SIRCSS – Machine Learning for Social Science

Lecture 2

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2025-12-02

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- Supervised learning
  - Learn a function $\hat{f}$ that maps input X to output Y

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- Beyond the standard linear model
  - *Polynomials*—enable $\hat{f}$ to pick up more patterns in data ($\downarrow$bias)
  - *Ridge*—penalizes $\hat{f}$ that picks up too much patterns ($\downarrow$variance)
  - *Cross-validation*—helps find a good level of complexity; balancing bias- and variance trade-off

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- Interpretable machine learning

Today

- Deep learning
  - The basics
  - Convolutional neural networks
- If time permits, a brief detour: unsupervised learning

Deep learning

Background: deep learning

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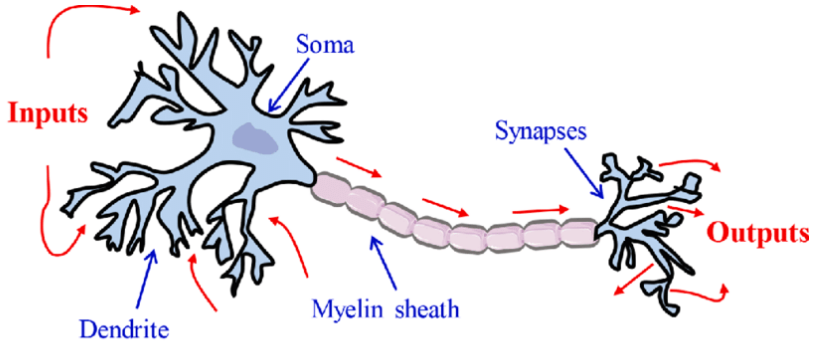
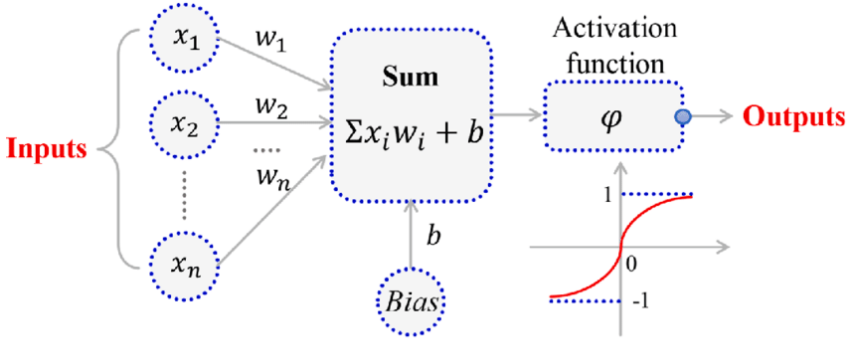
- New term, old tech? 1950-2010, we called them *"neural networks"*.
- Early research was inspired by *biological neurons*



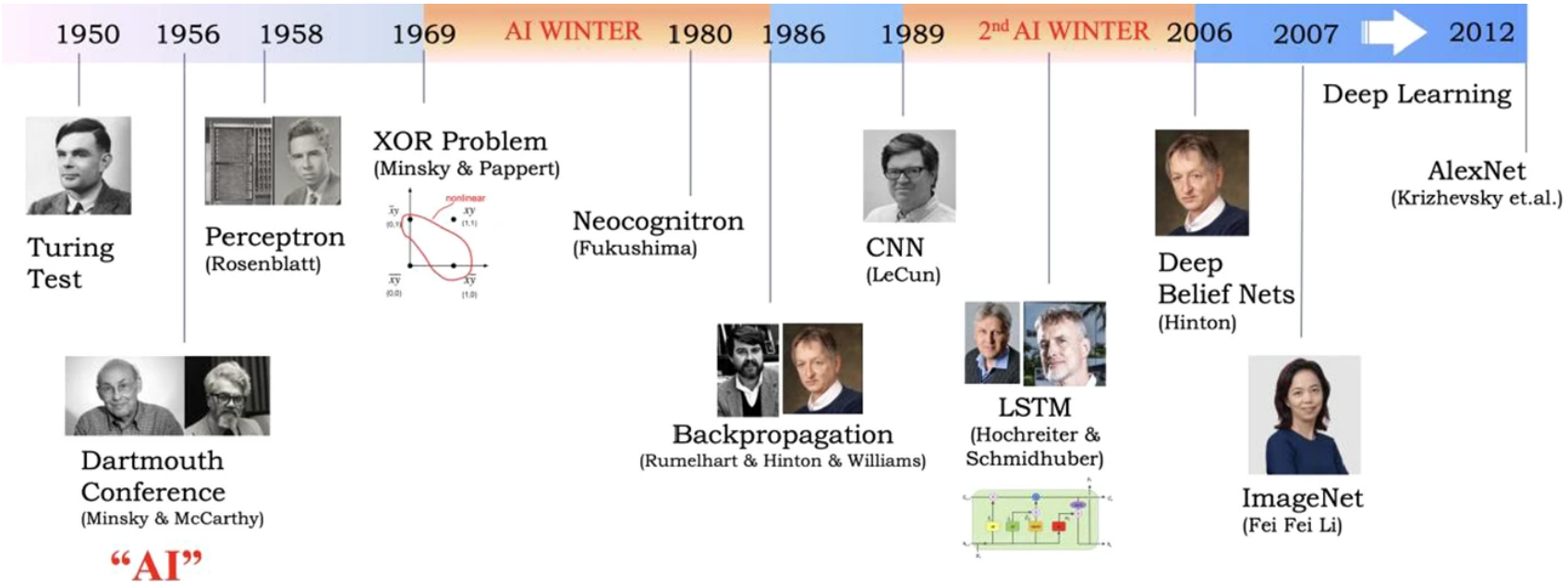
(a) Biological neuron



(b) Artificial neuron

Background: deep learning

- The neural networks timeline (people have been hyped before)



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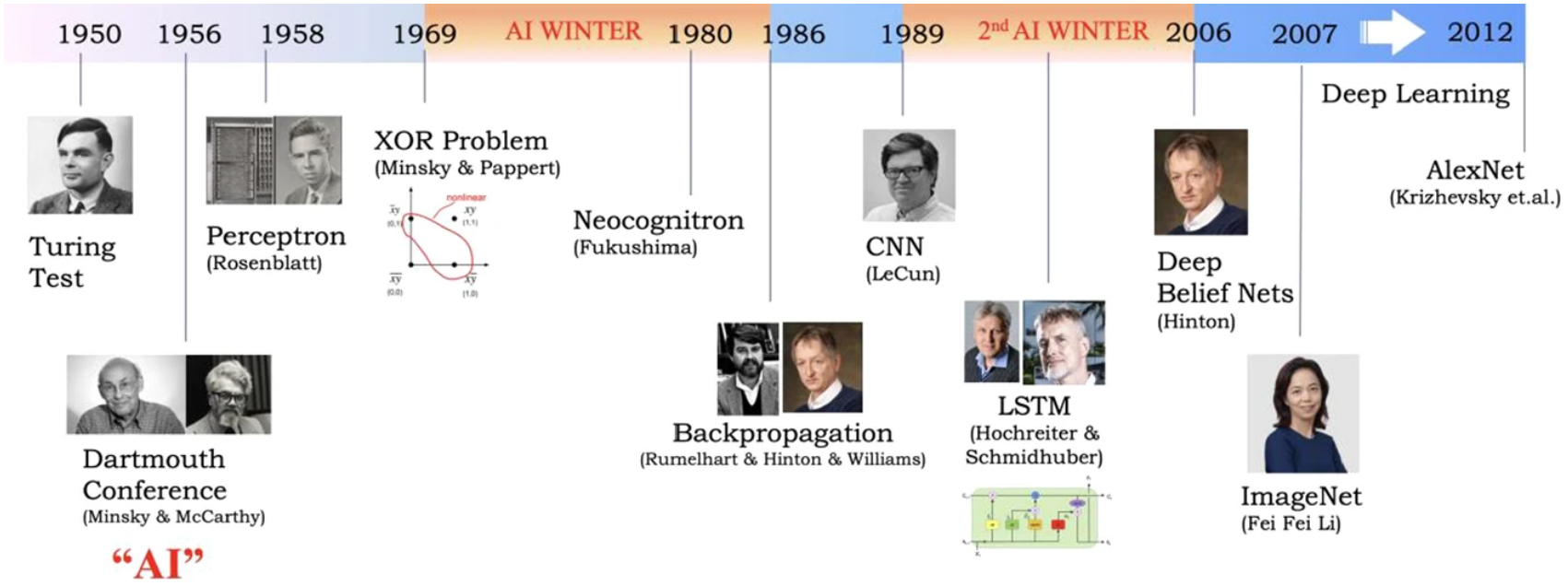

What's different now?

- Massive data and computational power.

Preview: what makes neural networks so powerful?

"Deep learning is a particular kind of machine learning that achieves great power and flexibility by representing the world as a nested hierarchy of concepts, with each concept defined in relation to simpler concepts, and more abstract representations computed in terms of less abstract ones." — Ian Goodfellow and Yoshua Bengio (2016)

The importance of representation

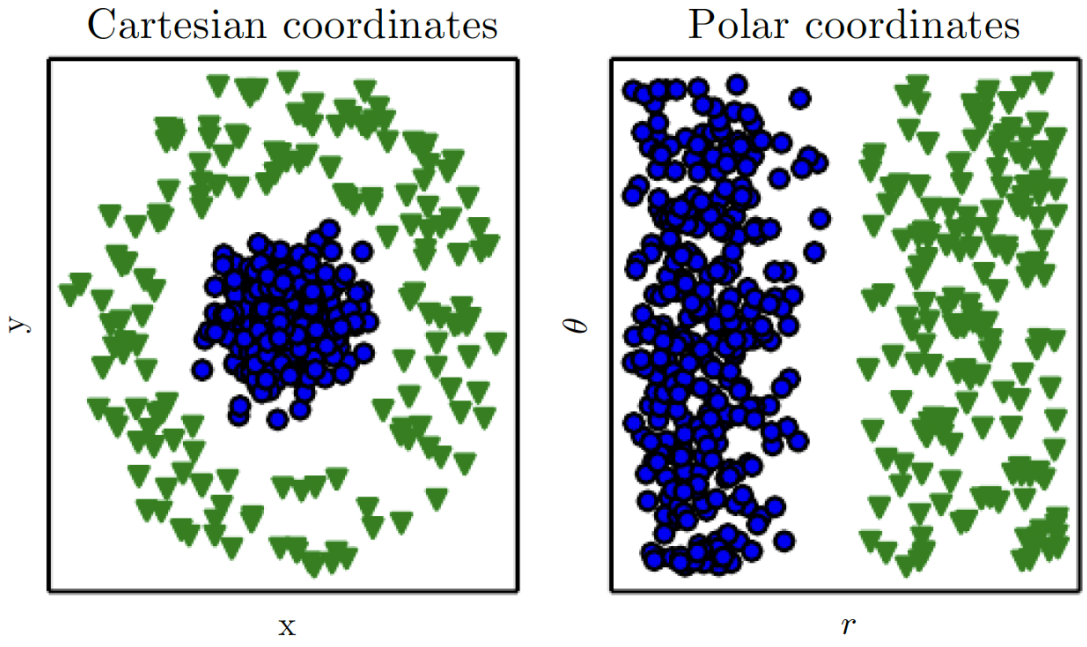

Figure 1.1: Example of different representations: suppose we want to separate two categories of data by drawing a line between them in a scatterplot. In the plot on the left, we represent some data using Cartesian coordinates, and the task is impossible. In the plot on the right, we represent the data with polar coordinates and the task becomes simple to solve with a vertical line. (Figure produced in collaboration with David Warde-Farley.)

Figure 1: Goodfellow and Bengio (2016)

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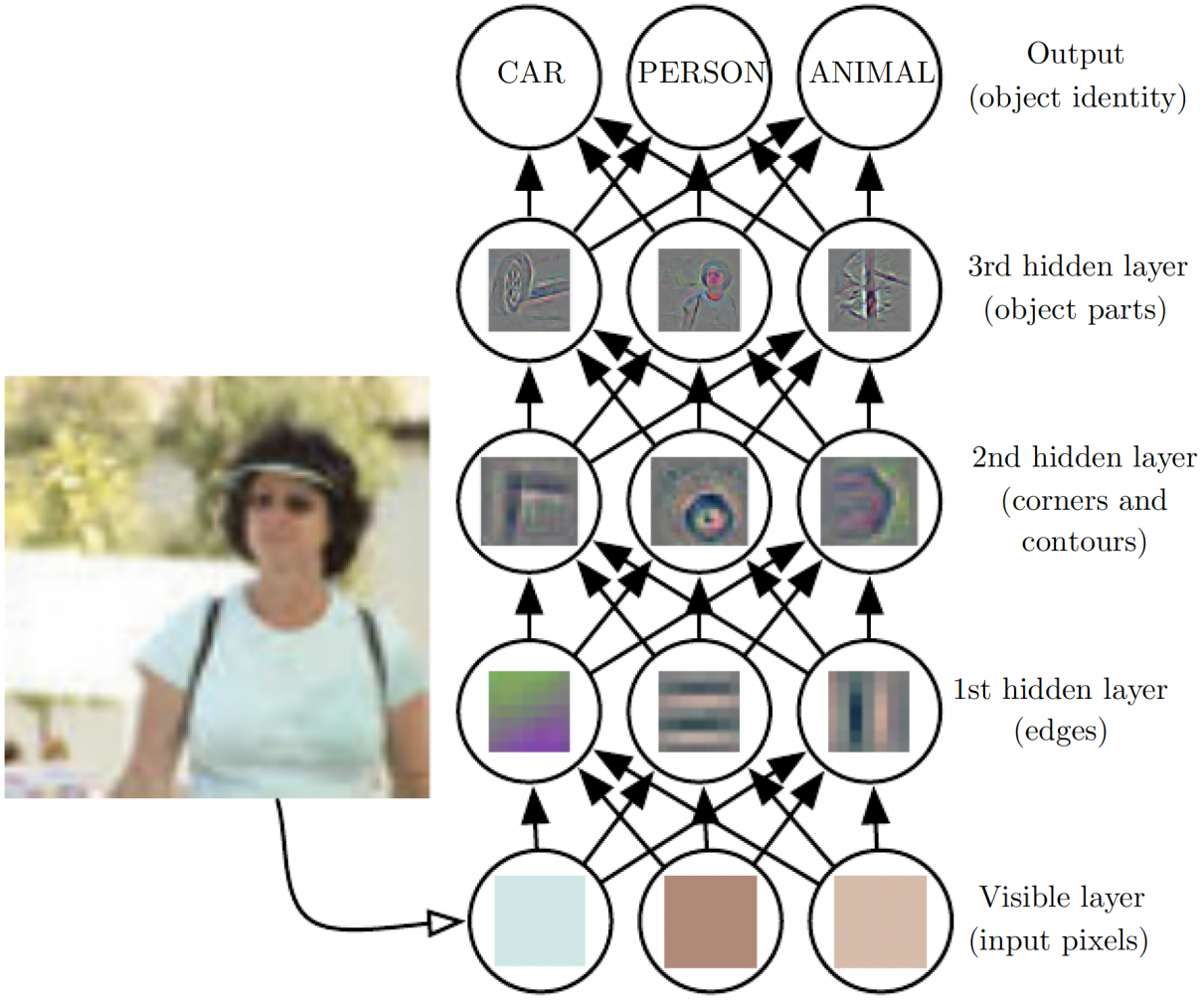


Figure 2: Goodfellow and Bengio (2016)

Contrasting with other stats/ml methods

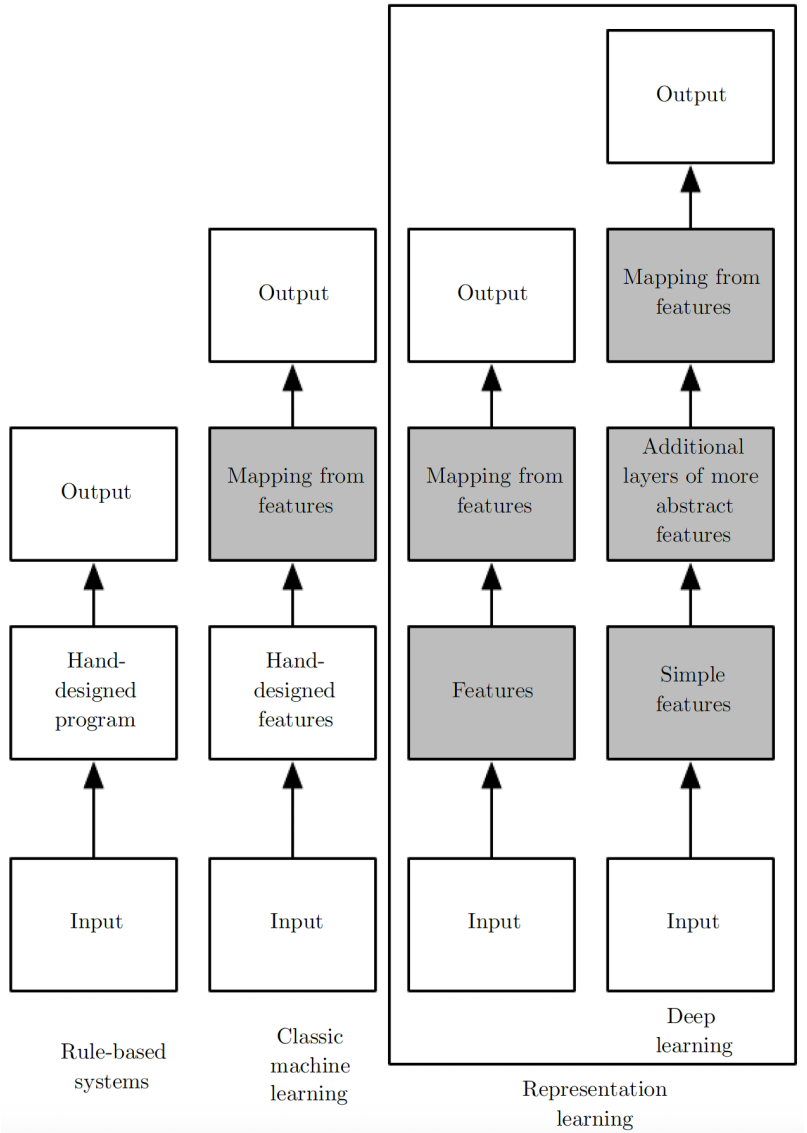


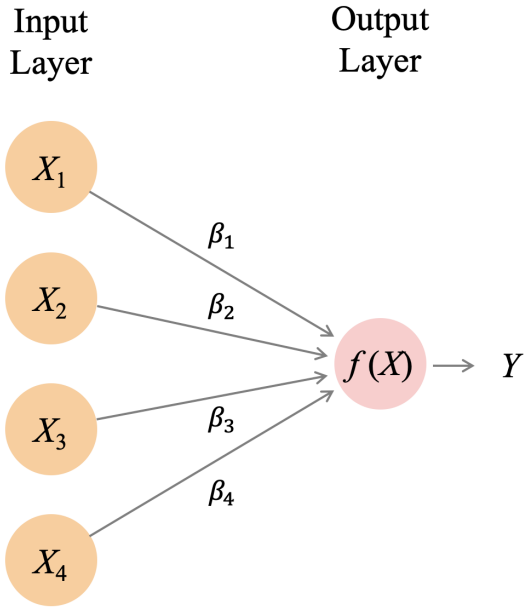
Figure 3: Goodfellow and Bengio (2016)

The basics

The simplest neural network

- Consists of two 'layers'
  - Input layer: X
  - Output layer: Y
- The output layer combines the inputs in a linear combination

$$f(X) = \sum_{j}^{P} X_j \beta_j$$



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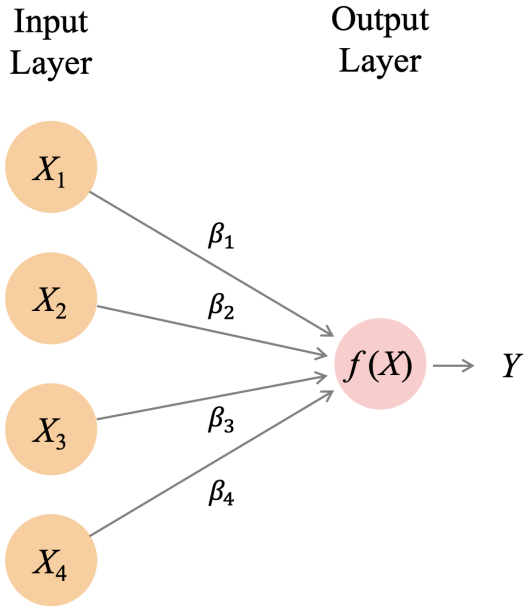

Note — this is just a standard linear regression!

