SIRCSS – Machine Learning for Social Science

Lecture 3

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Recap of yesterday

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Neural networks

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- Early layers: *basic* features (edges); deeper layers: *complex* objects
- Conv-outputs feed into *regular* (*dense*) layers for prediction

Today

- Recap causal inference basics + limitations of standard paradigm
- Machine learning for causal inference
    - Addressing non-linearity
    - Heterogeneous treatment effects

Causal inference basics

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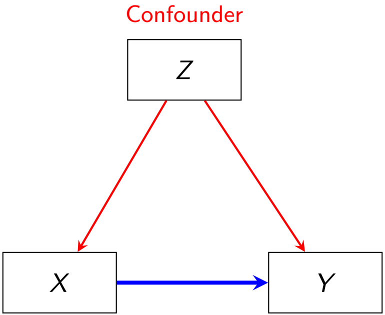
This can work — sometimes. When:

- No other variable Z that affects both X and Y (no confounder)
- Or, X is randomized.

When we have confounders



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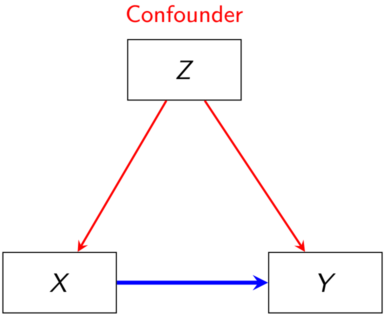

- If we have a confounder (Z), we must account for it somehow.
- Commonly — include as ‘control variable’ in regression.

$$Y = \beta_0 + \beta_1 X + \beta_2 Z$$

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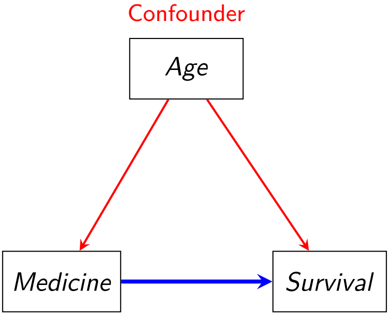

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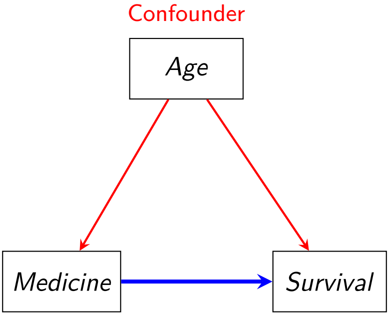

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- More likely to take medicine
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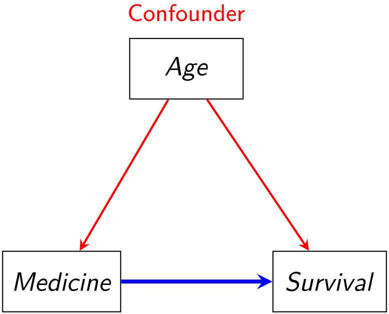

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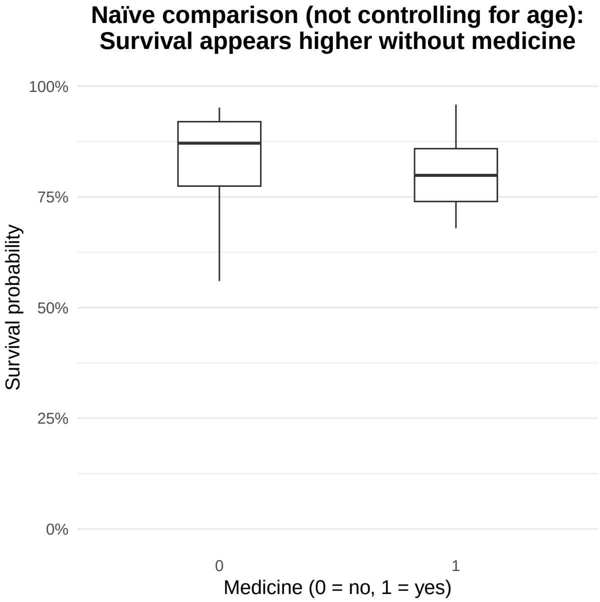
$\rightarrow$ If we don't control for age, it may look like medicine hurts survival.

