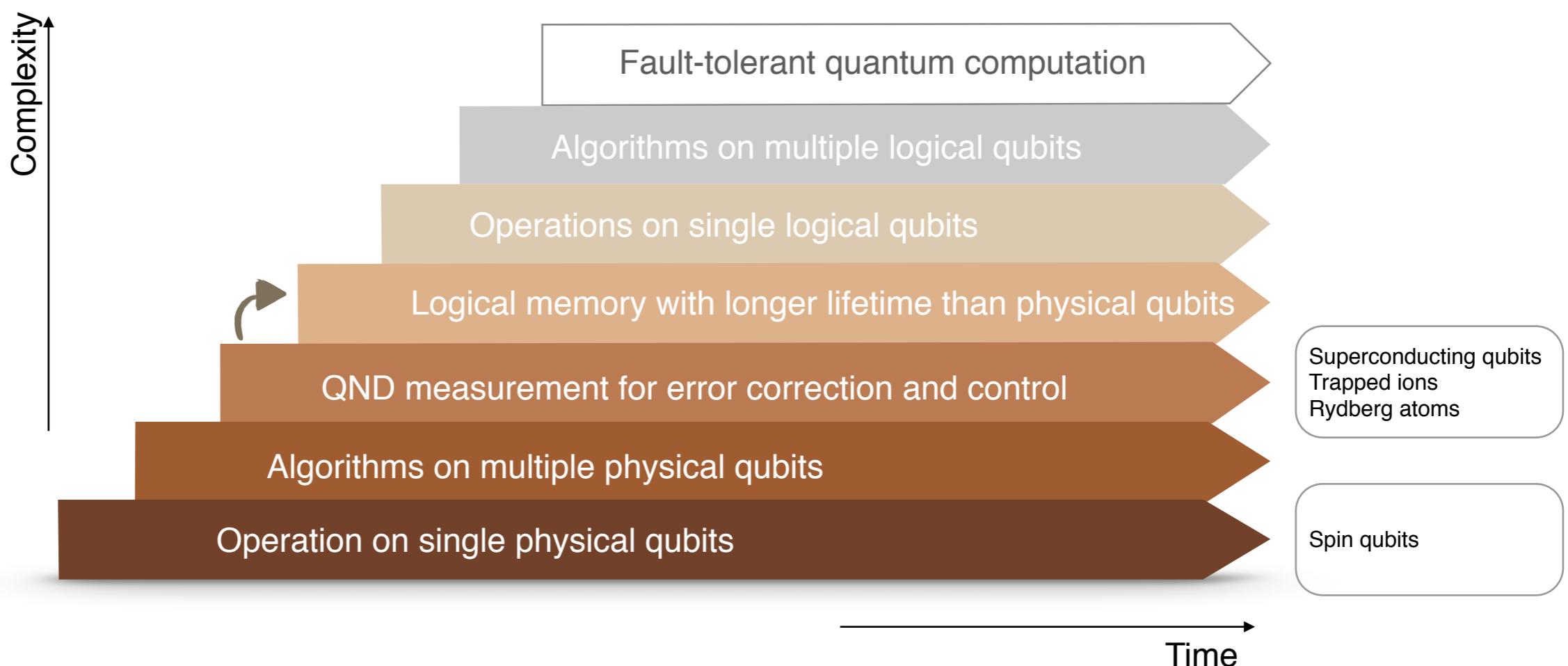
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Yale: Wallraff et al., Nature **431**, 162 (2004)  
Delft: Chiorescu et al., Nature **431**, 159 (2004)

Quantum optics on a chip  
with artificial atoms

Quantum information  
processing

# Past, present and future



# High-fidelity gates and readout

## Gates

### Single-qubit gate

Average fidelity: > 99.92%

Error when simultaneously operating neighbour qubits:  $< 10^{-4}$

UCSB: Nature **508**, 500 (2014)

### Two-qubit gate (direct)

Average fidelity: up to 99.4%

UCSB: Nature **508**, 500 (2014)

### Two-qubit gate (via bus)

Average fidelity: > 96.75%

IBM: PRL **109**, 060501 (2012)

## Readout

### Single-qubit readout

Fidelity: up to 99.8% in 140 ns

Multiplexed readout of 4 qubits

UCSB: PRL **112**, 190504 (2014)

### Two-qubit readout (logical basis)

Fidelity: > 90%

ETH Zurich: Nature **500**, 319 (2013)