A more complex neural network: adds a 'hidden' layer

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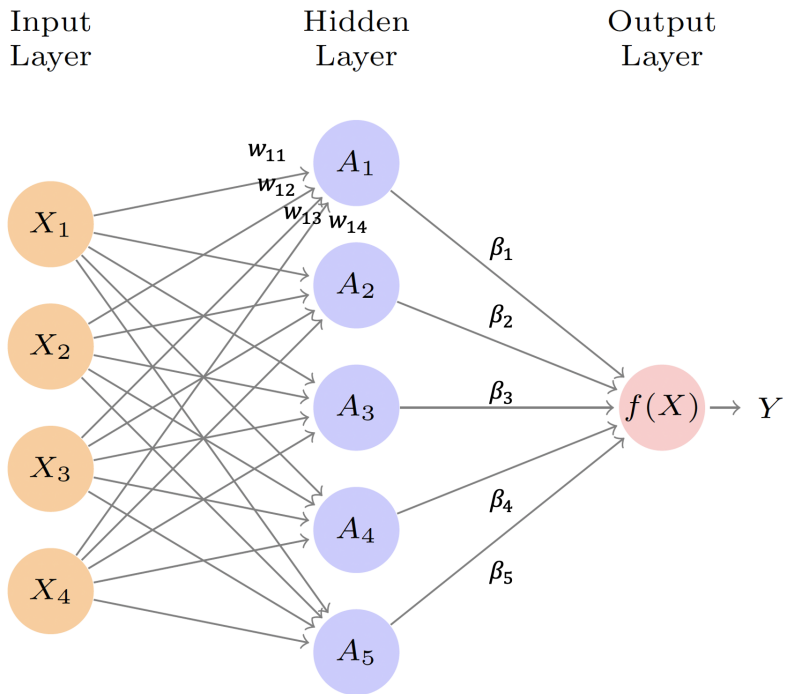

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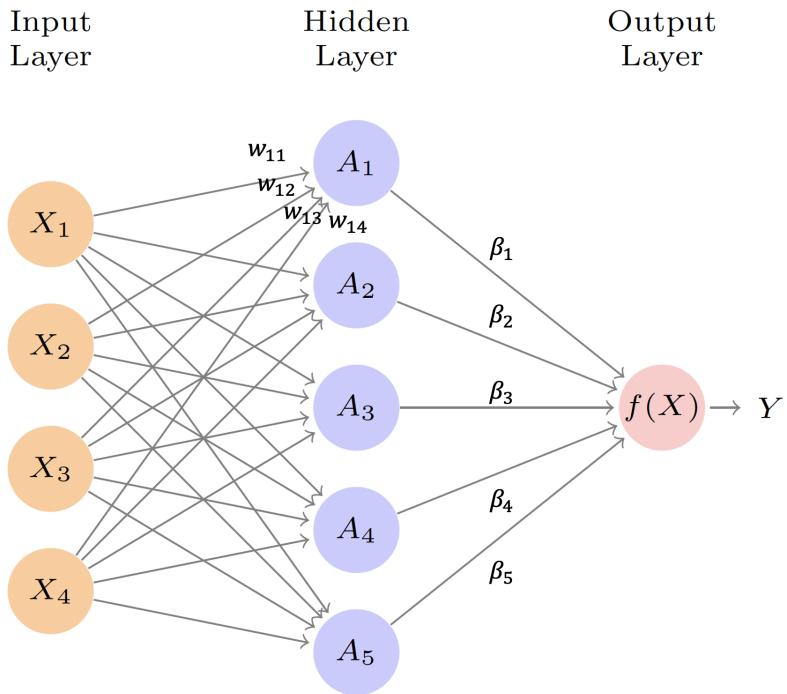

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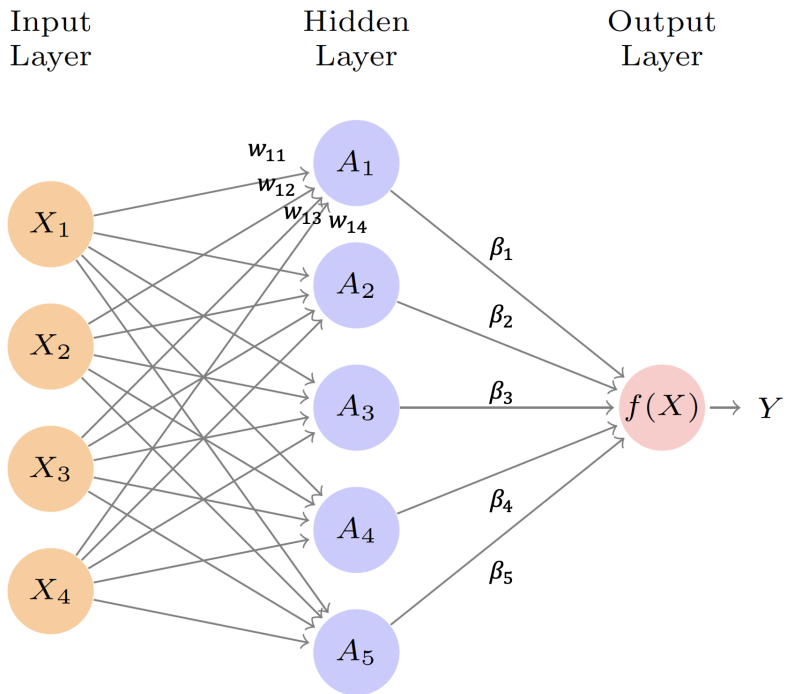

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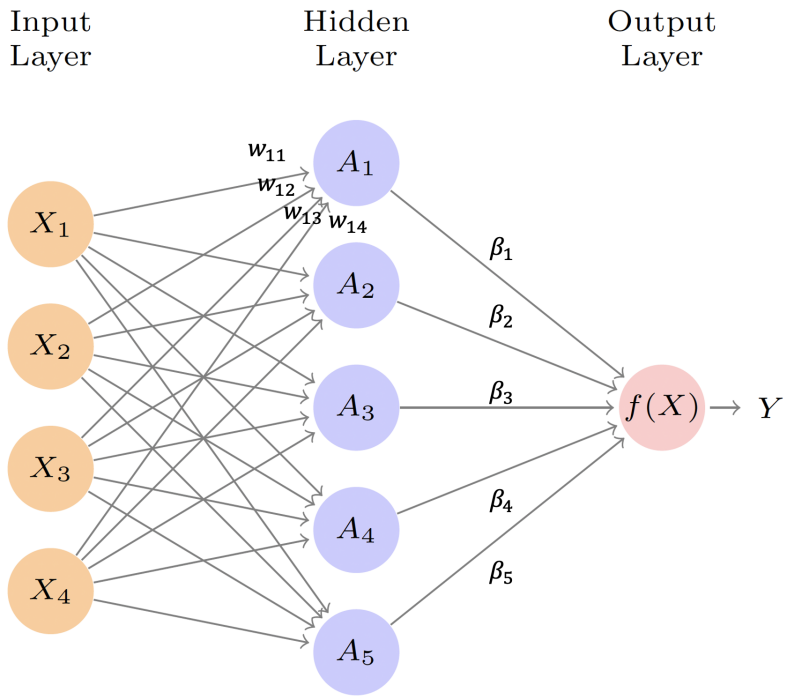

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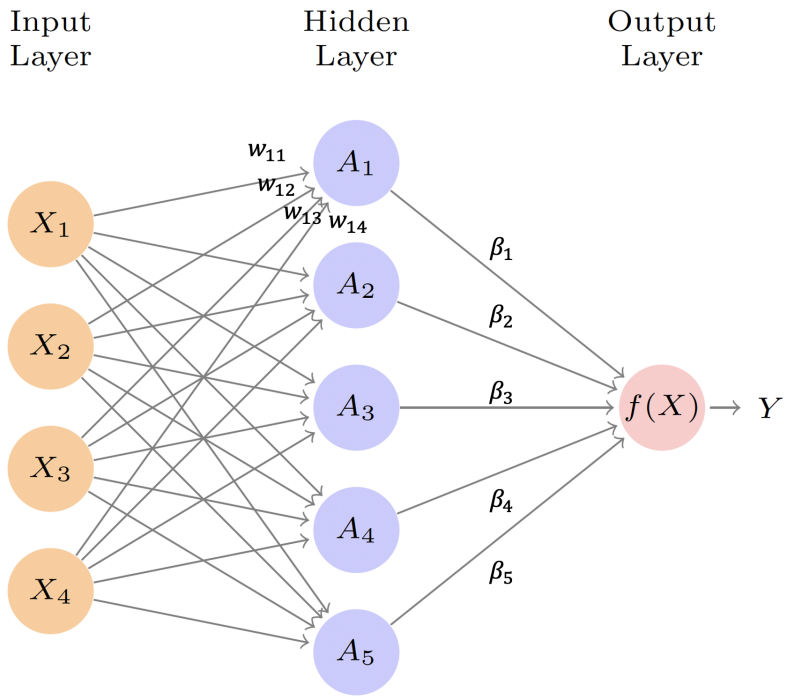
- **Goal**: learn β, w minimizing the *loss function* (eg squared loss).



Why is this powerful?

Recall **polynomial functions**

- Extend standard linear model by including non-linear transformations of individual variables X_j of choice: X_j^2, X_j^3 etc.
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Neural networks address both these limitations:

- Learns non-linearity and interactions of X automatically through the hidden layer.

A note on the non-linearity of $g()$

- When combining linear transformations, all weights merge into one slope per input $\rightarrow$ no curves, no interactions.

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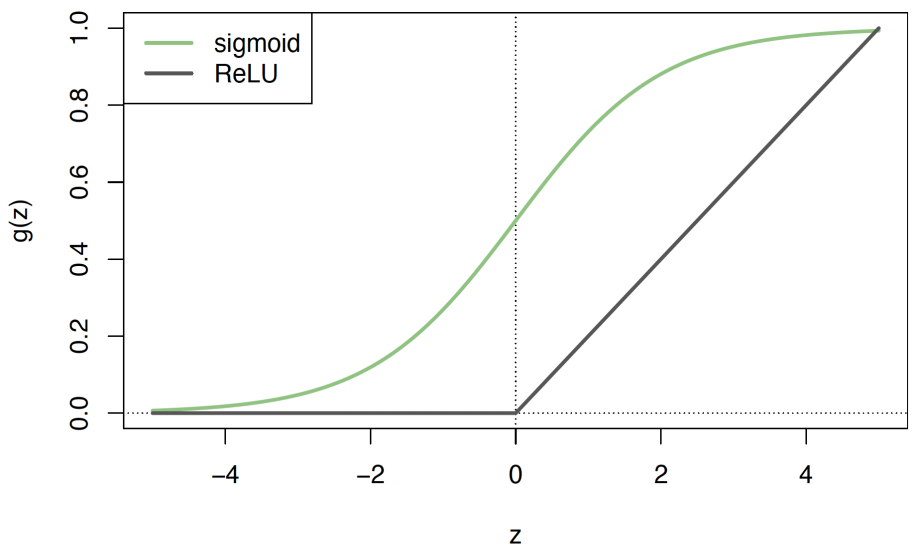

Can we have multiple hidden layers? Yes!

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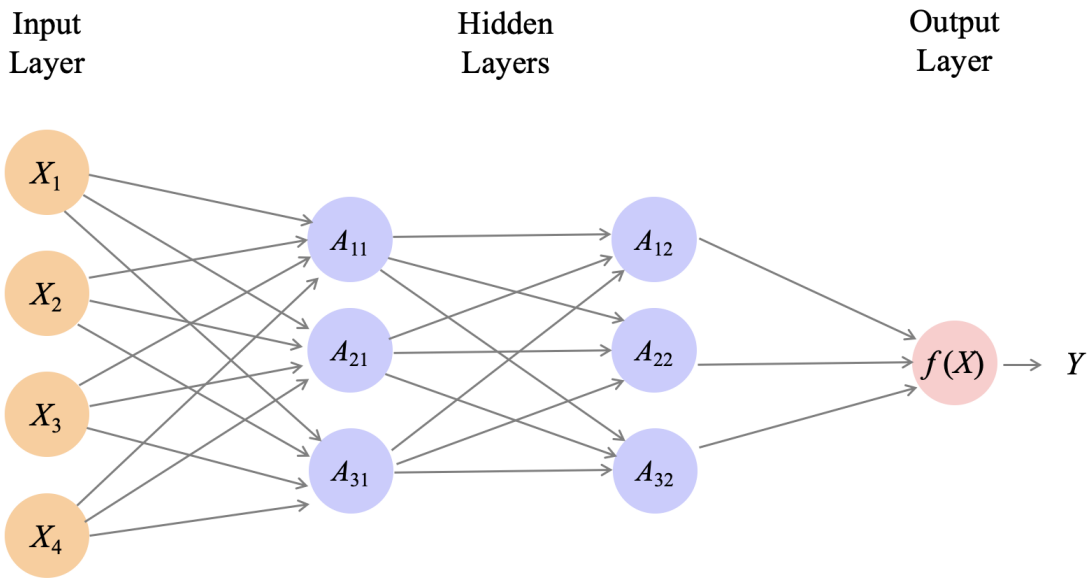
- Can learn richer representations, more effectively.



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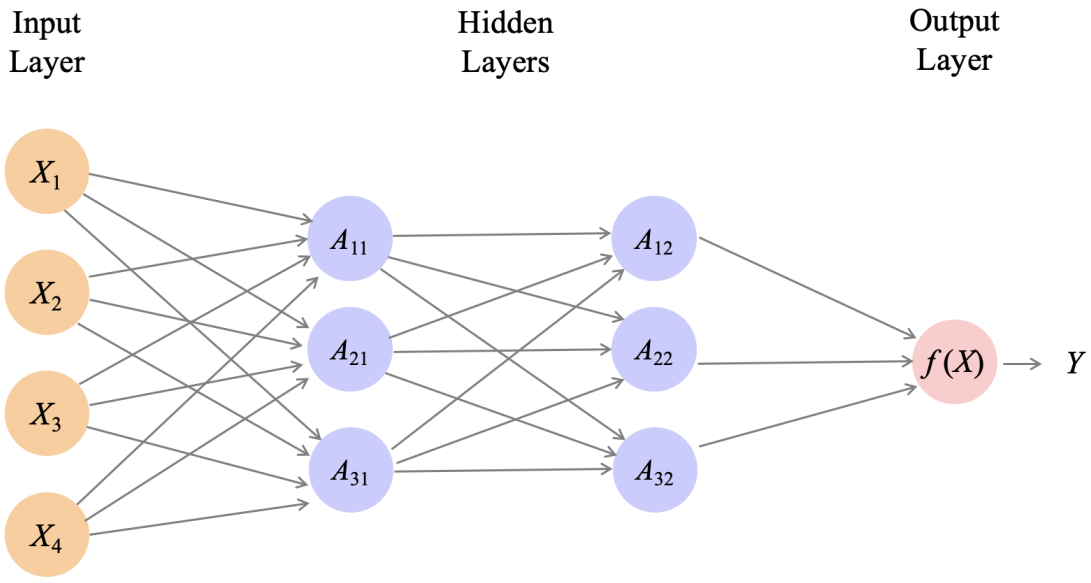

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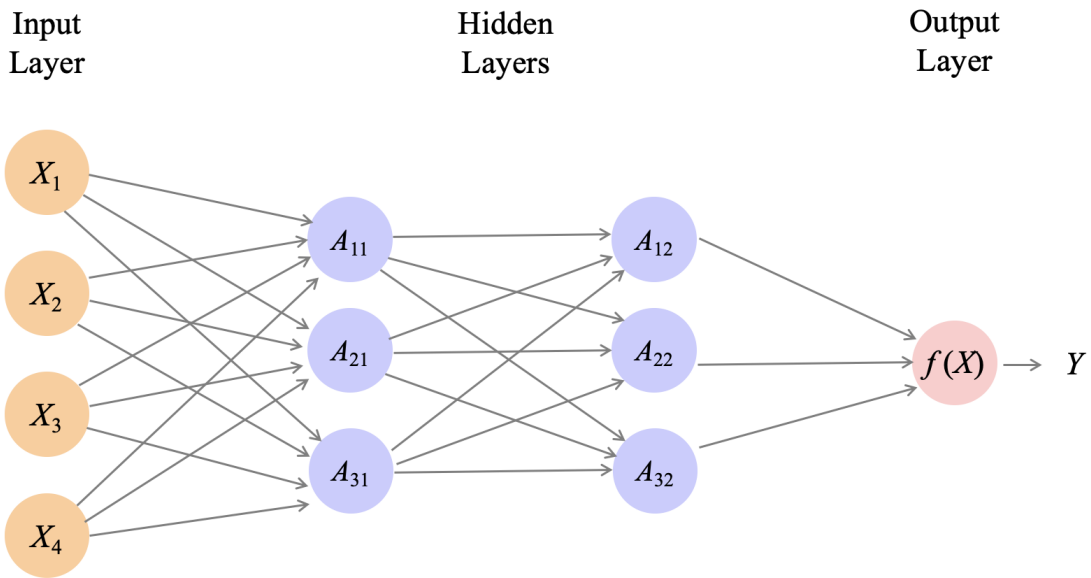

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For neural networks — several approaches for addressing overfitting.

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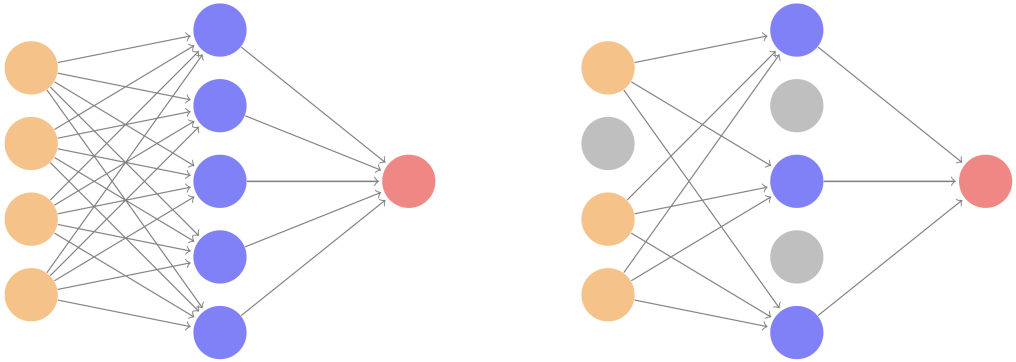

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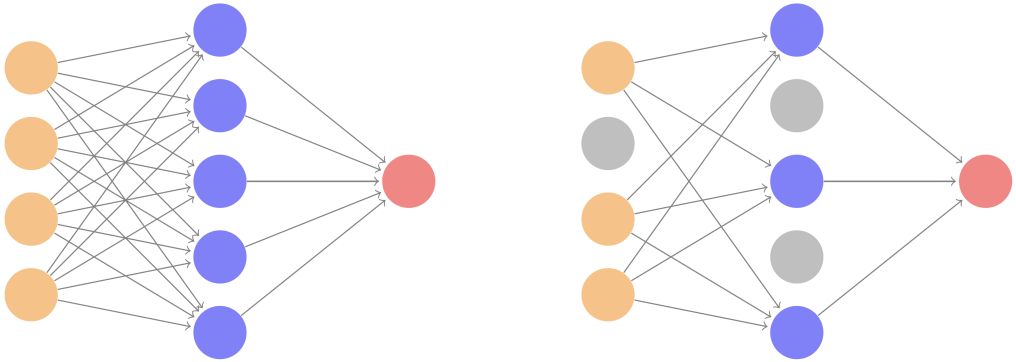

Early stopping

- Training error improves with the number of optimization rounds.
- Monitor validation loss and stop when it starts to increase.

Example application: predict handwritten digits

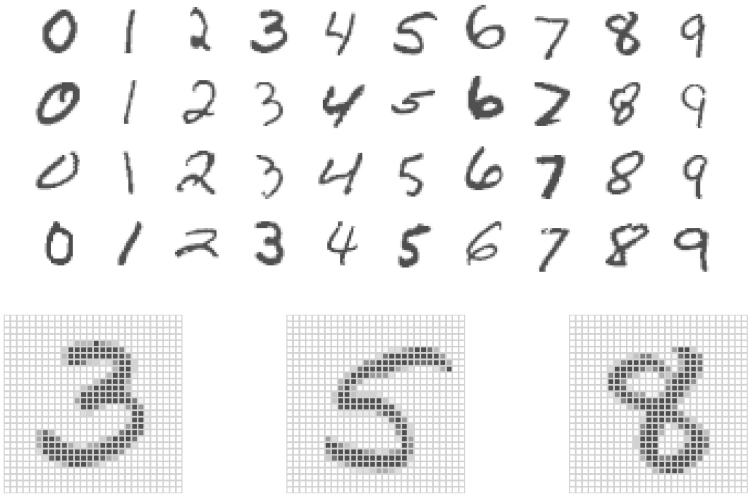

Figure 4: ISL, Fig 10.3

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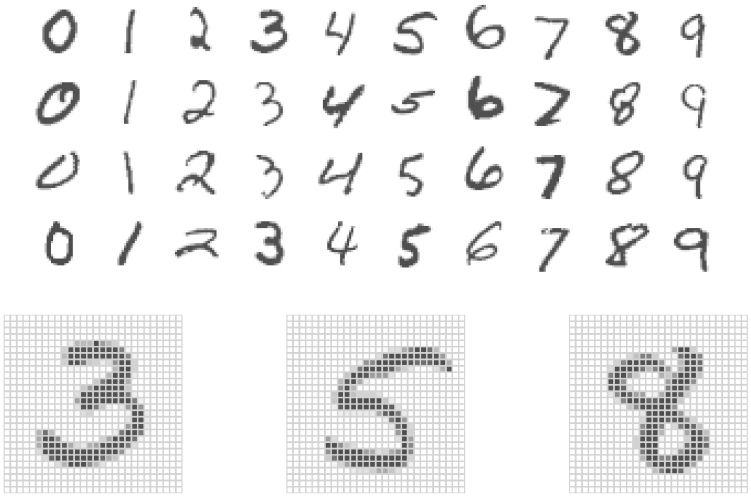

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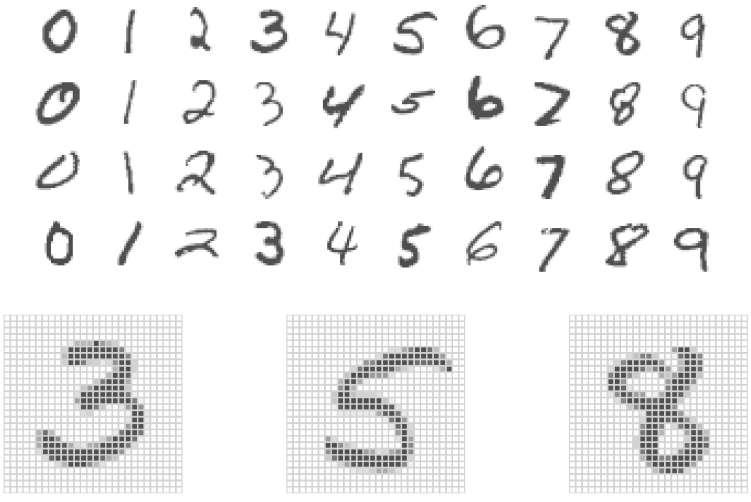

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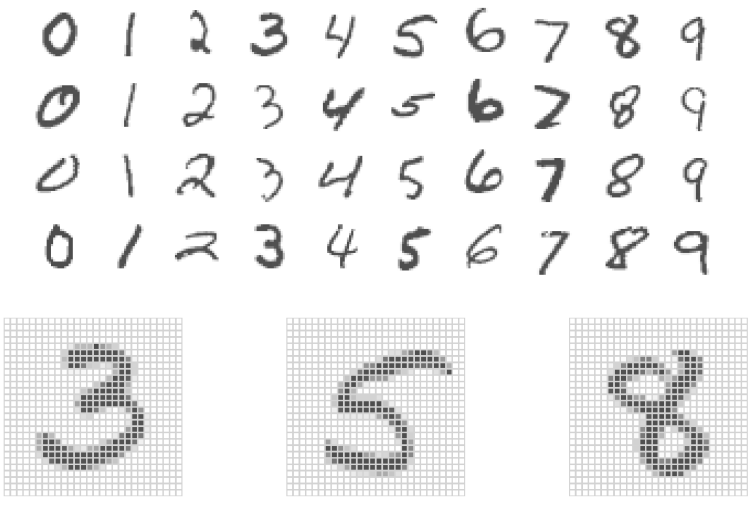

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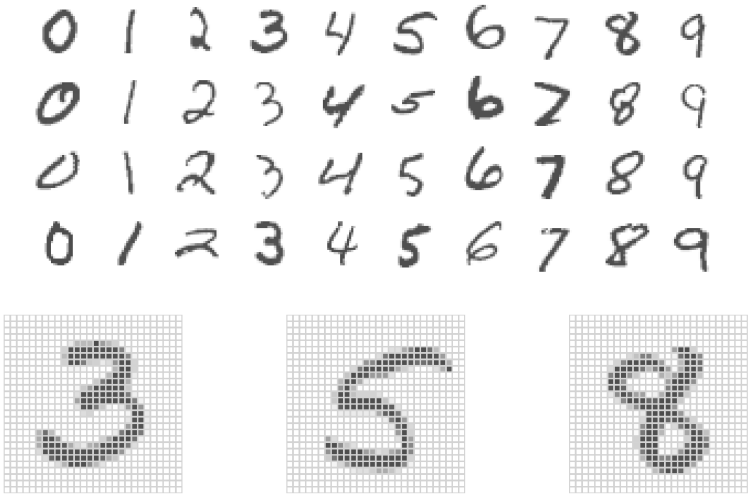

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- Outcome, y, is the class label: represented by 10 dummy variables.