Illustration – not controlling for age



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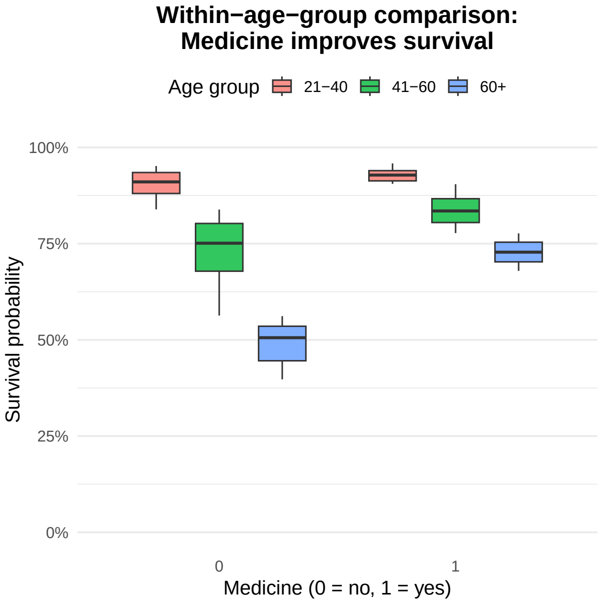

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(C) Struggles with 'complex' data like images, text, and networks.

- Often contain rich latent structures.
- Not accounted for by standard paradigm.

Non-linearity

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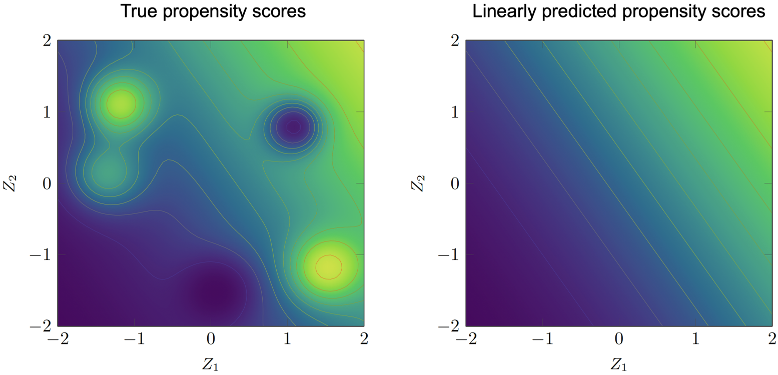

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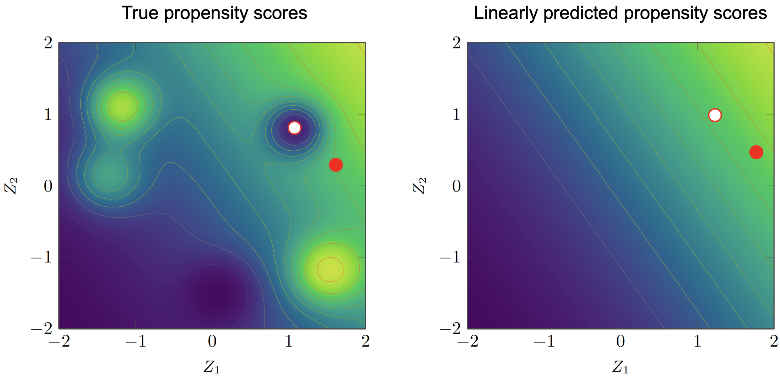

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- Can ML still be useful?
  —Yes, by reframing causal inference as prediction problems.

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Today, focus on two key ones:

- T-learner
- Orthogonal learner

T-learner

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T-learner: predict potential outcomes ("fill the gaps")

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Note: the name 'T-learner' stems from it learning *two* prediction models.[3]

[3] There exists an S-learner which only learns a single prediction model.