### Two-qubit readout (Bell basis)

Bell state concurrence: ~ 35%

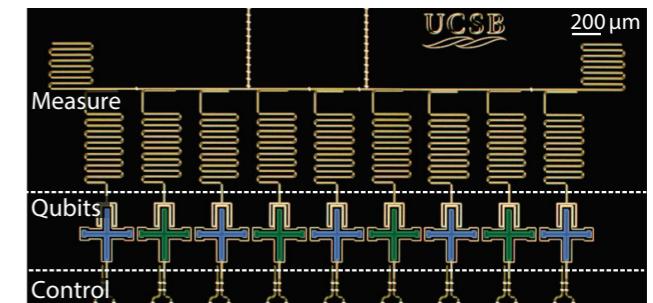
UCB: PRL **112**, 170501 (2014)

Delft: Nature **502**, 350 (2013)

## Complexity

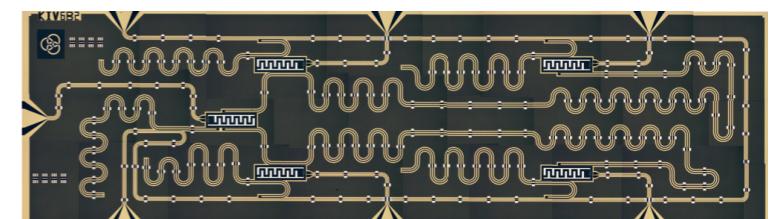
### Max number of qubits and resonators

Direct: 9 qubits and 10 resonators



UCSB: 1411.7403

Bus: 5 qubits and 7 resonators



Delft: 1411.5542

# Recent realizations

## Simple algorithms

### Deutsch–Jozsa

Yale: Nature **460**, 240 (2009)

### Grover ( $N=4$ )

Yale: Nature **460**, 240 (2009)

Saclay: PRB **85**, 140503 (2012)

### QFT

UCBS: Science **334**, 61 (2011)

### Shor (15; compiled)

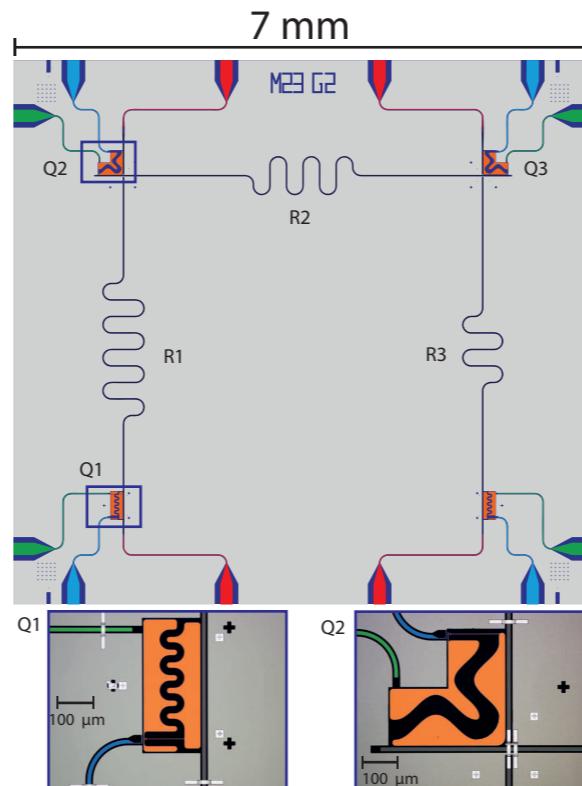
UCBS: Nature Physics **8**, 719 (2012)