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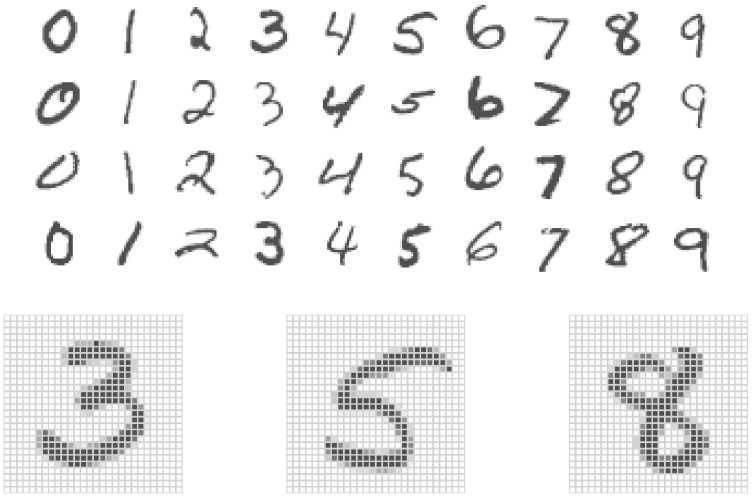

Figure 4: ISL, Fig 10.3

- **Goal**: build model to classify images into their correct class (0–9).
- Every image has 28*28=784 pixels.
- Each pixel is an 8-bit gray scale value between 0-255.
- Thus, X, consist of 784 variables, one for each pixel value.
- Outcome, y, is the class label: represented by 10 dummy variables.
- *Training* set: 50,000 images. *Test* set: 10,000 images.

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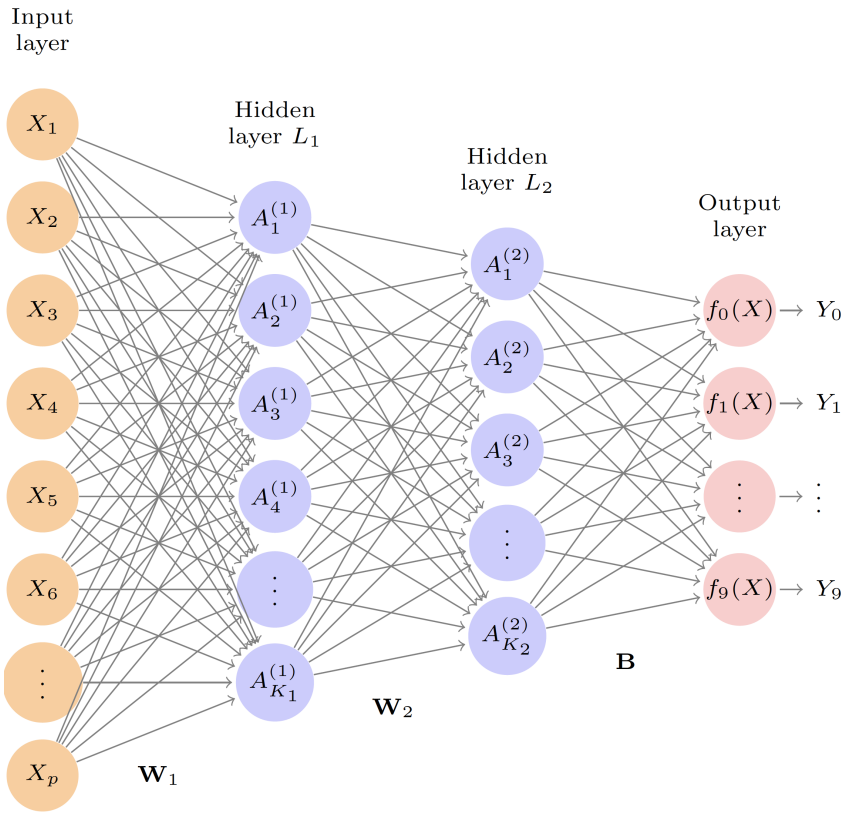

Figure 5: Model estimated in ISL (Fig 10.4)

- Two hidden layers (256, 128 units).
- Implies: (784*256) + (256*128) + (128*10) $\approx$ 235,000 parameters.

Results

Method	Test Error
Neural Network + Ridge Regularization	2.3%
Neural Network + Dropout Regularization	1.8%
Multinomial Logistic Regression	7.2%
Linear Discriminant Analysis	12.7%

Figure 6: ISL, Table 10.1

- Significant improvement over standard linear model(s).
- And — impressive accuracy ($\approx$ 98%)!

Example II: predicting salaries of baseball players

Data: 322 observations of major league players (20 input variables).

Comparing three models:

- Standard linear model: 20 parameters.
- Lasso regression (λ selected via CV): 12 non-zero parameters.
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Model	# Parameters	Mean Abs. Error	Test Set R^2
Linear Regression	20	254.7	0.56
Lasso	12	252.3	0.51
Neural Network	1345	257.4	0.54

Figure 7: ISL, Table 10.2

- On this (small N, small p) data, we see no improvement.

Considerations when applying neural nets

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    Early stopping almost always implemented

    Other: grid search (in conjunction with #hidden layers/units etc.)

Summary, thus far.

Pros

- Neural networks can learn highly *non-linear* and *complex interactions*
- Especially powerful for 'complex' and 'unstructured' data — where *representation learning* is key.
  - E.g., text, images, video, sound...
  - Rich library of architectures developed for such data.

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  - E.g., text, images, video, sound...
  - Rich library of architectures developed for such data.

Cons

- Require *a lot of data* to learn complex patterns.[1]
- *Many hyperparameters* that the analyst much choose.
- Lacking in *inference* and *interpretability*.

[1] When data is smaller / patterns simpler — usually not competitive.

Convolutional neural networks

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- Class of neural networks developed for image classification.

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- **Basic idea**:
  1. Identify low-level features in images (edges, patches of colors)
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  3. Learn association between high-level features and outcome variable.

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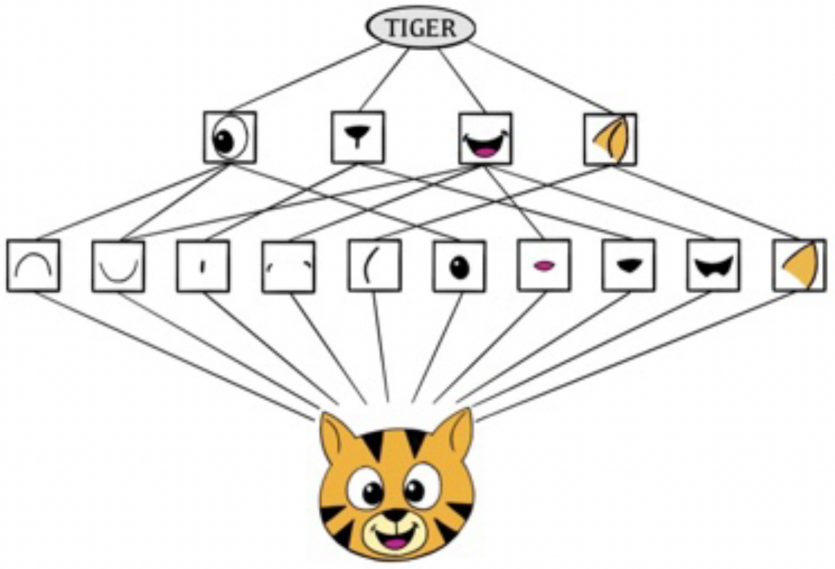

Figure 8: ISL, Fig 10.6

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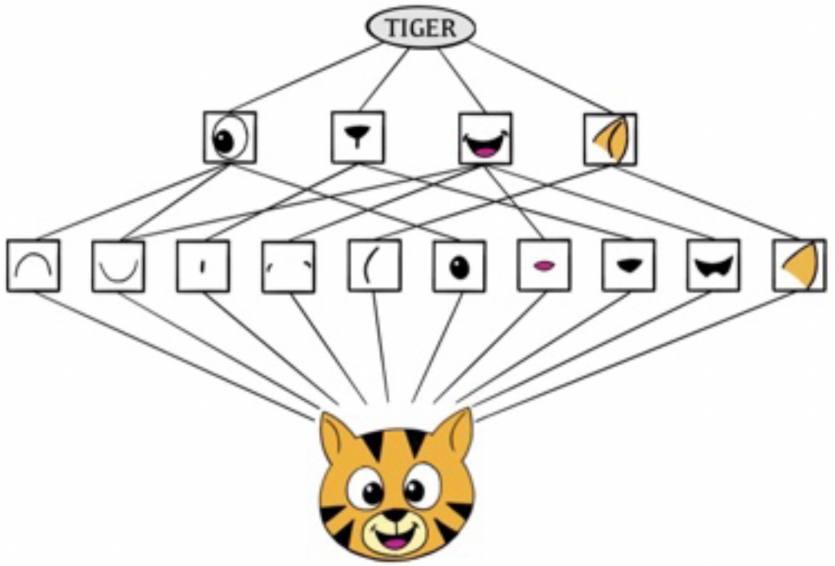

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This is achieved by adding a specialized type of hidden layer to the network.

Convolutional layers

- A convolutional layer contains a set of smaller matrices ('filters'), each representing a *particular visual feature*.[2]

[2] 1st layer: simple (eg lines); subsequent layers: combine → complex (eg objects).

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- This is repeated for all k filters, producing k feature maps.

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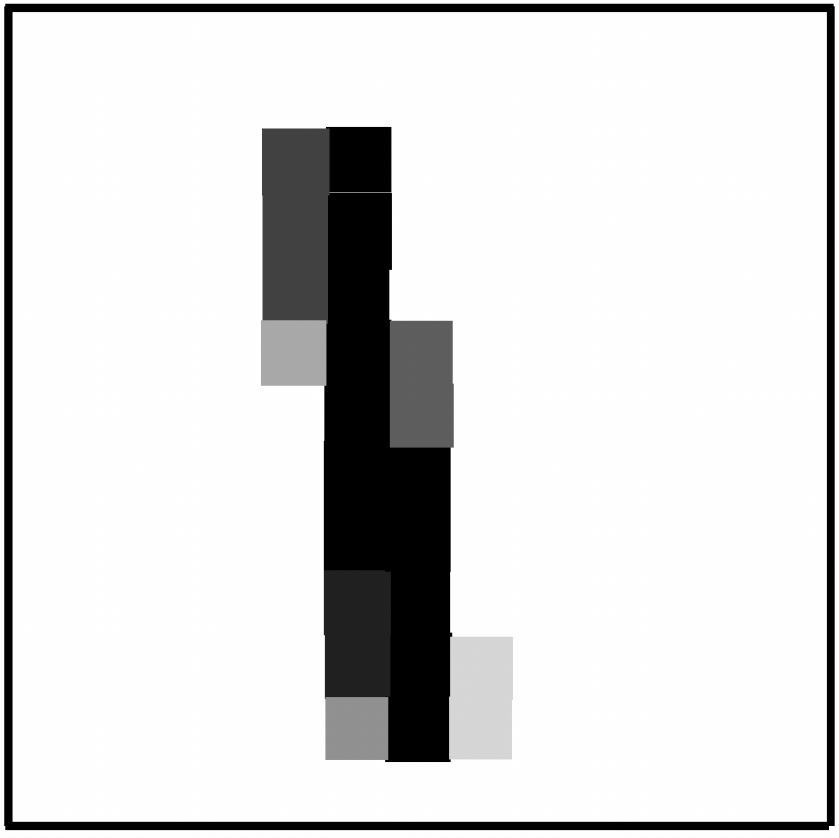
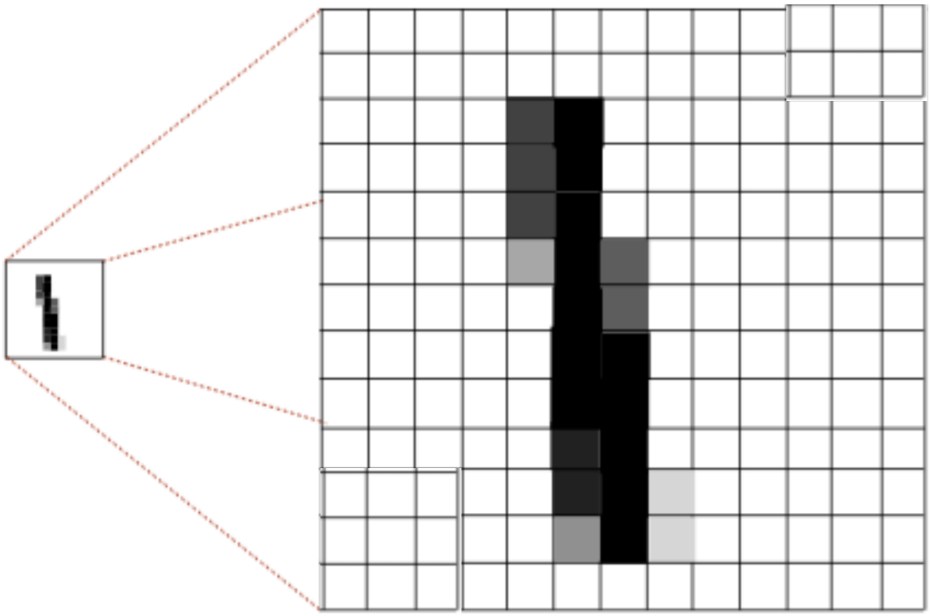
Illustration

Suppose this is our image:



Illustration

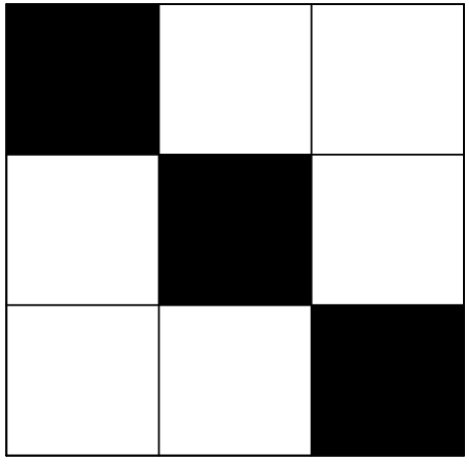
Which we map to a matrix of numbers (gray scale of each pixel)



0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	.6	1	0	0	0	0	0	0	0
0	0	0	0	.6	1	0	0	0	0	0	0	0
0	0	0	0	.6	1	0	0	0	0	0	0	0
0	0	0	0	.3	1	.5	0	0	0	0	0	0
0	0	0	0	0	1	.5	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	.8	.8	1	0	0	0	0	0	0
0	0	0	0	0	.8	1	.2	0	0	0	0	0
0	0	0	0	0	.3	1	.2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

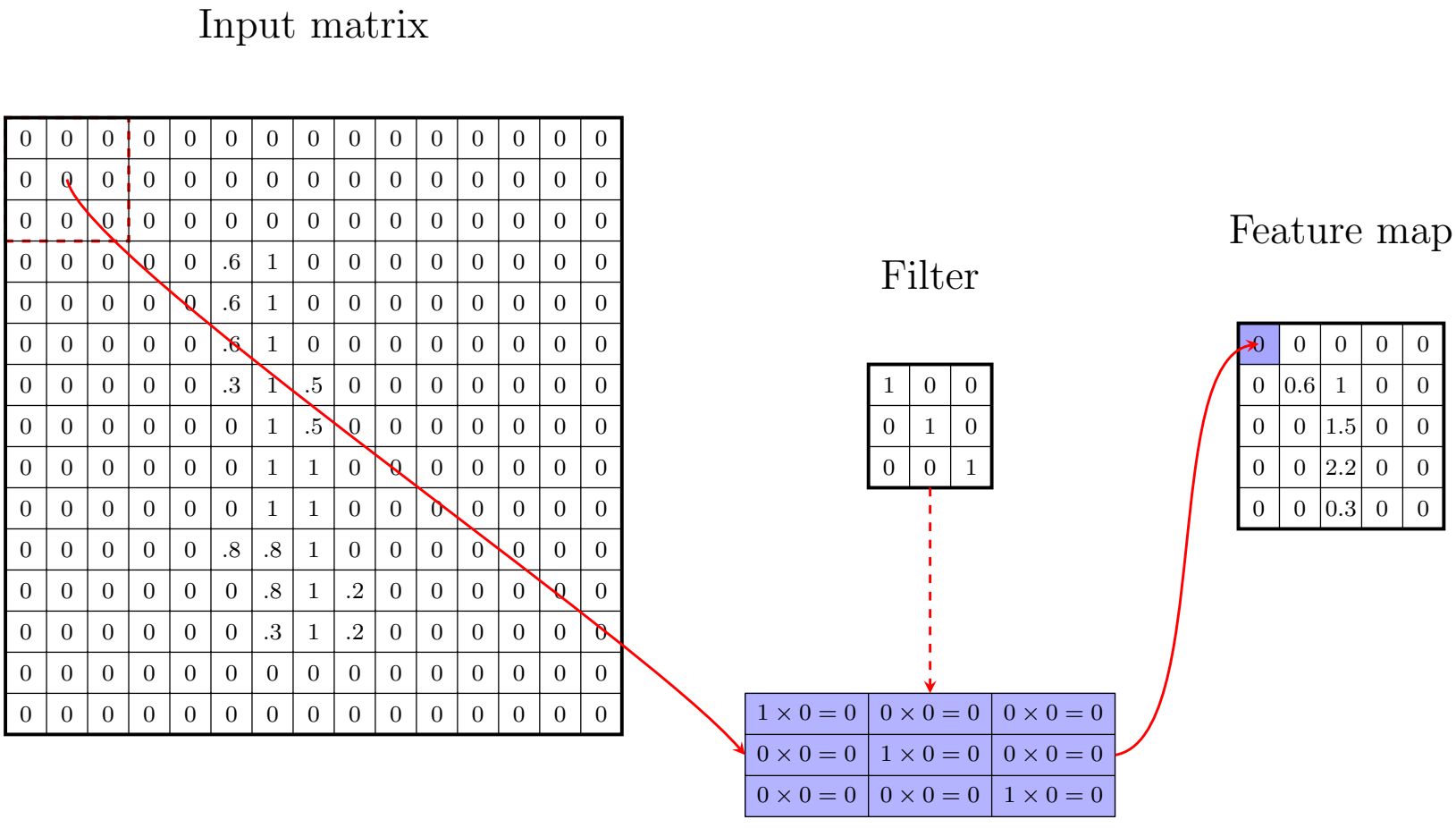
Illustration

And suppose this is our (3*3 sized) filter:



Illustration

We slide the filter through each 3x3 patch of the image — recording the presence of the filter-pattern:



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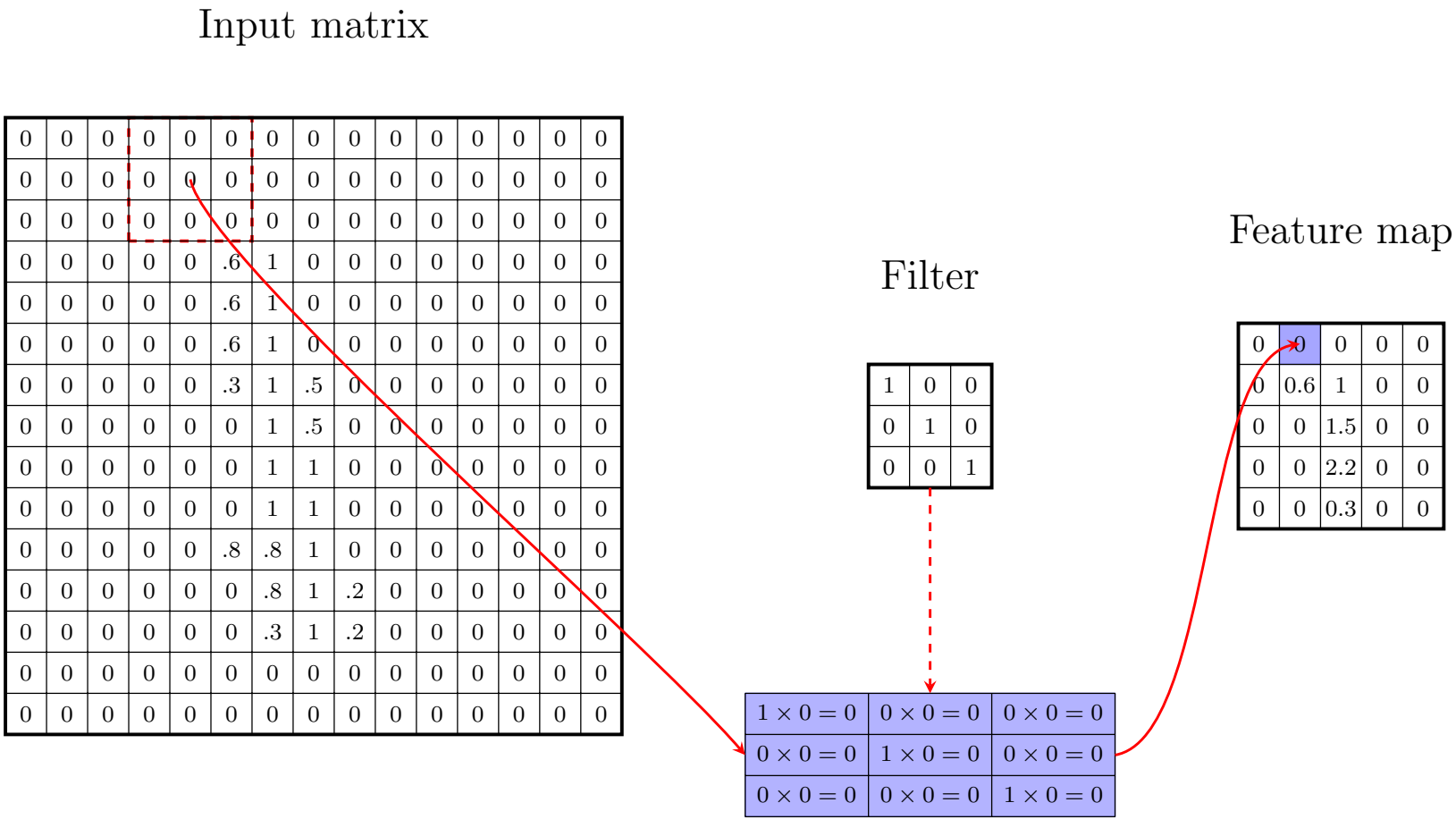