T-learner applied to the toy data

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$$\text{ATE} = \frac{1}{5}[(7-5)+(10-8)+(5-4)+(5-3)+(6-4)] = 1.8$$

Summary, T-learner

- Causal inference as prediction of potential outcomes.
- Makes use of supervised ML.
- Can *improve estimation* considerably when data is complex.
- But note: still relies on the assumption of no unobserved confounders.
- Limitation: splits data by *treatment status* $\Rightarrow$ data inefficiency[4]

[4] Less data to fit each model, and thus less able to learn patterns.

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Step 3: Estimate effect by regressing $\tilde{Y}$ on $\tilde{X}$

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- Orthogonal learner: residuals in hold-out not made smaller.

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Orthogonal learning procedure — with cross-fitting

0) Split data into K folds.

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- Compare: OLS, PS-matching, and the Orthogonal learner.[6]
- Repeat 50 times, and examine closeness to ground truth.

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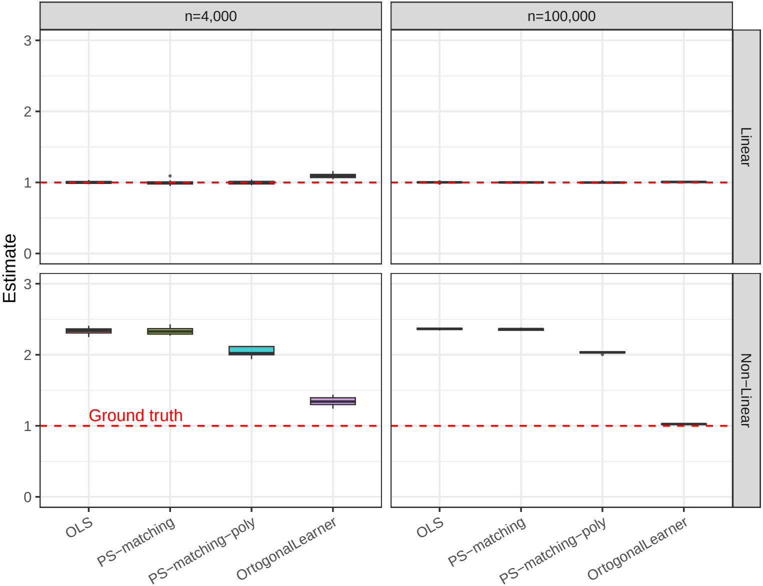
Simulation results



Summary, orthogonal learners

- Like the T-learner:
  - Reframes causal inference as a prediction task.
  - Enables the use of supervised ML and account for non-linearities.
  - Can improve estimation of causal effects substantially.
  - Assumes all confounders accounted for.
- But instead of predicting potential outcomes — it predicts the treatment (X) and the outcome (Y) based on confounders (Z) in order to remove their influence.
- More robust than the T-learner[7].

[7] Uses all data; connects with classic analytical results in economics

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Examples:

- A *medical drug*'s effectiveness often vary by *health condition*.
- *Returns to college* often vary by *social background*.
- *Policy interventions* often have varying impacts across *demographics*.

Traditional approach

- Include interactions representing subgroups of interest:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 X \times Z$$

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Limitations:

- Assumes we know moderators beforehand.
- We can do exploratory analysis—trying out interactions—but this (1) creates risk of p-hacking[8], and (2) is not feasible when data contain many variables.

[8]Unprincipled search for significance $\Rightarrow$ false positives.

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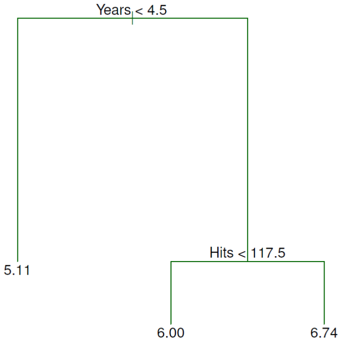

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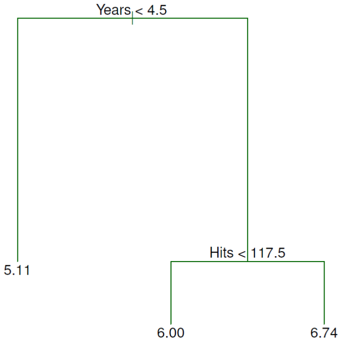

- But — not good at *inference* (also: leaves are about Y, not X).