⋮

## Protocols

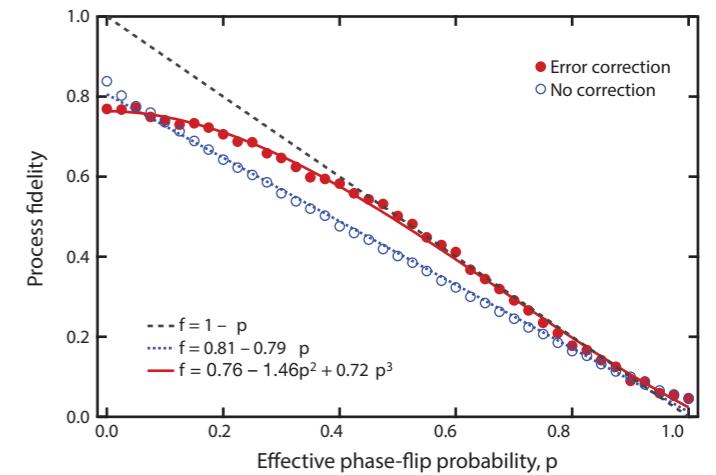
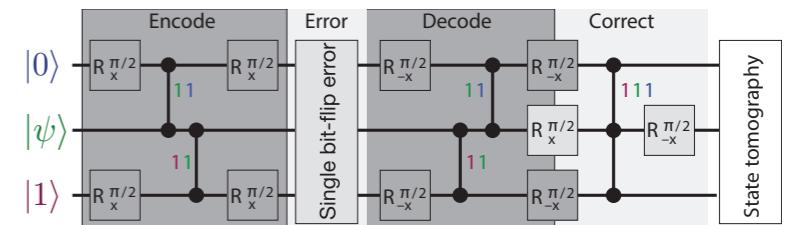
### Deterministic teleportation

ETH Zurich: Nature **500**, 319 (2013)



## Quantum error correction

### 3-qubit code



Yale: Nature **482**, 382 (2011)

# Recent realizations

## Simple algorithms

### Deutsch–Jozsa

Yale: Nature **460**, 240 (2009)

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Yale: Nature **460**, 240 (2009)

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### QFT

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### Shor (15; compiled)

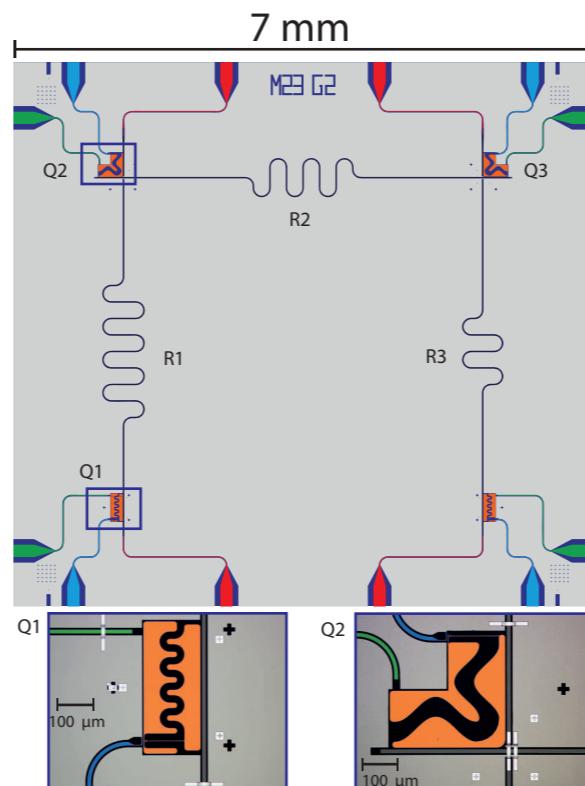
UCBS: Nature Physics **8**, 719 (2012)

⋮

## Protocols

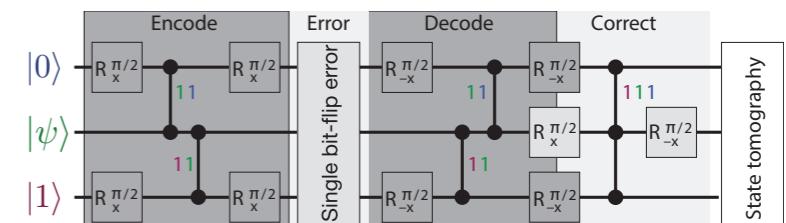
### Deterministic teleportation

ETH Zurich: Nature **500**, 319 (2013)



## Quantum error correction

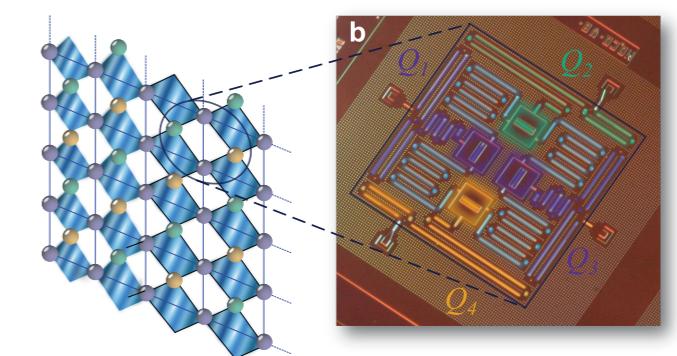
### 3-qubit code



Yale: Nature **482**, 382 (2011)