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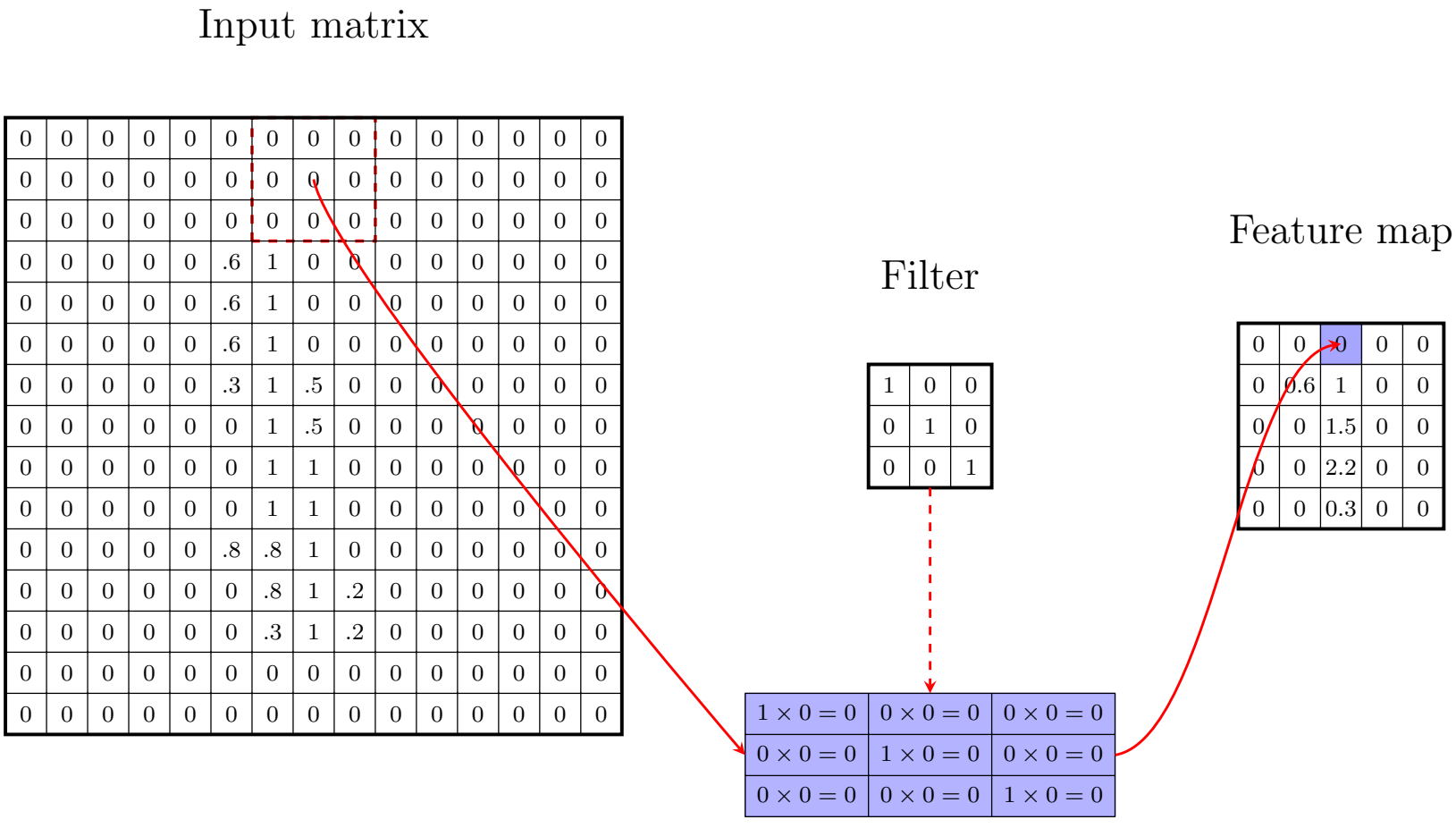

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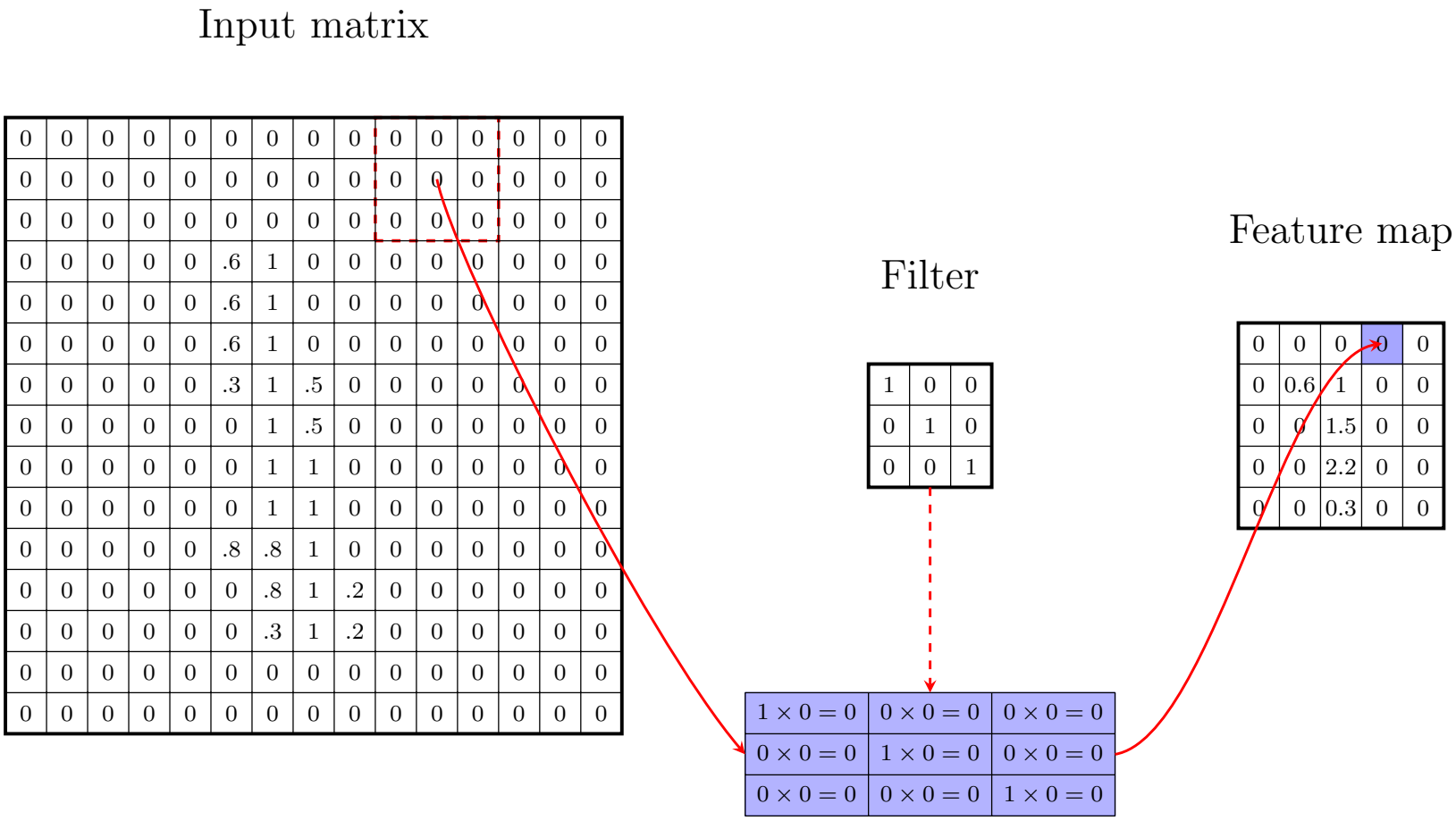

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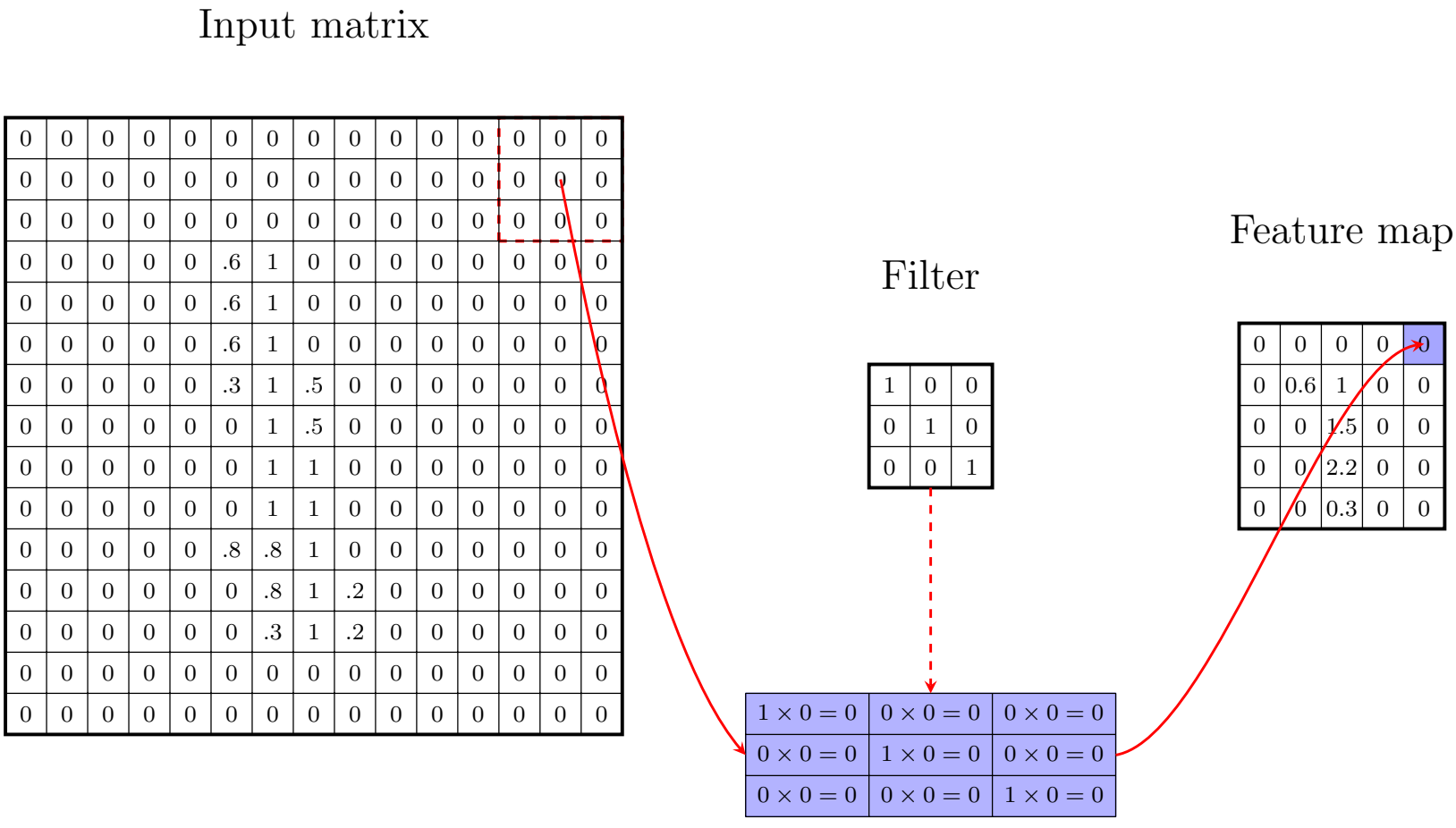

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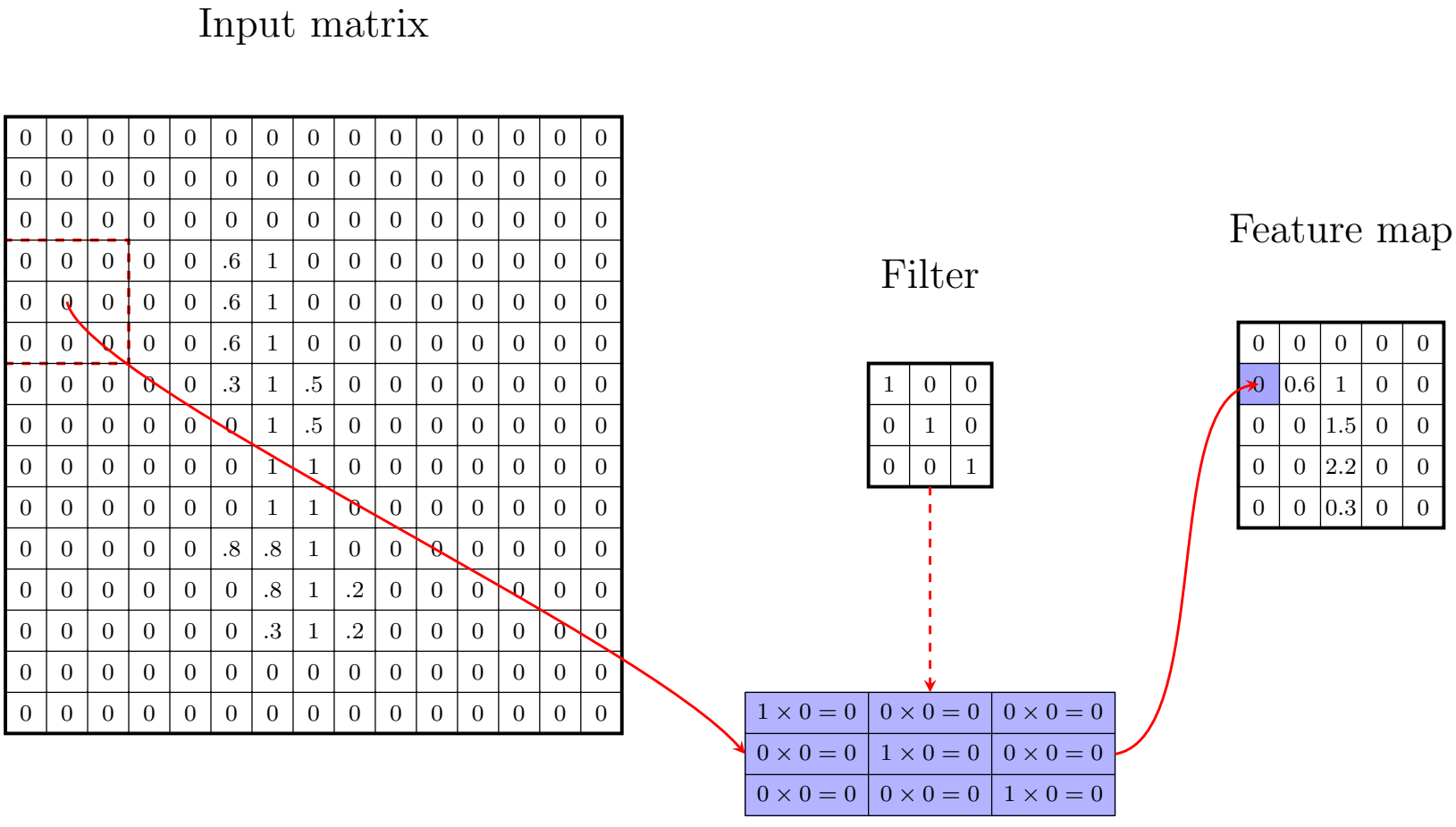

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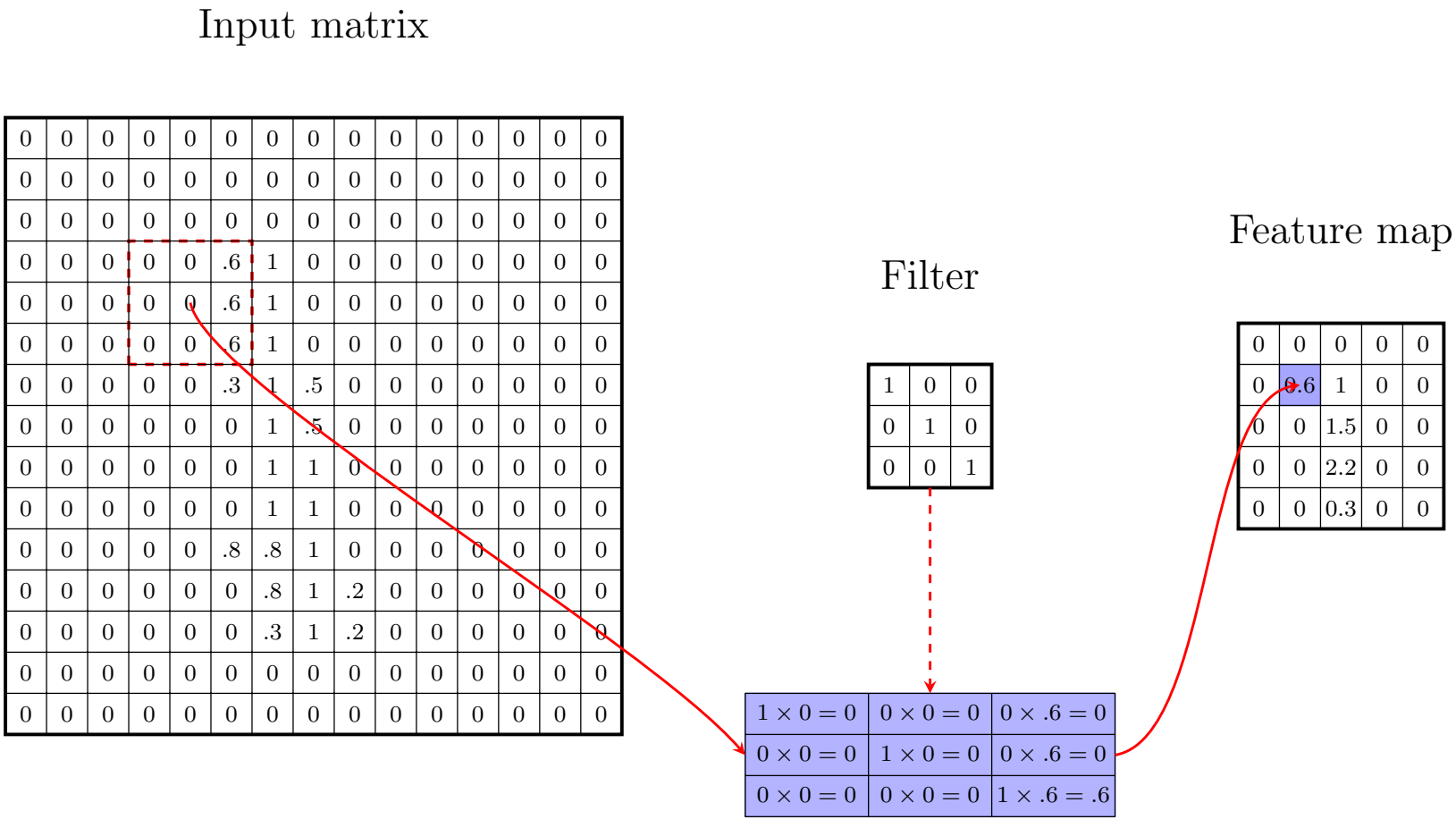

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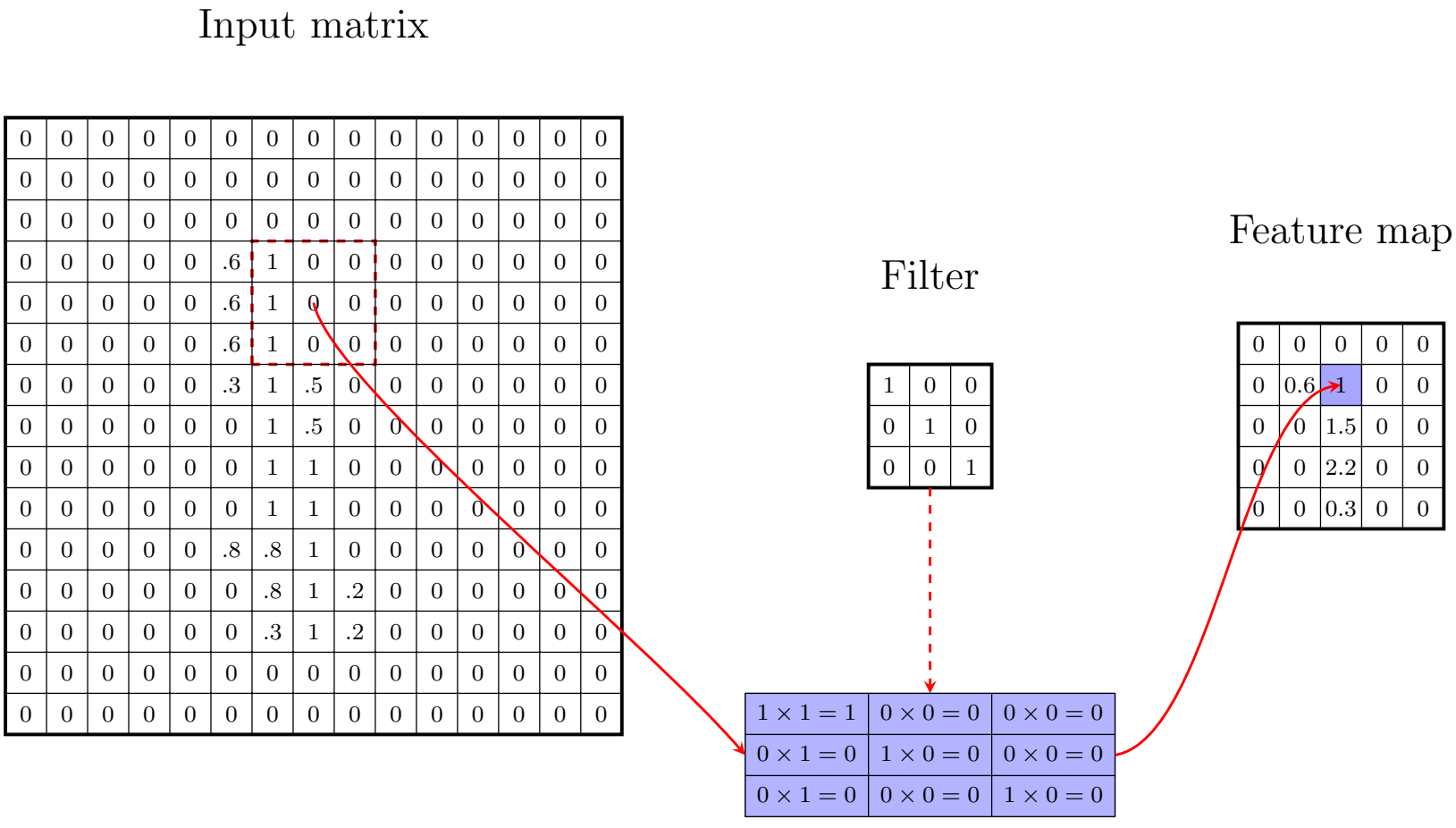