Causal trees

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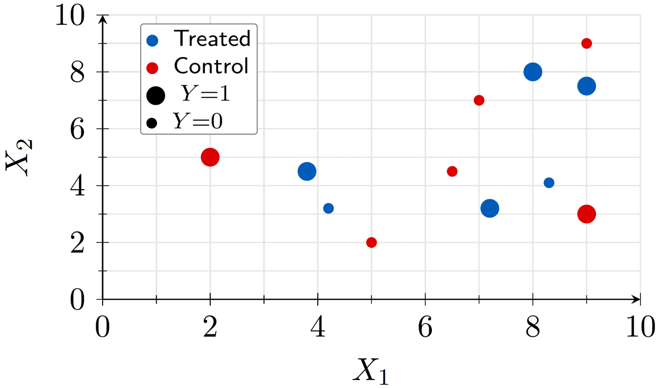
So, more precisely, causal trees:

- Identify splits that maximize across-leaf variation in the within-leaf treated–untreated outcome difference.

Illustration: causal tree splitting



Suppose we have the following dataset:

- 12 observations
- 2 variables: X_1,X_2

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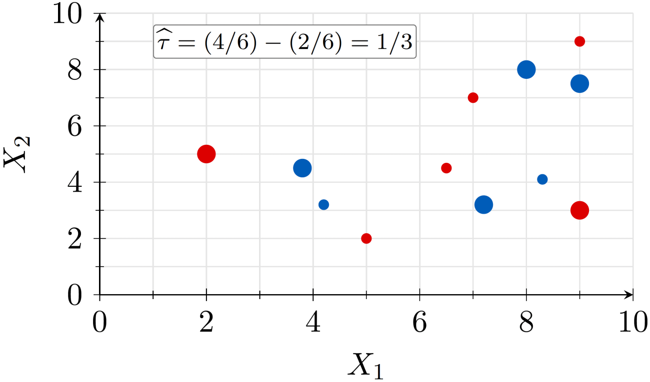

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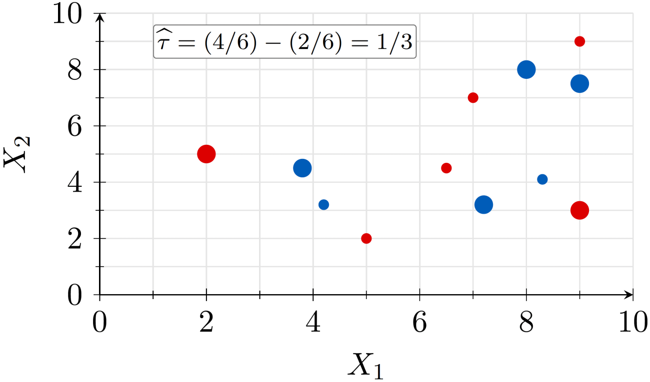

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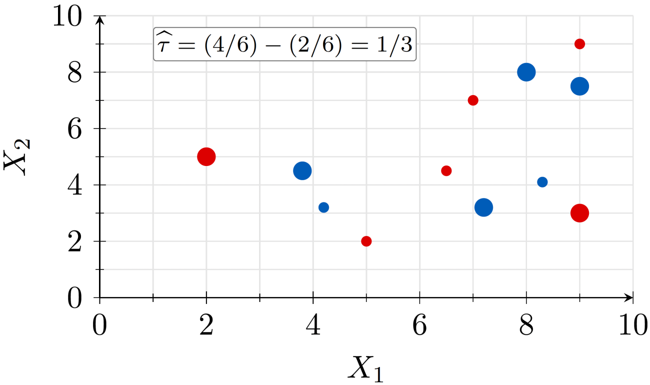
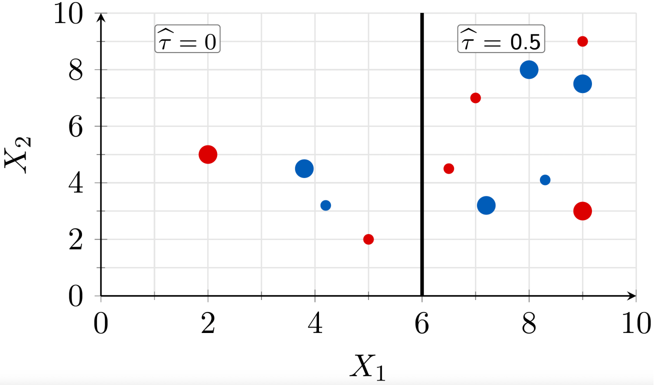