### Error detection via parity meas.

« ... using a two-by-two lattice of superconducting qubits to perform syndrome extraction and arbitrary error detection via simultaneous quantum non-demolition stabilizer measurements. This lattice represents a primitive tile for the surface code ... »



IBM: 1410.6419  
Delt: 1411.5542

# Recent realizations

## Simple algorithms

### Deutsch–Jozsa

Yale: Nature **460**, 240 (2009)

### Grover ( $N=4$ )

Yale: Nature **460**, 240 (2009)

Saclay: PRB **85**, 140503 (2012)

### QFT

UCBS: Science **334**, 61 (2011)

### Shor (15; compiled)

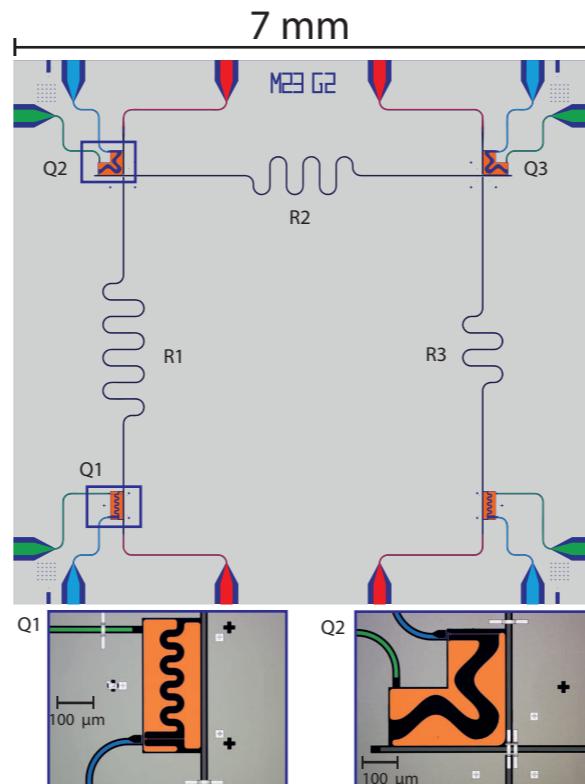
UCBS: Nature Physics **8**, 719 (2012)

⋮

## Protocols

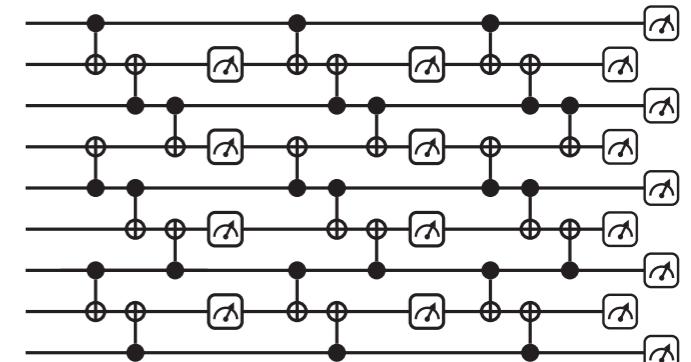
### Deterministic teleportation

ETH Zurich: Nature **500**, 319 (2013)