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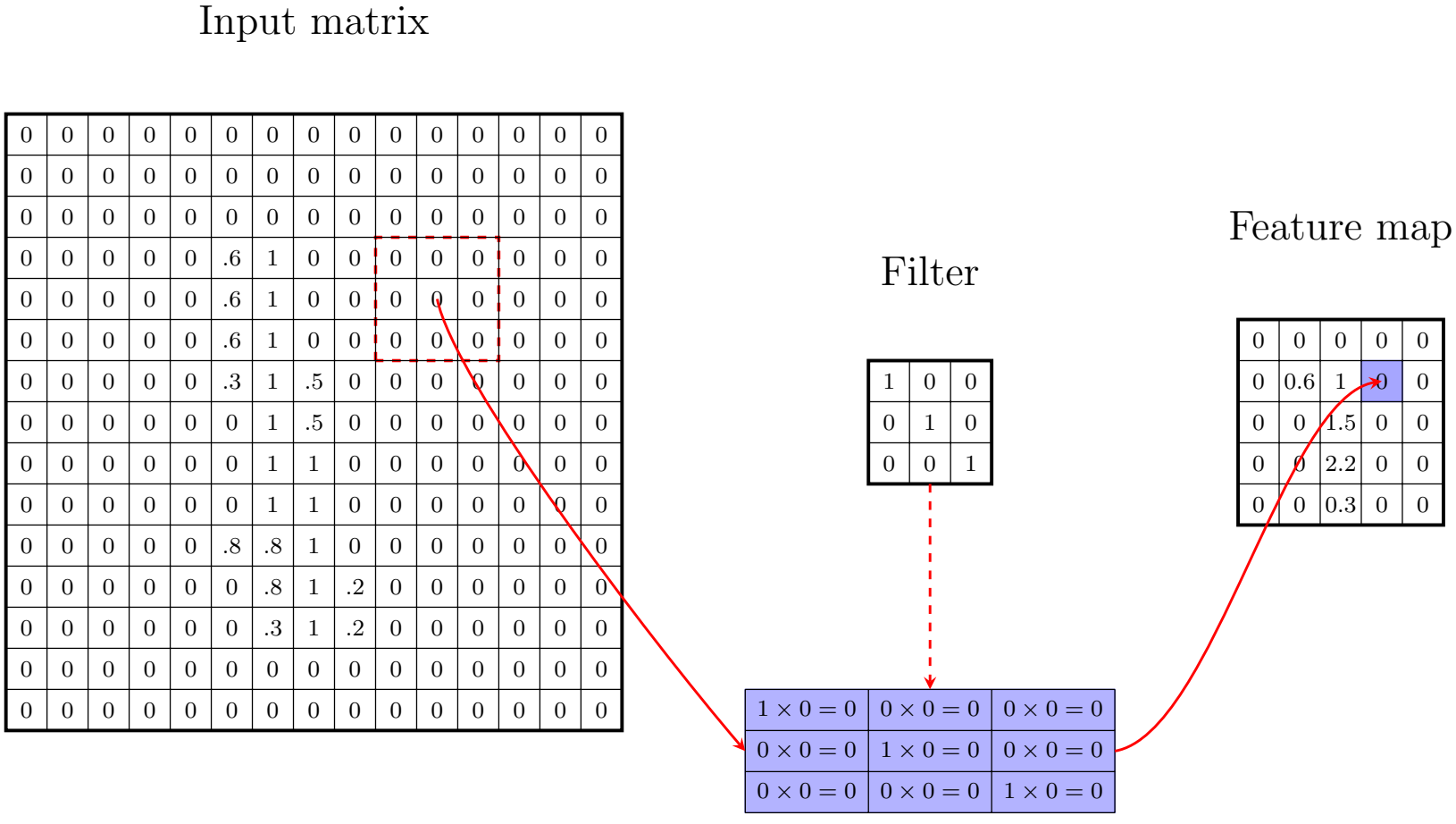

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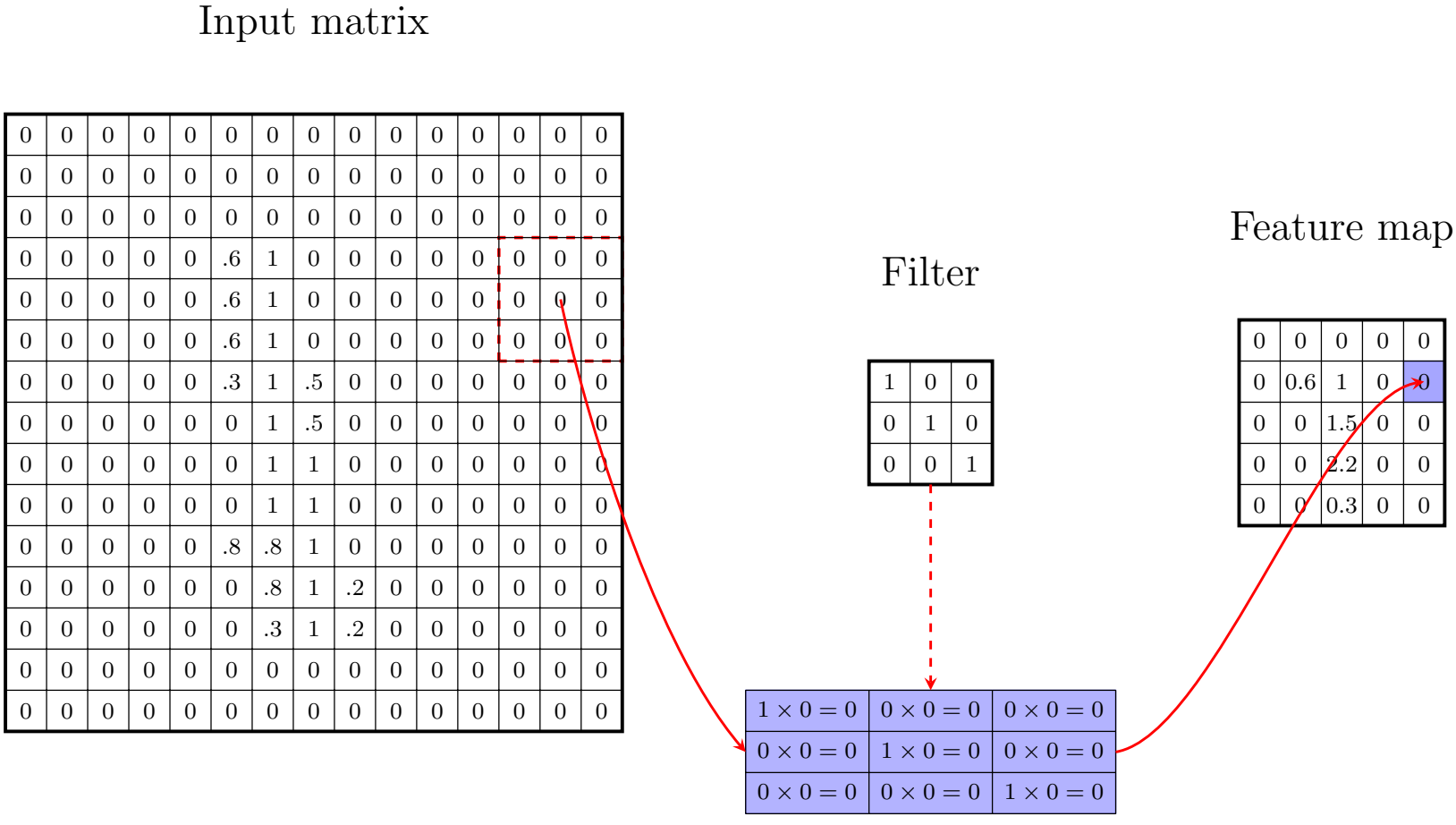
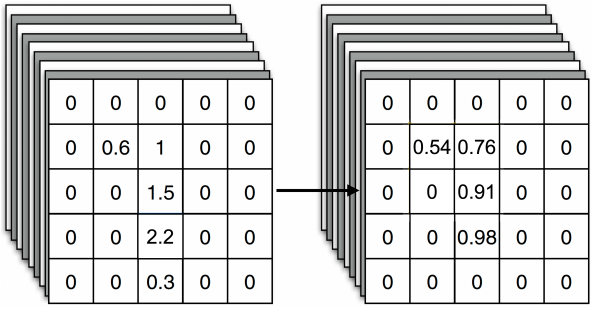


Processing of feature maps

1. **Non-linear transformation** — ensures non-redundancy of convolutional layers (extraction of more info).



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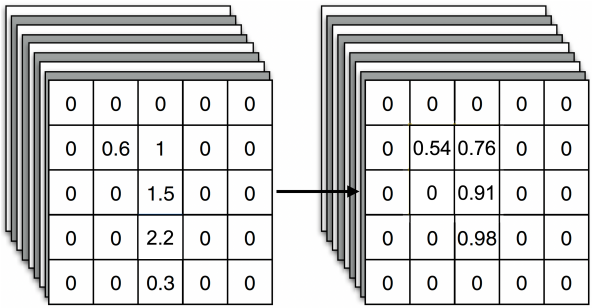

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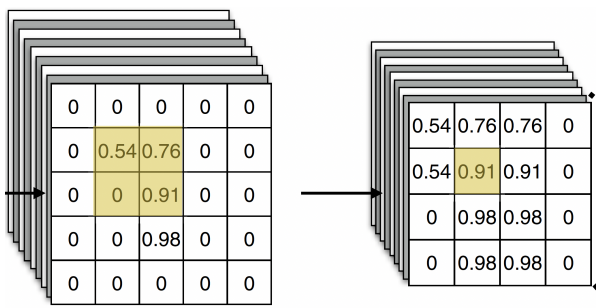

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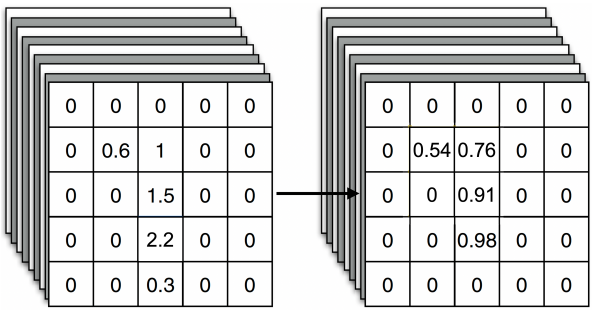

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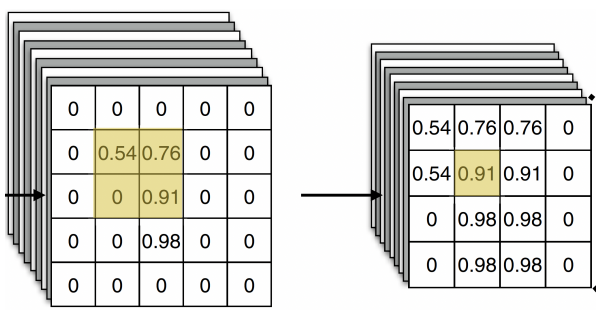

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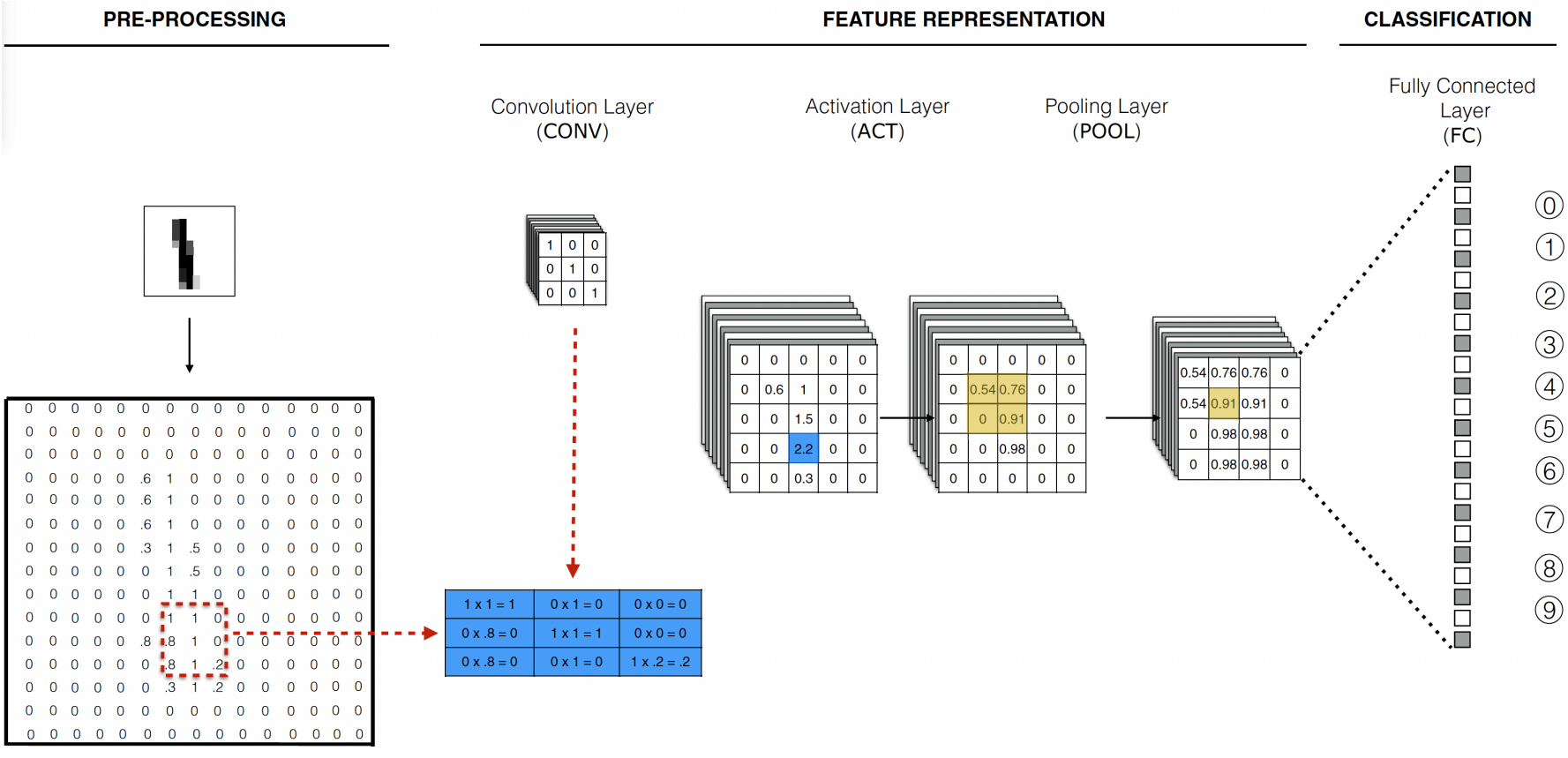

Figure 10: Overview of convolutional network structure (Torres & Cantu '22)

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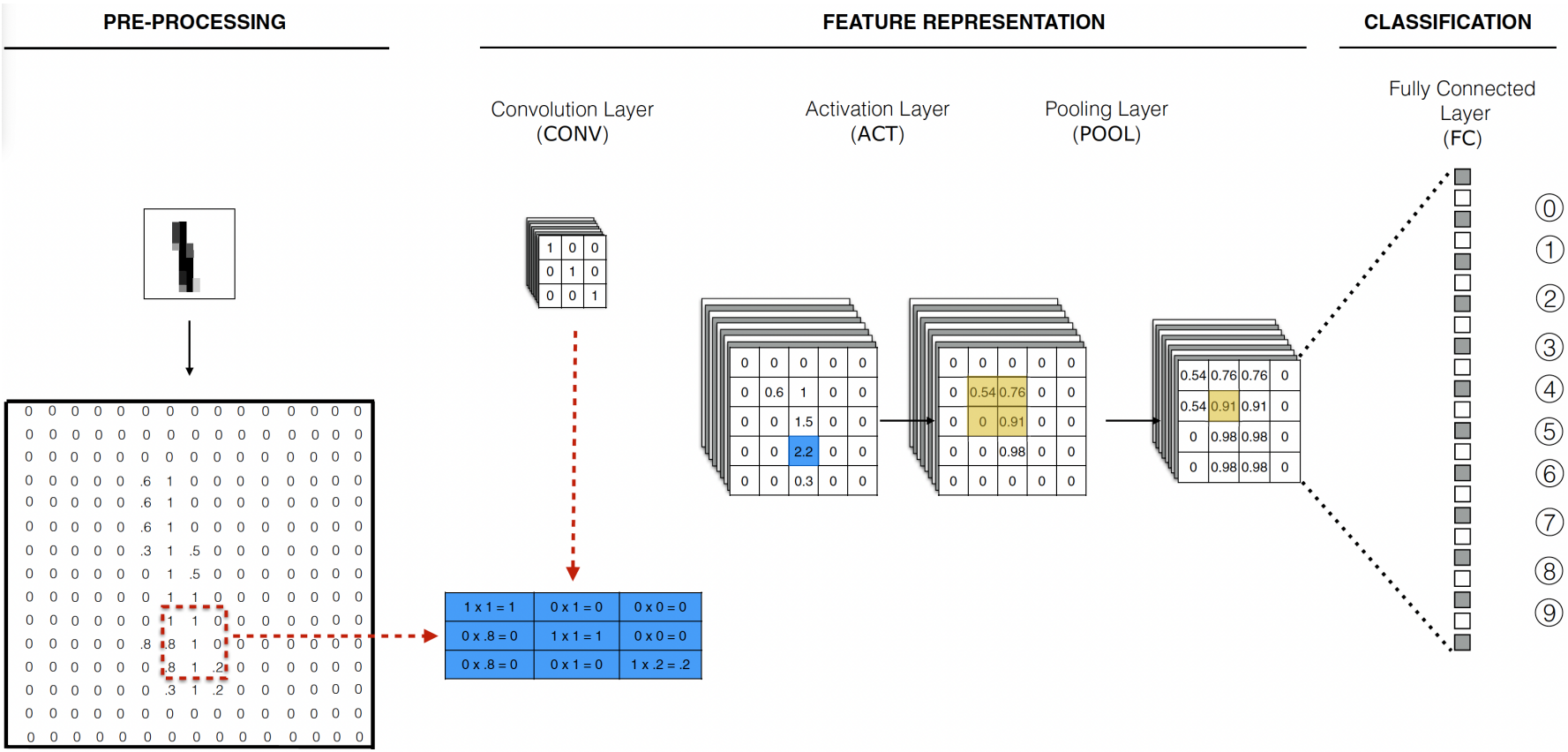

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Goal: learn weights in the CONV, FC that minimizes loss function.

Some practical considerations

- Choices to be made about the CNN architecture
- Data augmentation
- Transfer learning
- Validation

Choices to be made about the CNN architecture

The basic structure of a CNN:

Figure 11: Torres & Cantu (2022)

- M *convolutional* layers
- K *regular* hidden layers

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- Larger M: more *complex features* of image extracted.
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- No unique recipie — depends on data and problem.[6]
- Larger M: more *complex features* of image extracted.
- Larger K: more *complex classifier*.
- Starting point (Rosebrock 2017): $1 \leq M \leq 3$ and $K \geq 1$.
- Experimenting using grid search encouraged (Torres & Cantu 2022)

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Choices to be made about the CNN architecture

Additionally

- *Number of filters*:
  - Large # $\rightarrow$ network looks for more features of similar complexity.
  - Common procedure: increase # filters gradually through network.[7]
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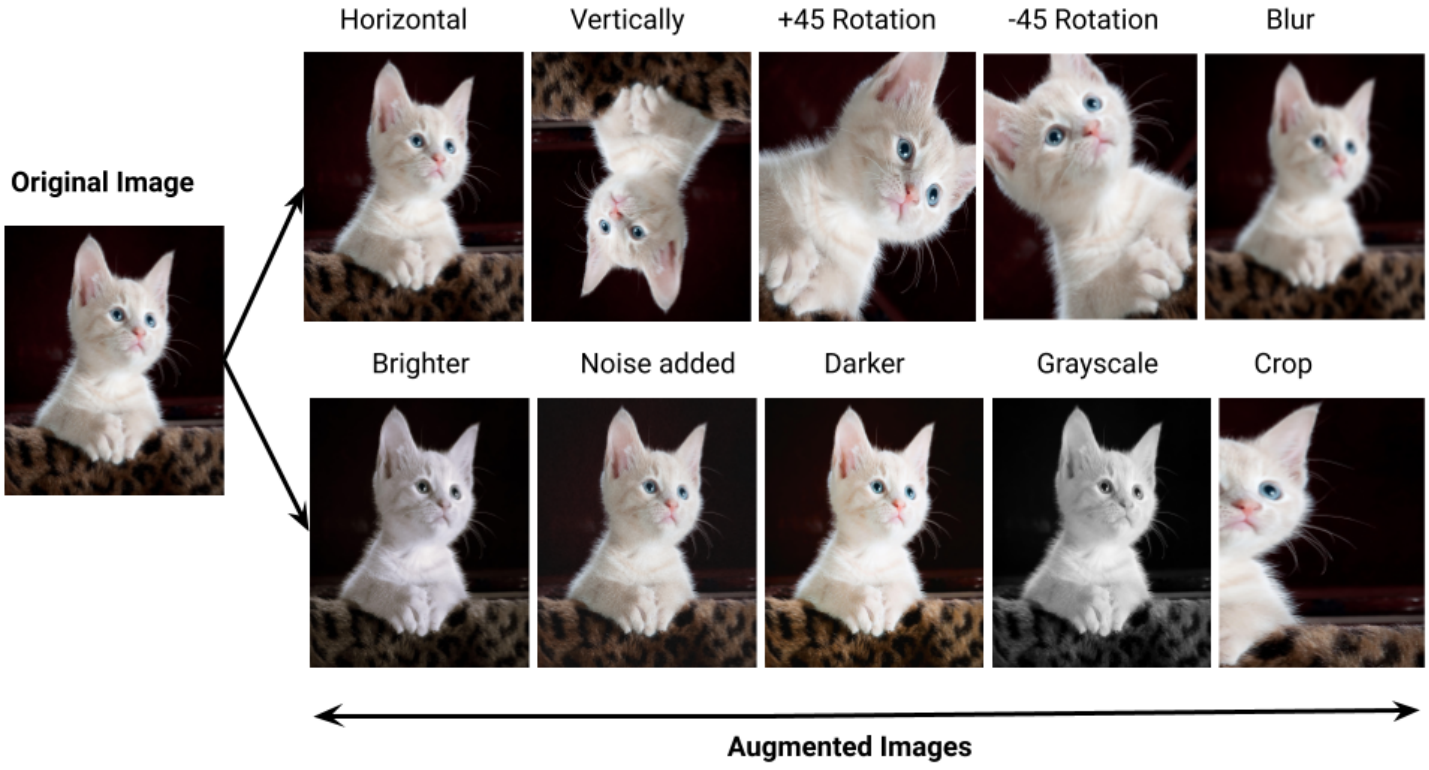
  - If goal is to find *more fine-grained features* → smaller filters.
- *Pooling*
  - Most common is max-pooling.
  - If very complex network: 3*3 in early stages and then 2*2 in later.
  - Else, 2*2 throughout.[8]

[7] x2 the no. filters every time we add an additional conv-layer (Shang 2016).
[8] With a stride of 2: ie, slide window 2 pixels each time → non-overlapping patches.

Data augmentation

- Produces random variations of the original training images by, e.g., *flipping*, *rotating*, *zooming out*.
- Forces model to pay less attention to location and orientation of a feature — instead: grasp relationship to other features.
- Generally leads to better generalizability.



Transfer learning

- Training a CNN *from scratch* can be computationally expensive and *require large data*.
- Alternative: *transfer learning* ; use (large) model trained on a *similar but different* data.
- How it works:
  - Freeze *initial k* layers of pre-trained model.
  - Re-train *last j* layers.

Validation

- When used for measurement purposes — '*validate*, *validate*, *validate*' (Grimmer & Stewart 2013; Torres & Cantu 2022)
- Recommended ways to improve model:
  - Review missclassified images in order to spot problems.
  - Visualize most important features (apsects of images) for predictions.

Application: coding electoral vote from vote tallies.