- Average treatment effect: $1/3$
- How can we split this data space (X_1, X_2) to maximize *difference treatment effect across regions*?

Illustration: causal tree splitting



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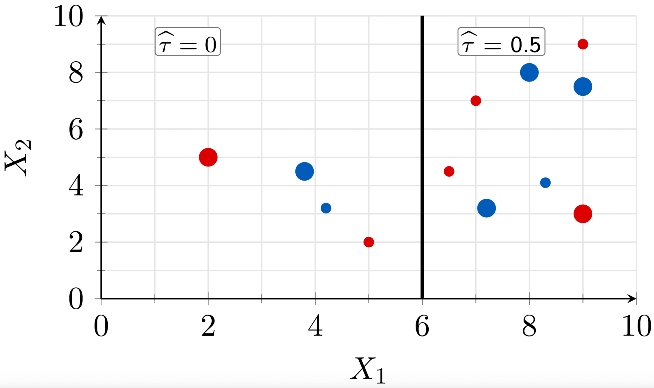
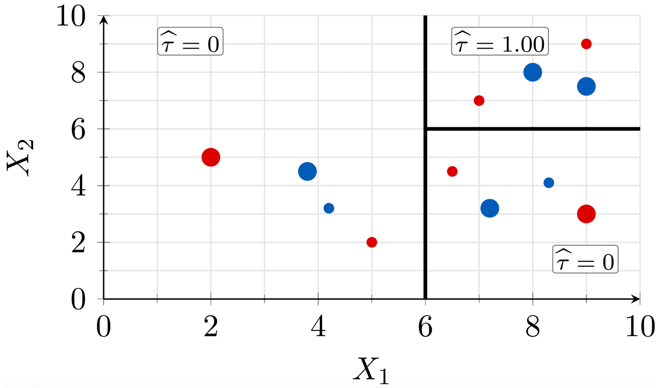


- Splitting at $X_1 = 6$ divides up the data into a "no effect" and "moderate effect" region.

Illustration: causal tree splitting



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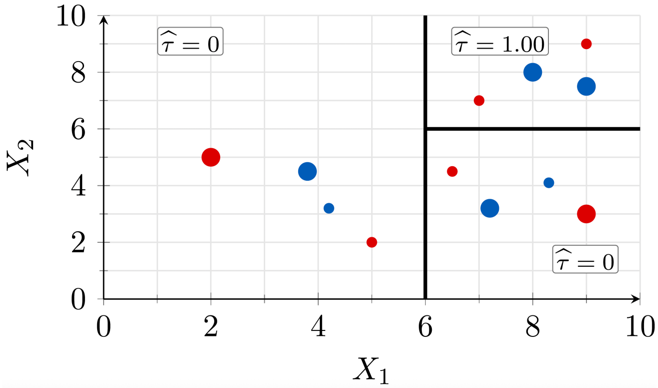

- If we additionally split the right region ($X_1 > 6$) at $X_2 = 6$, we further refine the regions, getting an even stronger treatment effect in the top-right corner, and 0 in the rest.