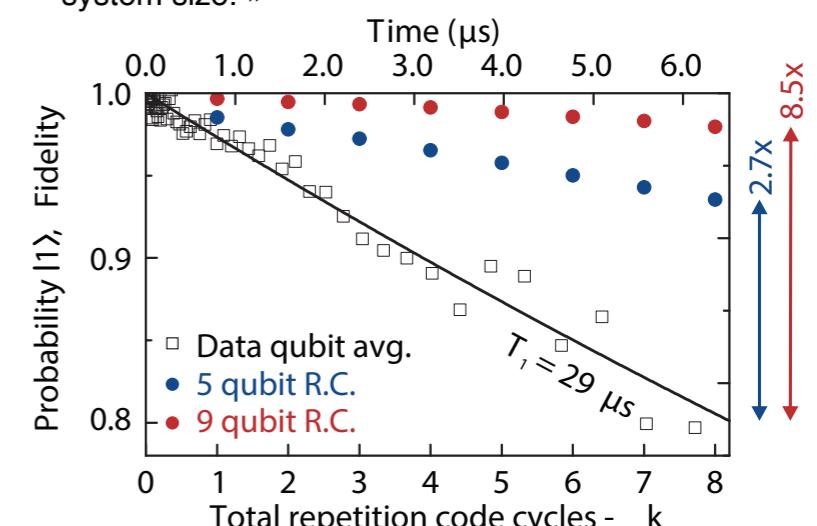


## Quantum error correction

### Error detection via parity meas.



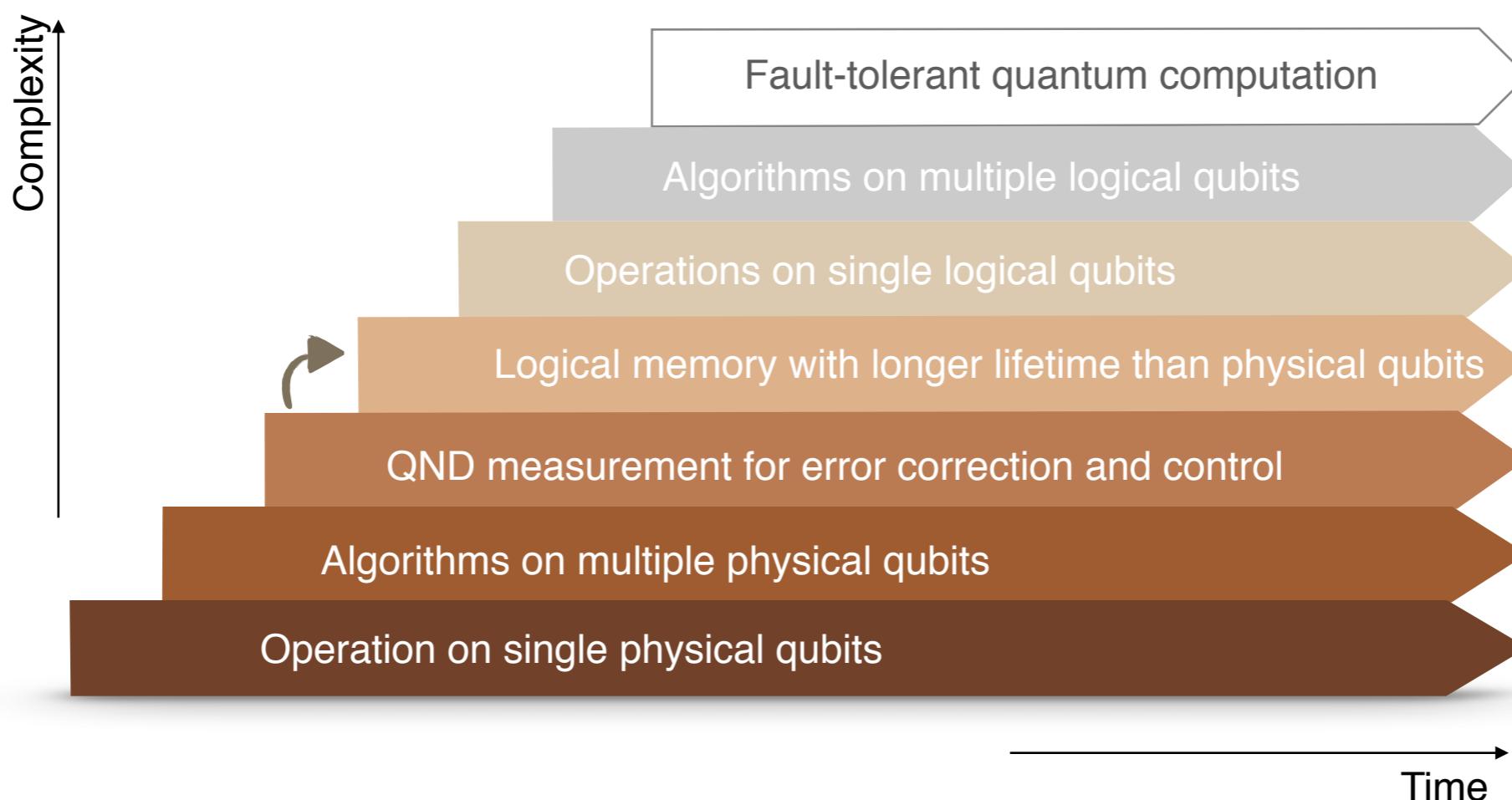
« ... we report the protection of **classical states** from environmental bit-flip errors and demonstrate the suppression of these errors with increasing system size. »