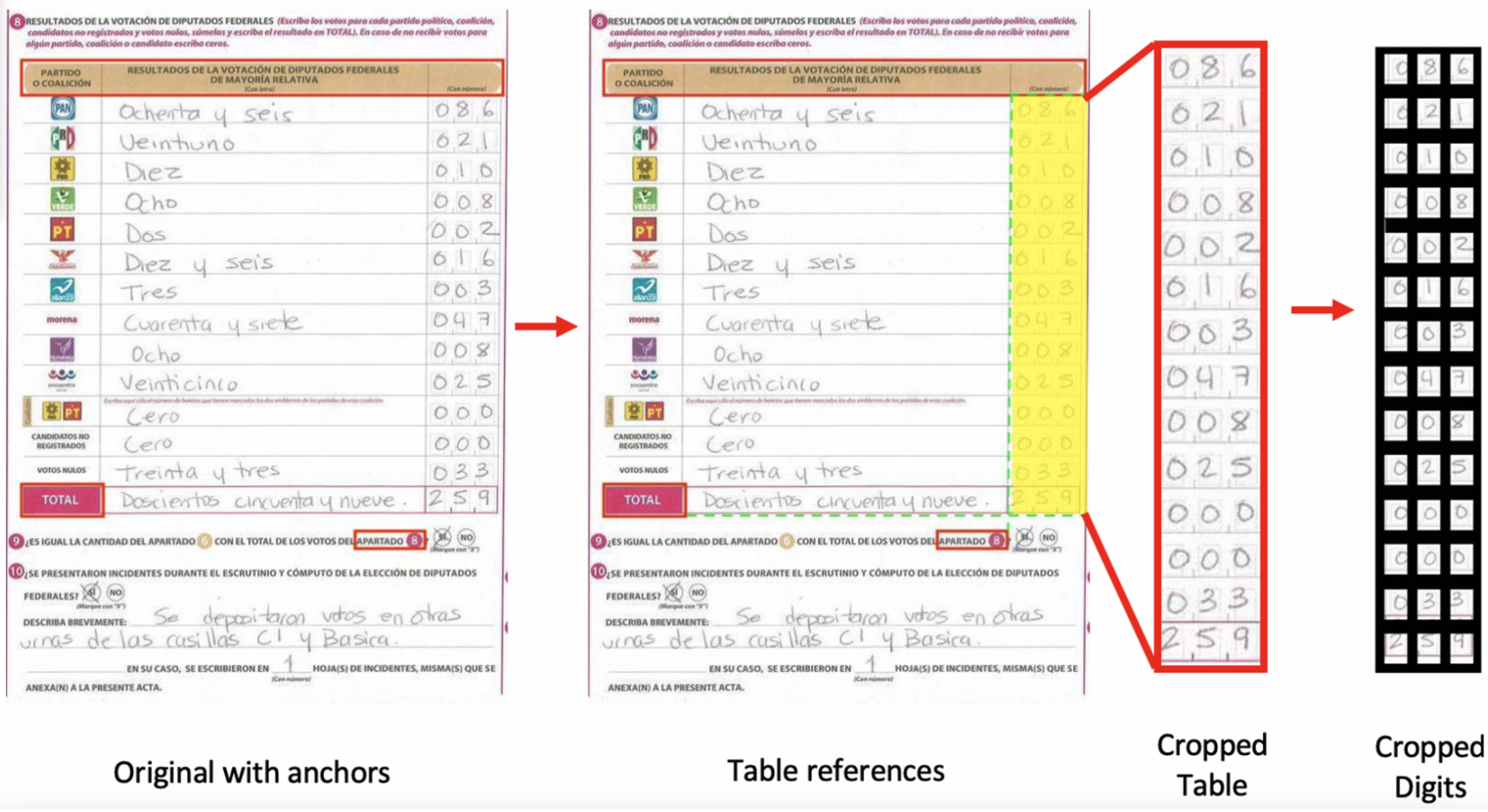


Figure 12: Vote-tally extraction, Mexico 2015 (Cantu and Torres 22)

Goal

- Correctly identify the handwritten digits.
- Sum predicted digits to arrive at electoral outcomes.

Application: coding electoral vote from vote tallies.

Basic setup

1. Train 'base' model on large *external data* (MNIST; 60K handwritten digits).
   - CNN with 2 CONV-layers, max-pooling, and 3 FC-layers.[9]
   - Select (via grid search) parameters to achieve *low validation error on Mexican sample*.

[9] Regular hidden layers

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3. Validate

[9] Regular hidden layers

Application: coding electoral vote from vote tallies.

Table A.2: Hyperparameter grid search set-up

Hyperparameter	Default	Min. value	Max. value	Interval	Selected
Number `CONV` blocks	2	1	3	1	3
Filters Layer 1	-	16	128	16	64
Filters Layer 2	-	16	128	16	96
Filters Layer 3	-	16	128	16	112
Dropout	0.5	0.1	0.5	0.1	0.1
Size `FC 1`	128	50	200	20	150
Size `FC 2`	50	10	100	10	70
Learning rate	0.0001	0.0001	0.01	-	$\approx$0.0001
Epochs	-	-	45	-	20

Figure 13: Grid search (Cantu and Torres 22)

Application: coding electoral vote from vote tallies.

Table A.3: Results from the hyperparameter search grid (selected)

Number CONV layers	Filters Layer 1	Filters Layer 2	Filters Layer 3	Dropout rate	Size FC 1	Size FC 2	Number Epochs	Validity Accuracy
3	64	32	32	0.200	110	20	45	0.332
3	96	32	112	0.300	70	50	5	0.335
3	112	80	80	0.200	70	50	45	0.338
3	80	64	96	0.200	50	20	5	0.340
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
3	96	112	48	0.300	130	50	45	0.352
3	80	112	112	0.500	170	30	15	0.350
3	32	32	112	0.200	110	30	5	0.346
3	96	32	112	0.300	70	50	15	0.343
3	32	32	112	0.200	110	30	15	0.342

Figure 14: Results: grid search (Cantu and Torres 22)

Application: coding electoral vote from vote tallies.

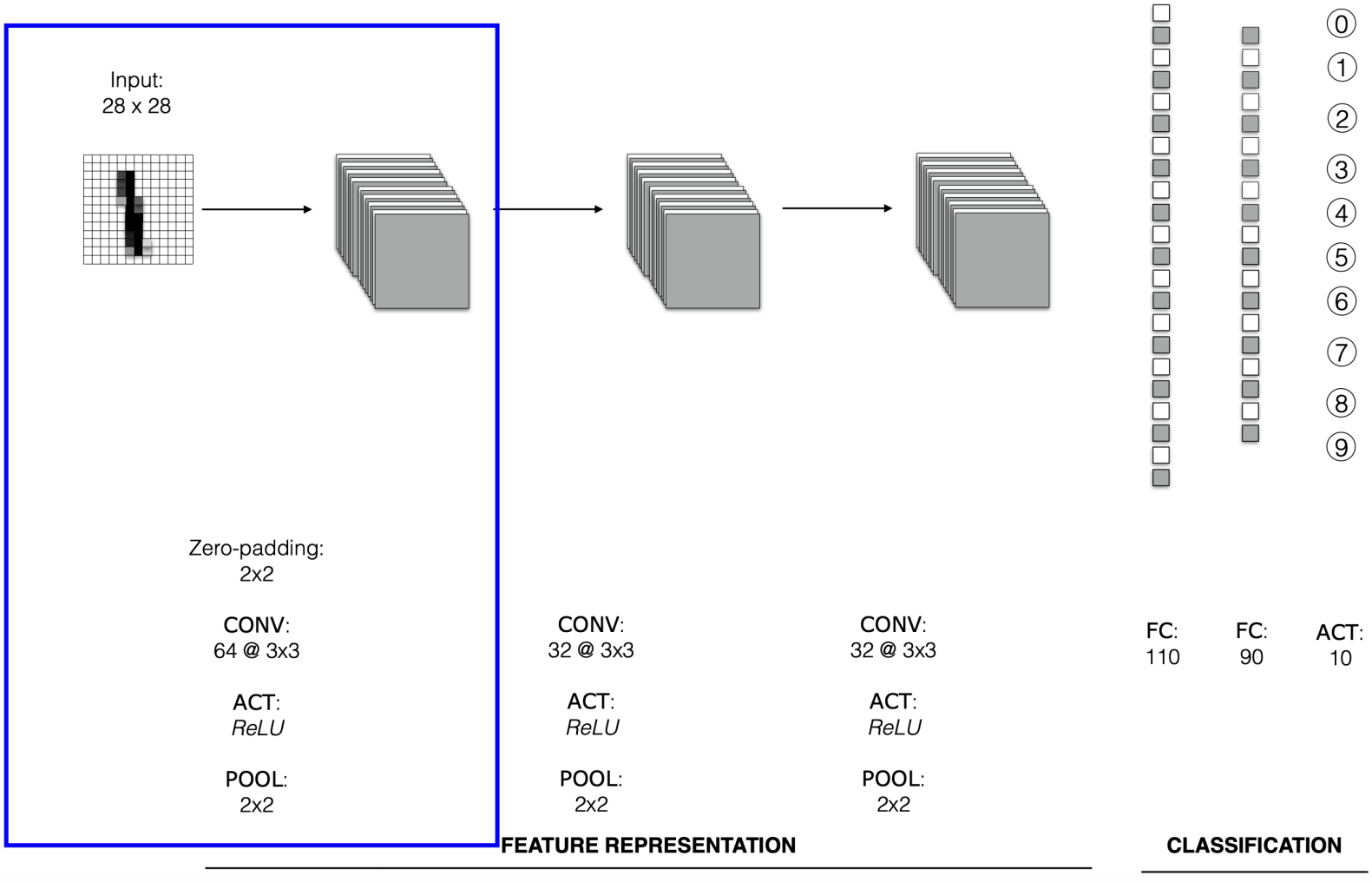
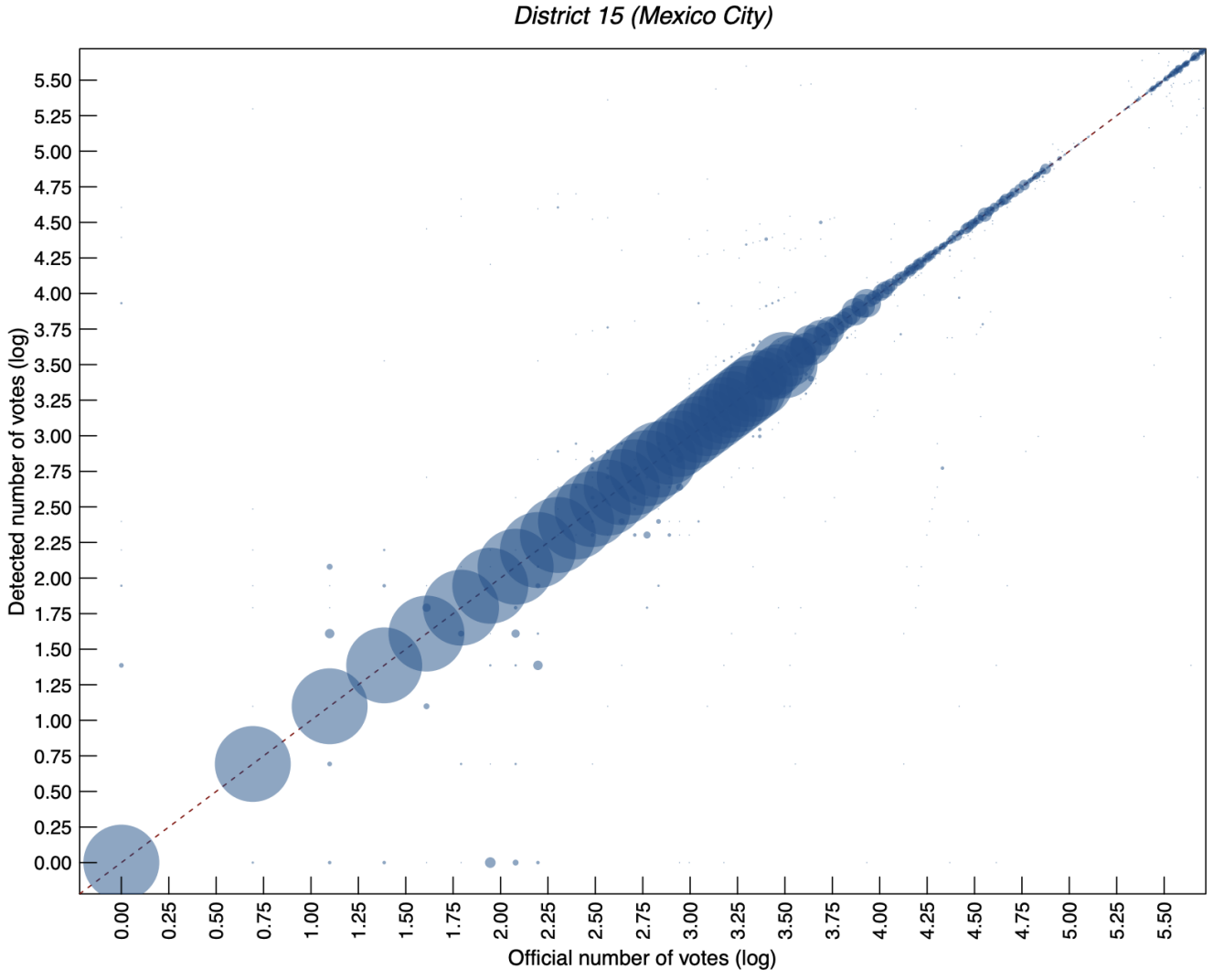


Figure 15: CNN-structure, Mexico 2015 (Cantu and Torres 22)

Application: coding electoral vote from vote tallies.



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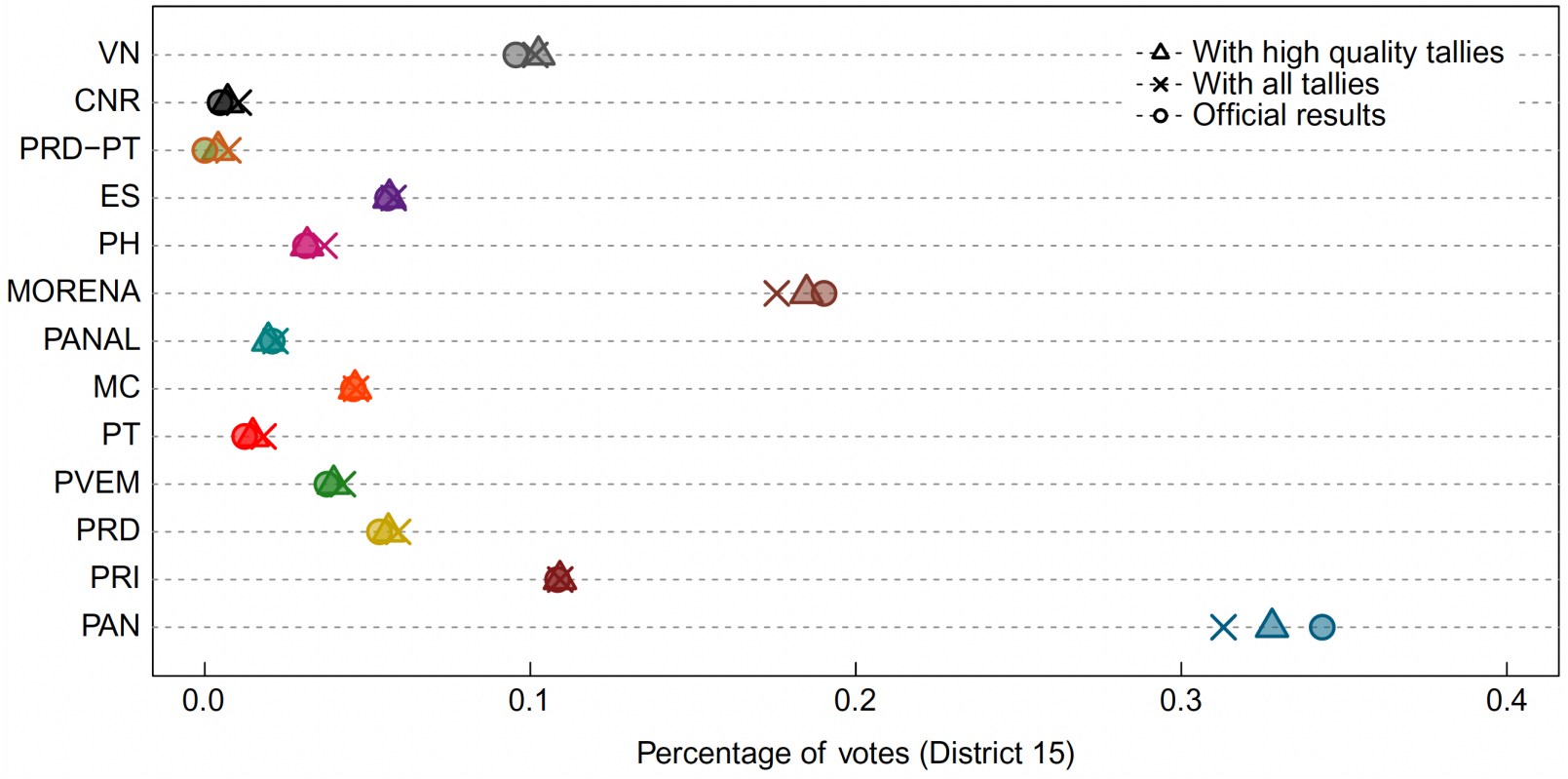

Figure 17: Final results (Cantu and Torres 22)

Misc

Summary

- CNNs are a powerful type of neural network, with an architecture attuned to image data.
- Learns simple and complex features of images (in *convolutional layers*) and then learns how to weight/combine them (in *regular hidden layers*) to effectively predict image classes.
- Like for regular networks:
  - Require lots of data to clearn complex patterns.
  - Even more hyperparameters to choose.
  - Lacks in inference and interpretability.

Further reading

- ISL, Ch. 10 (10.1-10.7; minus 10.5 and 10.7.1)
- Torres, M., & Cantú, F. (2022). Learning to see: Convolutional neural networks for the analysis of social science data. Political Analysis, 30(1), 113-131.
- Zhang, H., & Pan, J. (2019). Casm: A deep-learning approach for identifying collective action events with text and image data from social media. Sociological Methodology, 49(1), 1-57.

Unsupervised learning

Types of learning

Main focus of this course: branch of machine learning called **supervised learning**.

- For such problems, we have a *priviliged variabled* Y that we want to *predict* based on some other set of variables X—by estimating a function $\hat{f}(X)$.

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There exists two main—broad—types unsupervised learning:

- *Dimensionality reduction*: analyzes relationship between variables
- *Clustering*: analyzes relationship between observations.

Illustration—toy data






User A	0	0	0	1	1	1
User B	1	1	0	0	0	0
User C	0	0	0	1	1	0
User D	1	1	1	0	0	0
User E	0	0	0	0	1	1
User F	0	1	1	0	0	0
User G	1	1	0	0	0	0
User G	0	0	0	1	1	0

Can we spot (*a*) similar rows and/or (*b*) similar columns here?

Dimensionality reduction: *column patterns*



User A	0	0	0	1	1	1
User B	1	1	0	0	0	0
User C	0	0	0	1	1	0
User D	1	1	1	0	0	0
User E	0	0	0	0	1	1
User F	0	1	1	0	0	0
User G	1	1	0	0	0	0
User G	0	0	0	1	1	0

We can relatively easy spot two **subgroups of columns** (artists). They are deemed similar because they have high correlation / co-occurrence.

Clustering: *row patterns*






User A	0	0	0	1	1	1
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User E	0	0	0	0	1	1
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We can also spot **subgroups of rows** (users). They are deemed similar because they have adopted similar artists (i.e., have similar values on each column).

Clustering: *row patterns*






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When...

- We have just a couple of observations (rows) and just a couple of variables (columns),
- And, the data is made up of just a small number of simple & non-noisy patterns.

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Problem is—in reality, this is not how our data generally looks like.

- Many rows
- Many columns
- No simple separation between subgroups
- Complex patterns

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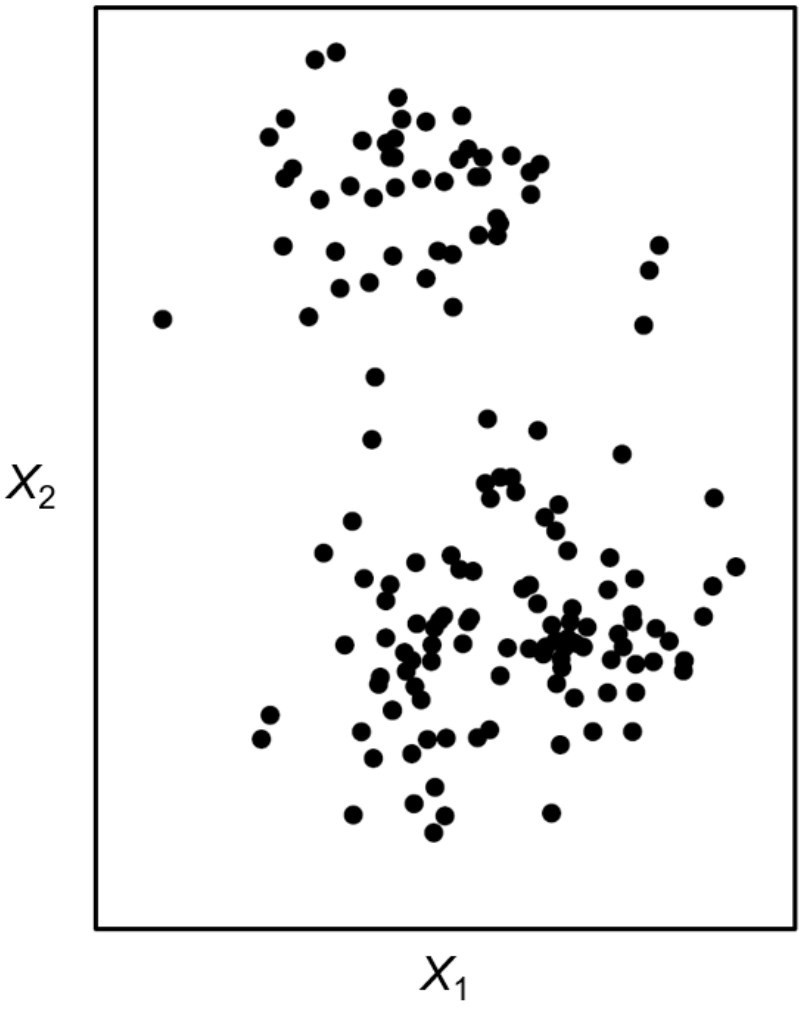
- This fuzziness can—in large part—be attributed to a **lack of a ground-truth** (which Y provides for supervised learning).
- We cannot simply check if our prediction was correct or not.