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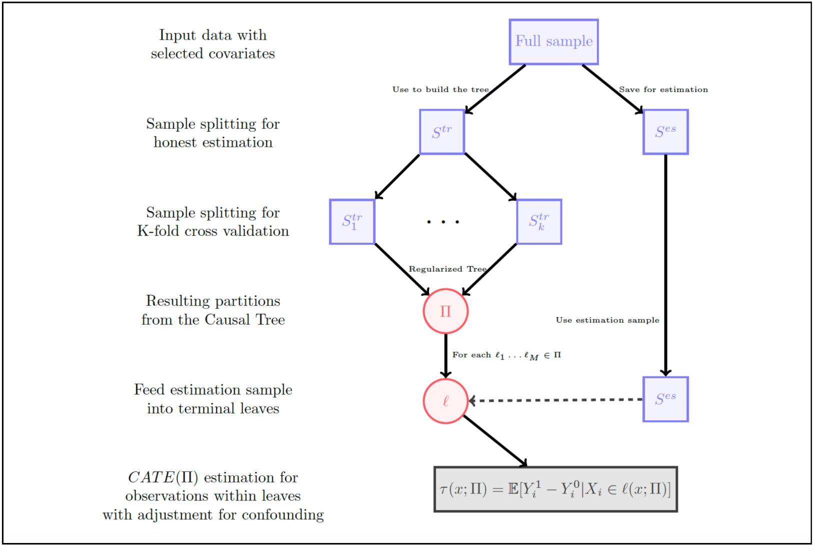
- **Exclude** non-significant predictors of p(treatment).

Causal tree pipeline (Brand et al. 2021)



Empirical illustration (Brand et al. 2021)

- **Data**: US longitudinal survey data (NLSY79)
- **Treatment**: college attendence
- **Outcome**: amount of low-wage work over a career.
- **Objective**: explore treatment effect heterogeneity
- **Method**: causal trees with IPW.

Empirical illustration (Brand et al. 2021)

Average treatment effect

Wage Outcome	Unadjusted	IPW
Proportion of time in low-wage work	–.223*** (.013)	–.189*** (.016)

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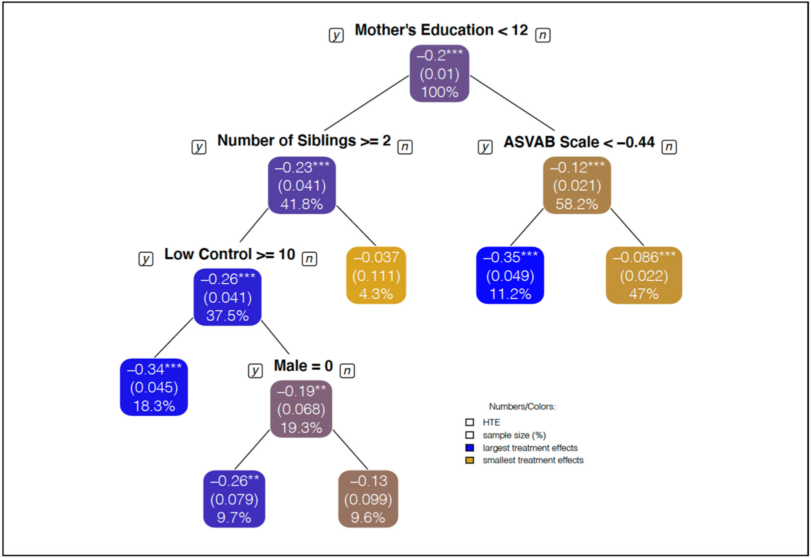