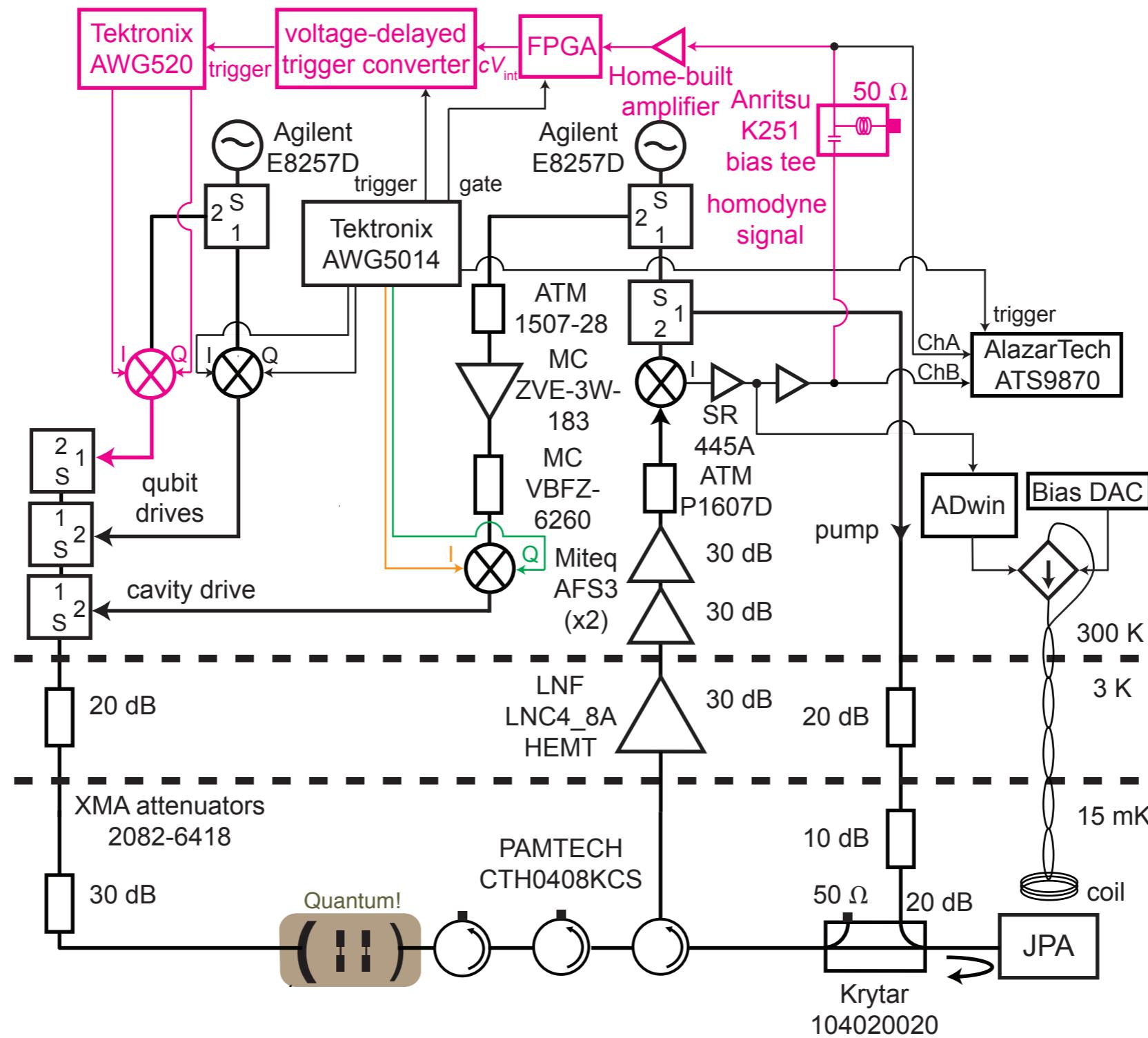
UCSB: 1411.7403

# Past, present and future

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# Under the rug...



# Summary

- Artificial atoms based on Josephson junctions
  - Low error per gate
  - Steady improvement
- Circuit QED
  - Resonator acts as bus for entangling gates
  - Dispersive regime: high-fidelity qubit readout
  - Basic protocols being implemented

- ❖ *Superconducting qubits and circuit QED*
- ❖ *Quantum error correction, quantum algorithms, ...*
- ❖ *Quantum dots, NV centers, ...*
- ❖ *Full counting statistics and quantum noise*

*Postdoc and PhD  
positions available!*