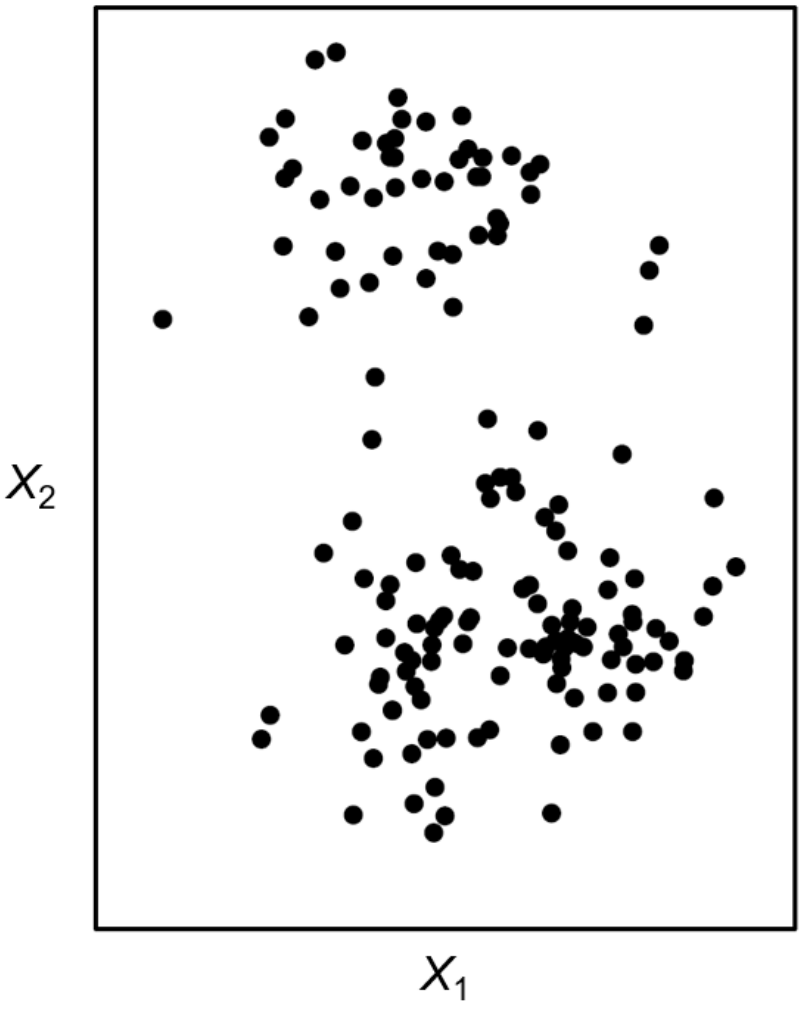
Clustering

Clustering



Objective of clustering: partition observations into distinct groups according to some *distance function* (e.g., euclidean distance) such that:

Clustering



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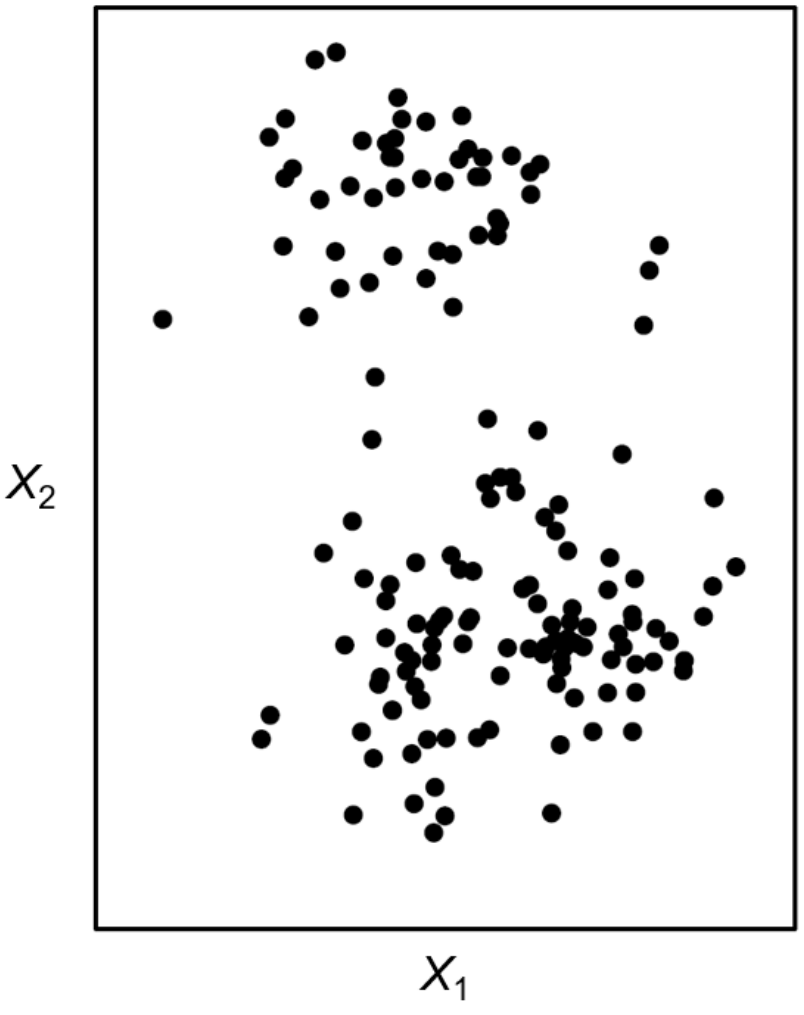

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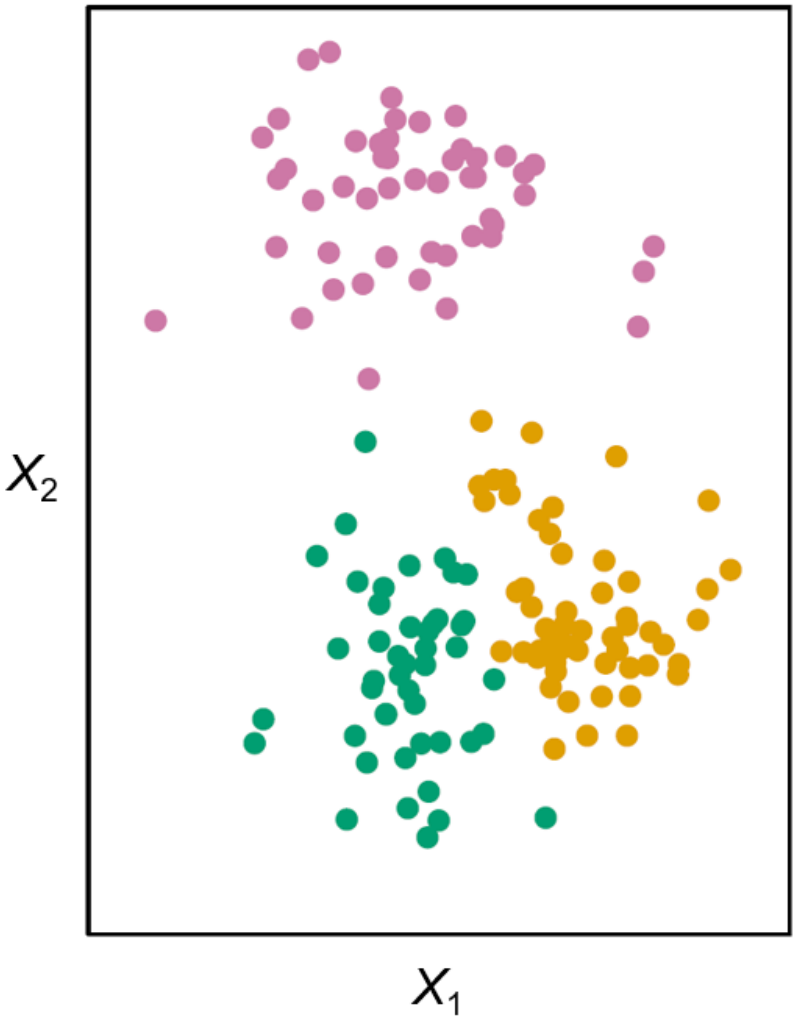

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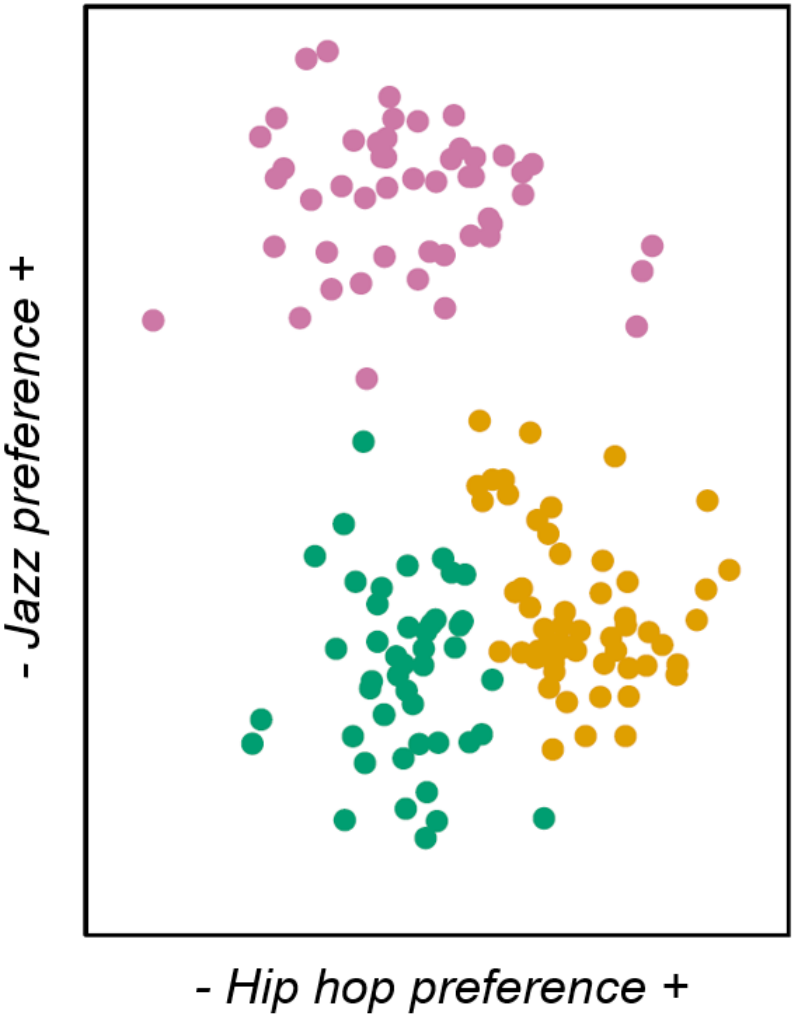
Here is one such partitioning of distinct groups.

How is this relevant for social scientists?



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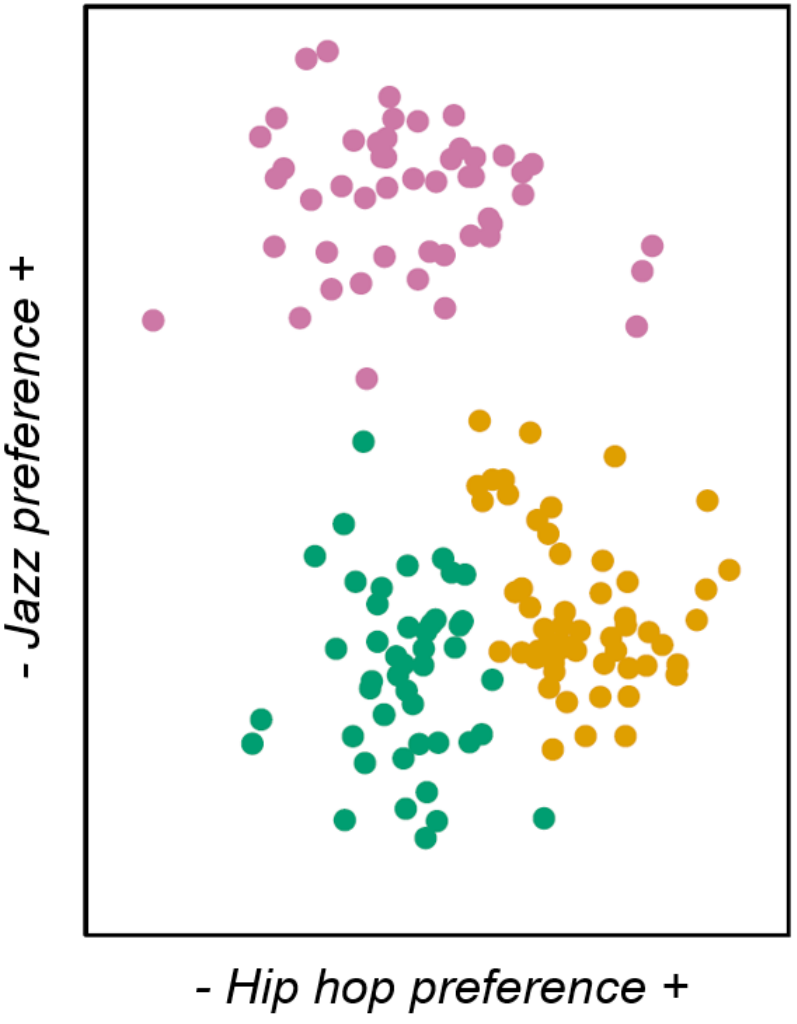

Can help us discover/measure heterogeneity in terms of known or unknown subpopulations that may be of social/theoretical relevance.

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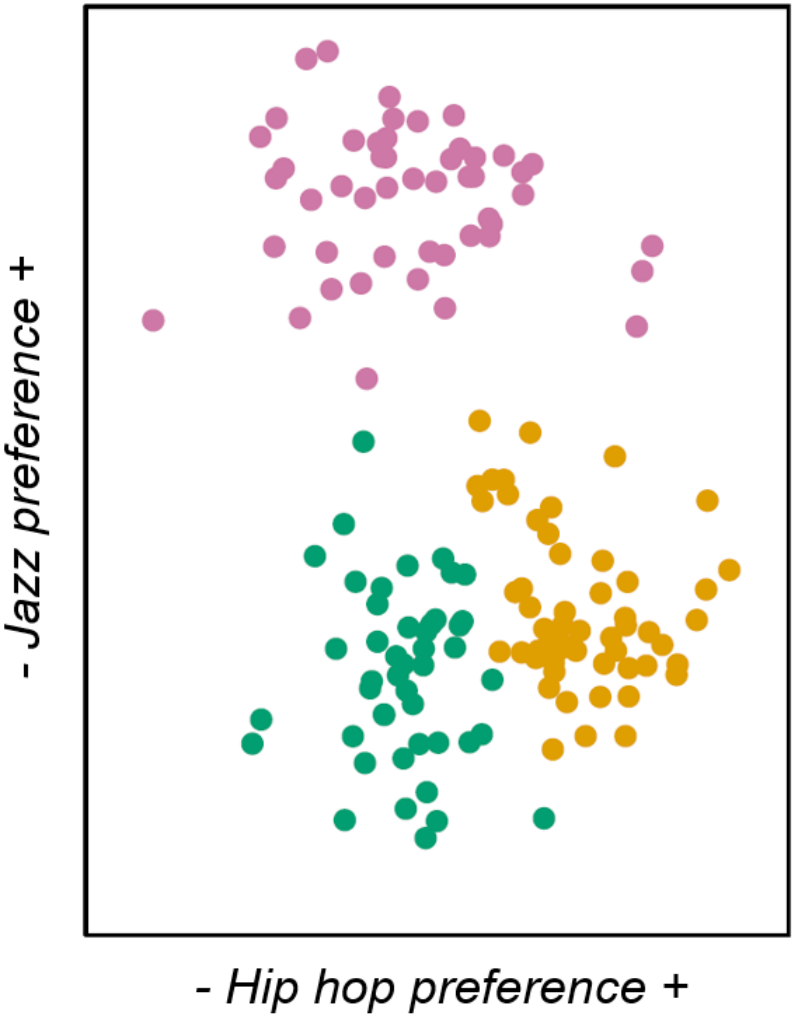

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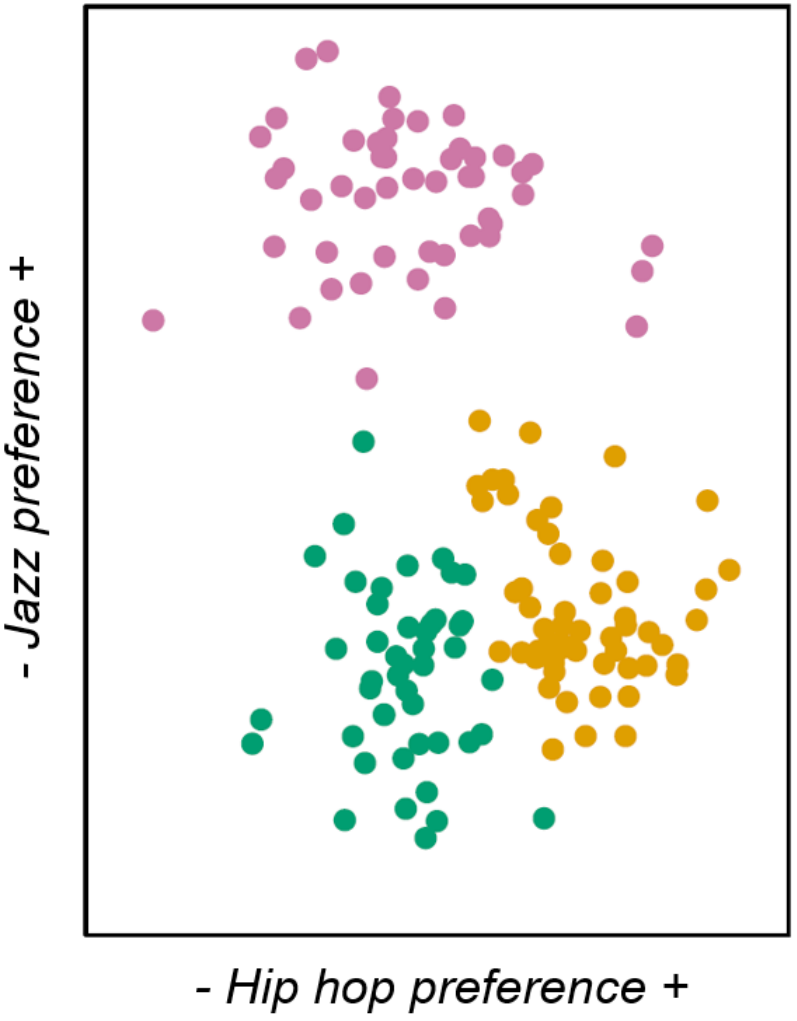

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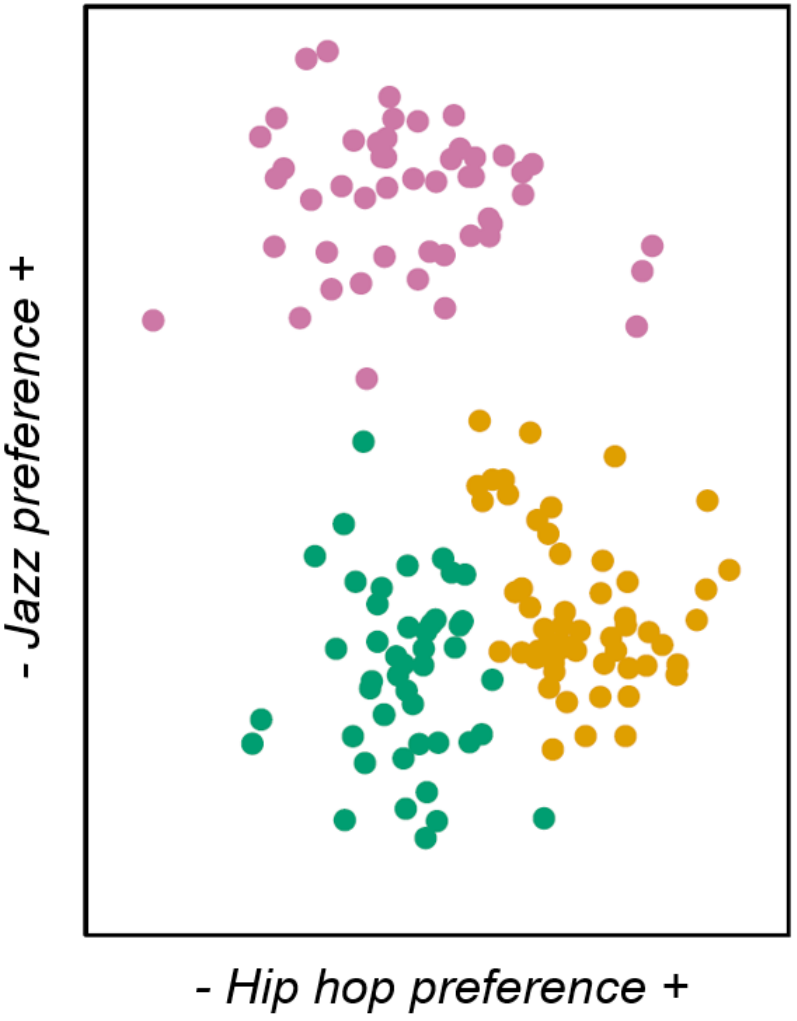

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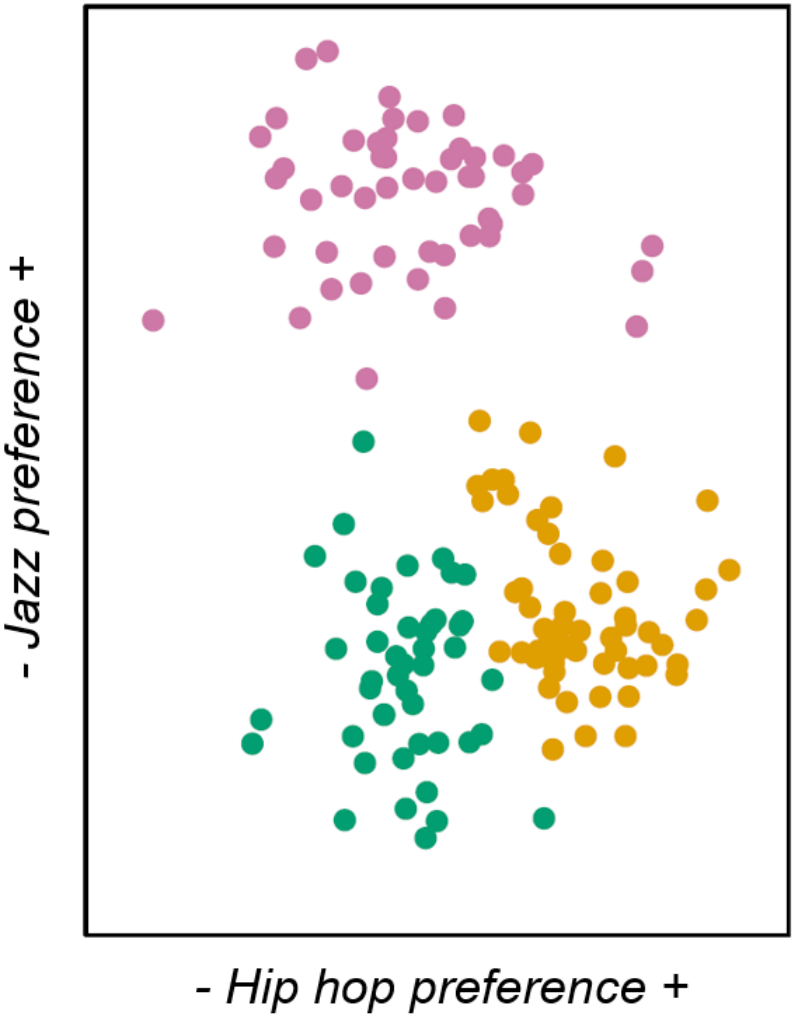

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- A third enjoys hip hop moderadely, and likes jazz a lot.

Types of clustering

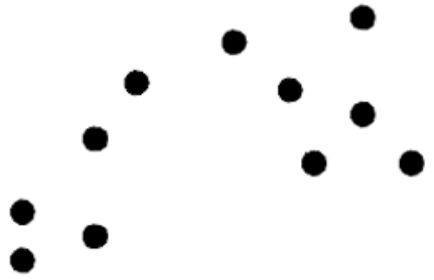
We can fruitfully categorize clustering according to three properties:

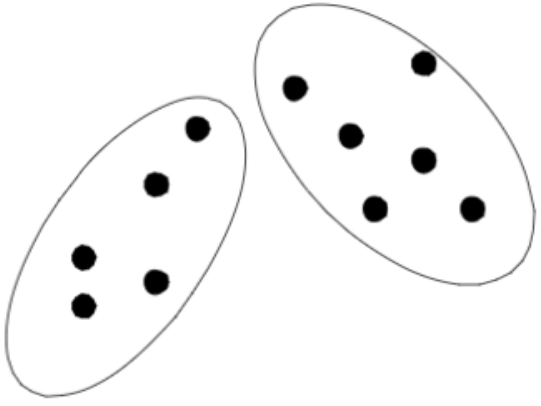
- *Flat* vs. *hiearchical*
- *Hard* vs. *soft*.
- *Model-based*[10] vs. *algorithmic*

[10] Also called *"Probabilistic"*.