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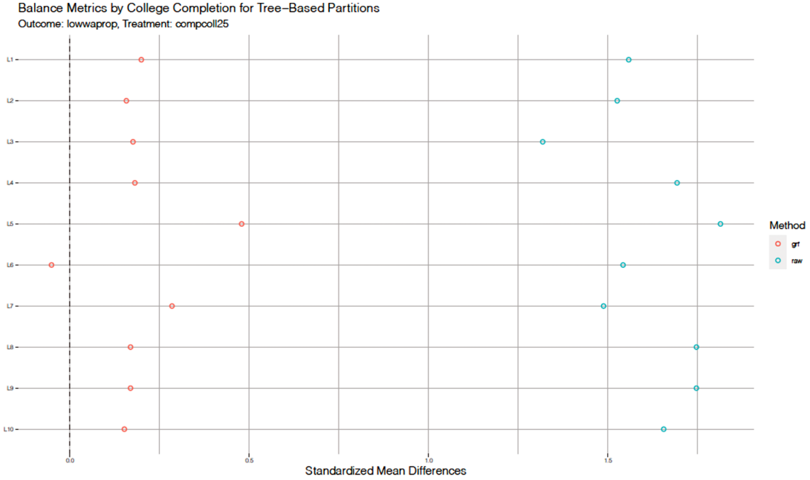
Balance in the different leaves:

- *Std mean difference* in *propensity of treatment* — by leaf.



Summary, causal trees

Pros

- Rigorous framework for discovering varying treatment effects[12]
- Highly interpretable tree-structure.<.->Consider leaves in terms of all variables, not just splits.
- Analytic guarantees for standard errors/CIs.

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- Analytic guarantees for standard errors/CIs.

Cons

- Causal trees, like decision trees, are high-variance estimators.
- Tree-structure can be sensitive to the sample used to train it.

[12] For social science: potential identification of new, important moderators.
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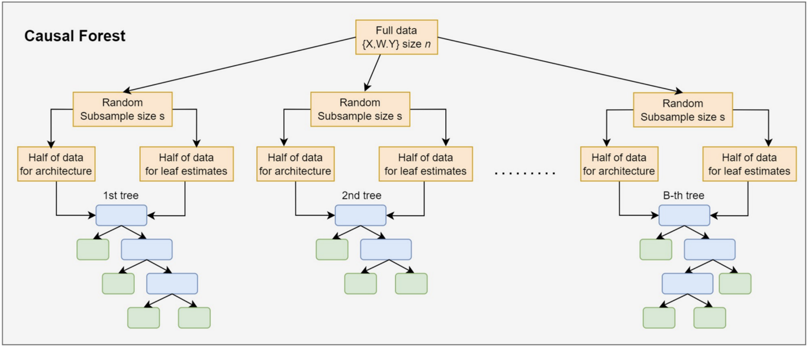

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    - Closeness is captured by a weight $\alpha_i(z)$—how often observation i and z fall in the same leaf *across trees*.
    - The CATE for z is obtained by a *weighted* regression of the residualized outcome on the residualized treatment (weights $\alpha_i(z)$).

What can do with the output?

Two basic questions we can investigate are:[14]

- Which vars account for most heterogeneity? Variable importance.
- How do they moderate the treatment effect? CATE by subset.

[14] There is a lot more one can do; see reading list.

Demonstration via simulation

Setup

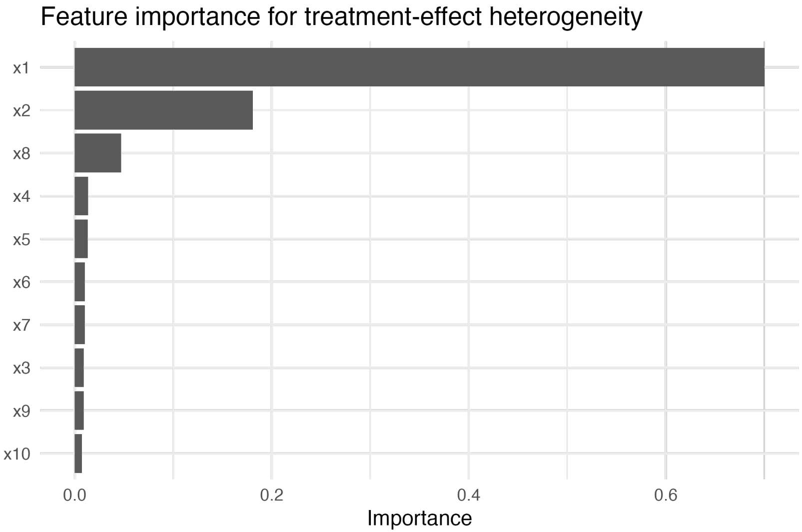
- Simulate data:
  - $n = 10,000$,
  - 10 features $X_1, \ldots, X_{10}$
  - Binary treatment W.
- True heterogeneity:

$$\tau(X) = 1.2 \sin(1.5\, X_1) + 0.8\, \mathbf{1}\{X_2{=}1\} + 0.2 X_8.$$

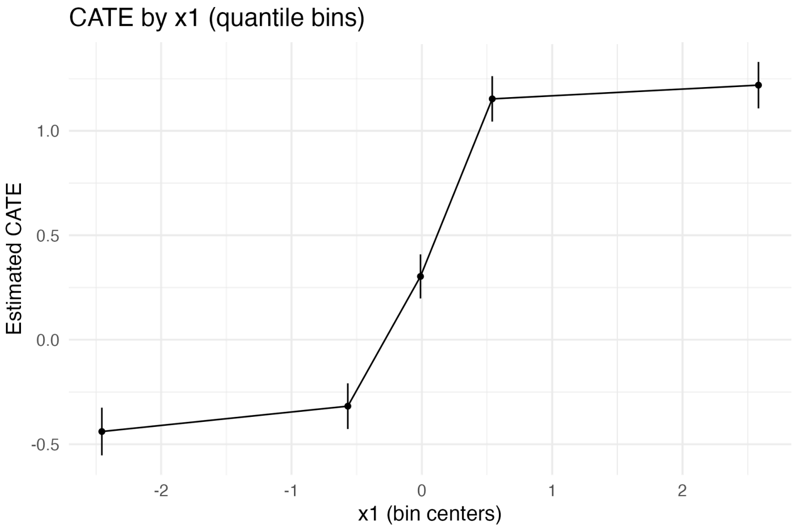
Objectives

- Identify key moderators.
- Quantify CATE by value ranges of top variables:

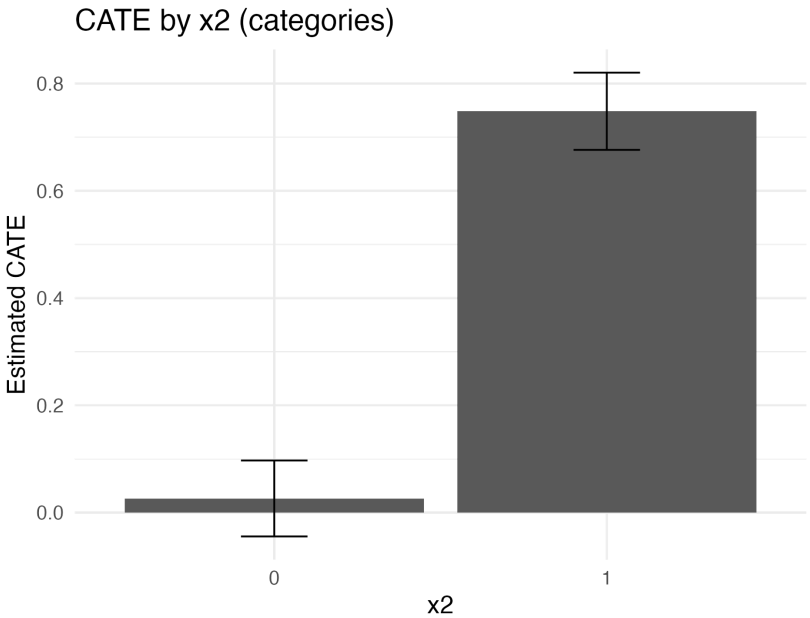
Variable importance



CATE for X_1


CATE by x1 (quantile bins)
Estimated CATE
1.0
0.5
0.0
-0.5
-2
-1
0
1
2
x1 (bin centers)

CATE for X_2



Summary

...

Further readings

Main:

- Brand, J. E., Xu, J., Koch, B., & Geraldo, P. (2021). Uncovering sociological effect heterogeneity using tree-based machine learning. Sociological Methodology, 51(2), 189-223.
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