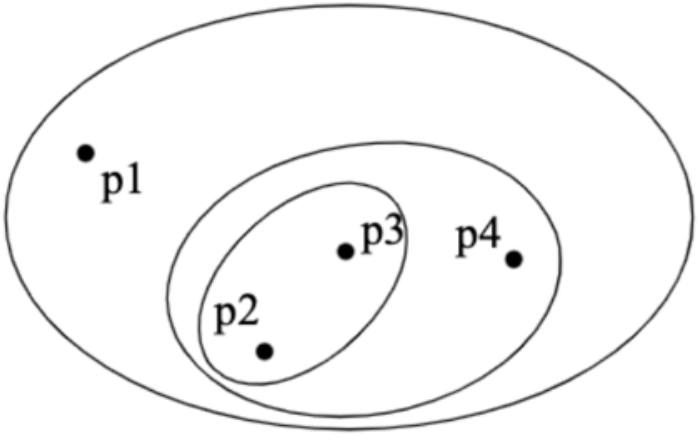
Flat vs. hiearchical

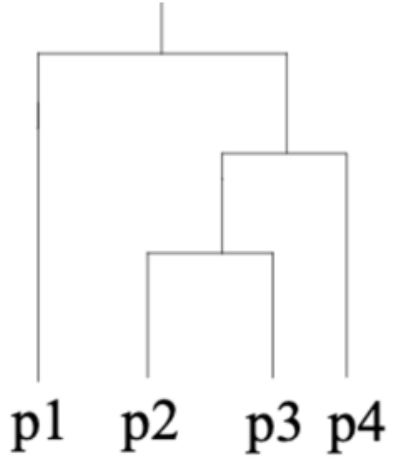
FLAT



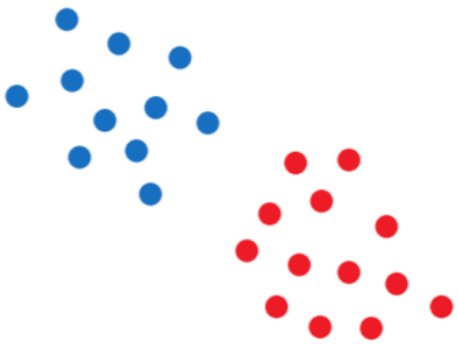


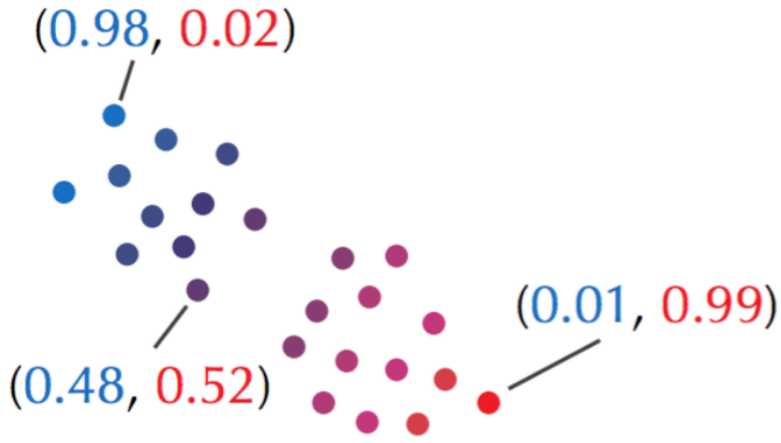
HIERARCHICAL





Hard vs. soft

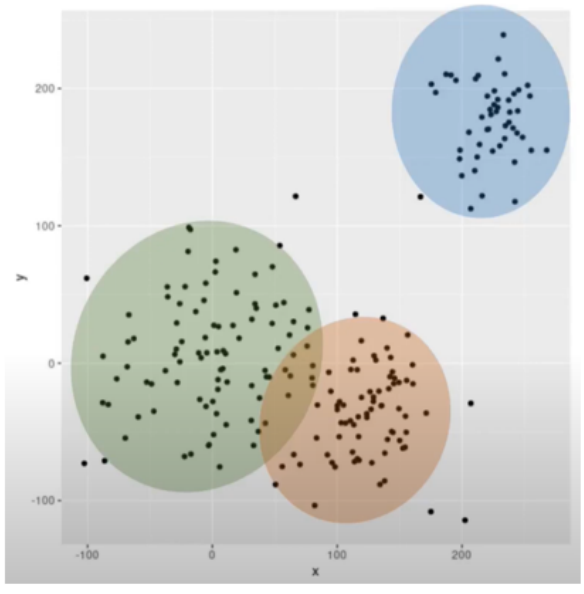




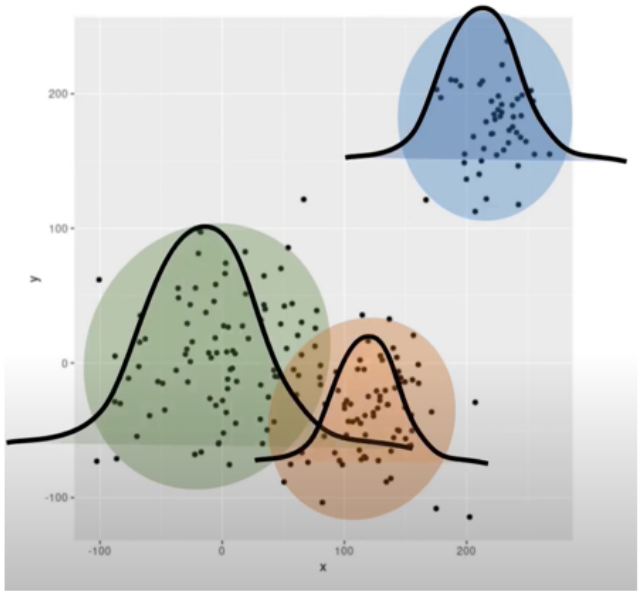
Model-based vs. algorithmic

Do we assume that data comes from some distribution?

NO



YES



What we will consider today

K-means

- Flat
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In bonus slides: **Mixture models**

- Flat
- Soft
- Model-based

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- How to measure *distance/variation*? Euclidian distance

$$\text{minimize} \quad \sum_{k=1}^{K} \underbrace{\frac{1}{|C_k|}}_{C_k \text{ size}} \underbrace{\sum_{i,i' \in C_k}}_{\text{All pairs } i,i' \text{ in } C_k} \underbrace{\sum_{j=1}^{p} (x_{ij} - x_{i',j})^2}_{\text{Euclidian dist.}}$$

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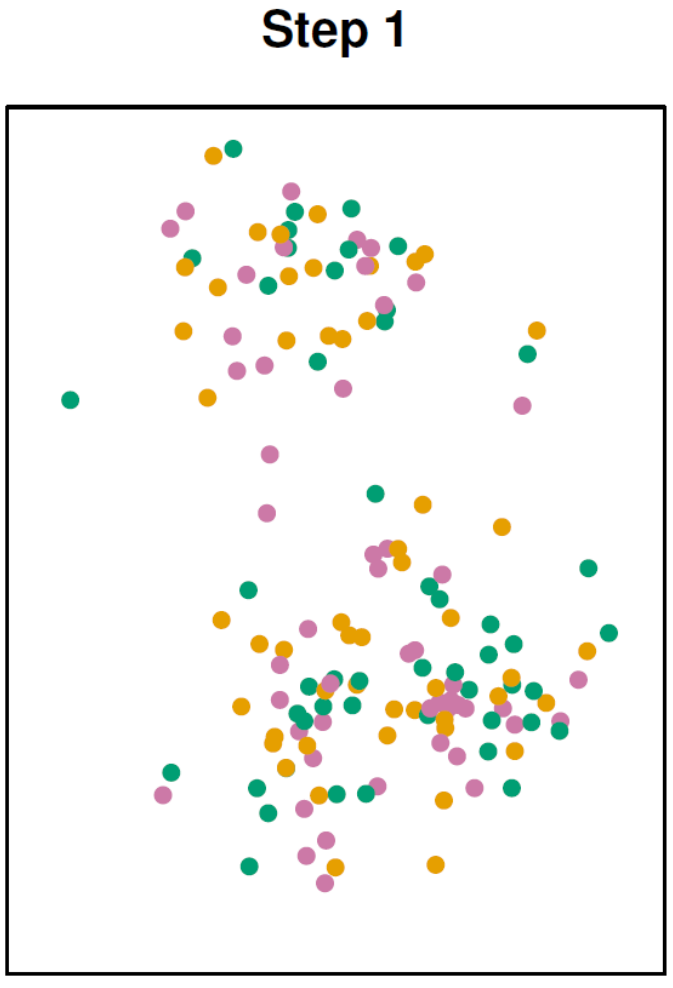
    b. Holding *cluster centroids fixed*—assign each observation to the cluster whose centroid is closest.

Illustration

The original data

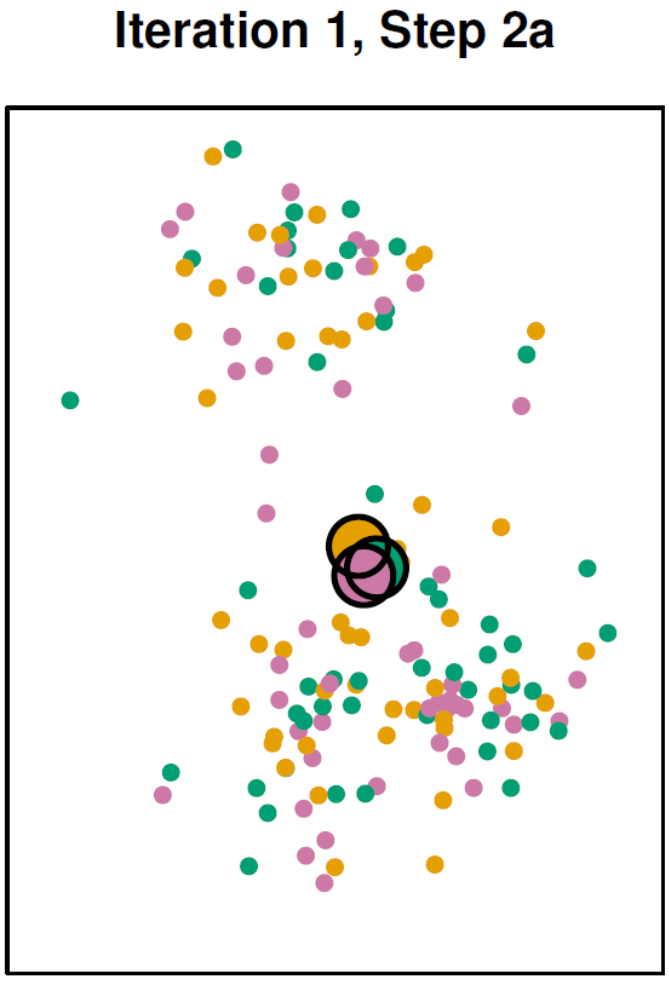
Illustration

Step 1—Random initialization of cluster assignments.



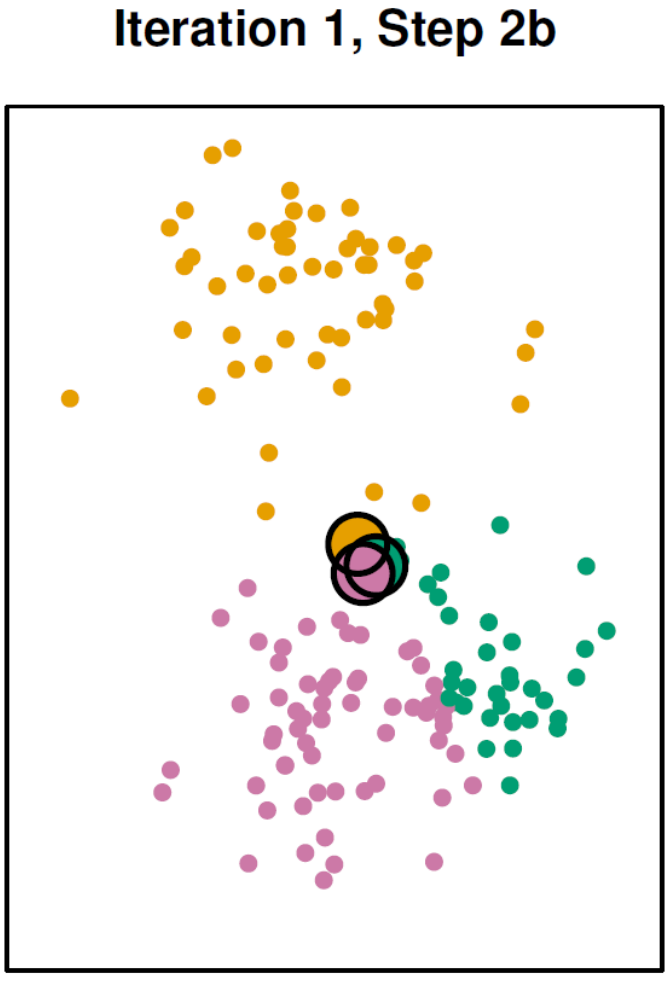
Illustration

Step 2a—Calculate centroids for each cluster.



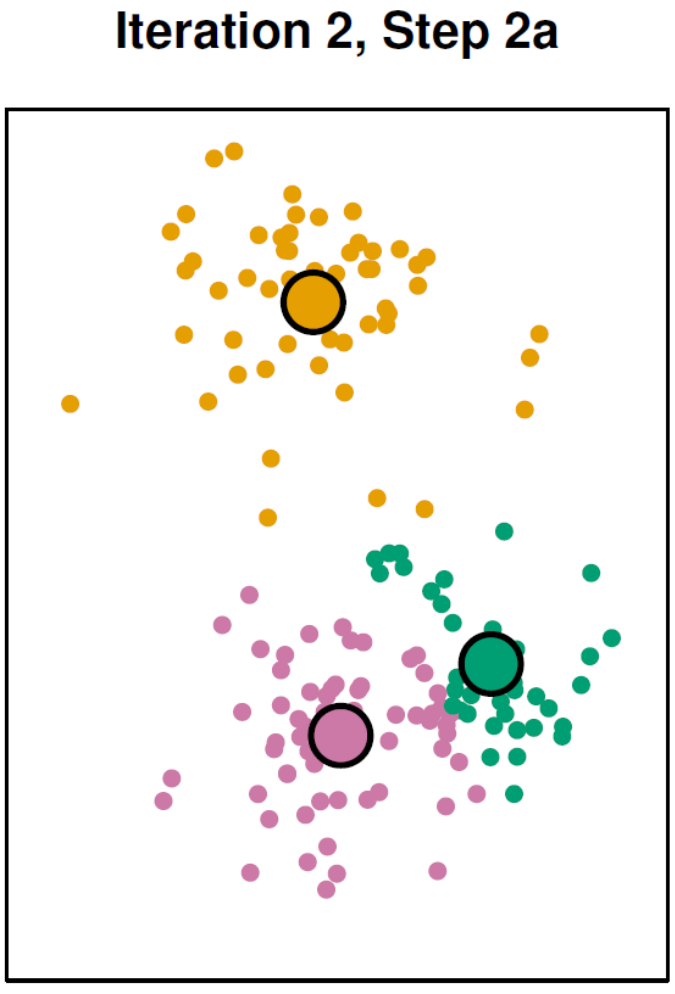
Illustration

Step 2b—Assign each observation to the nearest centroid.



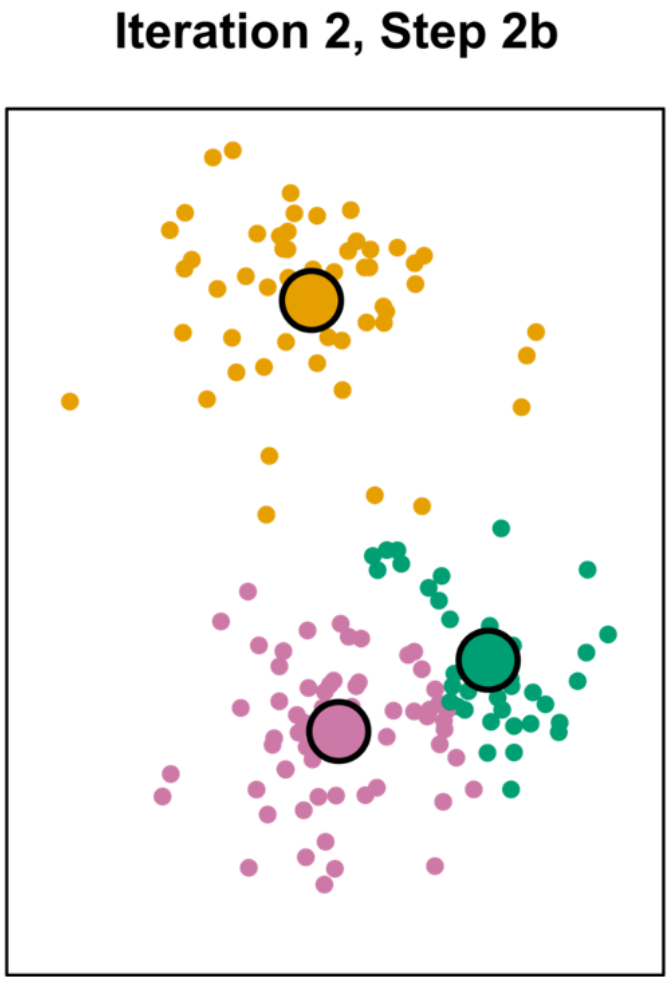
Illustration

Step 2a—Update position of cluster centroids.



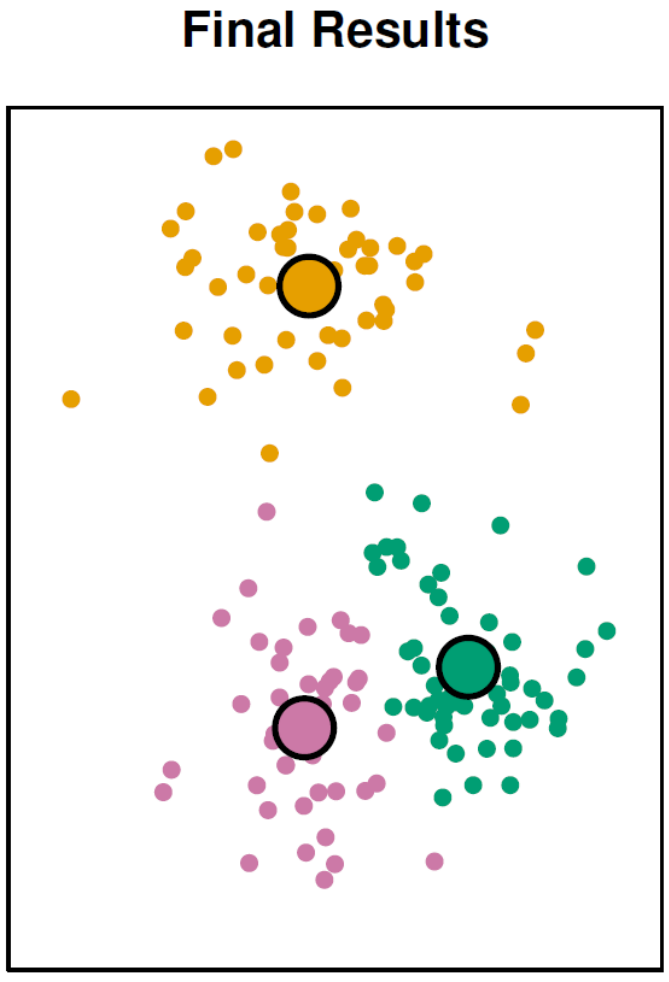
Illustration

Step 2b—Re-assign observations to nearest centroid.



Final results

After 10 iterations of 2a–2b, we get the following:



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$\rightarrow$ R re-runs several times with different initial values; then keep best.

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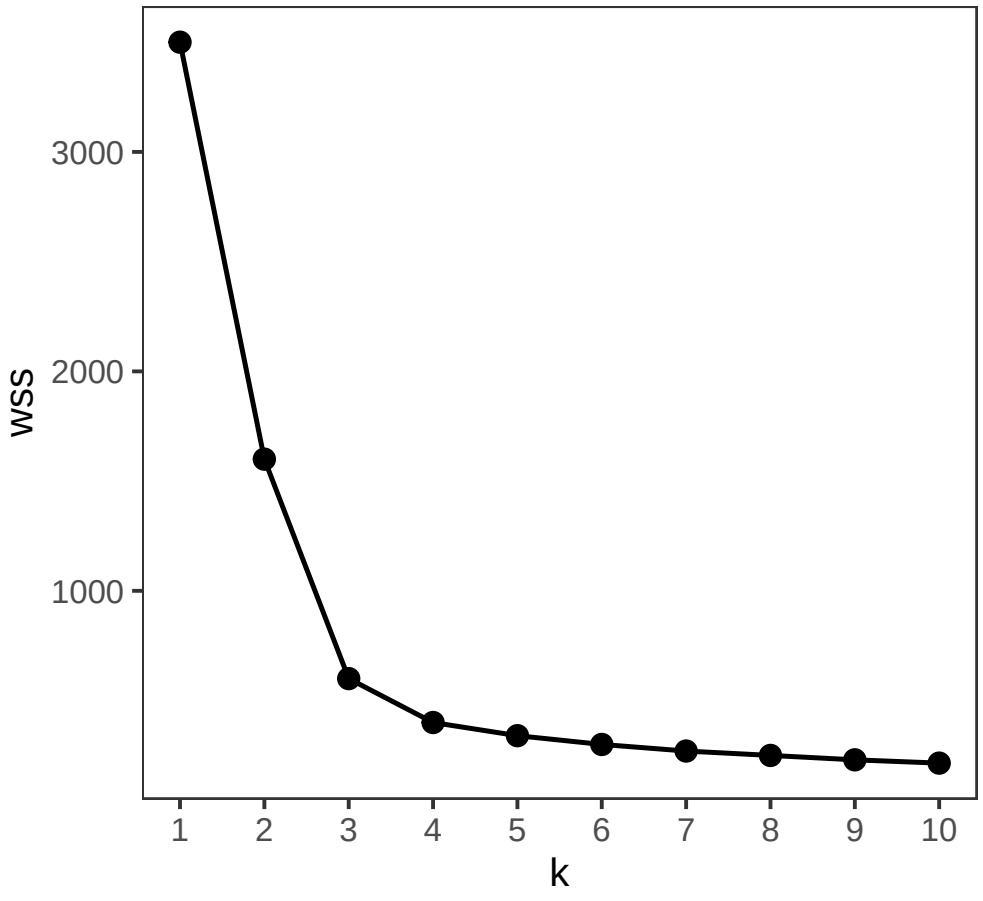
Identifying the elbow



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2. **Sampling observations** assigned to each cluster—and inspect them. If observations are text; read original texts.

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In the computational text analysis course (in the spring?) you will learn more sophisticated ways to cluster data.